Imperial College London

# Relativity – Lecture 5

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# Key concepts of lecture 4

## Lorentz transformations (1D):

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \frac{vx}{c^2})$$

- c is constant in any reference frame.
- LT give same result as length contraction & time dilation (if applicable)

## Velocity addition:

$$u' = \frac{u-v}{1-\frac{uv}{c^2}}$$

# What if *u* and *v* << *c*? What if *u'* → *c*?

$$u' = \frac{u-v}{1-\frac{uv}{c^2}}$$

$$\boxed{\frac{uv}{c^2} \ll 1}$$

$$u' = (u-v)\left(1+\frac{uv}{c^2}+\ldots\right)$$

Binomial expansion

$$= u - v$$

Just like Galilean

[Same result for $c \to \infty$]

$$u = \frac{u'+v}{1+\frac{u'v}{c^2}}$$

$$u' \to c$$

$$u = \frac{c+v}{1+\frac{cv}{c^2}} = \frac{c\cancel{\left(1+\frac{v}{c}\right)}}{\cancel{1+\frac{v}{c}}} = c$$

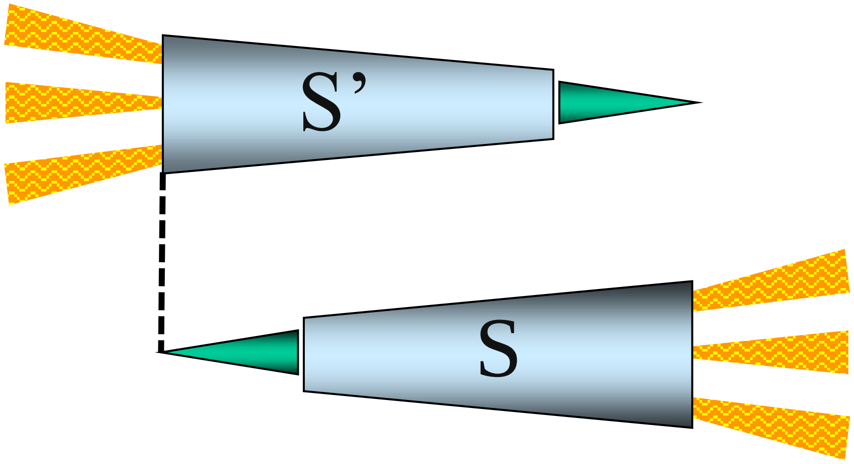
Maximum velocity = c

# Space fight

Two spacecraft of equal rest length $L_0$ = 100 m pass very, very close to each other as they travel in opposite directions at a relative speed of 3/5 c.

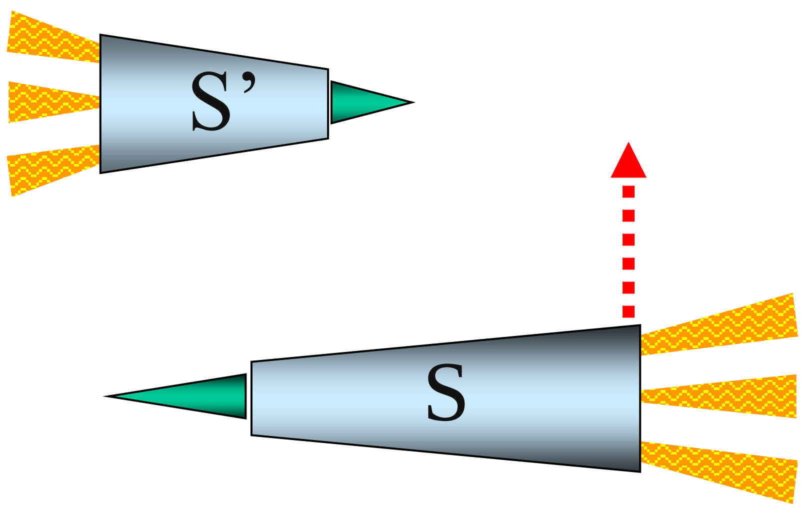
# Space fight

Ali, the captain of ship S, has a laser cannon at his tail that he plans to fire at the nose of Brenda's S' ship when he observes his nose lined up with her tail.



