Imperial College London

Relativity – Lecture 5

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Key concepts of lecture 4

Lorentz transformations (1D):

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \frac{vx}{c^2})$$

Velocity addition:

$$u' = \frac{u-v}{1-\frac{uv}{c^2}}$$

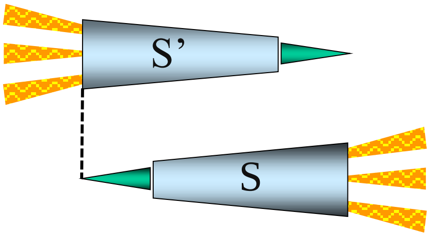
What if u and $v \ll c$? What if $u' \rightarrow c$?

Space fight

Two spacecraft of equal rest length L_0 = 100 m pass very, very close to each other as they travel in opposite directions at a relative speed of 3/5 c.

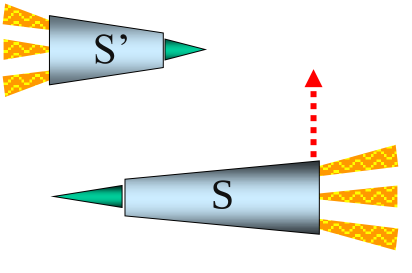
Space fight

Ali, the captain of ship S, has a laser cannon at his tail that he plans to fire at the nose of Brenda's S' ship when he observes his nose lined up with her tail.



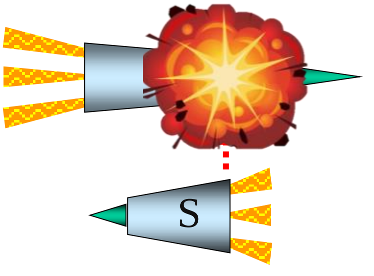
Space fight

It is only supposed to be a warning shot across nose and he figures it won't hit because Brenda's S' ship is length contracted.



Space fight

However, his co-pilot says that the shot will hit because Brenda sees that the length of ship S is shortened.



Who is right?

Brenda's view

Order of events

Ali:

1: Ali's nose lines up with Brenda's tail.

2: Ali shoots laser from his ship's tail.

3: Ali's tail lines up with Brenda's nose.

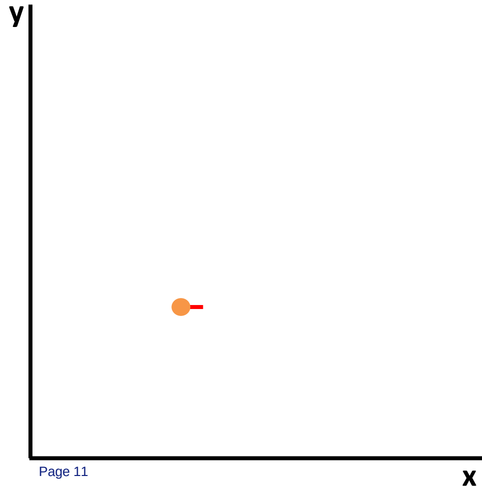
Brenda:

2: Ali shoots laser from his ship's tail.

3: Ali's tail lines up with Brenda's nose.

1: Ali's nose lines up with Brenda's tail.

X-Y diagram



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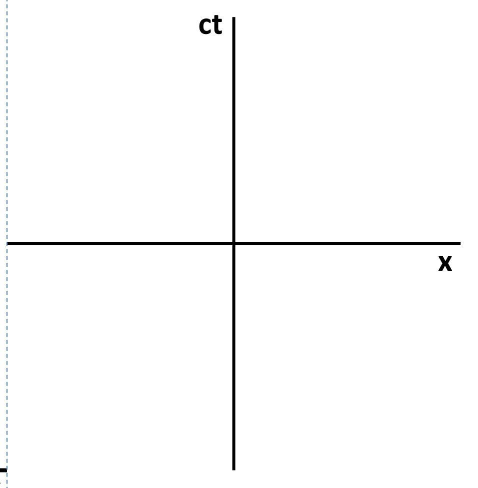
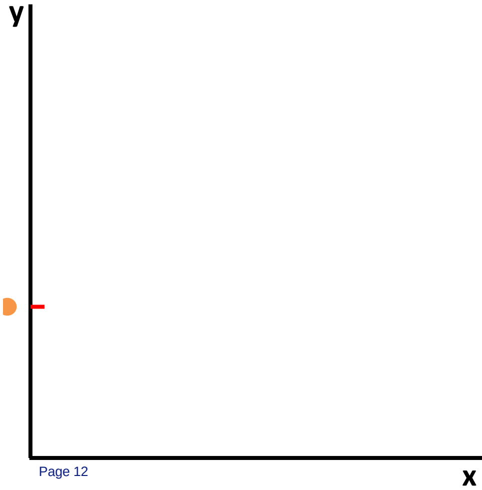
Spacetime diagram



