Imperial College London

Relativity – Lecture 3

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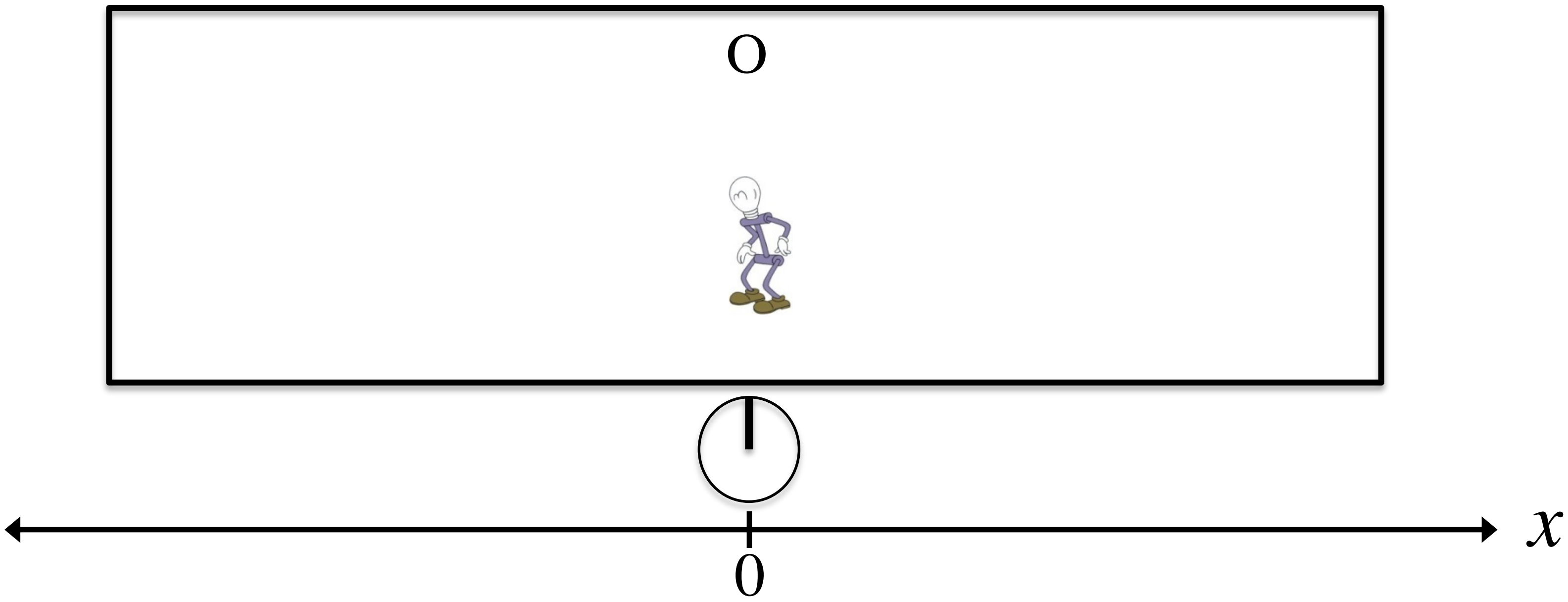
Key points of Lecture 2

- An inertial frame is a frame in which the law of inertia holds.
- Each reference frame contains an infinite number of observers with synchronised clocks who know their position.
- Events happen at a particular position AND at a particular time.
- The fact that observers in all inertial frames measure the same speed of light, *c*, leads to counterintuitive effects.

Synchronised clocks

Oscar sits at the origin of reference frame S ($x = 0$).



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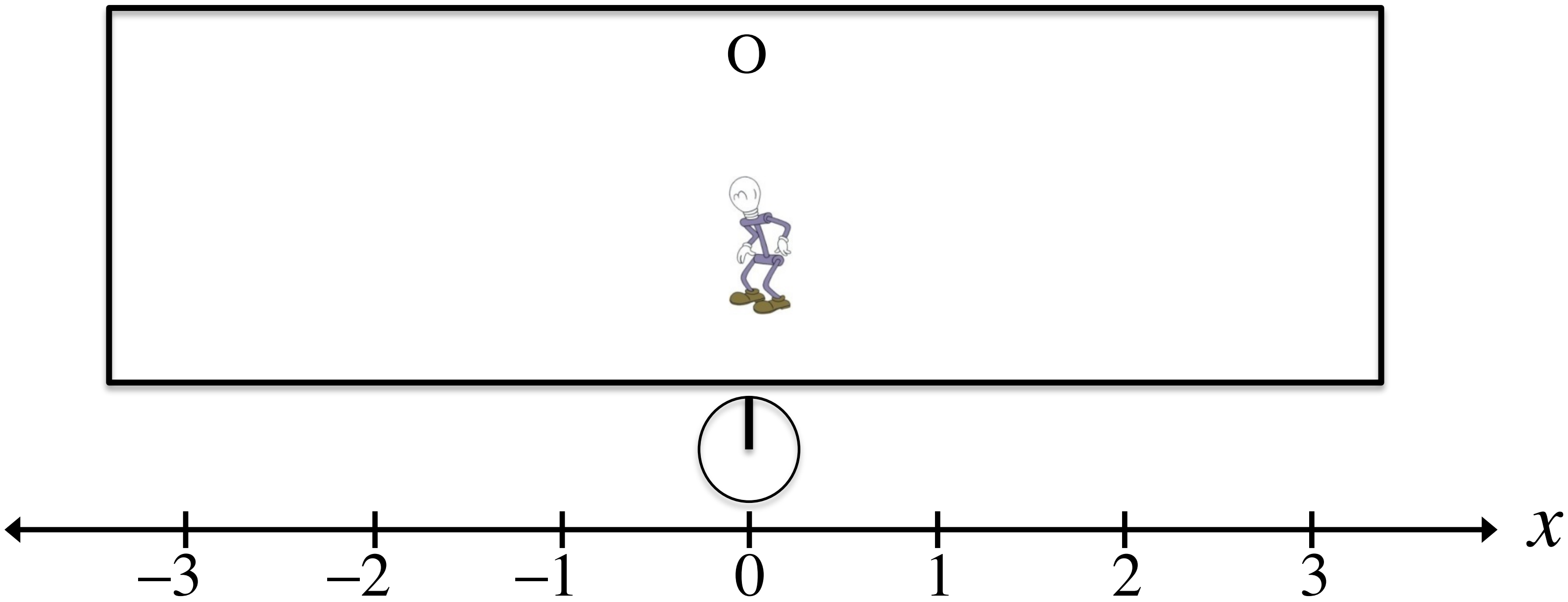
Oscar sits at the origin of reference frame *S* ($x = 0$).

Meter sticks establish distances in *S*.



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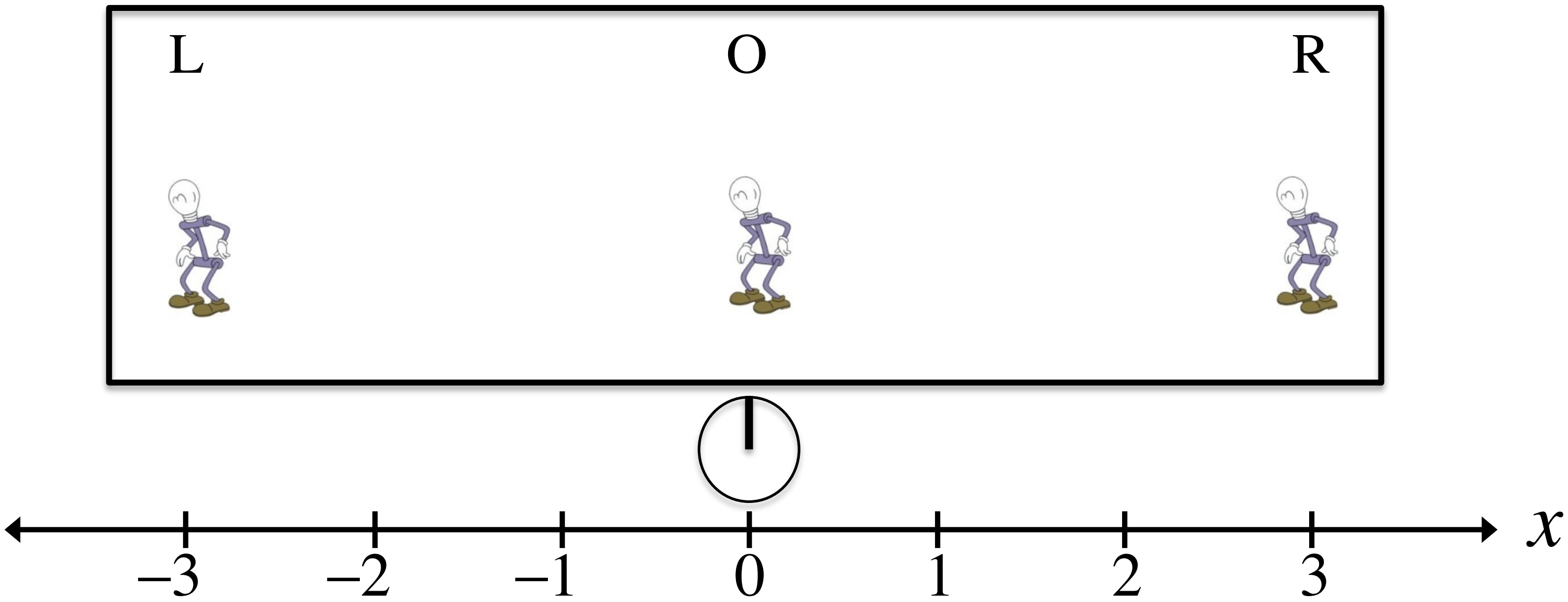
Local observers at $x = -3$ m (Lucy) & $x = +3$ m (Ricky).