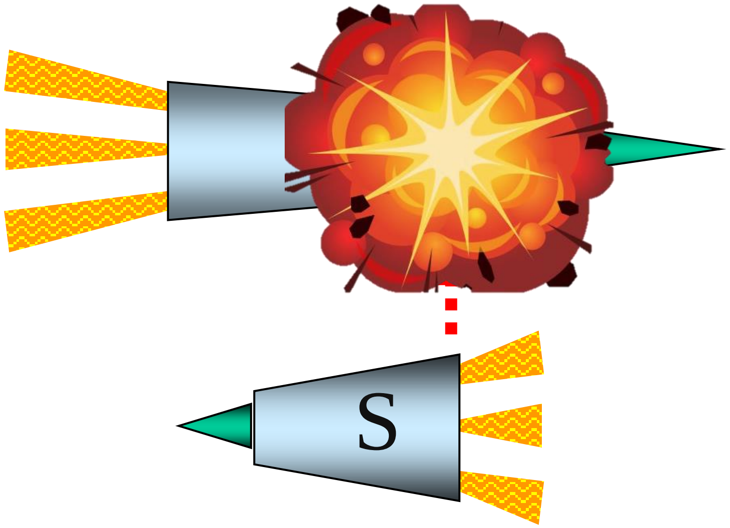
# Space fight

It is only supposed to be a warning shot across nose and he figures it won’t hit because Brenda’s S’ ship is length contracted.



# Space fight

However, his co-pilot says that the shot will hit because Brenda sees that the length of ship S is shortened.



# Who is right?

Event 1: Ali's nose lines up with Brenda's tail

$$x = 0 \qquad x' = 0$$
$$t = 0 \qquad t' = 0$$

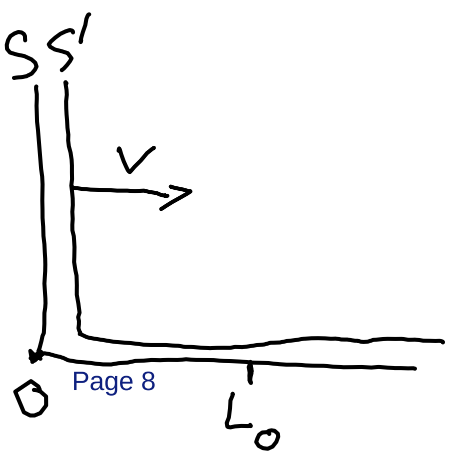
Event 2: Ali shoots laser from his ship's tail

$$x = L_0 \qquad x' = \gamma(x - vt) = \gamma L_0$$
$$t = 0 \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right) = -\frac{v}{c^2}\gamma L_0$$

So Brenda sees shot at:

$$(x', t') = \left(\gamma L_0, -\frac{v}{c^2}\gamma L_0\right)$$

So <u>in front</u> of her nose, and <u>before</u> her tail lines up with Ali's nose.



# Brenda's view

# Order of events

Ali:

1: Ali's nose lines up with Brenda's tail.

2: Ali shoots laser from his ship's tail.

3: Ali's tail lines up with Brenda's nose.

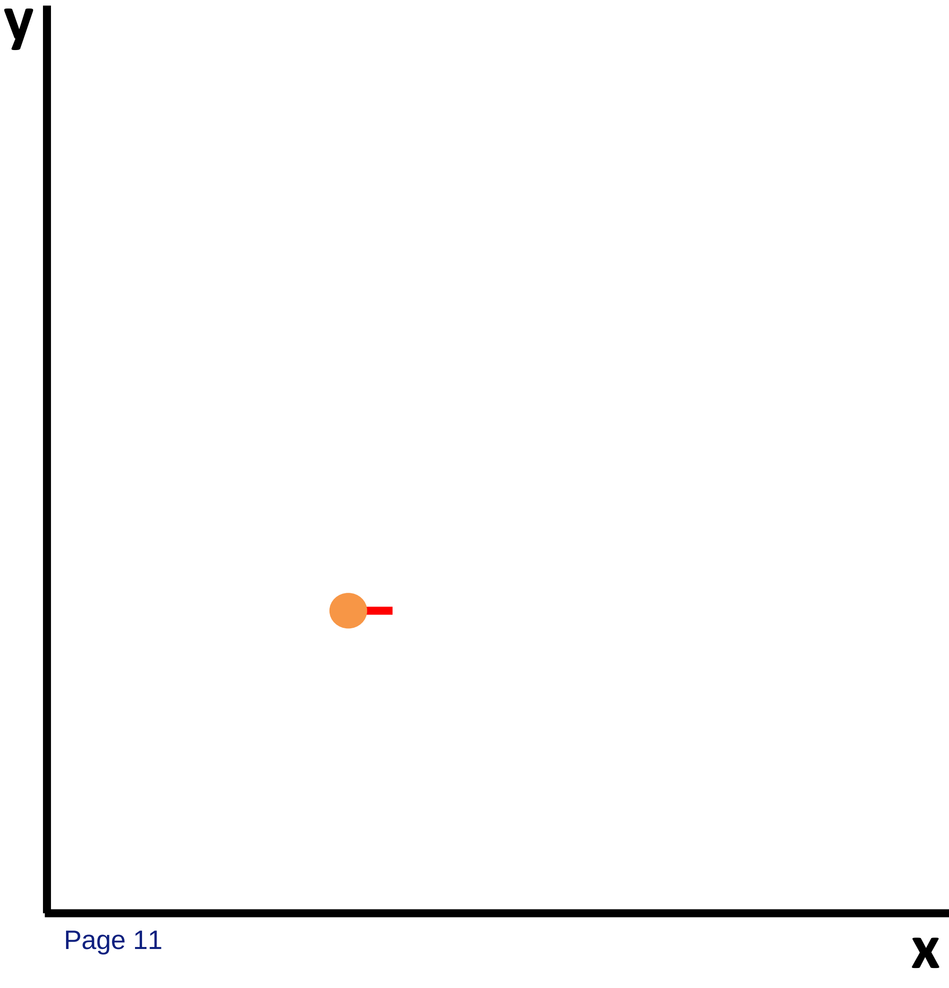
Brenda:

