Вывод ур-я относ. массового расхода горючего, отнесенного к массовому расходу на входе в I контур, через относ. массовый расход горючего, отнесенного к массовому расходу на входе в КС.

$$g_{охл\_КС3} = \frac{G_{охл}}{G_{КС3}} = \frac{G_{охл}}{G_I - G_{охл} - G_{ут} + G_{гор}}$$

$$g_{охл\_КС3} = \frac{G_{охл}}{G_I - G_{охл} - G_{ут} + G_{гор}} \ \text{solve}, G_{охл} \ = \frac{g_{охл\_КС3}\cdot(G_I - G_{ут} + G_{гор})}{g_{охл\_КС3} + 1}$$ (Тихонов)

$$g_{гор\_КС1} = \frac{G_{гор}}{G_{КС1}} = \frac{G_{гор}}{G_I - G_{охл} - G_{ут}}$$

$$g_{гор\_КС1} = \frac{G_{гор}}{G_I - G_{охл} - G_{ут}} \ \text{solve}, G_{гор} \ = -g_{гор\_КС1}\cdot(G_{ут} - G_I + G_{охл})$$

$$g_{гор\_КС1} = \frac{G_{гор}}{G_I - G_{охл} - G_{ут}} \ \text{substitute}, G_{охл} = \frac{g_{охл\_КС3}\cdot(G_I - G_{ут} + G_{гор})}{g_{охл\_КС3} + 1} \ = g_{гор\_КС1} = -\frac{G_{гор}\cdot(g_{охл\_КС3} + 1)}{G_{ут} - G_I + G_{гор}\cdot g_{охл\_КС3}}$$

$$g_{гор\_КС1} = -\frac{G_{гор}\cdot(g_{охл\_КС3} + 1)}{G_{ут} - G_I + G_{гор}\cdot g_{охл\_КС3}} \ \text{solve}, G_{гор} \ = \frac{G_I\cdot g_{гор\_КС1} - G_{ут}\cdot g_{гор\_КС1}}{g_{охл\_КС3} + g_{гор\_КС1}\cdot g_{охл\_КС3} + 1}$$

$$G_{гор} = \frac{G_I\cdot g_{гор\_КС1} - G_{ут}\cdot g_{гор\_КС1}}{g_{охл\_КС3} + g_{гор\_КС1}\cdot g_{охл\_КС3} + 1} = G_I\cdot\frac{g_{гор\_КС1} - g_{ут}\cdot g_{гор\_КС1}}{g_{охл\_КС3} + g_{гор\_КС1}\cdot g_{охл\_КС3} + 1}$$

$$g_{гор} = \frac{G_{гор}}{G_I} = \frac{G_I\cdot\dfrac{g_{гор\_КС1} - g_{ут}\cdot g_{гор\_КС1}}{g_{охл\_КС3} + g_{гор\_КС1}\cdot g_{охл\_КС3} + 1}}{G_I} = \frac{g_{гор\_КС1} - g_{ут}\cdot g_{гор\_КС1}}{g_{охл\_КС3} + g_{гор\_КС1}\cdot g_{охл\_КС3} + 1}$$

Вывод ур-я Тихонова относ. массового расхода на охлаждение, отнесенного к массовому расходу на входе в I контур, через относ. массовый расход на охлаждение, онесенного к массовому расходу на выходе из КС.

$$g_{охл} = \frac{G_{охл}}{G_I} = \frac{\dfrac{g_{охл\_КС3}\cdot(G_I - G_{ут} + G_{гор})}{g_{охл\_КС3} + 1}}{G_I} = \frac{G_I\cdot\dfrac{g_{охл\_КС3}\cdot(1 - g_{ут} + g_{гор})}{g_{охл\_КС3} + 1}}{G_I} = \frac{g_{охл\_КС3}\cdot(1 - g_{ут} + g_{гор})}{g_{охл\_КС3} + 1}$$

Вывод термодинамических парамтеров смешения

Ур-е теплового баланса: $Q + Q_{add} = Q_{СМ}$ или $G\cdot Cp\cdot T + G_{add}\cdot Cp_{add}\cdot T_{add} = G_{СМ}\cdot Cp_{СМ}\cdot T_{СМ} = (G + G_{add})\cdot Cp_{СМ}\cdot T_{СМ}$ =>

Температура СМ [К]: $G\cdot Cp\cdot T + G_{add}\cdot Cp_{add}\cdot T_{add} = (G + G_{add})\cdot Cp_{СМ}\cdot T_{СМ} \ \text{solve}, T_{СМ} \ = \dfrac{Cp\cdot G\cdot T + Cp_{add}\cdot G_{add}\cdot T_{add}}{Cp_{СМ}\cdot(G + G_{add})}$ $\quad T_{СМ} = \dfrac{Cp\cdot g\cdot T + Cp_{add}\cdot g_{add}\cdot T_{add}}{Cp_{СМ}\cdot(g + g_{add})}$

Давление СМ [Па] как среднемассовое: $P_{СМ} = \dfrac{G\cdot P + G_{add}\cdot P_{add}}{G + G_{add}} = \dfrac{g\cdot P + g_{add}\cdot P_{add}}{g + g_{add}}$

Коэф. избытка окислителя СМ []: $\alpha_{СМ} = \dfrac{G_{ок\Sigma}}{G_{гор\Sigma}} = \dfrac{G_{ок} + G_{add}}{G_{гор}\cdot l_0(Fuel)} = \dfrac{g_{ок} + g_{add}}{g_{гор}\cdot l_0(Fuel)}$