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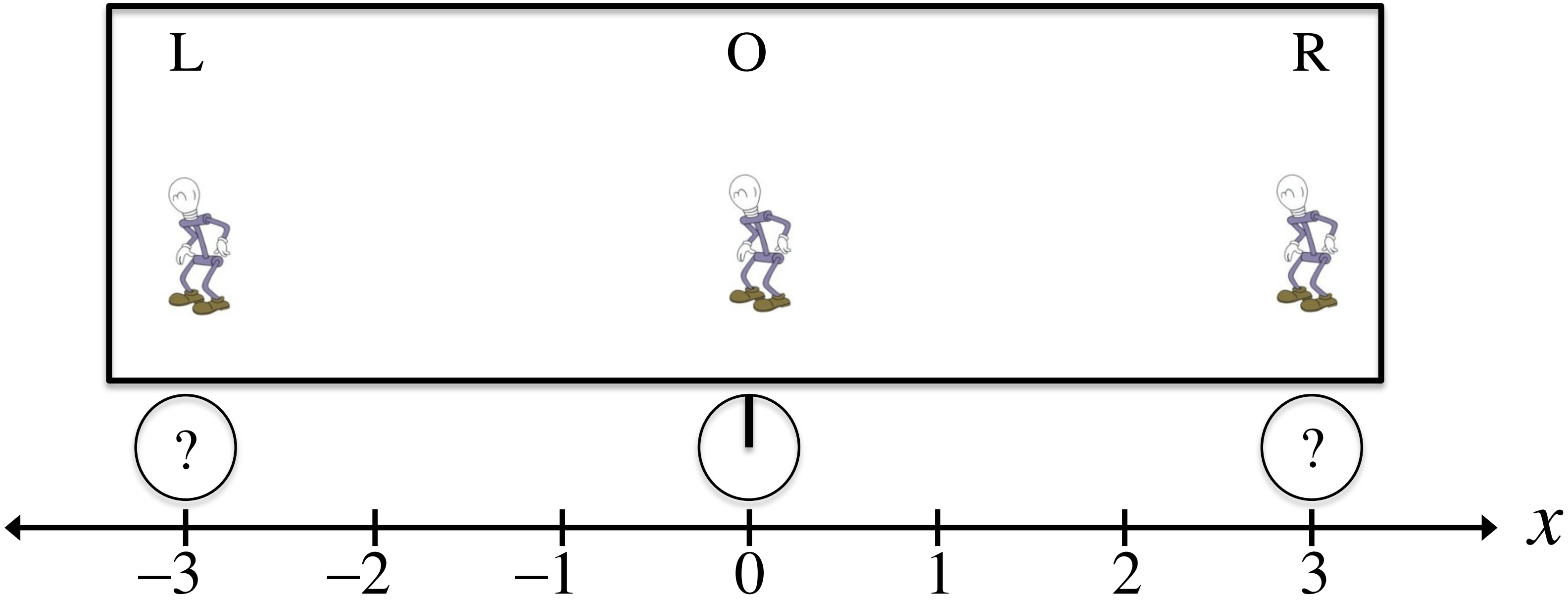
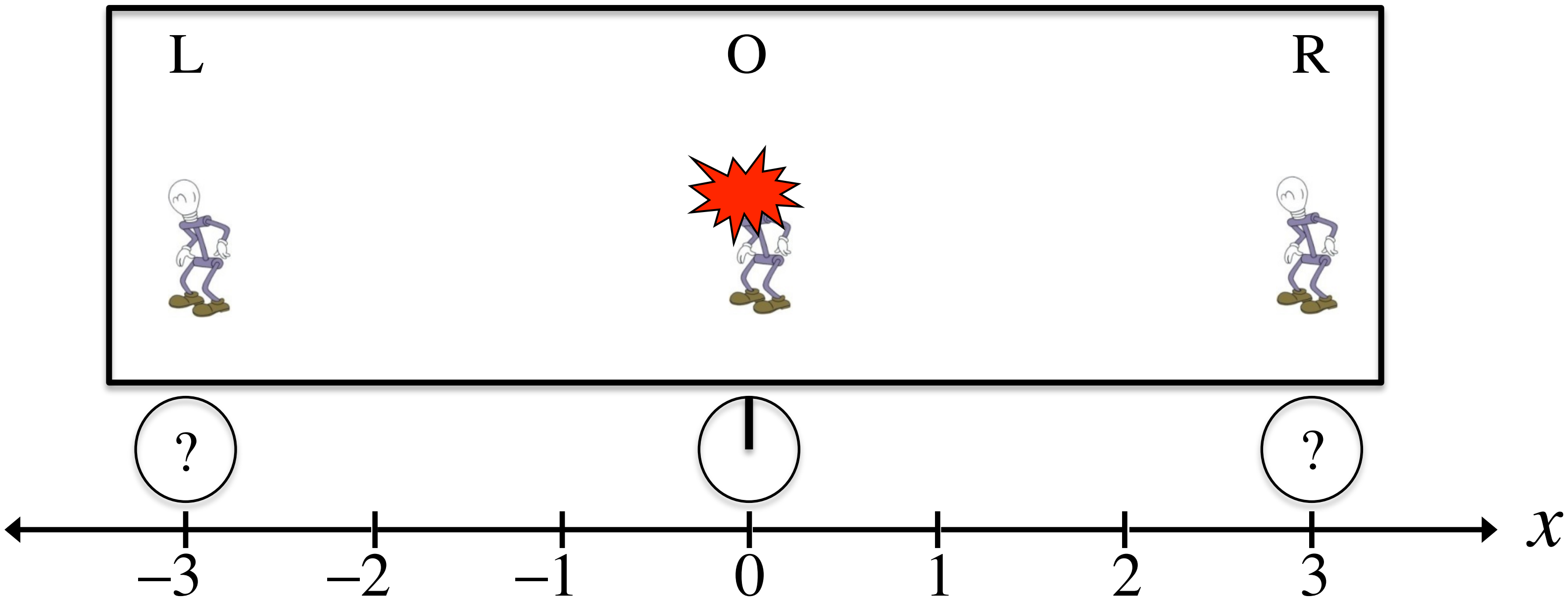
Procedure to synchronize all clocks in *S* ?



Procedure to synchronize all clocks in frame *S*

Oscar emits a light flash at $t = 0$.