2: Ali shoots laser from his ship's tail.

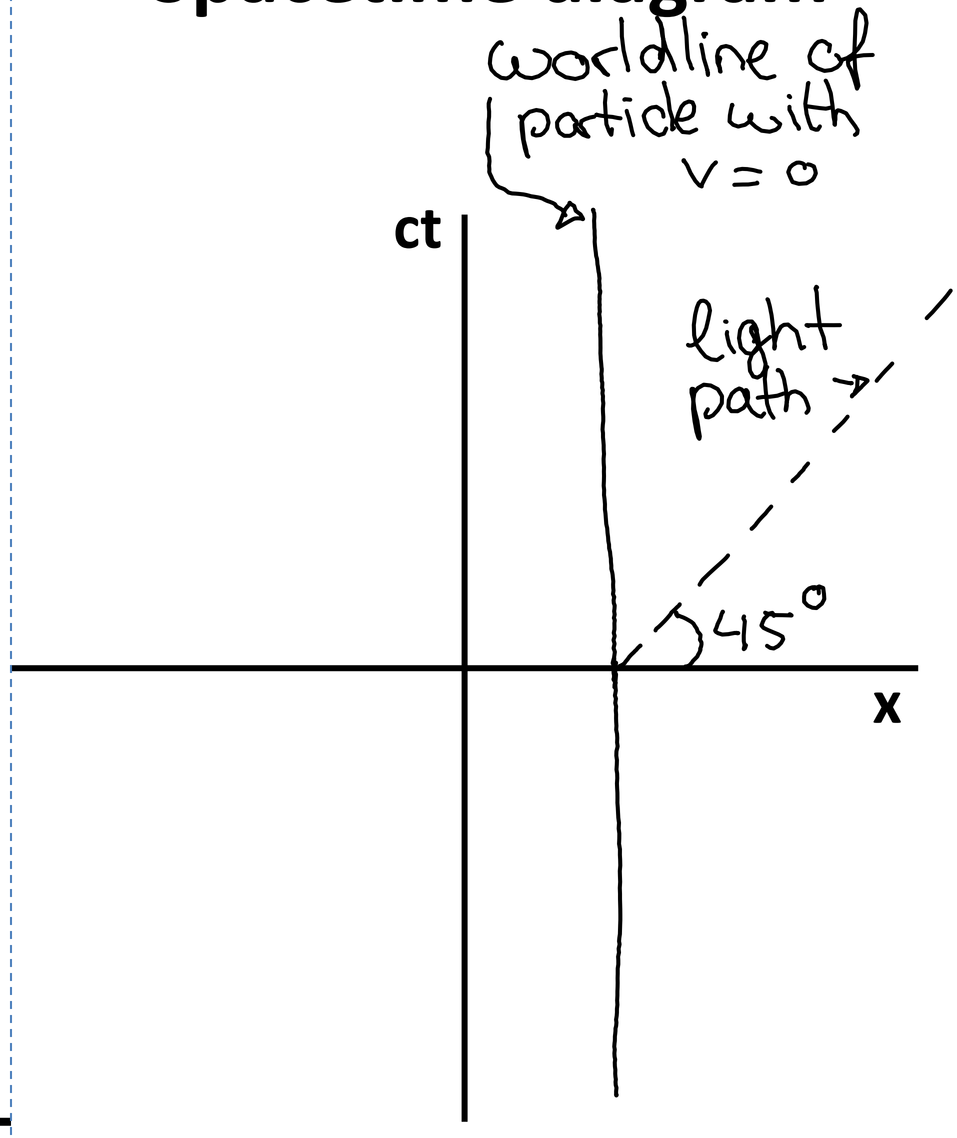
3: Ali's tail lines up with Brenda's nose.