Вывод оптимальной скорости выхода из С I контура

$$L_e = \frac{L_B}{\eta_{PB}\cdot\eta_{мех}} + \frac{c_{CI}^2 - \upsilon^2}{2} + \frac{c_{CII}^2 - \upsilon^2}{2} \text{ solve}, L_B \rightarrow \eta_{PB}\cdot\eta_{мех}\cdot\left(\upsilon^2 - \frac{c_{CI}^2}{2} - \frac{c_{CII}^2}{2} + L_e\right)$$

$$L_{тяг}(\upsilon, c_{CI}, c_{CII}, L_e, \eta_{мех}, \eta_{PB}, \eta_B, L_B) = L_B\cdot\eta_B + (c_{CI} - \upsilon)\cdot\upsilon + (c_{CII} - \upsilon) \text{ substitute}, L_B = \eta_{PB}\cdot\eta_{мех}\cdot\left(\upsilon^2 - \frac{c_{CI}^2}{2} - \frac{c_{CII}^2}{2} + L_e\right) = c_{CII} - \upsilon - \upsilon^2 + \upsilon\cdot c_{CI} + \upsilon^2\cdot\eta_B\cdot\eta_{PB}\cdot\eta_{мех} - \frac{c_{CI}^2\cdot\eta_B\cdot\eta_{PB}\cdot\eta_{мех}}{2} - \frac{\eta_B\cdot c_{CII}^2\cdot\eta_{PB}\cdot\eta_{мех}}{2} + L_e\cdot\eta_B\cdot\eta_{PB}\cdot\eta_{мех}$$

$$\frac{d}{dc_{CI}}L_{тяг}(\upsilon, c_{CI}, c_{CII}, L_e, \eta_{мех}, \eta_{PB}, \eta_B, L_B) \rightarrow \upsilon - c_{CI}\cdot\eta_B\cdot\eta_{PB}\cdot\eta_{мех}$$

$$\frac{d}{dc_{CI}}L_{тяг}(\upsilon, c_{CI}, c_{CII}, L_e, \eta_{мех}, \eta_{PB}, \eta_B, L_B) = 0 \text{ solve}, c_{CI} = \frac{\upsilon}{\eta_B\cdot\eta_{PB}\cdot\eta_{мех}}$$

$$L_e(\upsilon, c_{CI}, c_{CII}, m2, L_{ВлII}) = m2\cdot L_{ВлII} + \frac{c_{CI}(L_{ВлII})^2 - \upsilon^2}{2}$$

$$\frac{d}{dL_{ВлII}}L_e(\upsilon, c_{CI}, c_{CII}, m2, L_{ВлII}) = m2 + c_{CI}(L_{ВлII})\cdot\frac{d}{dL_{ВлII}}c_{CI}(L_{ВлII}) \text{ solve}, \frac{d}{dL_{ВлII}}c_{CI}(L_{ВлII}) = -\frac{m2}{c_{CI}(L_{ВлII})}$$

Баланс энергии II контура: $$L_{ВлII}\cdot\eta_{II} = \frac{c_{CII}(\upsilon, \eta_{II}, L_{ВлII})^2 - \upsilon^2}{2} \text{ solve}, c_{CII}(\upsilon, \eta_{II}, L_{ВлII}) = \begin{pmatrix} \sqrt{\upsilon^2 + 2\cdot\eta_{II}\cdot L_{ВлII}} \\ -\sqrt{\upsilon^2 + 2\cdot\eta_{II}\cdot L_{ВлII}} \end{pmatrix}$$

$$c_{CII}(\upsilon, \eta_{II}, L_{ВлII}) = \sqrt{\upsilon^2 + 2\cdot\eta_{II}\cdot L_{ВлII}}$$

$$\frac{d}{dL_{ВлII}}c_{CII}(\upsilon, \eta_{II}, L_{ВлII}) = \frac{\eta_{II}}{\sqrt{\upsilon^2 + 2\cdot\eta_{II}\cdot L_{ВлII}}}$$

Удельная тяга двигателя: $$R_{уд}(\upsilon, c_{CI}, c_{CII}, m2, L_{ВлII}, \eta_{II}) = \frac{R_{удI} + m2\cdot R_{удII}}{1 + m2} \left| \begin{array}{l} \text{substitute}, R_{удI} = c_{CI}(L_{ВлII}, m2) - \upsilon \\ \text{substitute}, R_{удII} = c_{CII}(\upsilon, \eta_{II}, L_{ВлII}) - \upsilon \end{array} \right. = -\frac{\upsilon - m2\cdot c_{CII}(\upsilon, \eta_{II}, L_{ВлII}) + \upsilon\cdot m2 - c_{CI}(L_{ВлII}, m2)}{m2 + 1}$$

Условие получения max тяги: $$\frac{d}{dL_{ВлII}}R_{уд}(\upsilon, c_{CI}, c_{CII}, m2, L_{ВлII}, \eta_{II}) = 0$$

$$\frac{d}{dL_{ВлII}}R_{уд}(\upsilon, c_{CI}, c_{CII}, m2, L_{ВлII}, \eta_{II}) = \frac{\frac{d}{dL_{ВлII}}c_{CI}(L_{ВлII}, m2) + \frac{m2\cdot\eta_{II}}{\sqrt{\upsilon^2 + 2\cdot\eta_{II}\cdot L_{ВлII}}}}{m2 + 1} \left| \begin{array}{l} \text{substitute}, \frac{d}{dL_{ВлII}}c_{CI}(L_{ВлII}, m2) = -\frac{m2}{c_{CI}(L_{ВлII}, m2)} \\ \text{substitute}, \frac{d}{dL_{ВлII}}c_{CII}(\upsilon, \eta_{II}, L_{ВлII}) = \frac{\eta_{II}}{c_{CII}(L_{ВлII})} \end{array} \right. = -\frac{m2\cdot\left(\sqrt{\upsilon^2 + 2\cdot\eta_{II}\cdot L_{ВлII}} - \eta_{II}\cdot c_{CI}(L_{ВлII}, m2)\right)}{\sqrt{\upsilon^2 + 2\cdot\eta_{II}\cdot L_{ВлII}}\cdot(m2 + 1)\cdot c_{CI}(L_{ВлII}, m2)} \text{ solve}, c_{CI}(L_{ВлII}, m2) = \frac{\sqrt{\upsilon^2 + 2\cdot\eta_{II}\cdot L_{ВлII}}}{\eta_{II}}$$