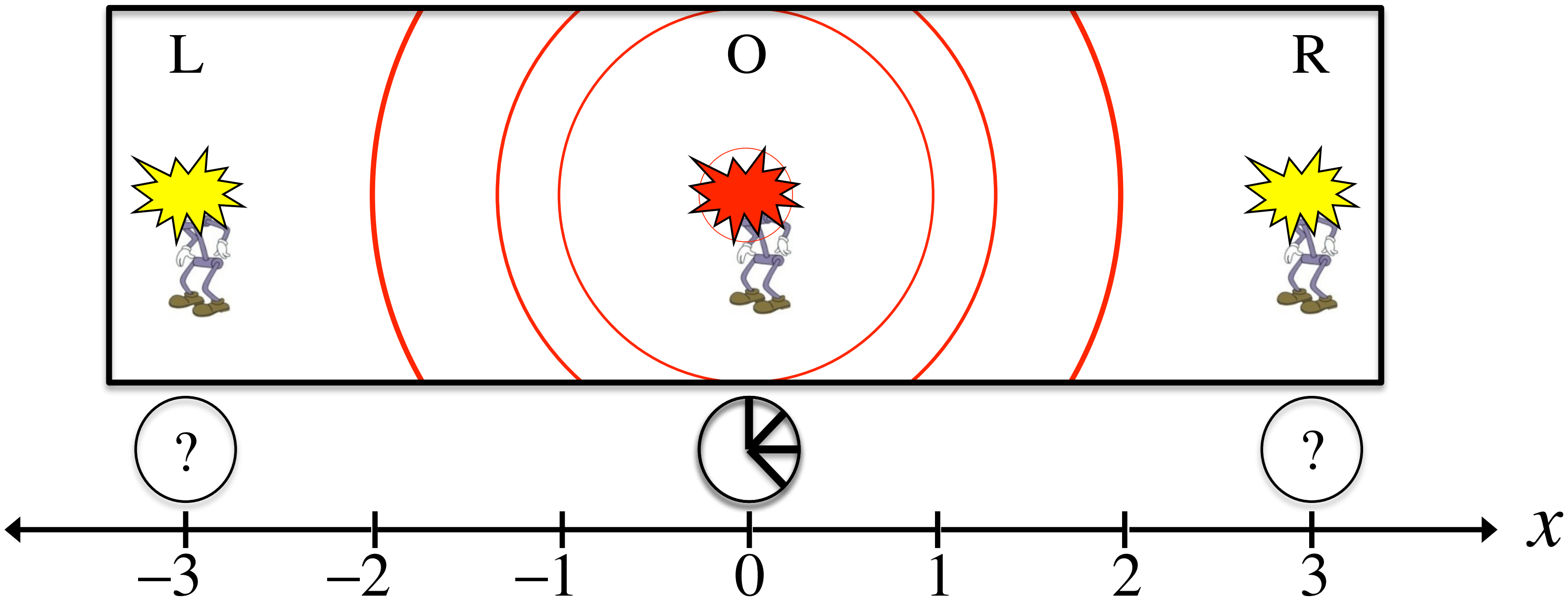


Procedure to synchronize all clocks in frame *S*

Oscar emits a light flash at $t = 0$.

Light spreads outwards in a spherical wavefront.

At $\Delta t = (+3\ \mathrm{m})/c$ the wavefront reaches Lucy and Ricky.



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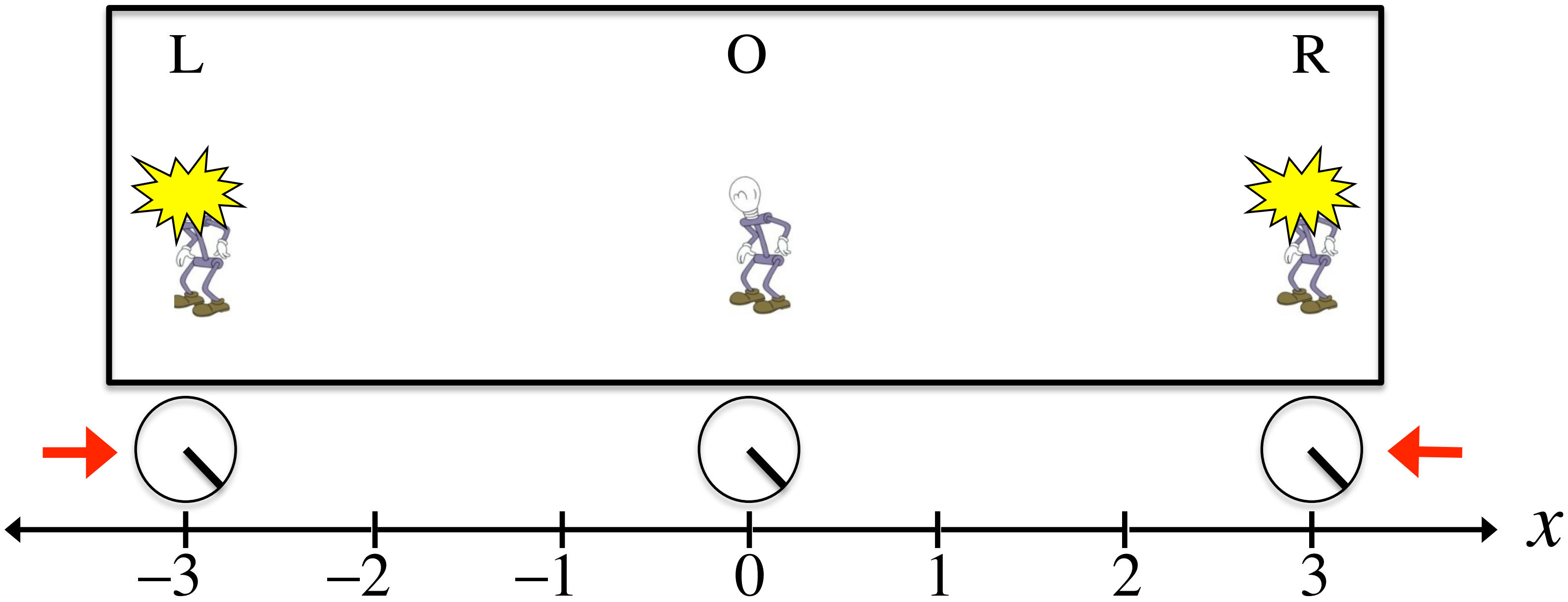
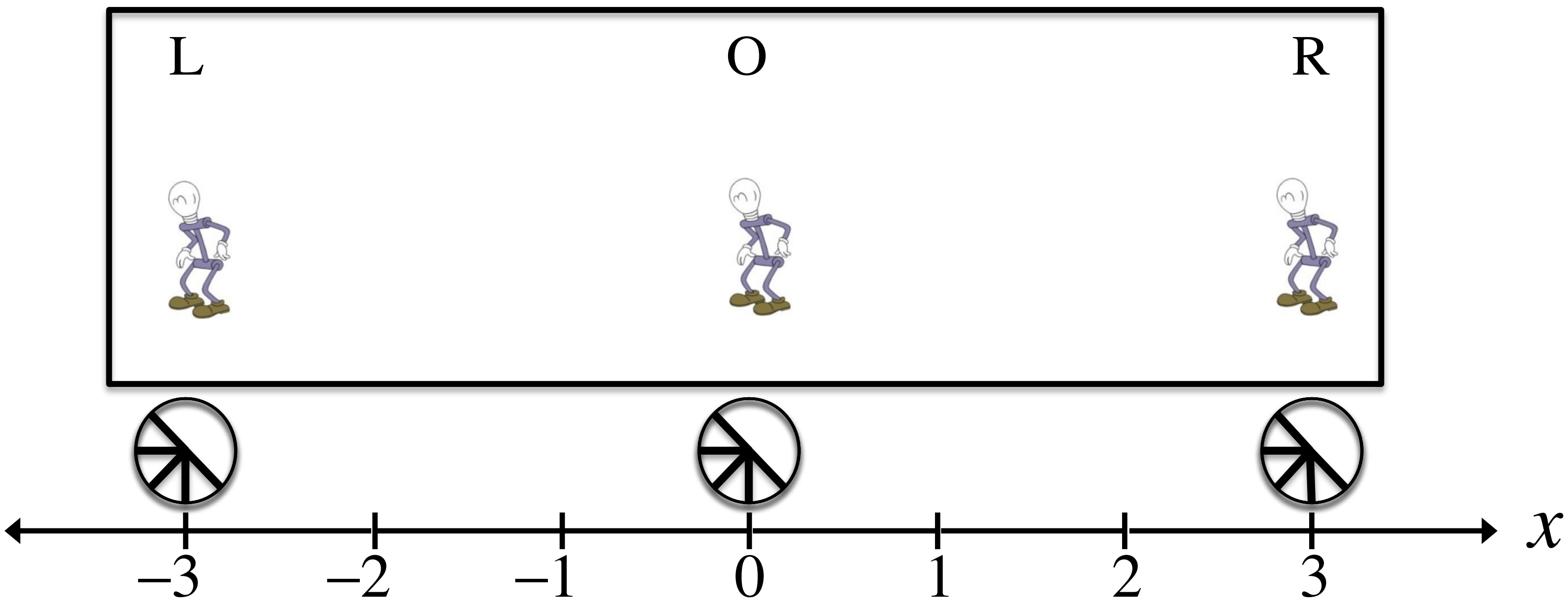


Lucy & Ricky know to set their clocks to $t=(0+3\ \mathrm{m})/c$!