1: Ali's nose lines up with Brenda's tail.

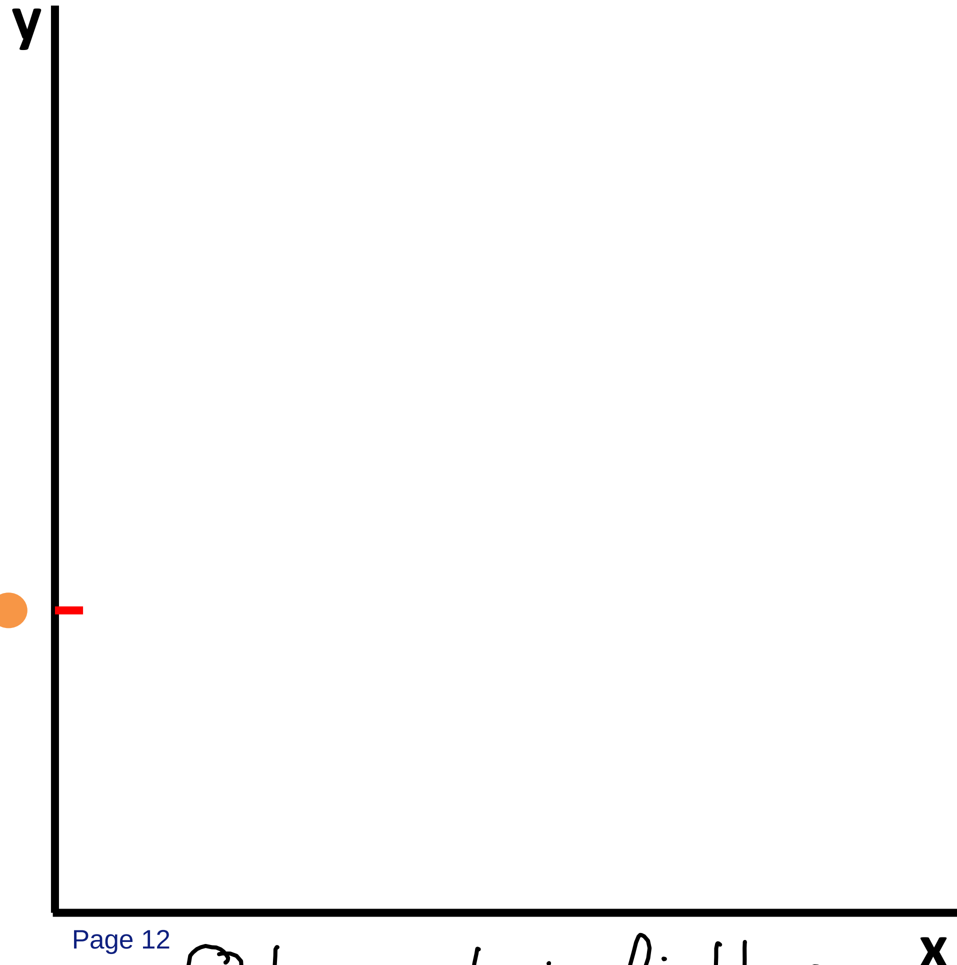
# X-Y diagram