X-Y diagram



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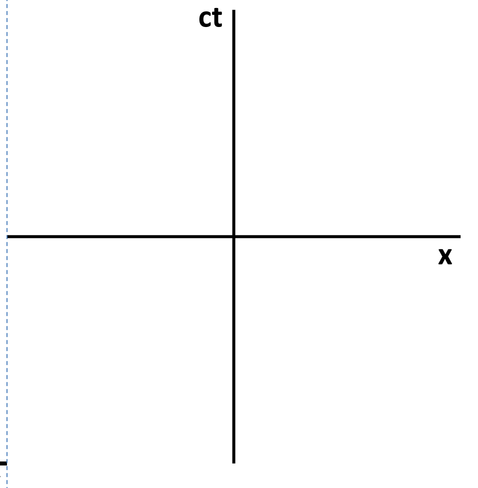
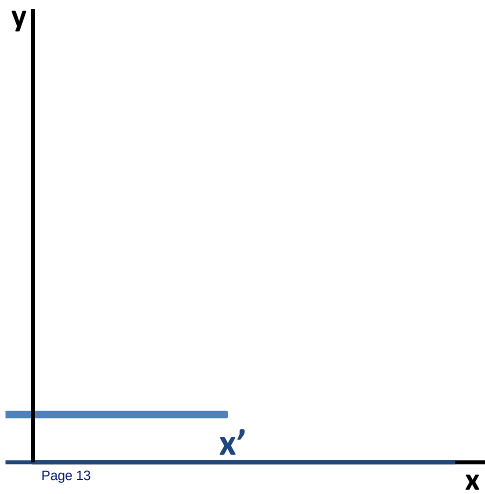
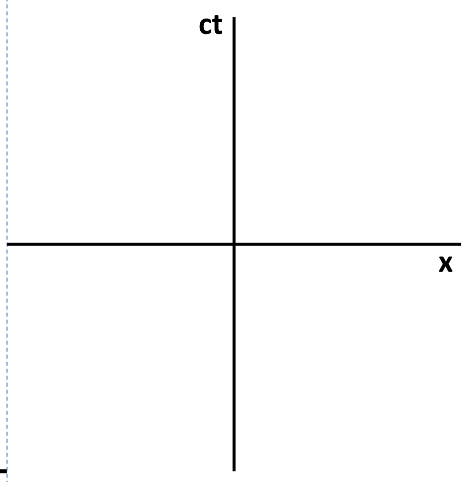
Spacetime diagram



X-Y diagram | **Spacetime diagram**





The position four-vector and the invariant interval

Events are expressed in 4 coordinates.

(ct, x, y, z) is called the position four-vector,
or 4-position.

$s^2 = c^2\Delta t^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2)$ is the invariant interval.

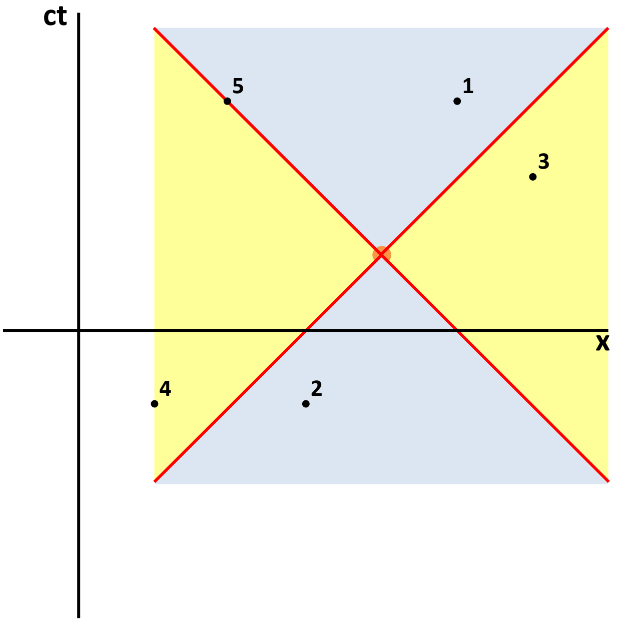
For light, $s^2 = 0$: the separation between two events is lightlike.