Procedure to synchronize all clocks in frame *S*

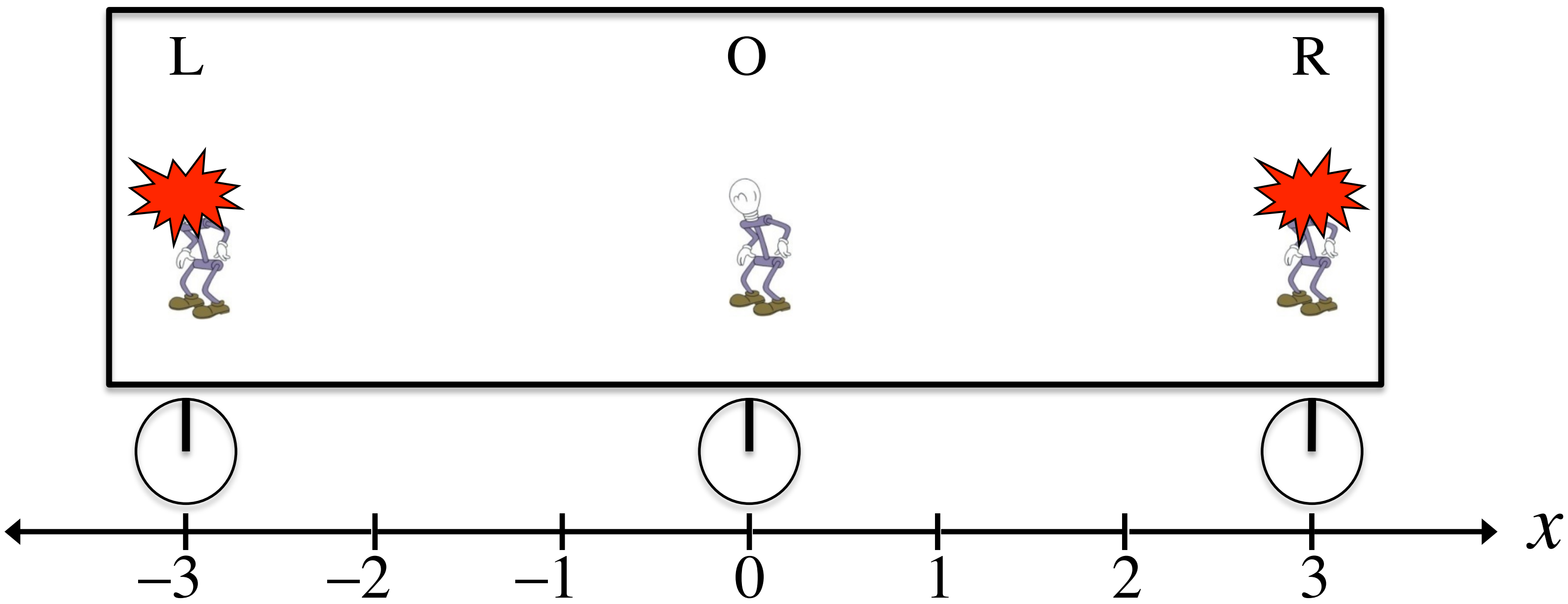
Now, all three clocks run in synch with each other!



Procedure to synchronize all clocks in frame *S*

Now, all three clocks run in synch with each other!

To check, Lucy & Ricky both emit light flashes at $t = 0$



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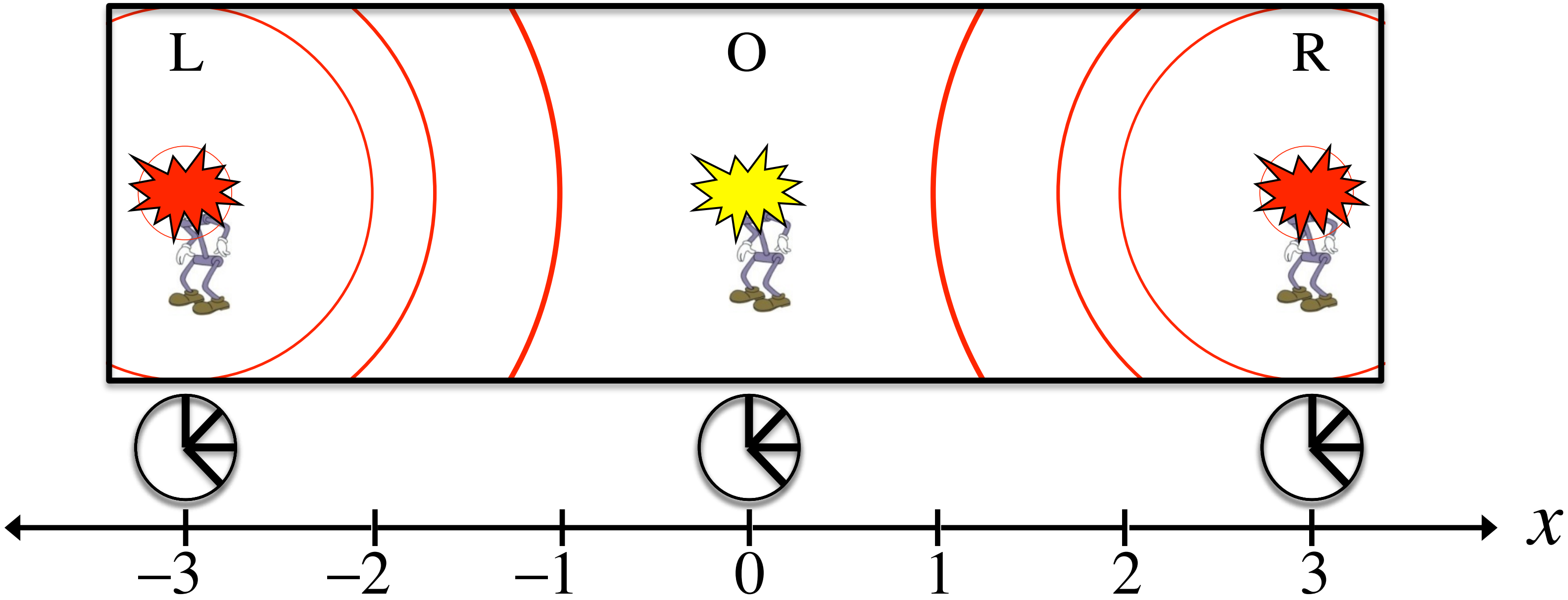
Oscar receives both light flashes at $t = (0 + 3\ \mathrm{m})/c$