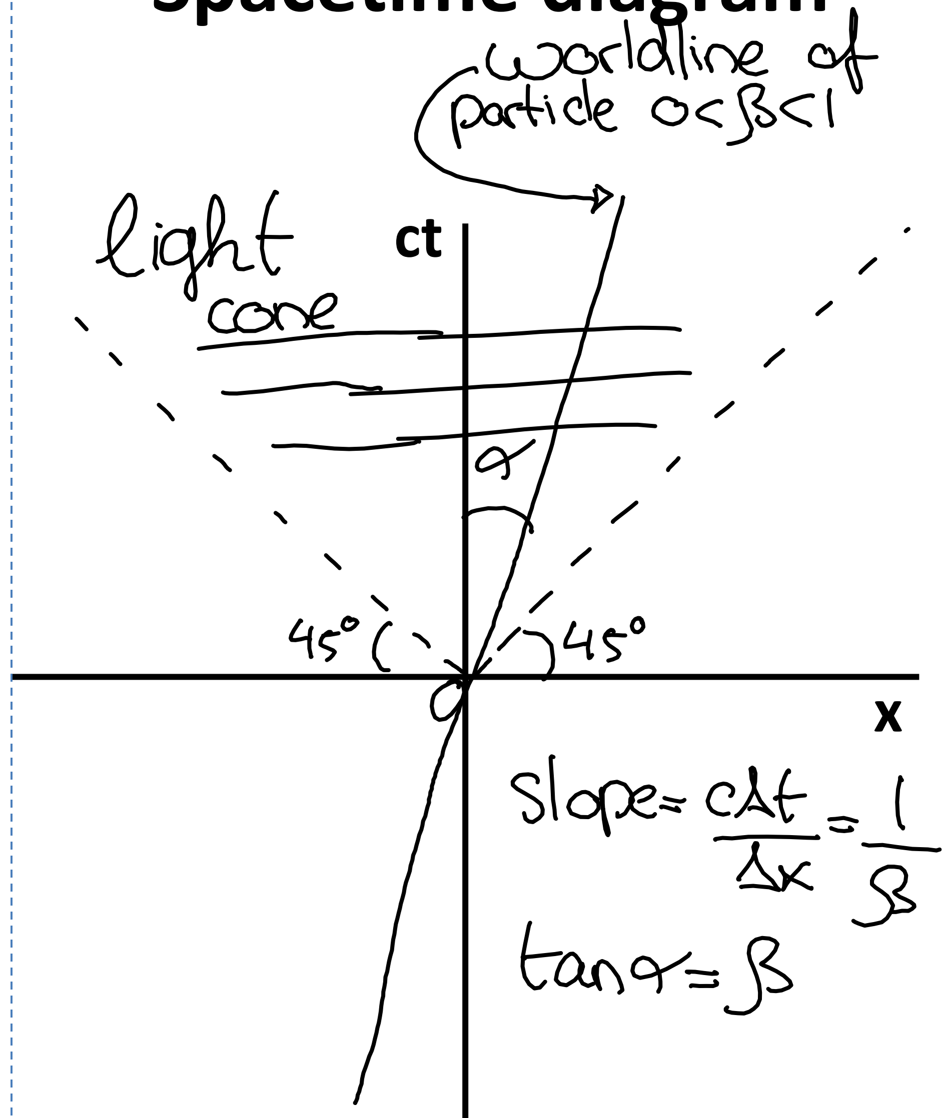

# Spacetime diagram



## X-Y diagram



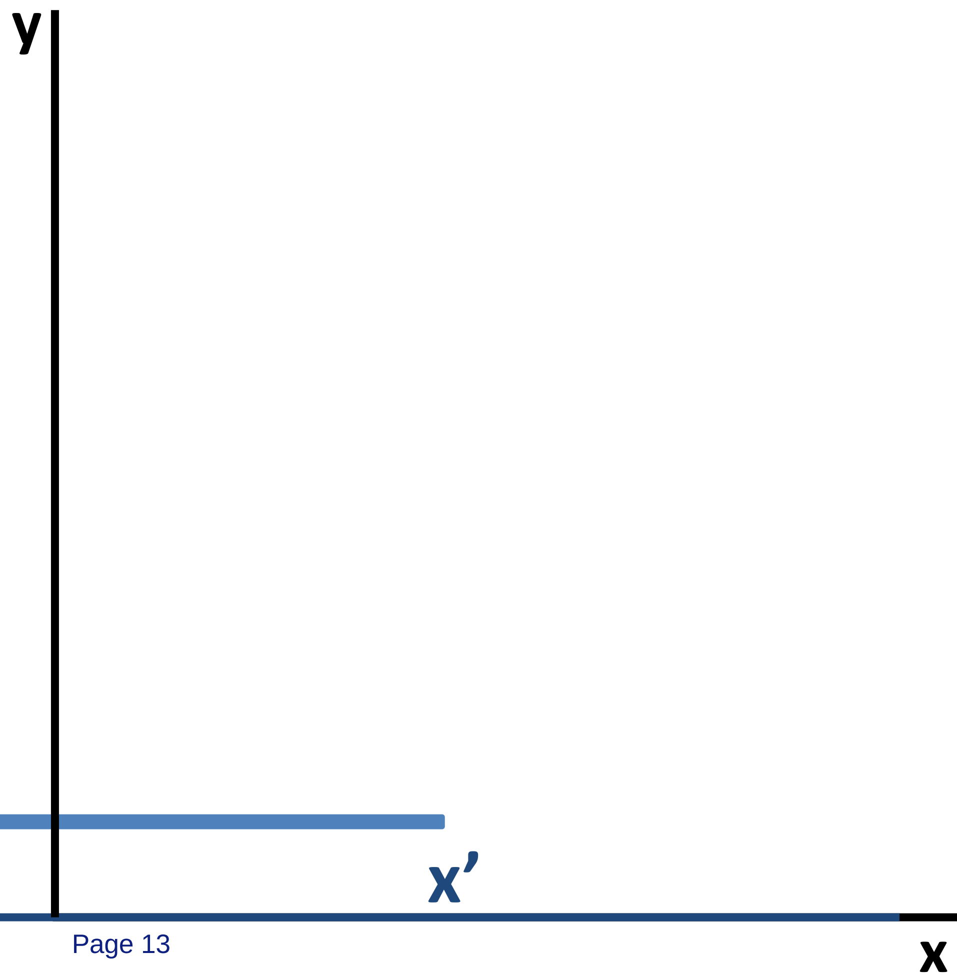