Вывод общего ур-я профилирования Л цилиндрической и конической ступени К

Дано:

(0) $\frac{d}{dr}H_T = \frac{d}{dr}\left(\frac{c_a^2}{2}\right) + c_u\cdot\left(\frac{d}{dr}c_u\right) + \frac{c_u^2}{r} = 0$ => $\frac{d}{dr}\left(\frac{c_a^2}{2}\right) = -c_u\cdot\left(\frac{d}{dr}c_u\right) - \frac{c_u^2}{r}$ (00) Домножим на 2 => $\frac{d}{dr}\left(c_a^2\right) = -2\cdot c_u\cdot\left(\frac{d}{dr}c_u\right) - \frac{2\cdot c_u^2}{r}$ (000)

(1) $\frac{c_{1u}+c_{2u}}{2}\cdot r^m = const$

(2) $H_T = \left(c_{2u}\cdot u_2 - c_{1u}\cdot u_1\right) = \left(c_{2u} - c_{1u}\right)u$

(3) $R = 1 - \frac{c_{1u}+c_{2u}}{u_1+u_2} = 1 - \frac{c_{1u}+c_{2u}}{2\cdot u}$

$u = \omega\cdot r$ $u_1 = \omega\cdot r_1$ $u_2 = \omega\cdot r_2$ $\omega = const$ $\frac{u_1}{r_1} = \frac{u_2}{r_2}$

$R = 1 - \frac{c_{1u}+c_{2u}}{2\cdot u} - \frac{c_{a2}^2 - c_{a1}^2}{2\cdot H_T}$ без допущений

Вывод:

(2) (3) =>

$c_{1u} = (1-R)\cdot\omega\cdot r - \frac{H_T}{2\cdot\omega\cdot r}$ (c1u)

$c_{2u} = (1-R)\cdot\omega\cdot r + \frac{H_T}{2\cdot\omega\cdot r}$ (c2u)

(3) => $\frac{c_{1u}+c_{2u}}{2} = (1-R)\cdot\omega\cdot r$

(1): $const = \frac{c_{1u}+c_{2u}}{2}\cdot r^m = (3) = [(1-R)\omega\cdot r]\cdot r^m = (1-R)\omega\cdot r^{m+1} = \left(1-R_{cp}\right)\omega\cdot r_{cp}^{m+1}$ => $(1-R)\omega = \frac{\left[\left(1-R_{cp}\right)\cdot\omega\cdot r_{cp}^{m+1}\right]}{r^{m+1}}$ (&)

(&)->(c1u) $c_{1u} = (1-R)\cdot\omega\cdot r - \frac{H_T}{2\cdot\omega\cdot r} = \frac{\left[\left(1-R_{cp}\right)\cdot\omega\cdot r_{cp}^{m+1}\right]}{r^{m+1}}\cdot r - \frac{H_T}{2\cdot\omega\cdot r} = \frac{\left[\left(1-R_{cp}\right)\cdot\omega\cdot r_{cp}^{m+1}\right]}{r^m} - \frac{\left(\frac{H_T}{2\cdot\omega}\right)}{r}$ (AB1) $A = \left[\left(1-R_{cp}\right)\cdot\omega\cdot r_{cp}^{m+1}\right]$ $c_{1u}(r,m,A,B) = \frac{(A)}{r^m} - \frac{(B)}{r}$

(&)->(c2u) $c_{2u} = (1-R)\cdot\omega\cdot r + \frac{H_T}{2\cdot\omega\cdot r} = \frac{\left[\left(1-R_{cp}\right)\cdot\omega\cdot r_{cp}^{m+1}\right]}{r^{m+1}}\cdot r + \frac{H_T}{2\cdot\omega\cdot r} = \frac{\left[\left(1-R_{cp}\right)\cdot\omega\cdot r_{cp}^{m+1}\right]}{r^m} + \frac{\left(\frac{H_T}{2\cdot\omega}\right)}{r}$ (AB2) $B = \left(\frac{H_T}{2\cdot\omega}\right)$ $c_{2u}(r,m,A,B) = \frac{(A)}{r^m} + \frac{(B)}{r}$ Для цилиндрической ступени

$c_{2u} = \frac{H_T + c_{1u}\cdot u_1}{u_2} = \frac{H_T + c_{1u}\cdot r_1}{r_2}$ Для конической ступни ступени (см. (3))

(000) $\int_{r_{cp}}^{r}\frac{d}{dr}\left(c_a^2\right)dr = c_a^2 - c_{a.cp}^2 = \int_{r_{cp}}^{r} -2\cdot c_u\cdot\left(\frac{d}{dr}c_u\right) - \frac{2\cdot c_u^2}{r}\,dr$ =>

$c_{a1}^2 - c_{a1.cp}^2 = \int_{r_{cp}}^{r} -2\cdot c_{u1}\cdot\left(\frac{d}{dr}c_{u1}\right) - \frac{2\cdot c_{u1}^2}{r}\,dr$

$c_{a2}^2 - c_{a2.cp}^2 = \int_{r_{cp}}^{r} -2\cdot c_{u2}\cdot\left(\frac{d}{dr}c_{u2}\right) - \frac{2\cdot c_{u2}^2}{r}\,dr$

=>

$c_{a1} = \sqrt{c_{a1.cp}^2 + \int_{r_{cp}}^{r} -2\cdot c_{u1}\cdot\left(\frac{d}{dr}c_{u1}\right) - \frac{2\cdot c_{u1}^2}{r}\,dr}$

$c_{a2} = \sqrt{c_{a2.cp}^2 + \int_{r_{cp}}^{r} -2\cdot c_{u2}\cdot\left(\frac{d}{dr}c_{u2}\right) - \frac{2\cdot c_{u2}^2}{r}\,dr}$