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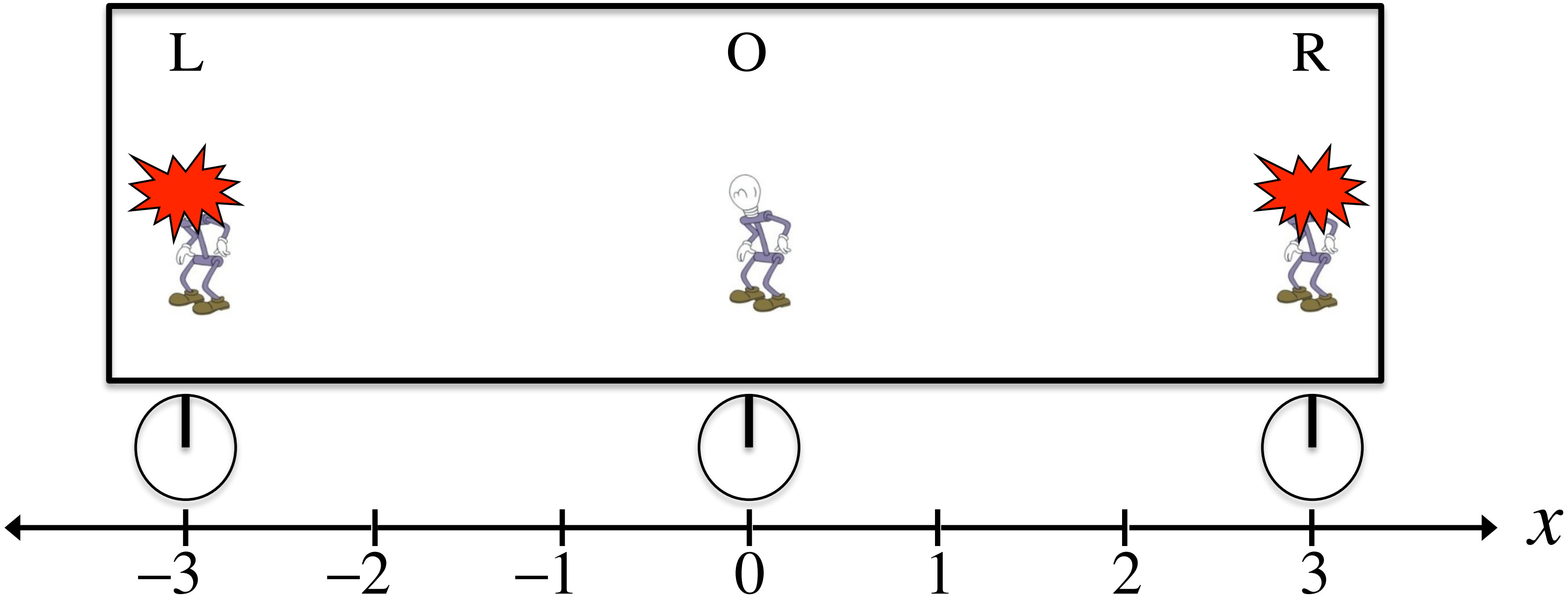
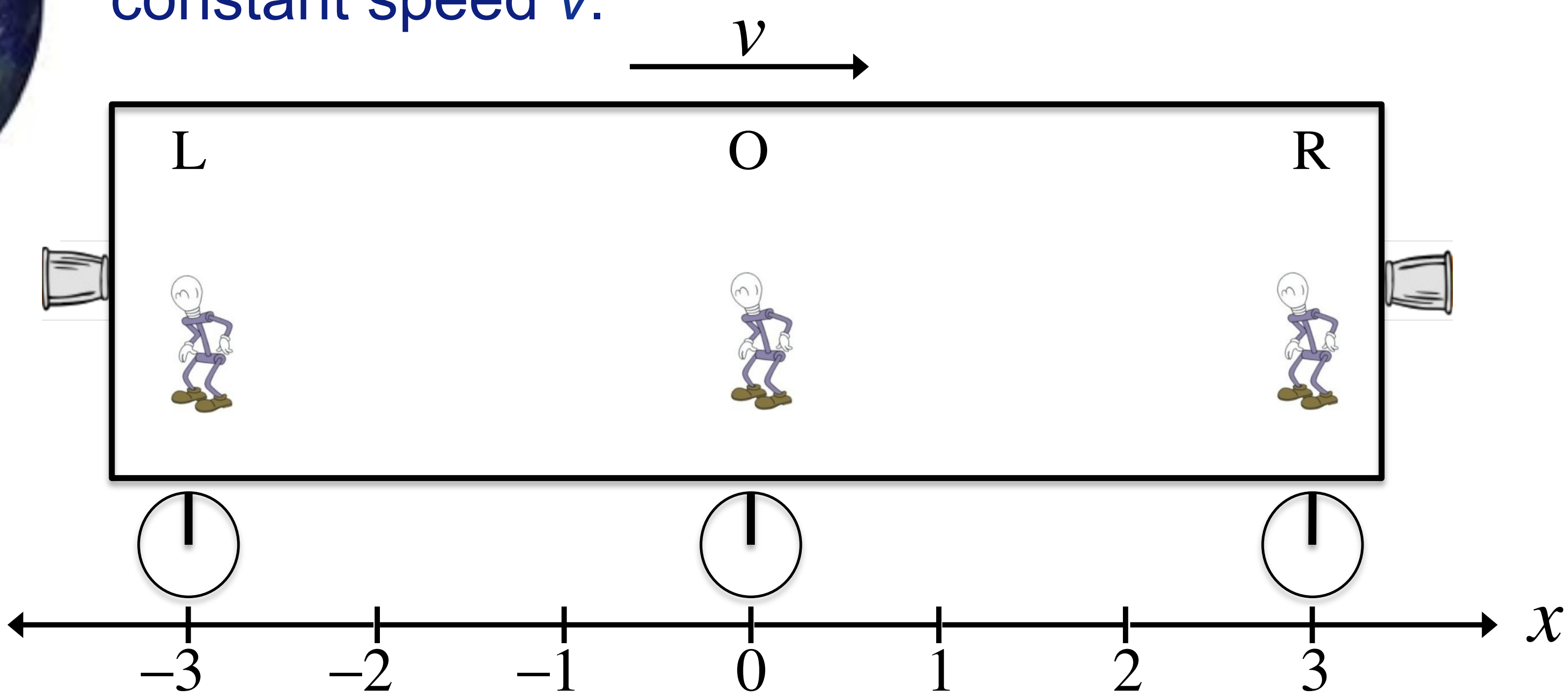


Oscar concludes both flashes were sent simultaneously.

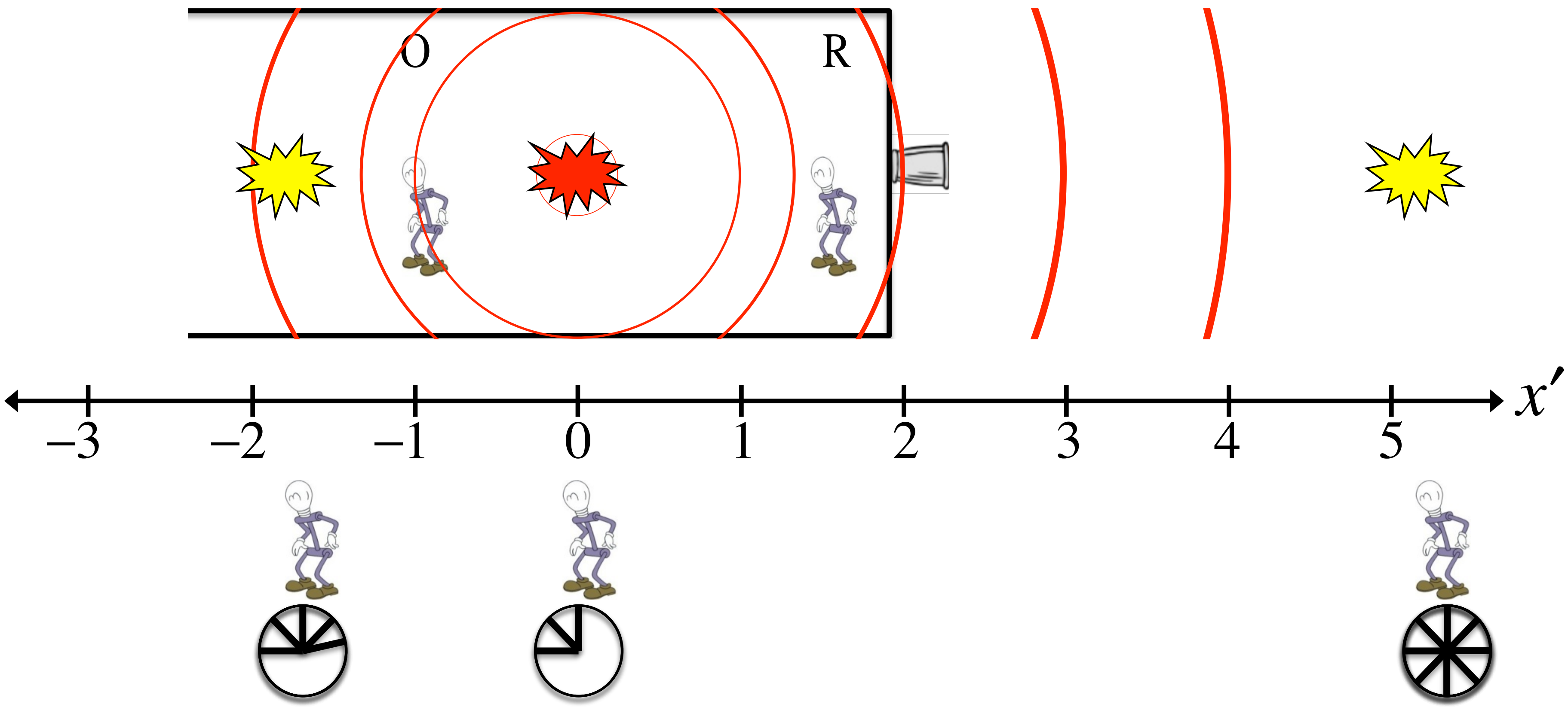
Changing to a different inertial frame

It's revealed that Lucy, Oscar and Ricky are actually in a spaceship moving away from the Earth at a constant speed *v*.