## Spacetime diagram



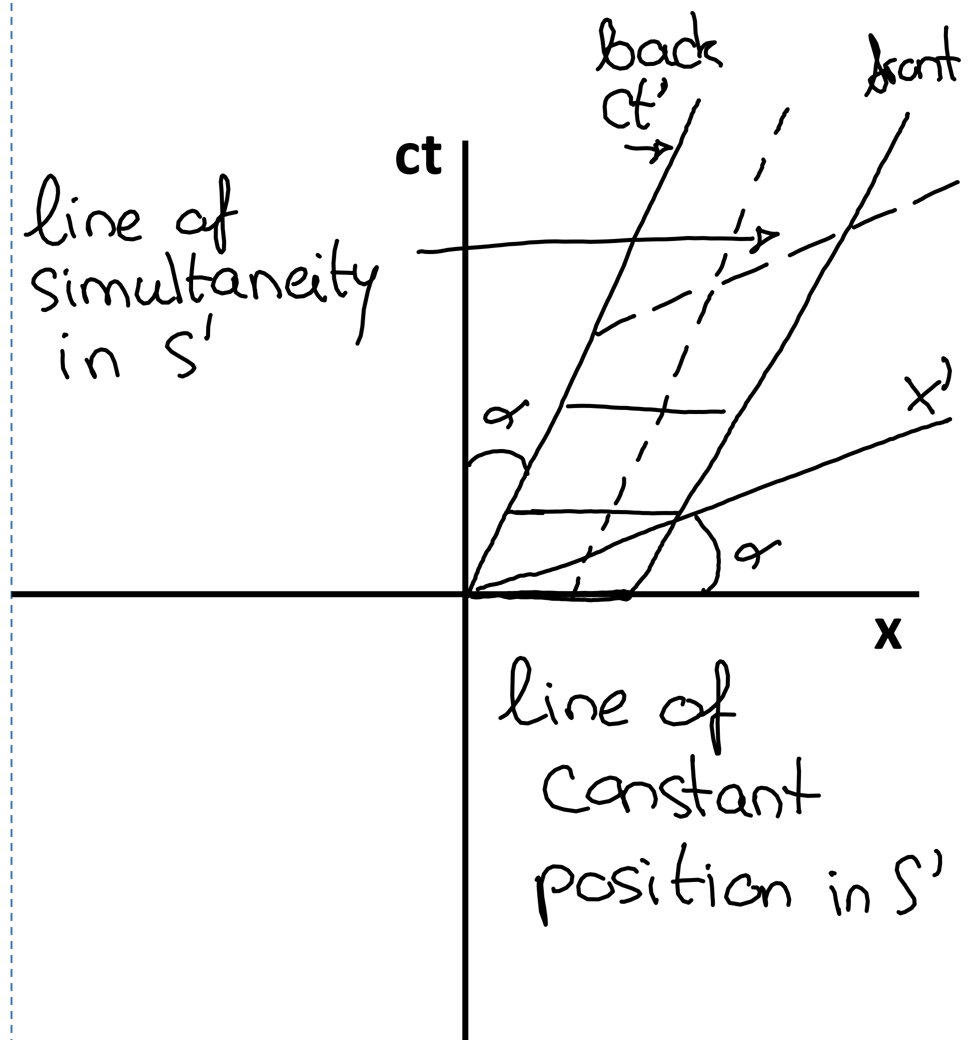



Only events in light cone can be reached from O.