(0000)

$$\sqrt{c_{a1.cp}{}^{2} + \int_{r_{cp}}^{r} -2\,c_{1u}(r,m,A,B)\cdot\left(\frac{d}{dr}c_{1u}(r,m,A,B)\right) - \frac{2\,c_{1u}(r,m,A,B)^{2}}{r}\,dr} \;\left|\begin{array}{l}\text{assume}, r_{cp} > 0, r \geq 0, m = -1\\ \text{substitute}, m = -1\end{array}\right. = \sqrt{c_{a1.cp}{}^{2} - 2\cdot A^{2}\cdot r^{2} + 2\cdot A^{2}\cdot r_{cp}{}^{2} + 4\cdot A\cdot B\cdot \ln(r) - 4\cdot A\cdot B\cdot \ln\left(r_{cp}\right)}$$

$$\sqrt{c_{a2.cp}{}^{2} + \int_{r_{cp}}^{r} -2\,c_{2u}(r,m,A,B)\cdot\left(\frac{d}{dr}c_{2u}(r,m,A,B)\right) - \frac{2\,c_{2u}(r,m,A,B)^{2}}{r}\,dr} \;\left|\begin{array}{l}\text{assume}, r_{cp} > 0, r \geq 0, m = -1\\ \text{substitute}, m = -1\end{array}\right. = \sqrt{c_{a2.cp}{}^{2} - 2\cdot A^{2}\cdot r^{2} + 2\cdot A^{2}\cdot r_{cp}{}^{2} - 4\cdot A\cdot B\cdot \ln(r) + 4\cdot A\cdot B\cdot \ln\left(r_{cp}\right)}$$

$$\sqrt{c_{a1.cp}{}^{2} + \int_{r_{cp}}^{r} -2\,c_{1u}(r,m,A,B)\cdot\left(\frac{d}{dr}c_{1u}(r,m,A,B)\right) - \frac{2\,c_{1u}(r,m,A,B)^{2}}{r}\,dr} \text{ assume}, r_{cp} > 0, r \geq 0, m = 0 \;=\; \sqrt{2\cdot A^{2}\cdot \ln\left(r_{cp}\right) - 2\cdot A^{2}\cdot \ln(r) + c_{a1.cp}{}^{2} - \frac{2\cdot A\cdot B}{r} + \frac{2\cdot A\cdot B}{r_{cp}}}$$

$$\sqrt{c_{a2.cp}{}^{2} + \int_{r_{cp}}^{r} -2\,c_{2u}(r,m,A,B)\cdot\left(\frac{d}{dr}c_{2u}(r,m,A,B)\right) - \frac{2\,c_{2u}(r,m,A,B)^{2}}{r}\,dr} \text{ assume}, r_{cp} > 0, r \geq 0, m = 0 \;=\; \sqrt{2\cdot A^{2}\cdot \ln\left(r_{cp}\right) - 2\cdot A^{2}\cdot \ln(r) + c_{a2.cp}{}^{2} + \frac{2\cdot A\cdot B}{r} - \frac{2\cdot A\cdot B}{r_{cp}}}$$

$$\sqrt{c_{a1.cp}{}^{2} + \int_{r_{cp}}^{r} -2\,c_{1u}(r,m,A,B)\cdot\left(\frac{d}{dr}c_{1u}(r,m,A,B)\right) - \frac{2\,c_{1u}(r,m,A,B)^{2}}{r}\,dr} \text{ assume}, r_{cp} > 0, r \geq 0, -1 < m \leq 1, m \neq 0 \;=\; \sqrt{c_{a1.cp}{}^{2} + \frac{A\cdot(m-1)\cdot\left(A\cdot r\cdot r^{2\cdot m}\cdot r_{cp} - A\cdot r\cdot r_{cp}\cdot r_{cp}{}^{2\cdot m} - 2\cdot B\cdot m\cdot r\cdot r^{2\cdot m}\cdot r_{cp}{}^{m} + 2\cdot B\cdot m\cdot r^{m}\cdot r_{cp}\cdot r_{cp}{}^{2\cdot m} + A\cdot m\cdot r\cdot r^{2\cdot m}\cdot r_{cp} - A\cdot m\cdot r\cdot r_{cp}\cdot r_{cp}{}^{2\cdot m}\right)}{m\cdot(m+1)\cdot\left(r\cdot r_{cp}\right)^{2\cdot m+1}}}$$

$$\sqrt{c_{a2.cp}{}^{2} + \int_{r_{cp}}^{r} -2\,c_{2u}(r,m,A,B)\cdot\left(\frac{d}{dr}c_{2u}(r,m,A,B)\right) - \frac{2\,c_{2u}(r,m,A,B)^{2}}{r}\,dr} \text{ assume}, r_{cp} > 0, r \geq 0, -1 < m \leq 1, m \neq 0 \;=\; \sqrt{c_{a2.cp}{}^{2} + \frac{A\cdot(m-1)\cdot\left(A\cdot r\cdot r^{2\cdot m}\cdot r_{cp} - A\cdot r\cdot r_{cp}\cdot r_{cp}{}^{2\cdot m} + 2\cdot B\cdot m\cdot r\cdot r^{2\cdot m}\cdot r_{cp}{}^{m} - 2\cdot B\cdot m\cdot r^{m}\cdot r_{cp}\cdot r_{cp}{}^{2\cdot m} + A\cdot m\cdot r\cdot r^{2\cdot m}\cdot r_{cp} - A\cdot m\cdot r\cdot r_{cp}\cdot r_{cp}{}^{2\cdot m}\right)}{m\cdot(m+1)\cdot\left(r\cdot r_{cp}\right)^{2\cdot m+1}}}$$