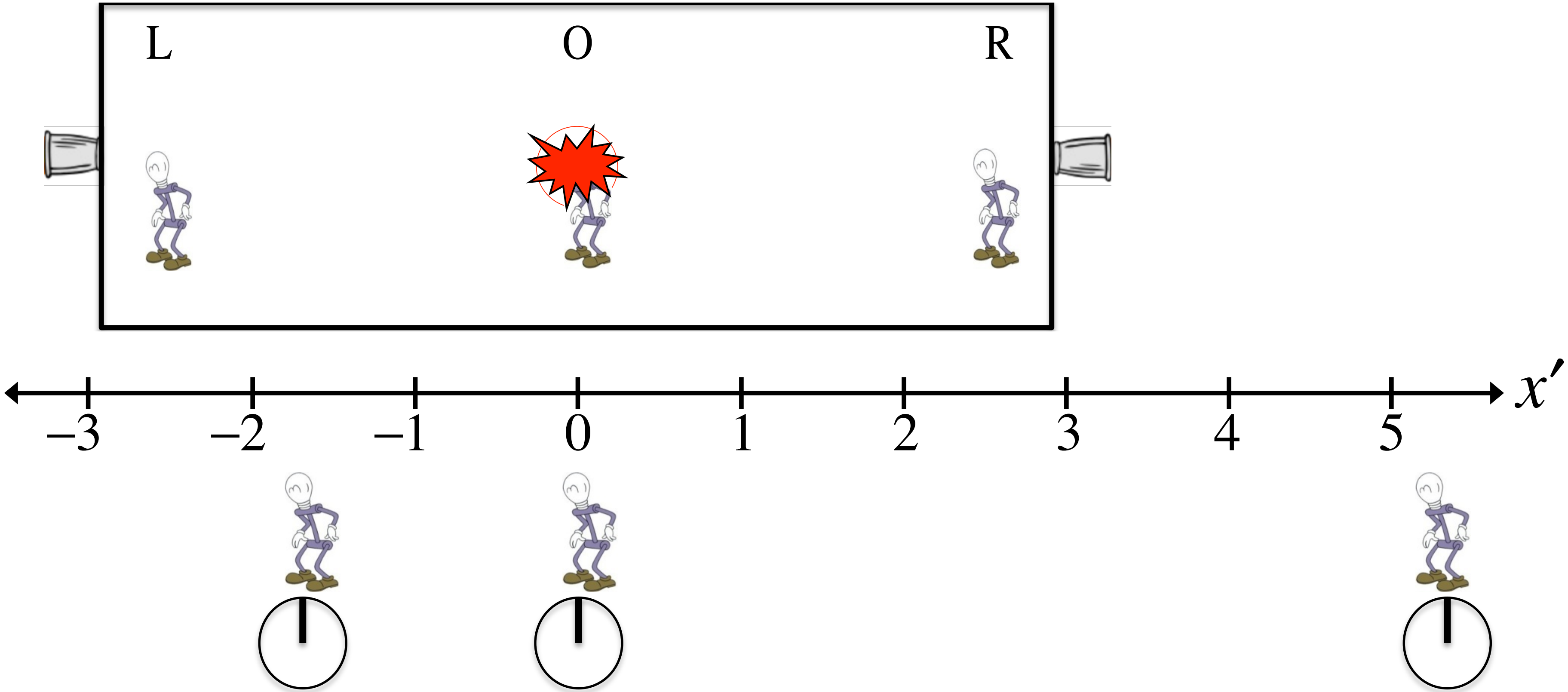
Changing to a different inertial frame

What does this procedure look like in a different frame S'?
Frame S moves to the right with speed *v* relative to frame S'.



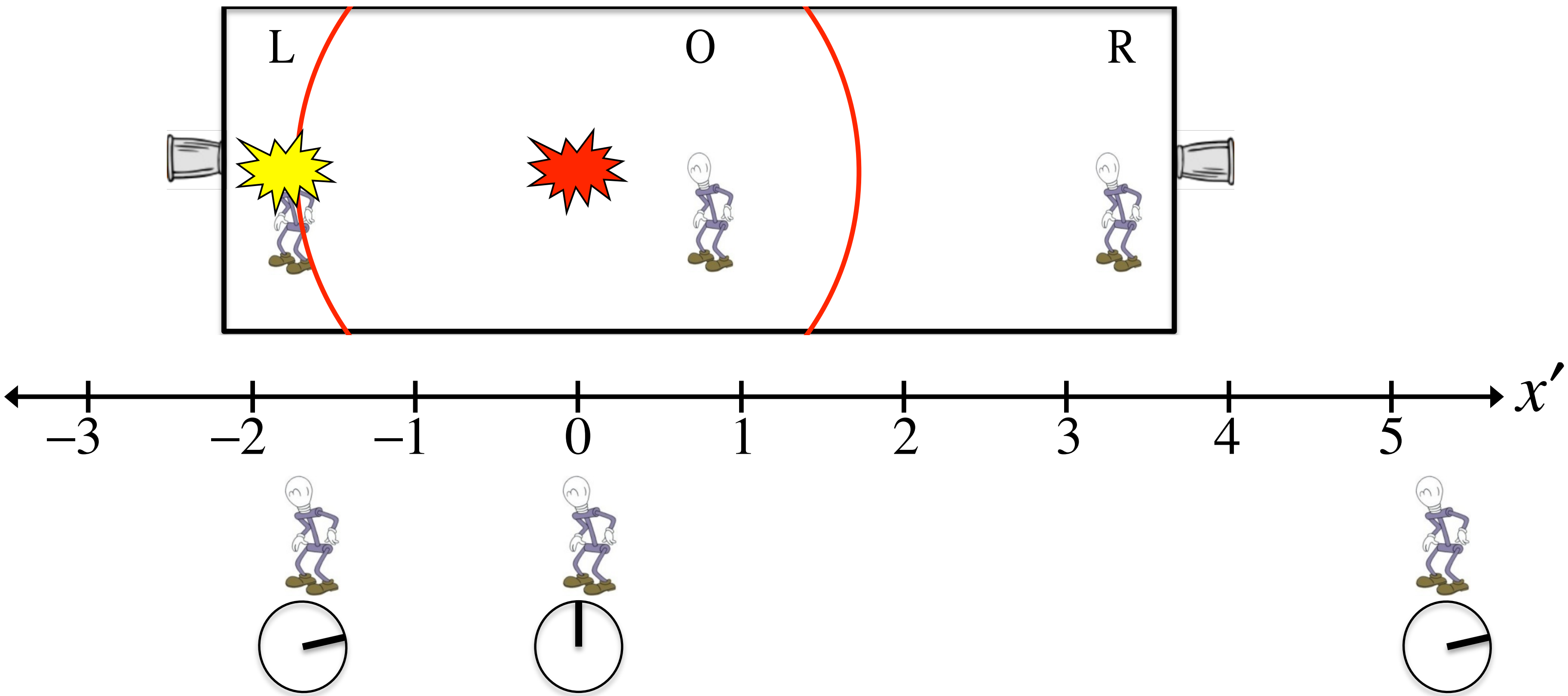
What does this procedure look like in a different frame S'?
Frame S moves to the right with speed *v* relative to frame S'.

- Oscar emits a light flash at $t = t' = 0$ & $x = x' = 0$.



What does this procedure look like in a different frame S'?
Frame S moves to the right with speed *v* relative to frame S'.

- Oscar emits a light flash at $t = t' = 0$ & $x = x' = 0$.
- Light spreads outwards in a spherical wavefront.
- The wavefront reaches Lucy.



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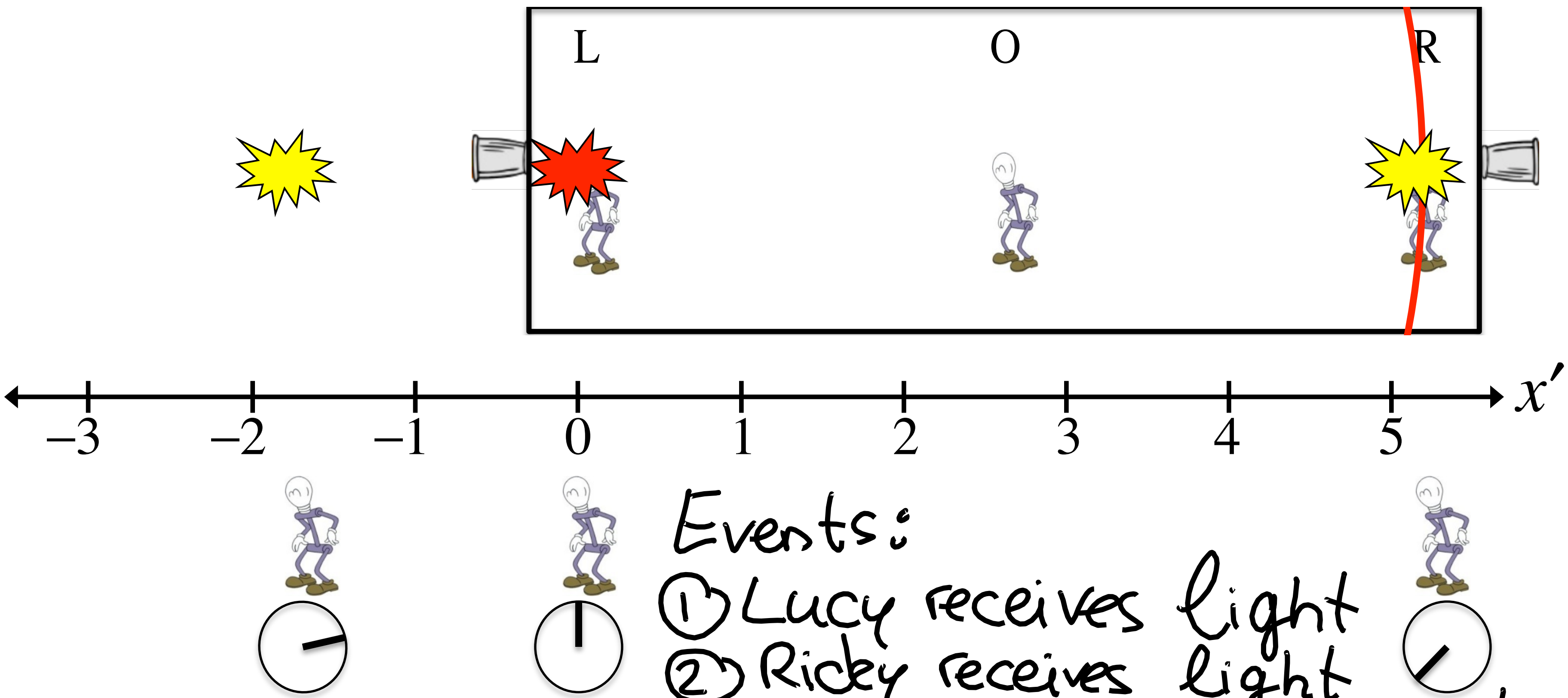
Frame S moves to the right with speed *v* relative to Frame S'.