# X-Y diagram




# Spacetime diagram



# The position four-vector and the invariant interval

Events are expressed in 4 coordinates.

($ct, x, y, z$) is called the position four-vector,
or 4-position.

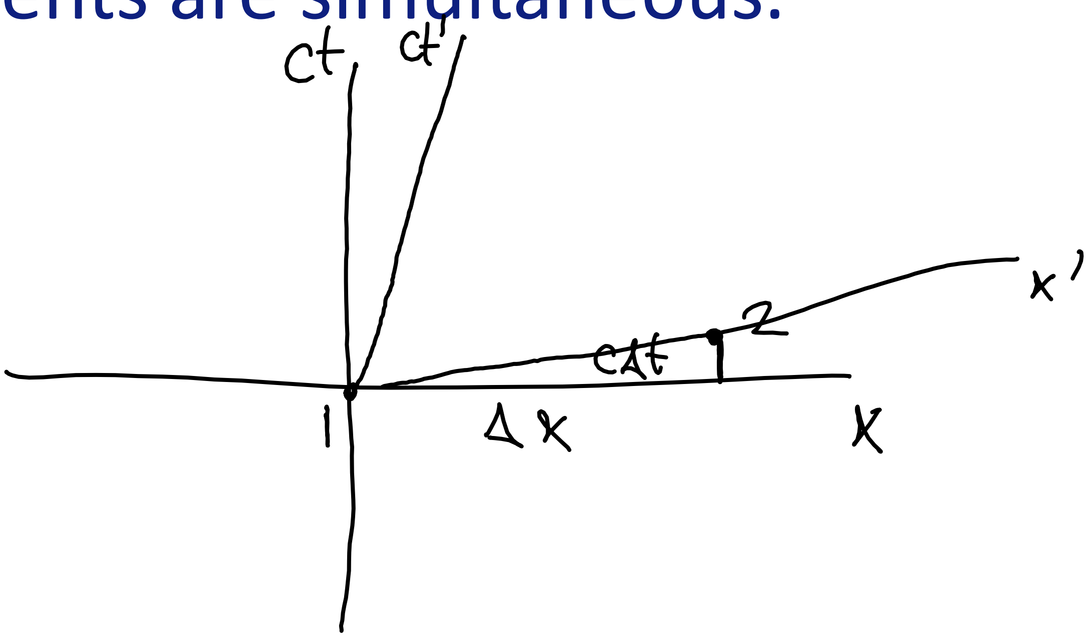
$s^2 = c^2\Delta t^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2)$ is the <u>invariant</u> interval.

$s^2$ is the same in any frame!

For light, $s^2 = 0$: the separation between two events is lightlike.