Вывод общего ур-я профилирования Л цилиндрической и конической ступени Т

(1) $\frac{c_{1u}+c_{2u}}{2}\cdot r^m = const$

$const = \frac{c_{1u}+c_{2u}}{2}\cdot r^m = (3) = (1-R)\omega\cdot r\cdot r^m = (1-R)\omega\cdot r^{m+1} = (1-R_{cp})\omega\cdot r_{cp}^{m+1}$ => $(1-R)\omega\cdot r = \frac{\left[(1-R_{cp})\cdot\omega\cdot r_{cp}^{m+1}\right]}{r^m}$

$A = \left[(1-R_{cp})\cdot\omega\cdot r_{cp}^{m+1}\right]$

(2) $L_u = c_{2u}\cdot u_2 + c_{1u}\cdot u_1 = (c_{2u}+c_{1u})u$ $\quad u = \omega\cdot r \quad$ $L_u = c_{2u}\cdot\omega\cdot r_2 + c_{1u}\cdot\omega\cdot r_1 = (c_{2u}+c_{1u})\omega\cdot r$

$c_{1u} = c_{2u} + \frac{L_u}{\omega\cdot r}$

$c_{2u} = c_{1u} + \frac{L_u}{\omega\cdot r}$

(3) $R = 1 - \frac{c_{1u}-c_{2u}}{2\cdot u}$ $\quad u = \omega\cdot r \quad$ $R = 1 - \frac{c_{1u}-c_{2u}}{2\cdot\omega\cdot r}$

$c_{1u} = (1-R)\cdot 2\cdot\omega\cdot r - c_{2u}$

$c_{2u} = -(1-R)\cdot 2\cdot\omega\cdot r - c_{1u}$

$c_{1u} = (1-R)\cdot\omega\cdot r + \frac{L_u}{2\cdot\omega\cdot r} = (1-R)\cdot\omega\cdot r + \frac{\left(\frac{L_u}{2\cdot\omega}\right)}{r}$

$c_{2u} = -(1-R)\cdot\omega\cdot r + \frac{L_u}{2\cdot\omega\cdot r} = -(1-R)\cdot\omega\cdot r + \frac{\left(\frac{L_u}{2\cdot\omega}\right)}{r}$

$B = \left(\frac{L_u}{2\cdot\omega}\right)$

$c_{1u}(r,m,A,B) = \frac{(A)}{r^m} + \frac{(B)}{r}$

$c_{2u}(r,m,A,B) = -\frac{A}{r^m} + \frac{(B)}{r}$

Работа сил трения: $L_{тр} = \left(\frac{1}{\varphi^2}-1\right)\cdot\frac{c_1^2}{2} = \frac{c_1^2}{2\cdot\varphi^2} - \frac{c_1^2}{2}$

Работа гидравлических сил сопротивления: $L_{гид} = \left(\frac{1}{\varphi^2}-1\right)\cdot\frac{c_1^2}{2} + \left(\frac{1}{\psi^2}-1\right)\cdot\frac{w_2^2}{2} = \left(\frac{1}{\varphi^2}-1\right)\cdot\frac{c_{a1}^2+c_{u1}^2}{2} + \left(\frac{1}{\psi^2}-1\right)\cdot\frac{(c_{2u}+u_2)^2+c_{2a}^2}{2}$

Ур-е радиадьного равновесия: $\frac{1}{\rho}\cdot\frac{d}{dr}P = \frac{c_u^2}{r}$

Ур-е Бернулли:

$\frac{1}{\rho}\cdot dP + d\left(\frac{c_1^2}{2}\right) + d(L_{тр}) = d(H_т)$ $\qquad 0 = \frac{c_1^2}{2} + \int_0^1 \frac{1}{\rho}\,dP + L_{тр}$

$\frac{1}{\rho}\cdot dP + d\left(\frac{c_2^2}{2}\right) + d(L_{гид}) + d(L_{ст}) = d(H_т)$ $\qquad 0 = \frac{c_2^2}{2} + \int_0^2 \frac{1}{\rho}\,dP + L_{гид} + L_{ст}$

Допущения:

$H_т(r) = const$ => $\frac{d}{dr}H_т(r) = 0$

$\varphi(r) = const$ => $\frac{d}{dr}\varphi(r) = 0$

$\psi(r) = const$ $\quad \frac{d}{dr}\psi(r) = 0$

Вывод:

$$0=\frac{c_1^2}{2}+\int_0^1\frac{1}{\rho}\,dP+L_{\text{тр}}=\frac{c_1^2}{2}+\int_0^1\frac{1}{\rho}\,dP+\frac{c_1^2}{2\cdot\varphi^2}-\frac{c_1^2}{2}=\frac{c_1^2}{2\cdot\varphi^2}+\int_0^1\frac{1}{\rho}\,dP=\frac{c_{a1}^2+c_{u1}^2}{2\cdot\varphi^2}+\int_0^1\frac{1}{\rho}\,dP=\frac{1}{\varphi^2}\cdot\left(\frac{c_{a1}^2}{2}+\frac{c_{u1}^2}{2}\right)+\int_0^1\frac{1}{\rho}\,dP$$

$$0=\frac{d}{dr}\left[\frac{1}{\varphi^2}\cdot\left(\frac{c_{a1}^2}{2}+\frac{c_{u1}^2}{2}\right)+\int_0^1\frac{1}{\rho}\,dP\right]=\frac{1}{\varphi^2}\cdot\left(c_{a1}\cdot\frac{d}{dr}c_{a1}+c_{u1}\cdot\frac{d}{dr}c_{u1}\right)+\frac{1}{\rho}\cdot\frac{d}{dr}P=\frac{1}{\varphi^2}\cdot\left(c_{a1}\cdot\frac{d}{dr}c_{a1}+c_{u1}\cdot\frac{d}{dr}c_{u1}\right)+\frac{c_{u1}^2}{r}$$