Spacelike separation of events

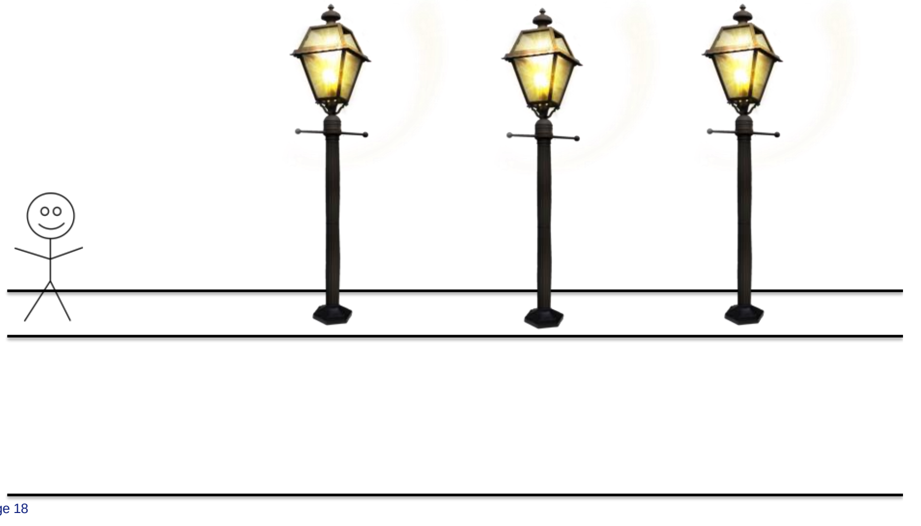
- $s^2 < 0$, so $\Delta r^2 > c^2 \Delta t^2$. Nothing can travel between the two events.
- A reference frame can be found where the two events are simultaneous.

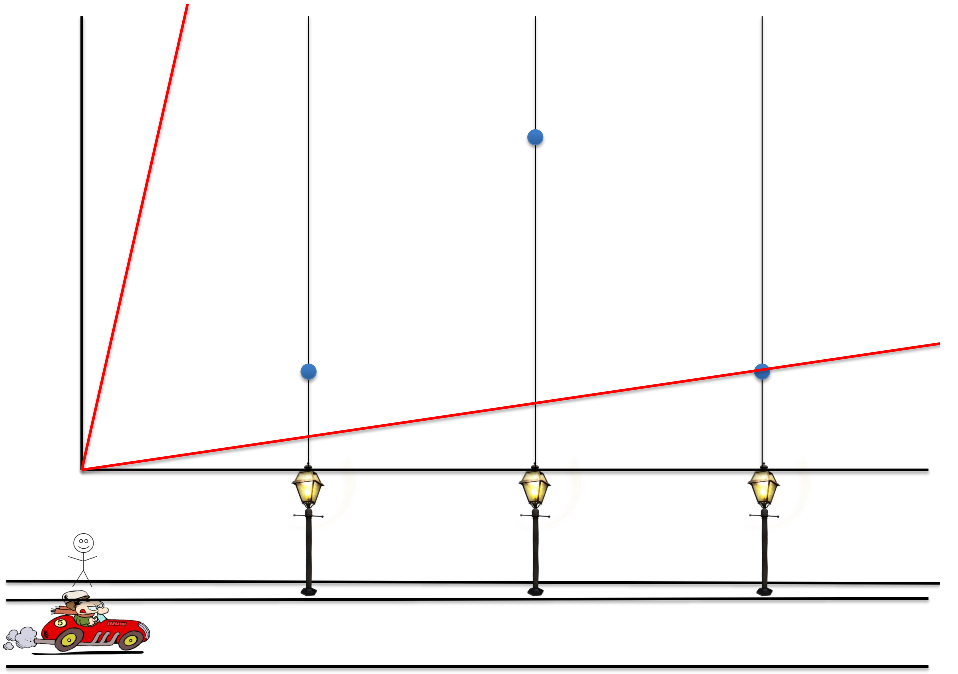
Timelike separation of events

- $s^2 > 0$, so $\Delta r^2 < c^2 \Delta t^2$. Information can be exchanged between the two events.
- Causality: the order of events is preserved.
- A reference frame can be found where the two events occur in the same position.



Example: street lights in a relativistic car





Questions

1. What is the order in which the lamps are turned on in the observer's frame?
2. What is the order in which the lamps are turned on in the car's frame?
3. Where is the car compared to the street lights when the light from lamp A reaches it?

Summary

1. Events show up as points in a spacetime diagram. Moving objects have a worldline in this diagram.
2. The 4-position contains the four coordinates of an event in time and space.
3. The invariant interval $s^2 = c^2\Delta t^2 - \Delta \mathbf{r}^2$ denotes the separation between events.
4. $s^2 < 0$, spacelike separation,
   $s^2 > 0$, timelike separation,
   $s^2 = 0$, timelike separation.

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