- Oscar emits a light flash at $t = t' = 0$ & $x = x' = 0$.
- Light spreads outwards in a spherical wavefront.
- The wavefront reaches Lucy.
- At some later time the wavefront reaches Ricky.

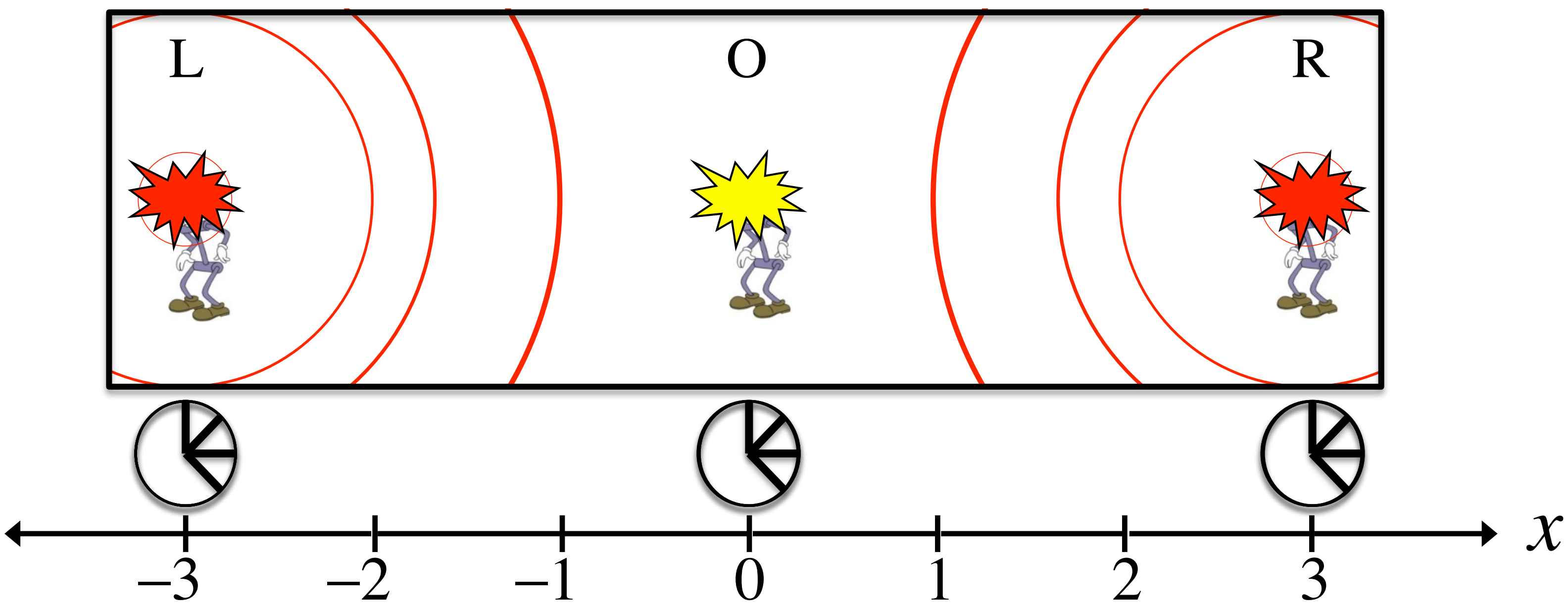


Events:
① Lucy receives light
② Ricky receives light

① and ② are non-simultaneous in frame S'!

Remember this situation? Lucy & Ricky both emit light flashes at $t = 0$ in frame S.

Oscar receives both light flashes at $t = (0 + 3\text{ m})/c$.



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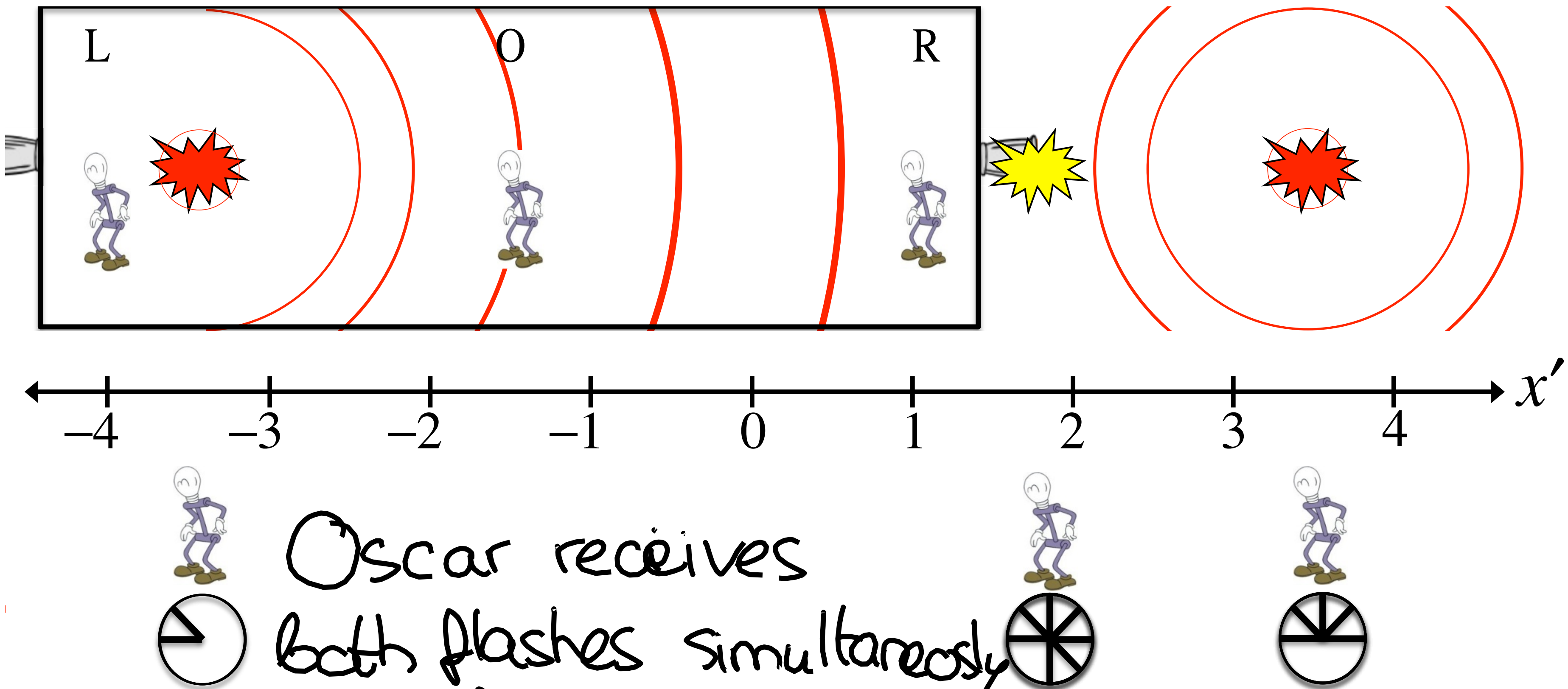


Oscar receives both flashes simultaneously in frame S'.