$$\frac{1}{\varphi^2}\cdot\left(c_{a1}\cdot\frac{d}{dr}c_{a1}+c_{u1}\cdot\frac{d}{dr}c_{u1}\right)+\frac{c_{u1}^2}{r}=0\qquad \frac{d}{dr}\left(\frac{c_{a1}^2}{2}\right)=-c_{u1}\cdot\left(\frac{d}{dr}c_{u1}\right)-\varphi^2\cdot\frac{c_{u1}^2}{r}\qquad \frac{d}{dr}c_{a1}^2=-2\cdot c_{u1}\cdot\left(\frac{d}{dr}c_{u1}\right)-2\cdot\varphi^2\cdot\frac{c_{u1}^2}{r}$$

$$\int_{r_{\text{ср}}}^{r}\frac{d}{dr}c_{a1}^2\,dr=c_{a1}^2-c_{a1.\text{ср}}^2=\int_{r_{\text{ср}}}^{r}-2\cdot c_{u1}\cdot\left(\frac{d}{dr}c_{u1}\right)-2\cdot\varphi^2\cdot\frac{c_{u1}^2}{r}\,dr\qquad c_{a1}=\sqrt{c_{a1.\text{ср}}^2+\int_{r_{\text{ср}}}^{r}-2\cdot c_{u1}\cdot\left(\frac{d}{dr}c_{u1}\right)-2\cdot\varphi^2\cdot\frac{c_{u1}^2}{r}\,dr}\qquad (c.a1)$$

$$0=\frac{d}{dr}c_{a2}^2+\frac{d}{dr}c_{u2}^2+\left(\frac{1}{\varphi^2}-1\right)\cdot\psi^2\cdot\left(\frac{d}{dr}c_{a1}^2+\frac{d}{dr}c_{u1}^2\right)+\psi^2\cdot\frac{2\cdot c_{u2}^2}{r}+\left(1-\psi^2\right)\cdot\frac{d}{dr}\left(2\cdot c_{u2}\cdot u+u^2\right)$$

$$\frac{d}{dr}c_{a2}^2=-\frac{d}{dr}c_{u2}^2-\left(\frac{1}{\varphi^2}-1\right)\cdot\psi^2\cdot\left(\frac{d}{dr}c_{a1}^2+\frac{d}{dr}c_{u1}^2\right)-\psi^2\cdot\frac{2\cdot c_{u2}^2}{r}-\left(1-\psi^2\right)\cdot\frac{d}{dr}\left[2\cdot c_{u2}\cdot\omega\cdot r+(\omega\cdot r)^2\right]$$

$$\int_{r_{\text{ср}}}^{r}\frac{d}{dr}c_{a2}^2\,dr=c_{a2}^2-c_{a2.\text{ср}}^2=\int_{r_{\text{ср}}}^{r}-\frac{d}{dr}c_{u2}^2-\left(\frac{1}{\varphi^2}-1\right)\cdot\psi^2\cdot\left(\frac{d}{dr}c_{a1}^2+\frac{d}{dr}c_{u1}^2\right)-\psi^2\cdot\frac{2\cdot c_{u2}^2}{r}-\left(1-\psi^2\right)\cdot\frac{d}{dr}\left[2\cdot c_{u2}\cdot\omega\cdot r+(\omega\cdot r)^2\right]dr=\int_{r_{\text{ср}}}^{r}-\frac{d}{dr}c_{u2}^2-\left(\frac{1}{\varphi^2}-1\right)\cdot\psi^2\cdot\left(\frac{d}{dr}c_{a1}^2+\frac{d}{dr}c_{u1}^2\right)-\psi^2\cdot\frac{2\cdot c_{u2}^2}{r}-\left(1-\psi^2\right)\cdot\frac{d}{dr}\left[2\cdot c_{u2}\cdot\omega\cdot r+(\omega\cdot r)^2\right]dr$$

$$c_{a2}=\sqrt{c_{a2.\text{ср}}^2+\int_{r_{\text{ср}}}^{r}-\frac{d}{dr}c_{u2}^2-\left(\frac{1}{\varphi^2}-1\right)\cdot\psi^2\cdot\left(\frac{d}{dr}c_{a1}^2+\frac{d}{dr}c_{u1}^2\right)-\psi^2\cdot\frac{2\cdot c_{u2}^2}{r}-\left(1-\psi^2\right)\cdot\frac{d}{dr}\left[2\cdot c_{u2}\cdot\omega\cdot r+(\omega\cdot r)^2\right]dr}=\sqrt{c_{a2.\text{ср}}^2+\int_{r_{\text{ср}}}^{r}-\frac{d}{dr}c_{u2}^2-\left(\frac{1}{\varphi^2}-1\right)\cdot\psi^2\cdot\left[\frac{d}{dr}\left[c_{a1.\text{ср}}^2+\int_{r_{\text{ср}}}^{r}-2\cdot c_{u1}\cdot\left(\frac{d}{dr}c_{u1}\right)-2\cdot\varphi^2\cdot\frac{c_{u1}^2}{r}\,dr\right]+\frac{d}{dr}c_{u1}^2\right]-\psi^2\cdot\frac{2\cdot c_{u2}^2}{r}-\left(1-\psi^2\right)\cdot\frac{d}{dr}\left[2\cdot c_{u2}\cdot\omega\cdot r+(\omega\cdot r)^2\right]dr}$$