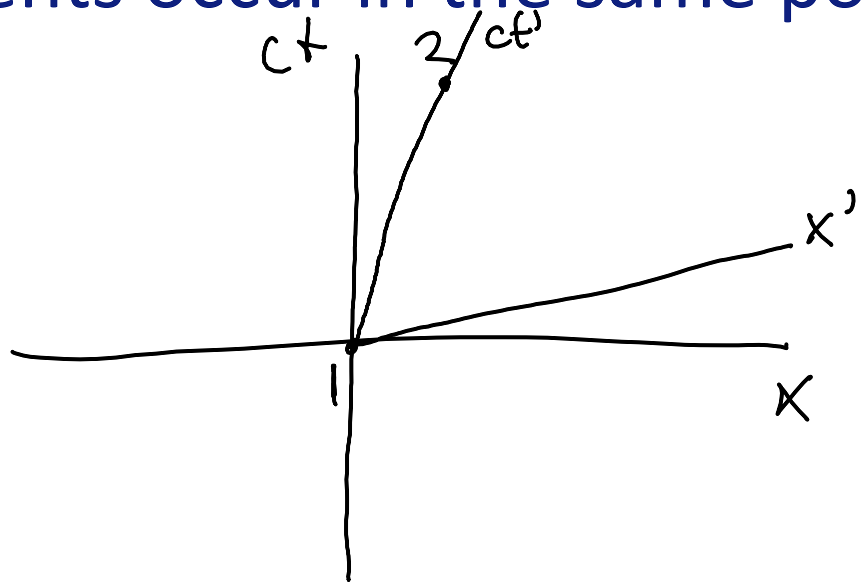
## Spacelike separation of events

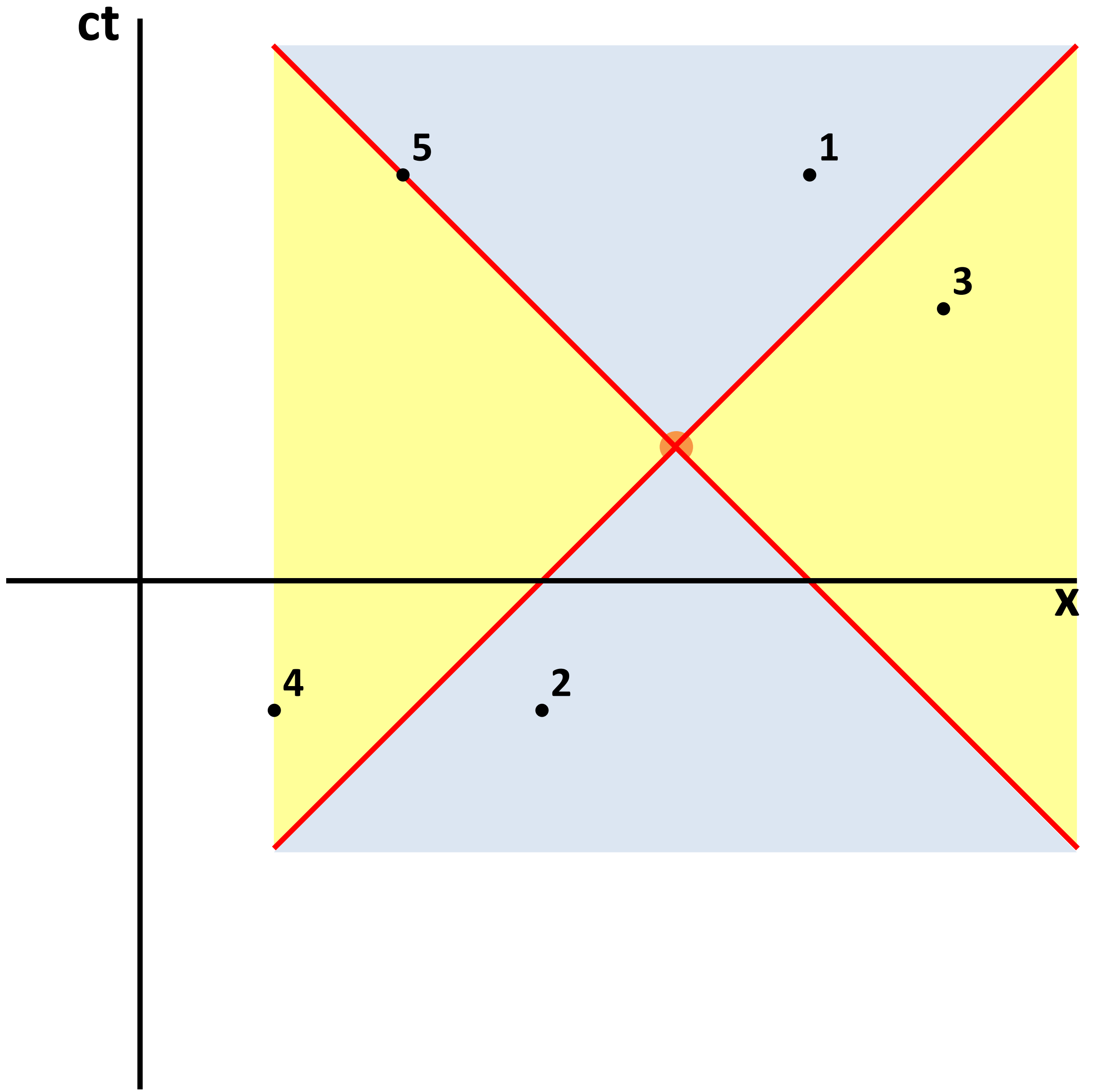
- $s^2 < 0$, so $\Delta r^2 > c^2\Delta t^2$. Nothing can travel between the two events.
- A reference frame can be found where the two events are simultaneous.



## Timelike separation of events

- $s^2 > 0$, so $\Delta r^2 < c^2 \Delta t^2$. Information can be exchanged between the two events.
- Causality: the order of events is preserved.
- A reference frame can be found where the two events occur in the same position.




ct
x
5
1
3
4
2

## Summary

1. Events show up as points in a spacetime diagram. Moving objects have a worldline in this diagram.
2. The 4-position contains the four coordinates of an event in time and space.
3. The invariant interval $s^2 = c^2\Delta t^2 - \Delta \boldsymbol{r}^2$ denotes the separation between events.
4. $s^2 < 0$, spacelike separation,
   $s^2 > 0$, timelike separation,
   $s^2 = 0$, timelike separation.