(c.a2)

$$\sqrt{c_{a1.cp}^{2} + \int_{r_{cp}}^{r} -2\cdot c_{1u}(r,m,A,B)\cdot\left(\frac{d}{dr}c_{1u}(r,m,A,B)\right) - 2\cdot\varphi^{2}\cdot\frac{c_{1u}(r,m,A,B)^{2}}{r}\,dr}\ \left|\ \begin{array}{l}\text{assume}, r_{cp}>0, r\ge 0, m=-1\\ \text{substitute}, m=-1\\ \text{simplify}\end{array}\right. = \frac{\sqrt{B^{2}\cdot r^{2} - B^{2}\cdot r_{cp}^{2} - B^{2}\cdot\varphi^{2}\cdot r^{2} + B^{2}\cdot\varphi^{2}\cdot r_{cp}^{2} + A^{2}\cdot r^{2}\cdot r_{cp}^{4} - A^{2}\cdot r^{4}\cdot r_{cp}^{2} + r^{2}\cdot r_{cp}^{2}\cdot c_{a1.cp}^{2} + A^{2}\cdot\varphi^{2}\cdot r^{2}\cdot r_{cp}^{4} - A^{2}\cdot\varphi^{2}\cdot r^{4}\cdot r_{cp}^{2} - 4\cdot A\cdot B\cdot\varphi^{2}\cdot r^{2}\cdot r_{cp}^{2}\cdot\ln(r) + 4\cdot A\cdot B\cdot\varphi^{2}\cdot r^{2}\cdot r_{cp}^{2}\cdot\ln(r_{cp})}}{r\cdot r_{cp}}$$

$$\sqrt{c_{a2.cp}^{2} + \int_{r_{cp}}^{r} -2\, c_{2u}(r,m,A,B)\cdot\left(\frac{d}{dr}c_{2u}(r,m,A,B)\right) - 2\cdot\varphi^{2}\cdot\frac{c_{2u}(r,m,A,B)^{2}}{r}\,dr}\ \left|\ \begin{array}{l}\text{assume}, r_{cp}>0, r\ge 0, m=-1\\ \text{substitute}, m=-1\\ \text{simplify}\end{array}\right. = \frac{\sqrt{B^{2}\cdot r^{2} - B^{2}\cdot r_{cp}^{2} - B^{2}\cdot\varphi^{2}\cdot r^{2} + B^{2}\cdot\varphi^{2}\cdot r_{cp}^{2} + A^{2}\cdot r^{2}\cdot r_{cp}^{4} - A^{2}\cdot r^{4}\cdot r_{cp}^{2} + r^{2}\cdot r_{cp}^{2}\cdot c_{a2.cp}^{2} + A^{2}\cdot\varphi^{2}\cdot r^{2}\cdot r_{cp}^{4} - A^{2}\cdot\varphi^{2}\cdot r^{4}\cdot r_{cp}^{2} + 4\cdot A\cdot B\cdot\varphi^{2}\cdot r^{2}\cdot r_{cp}^{2}\cdot\ln(r) - 4\cdot A\cdot B\cdot\varphi^{2}\cdot r^{2}\cdot r_{cp}^{2}\cdot\ln(r_{cp})}}{r\cdot r_{cp}}$$

$$\sqrt{c_{a1.cp}^{2} + \int_{r_{cp}}^{r} -2\, c_{1u}(r,m,A,B)\cdot\left(\frac{d}{dr}c_{1u}(r,m,A,B)\right) - 2\cdot\varphi^{2}\cdot\frac{c_{1u}(r,m,A,B)^{2}}{r}\,dr}\ \left|\ \begin{array}{l}\text{assume}, r_{cp}>0, r\ge 0, m=0\\ \text{simplify}\end{array}\right. = \frac{\sqrt{B^{2}\cdot r^{2} - B^{2}\cdot r_{cp}^{2} - B^{2}\cdot\varphi^{2}\cdot r^{2} + B^{2}\cdot\varphi^{2}\cdot r_{cp}^{2} + r^{2}\cdot r_{cp}^{2}\cdot c_{a1.cp}^{2} - 2\cdot A\cdot B\cdot r\cdot r_{cp}^{2} + 2\cdot A\cdot B\cdot r^{2}\cdot r_{cp} + 4\cdot A\cdot B\cdot\varphi^{2}\cdot r\cdot r_{cp}^{2} - 4\cdot A\cdot B\cdot\varphi^{2}\cdot r^{2}\cdot r_{cp} - 2\cdot A^{2}\cdot\varphi^{2}\cdot r^{2}\cdot r_{cp}^{2}\cdot\ln(r) + 2\cdot A^{2}\cdot\varphi^{2}\cdot r^{2}\cdot r_{cp}^{2}\cdot\ln(r\cdots}}{r\cdot r_{cp}}$$

$$\sqrt{c_{a2.cp}^{2} + \int_{r_{cp}}^{r} -2\, c_{2u}(r,m,A,B)\cdot\left(\frac{d}{dr}c_{2u}(r,m,A,B)\right) - 2\cdot\varphi^{2}\cdot\frac{c_{2u}(r,m,A,B)^{2}}{r}\,dr}\ \left|\ \begin{array}{l}\text{assume}, r_{cp}>0, r\ge 0, m=0\\ \text{simplify}\end{array}\right. = \frac{\sqrt{B^{2}\cdot r^{2} - B^{2}\cdot r_{cp}^{2} - B^{2}\cdot\varphi^{2}\cdot r^{2} + B^{2}\cdot\varphi^{2}\cdot r_{cp}^{2} + r^{2}\cdot r_{cp}^{2}\cdot c_{a2.cp}^{2} + 2\cdot A\cdot B\cdot r\cdot r_{cp}^{2} - 2\cdot A\cdot B\cdot r^{2}\cdot r_{cp} - 4\cdot A\cdot B\cdot\varphi^{2}\cdot r\cdot r_{cp}^{2} + 4\cdot A\cdot B\cdot\varphi^{2}\cdot r^{2}\cdot r_{cp} - 2\cdot A^{2}\cdot\varphi^{2}\cdot r^{2}\cdot r_{cp}^{2}\cdot\ln(r) + 2\cdot A^{2}\cdot\varphi^{2}\cdot r^{2}\cdot r_{cp}^{2}\cdot\ln(r\cdots}}{r\cdot r_{cp}}$$

$$\sqrt{c_{a1.cp}^{2} + \int_{r_{cp}}^{r} -2\, c_{1u}(r,m,A,B)\cdot\left(\frac{d}{dr}c_{1u}(r,m,A,B)\right) - 2\cdot\varphi^{2}\cdot\frac{c_{1u}(r,m,A,B)^{2}}{r}\,dr}\ \left|\ \begin{array}{l}\text{assume}, r_{cp}>0, r\ge 0, -1<m\le 1, m\ne 0\\ \text{simplify}\end{array}\right. = \sqrt{\frac{B^{2}\cdot m + B^{2}\cdot m^{2} + m\cdot r_{cp}^{2}\cdot c_{a1.cp}^{2} + A^{2}\cdot m\cdot r_{cp}^{2-2\cdot m} - B^{2}\cdot\varphi^{2}\cdot m^{2} + m^{2}\cdot r_{cp}^{2}\cdot c_{a1.cp}^{2} - A^{2}\cdot\varphi^{2}\cdot r_{cp}^{2-2\cdot m} + A^{2}\cdot m^{2}\cdot r_{cp}^{2-2\cdot m} - B^{2}\cdot\varphi^{2}\cdot m + 2\cdot A\cdot B\cdot m\cdot r_{cp}^{1-m} - A^{2}\cdot\varphi^{2}\cdots}{\cdots}}$$

$$\sqrt{c_{a2.cp}^{2} + \int_{r_{cp}}^{r} -2\, c_{2u}(r,m,A,B)\cdot\left(\frac{d}{dr}c_{2u}(r,m,A,B)\right) - 2\cdot\varphi^{2}\cdot\frac{c_{2u}(r,m,A,B)^{2}}{r}\,dr}\ \left|\ \begin{array}{l}\text{assume}, r_{cp}>0, r\ge 0, -1<m\le 1, m\ne 0\\ \text{simplify}\end{array}\right. = \sqrt{\frac{B^{2}\cdot m + B^{2}\cdot m^{2} + m\cdot r_{cp}^{2}\cdot c_{a2.cp}^{2} + A^{2}\cdot m\cdot r_{cp}^{2-2\cdot m} - B^{2}\cdot\varphi^{2}\cdot m^{2} + m^{2}\cdot r_{cp}^{2}\cdot c_{a2.cp}^{2} - A^{2}\cdot\varphi^{2}\cdot r_{cp}^{2-2\cdot m} + A^{2}\cdot m^{2}\cdot r_{cp}^{2-2\cdot m} - B^{2}\cdot\varphi^{2}\cdot m - 2\cdot A\cdot B\cdot m\cdot r_{cp}^{1-m} - A^{2}\cdot\varphi^{2}\cdots}{\cdots}}$$

$$\frac{\dfrac{m\cdot r_{cp}^{2-2\cdot m}+A^{2}\cdot\varphi^{2}\cdot r_{cp}^{2}\cdot\left(\frac{1}{r^{2}}\right)^{m}-A^{2}\cdot m^{2}\cdot r_{cp}^{2}\cdot\left(\frac{1}{r^{2}}\right)^{m}-\frac{B^{2}\cdot m^{2}\cdot r_{cp}^{2}}{r^{2}}+2\cdot A\cdot B\cdot m^{2}\cdot r_{cp}^{1-m}-A^{2}\cdot m\cdot r_{cp}^{2}\cdot\left(\frac{1}{r^{2}}\right)^{m}-\frac{B^{2}\cdot m\cdot r_{cp}^{2}}{r^{2}}-\frac{2\cdot A\cdot B\cdot m\cdot r_{cp}^{2}}{r^{m+1}}+\frac{B^{2}\cdot\varphi^{2}\cdot m^{2}\cdot r_{cp}^{2}}{r^{2}}-\frac{2\cdot A\cdot B\cdot m^{2}\cdot r_{cp}^{2}}{r^{m+1}}+A^{2}\cdot\varphi^{2}\cdot m\cdot r_{cp}^{2}\cdot\left(\frac{1}{r^{2}}\right)^{m}+\frac{B^{2}\cdot\varphi^{2}\cdot m\cdot r_{cp}^{2}}{r^{2}}-4\cdot A\cdot B\cdot\varphi^{2}\cdot m\cdot r_{cp}^{1-m}+\frac{4\cdot A\cdot B\cdot\varphi^{2}\cdot m\cdot r_{cp}^{2}}{r^{m+1}}}{m\cdot(m+1)}}{r_{cp}}$$

$$\frac{\dfrac{m\cdot r_{cp}^{2-2\cdot m}+A^{2}\cdot\varphi^{2}\cdot r_{cp}^{2}\cdot\left(\frac{1}{r^{2}}\right)^{m}-A^{2}\cdot m^{2}\cdot r_{cp}^{2}\cdot\left(\frac{1}{r^{2}}\right)^{m}-\frac{B^{2}\cdot m^{2}\cdot r_{cp}^{2}}{r^{2}}-2\cdot A\cdot B\cdot m^{2}\cdot r_{cp}^{1-m}-A^{2}\cdot m\cdot r_{cp}^{2}\cdot\left(\frac{1}{r^{2}}\right)^{m}-\frac{B^{2}\cdot m\cdot r_{cp}^{2}}{r^{2}}+\frac{2\cdot A\cdot B\cdot m\cdot r_{cp}^{2}}{r^{m+1}}+\frac{B^{2}\cdot\varphi^{2}\cdot m^{2}\cdot r_{cp}^{2}}{r^{2}}+\frac{2\cdot A\cdot B\cdot m^{2}\cdot r_{cp}^{2}}{r^{m+1}}+A^{2}\cdot\varphi^{2}\cdot m\cdot r_{cp}^{2}\cdot\left(\frac{1}{r^{2}}\right)^{m}+\frac{B^{2}\cdot\varphi^{2}\cdot m\cdot r_{cp}^{2}}{r^{2}}+4\cdot A\cdot B\cdot\varphi^{2}\cdot m\cdot r_{cp}^{1-m}-\frac{4\cdot A\cdot B\cdot\varphi^{2}\cdot m\cdot r_{cp}^{2}}{r^{m+1}}}{m\cdot(m+1)}}{r_{cp}}$$