Important Point

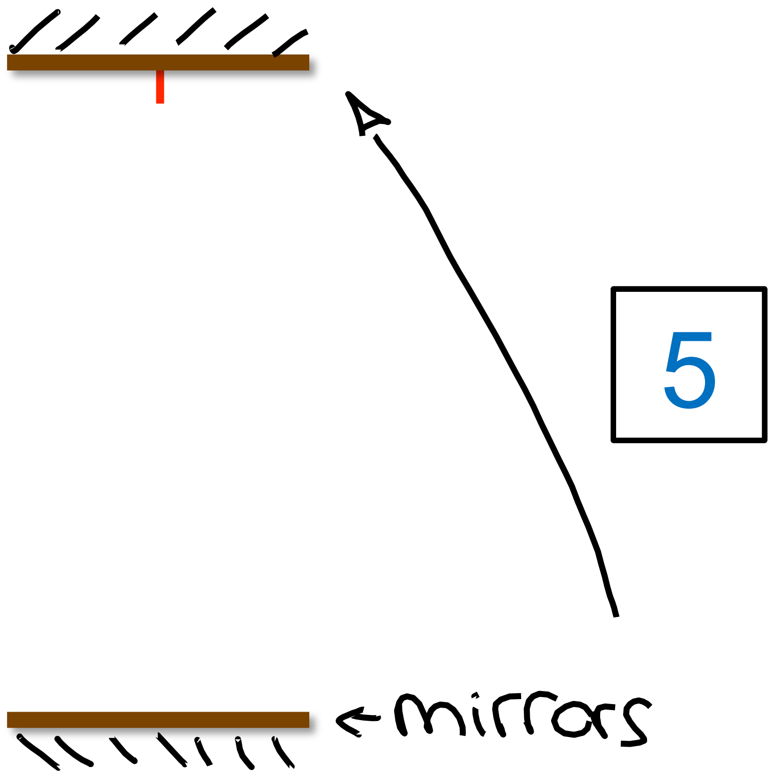
If two events occur at the same point in space *and also* at the same point in time in frame S:

Event 1: Light from Lucy reaches Oscar – $ct = +3 \quad \& \quad x = 0$

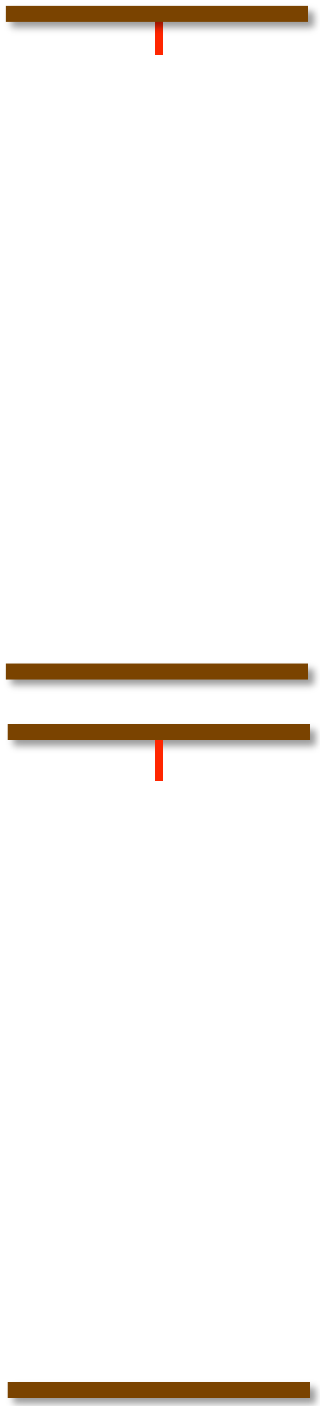
Event 2: Light from Ricky reaches Oscar – $ct = +3 \quad \& \quad x = 0$

…then they occur at the same point in spacetime in every inertial reference frame – only the coordinates are different.

A stationary light clock



A moving light clock



$\boxed{5}$

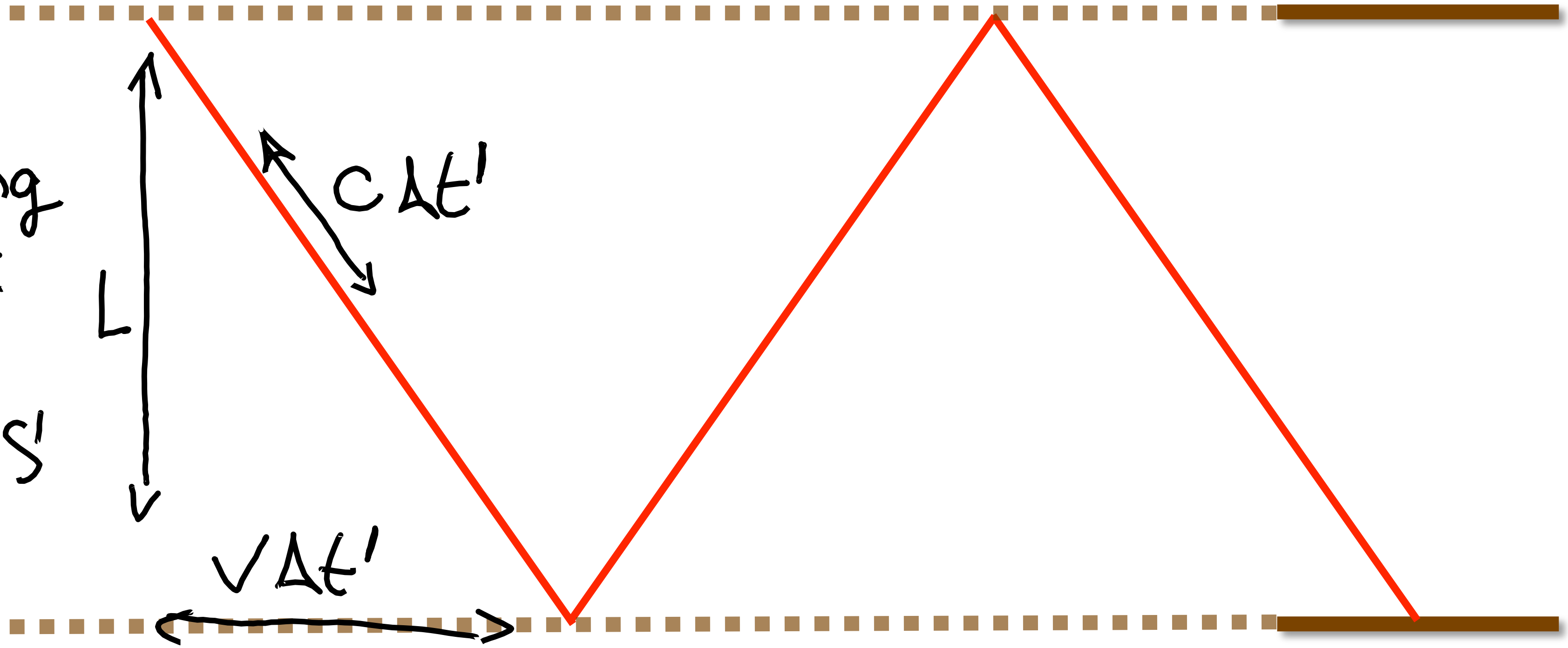
A moving light clock

Clock's rest frame: S

$$\Delta t = \frac{L}{c}$$

$\vec{v}$ = velocity of the clock in frame S'

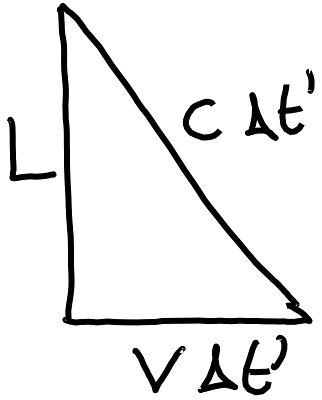
Moving Clock in frame S'



Time dilation

in S:

$$\Delta t = \frac{L}{c}$$



So: $L^2 + (v\Delta t')^2 = (c\Delta t')^2$

$$\Delta t'^2 (c^2 - v^2) = L^2$$

$$\Delta t'^2 = \frac{L^2}{c^2 - v^2} = \left(\frac{L}{c}\right)^2 \frac{1}{\left(1 - \left(\frac{v}{c}\right)^2\right)}$$

$$\Rightarrow \Delta t' = \Delta t \underbrace{\frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}_{\gamma}$$

$v < c$, so $\gamma > 1$

$$\boxed{\Delta t' = \gamma \Delta t}$$

Time dilation formula

γ: Lorentz factor (know by heart!)

Proper time

A stationary observer sees a moving clock run slow.

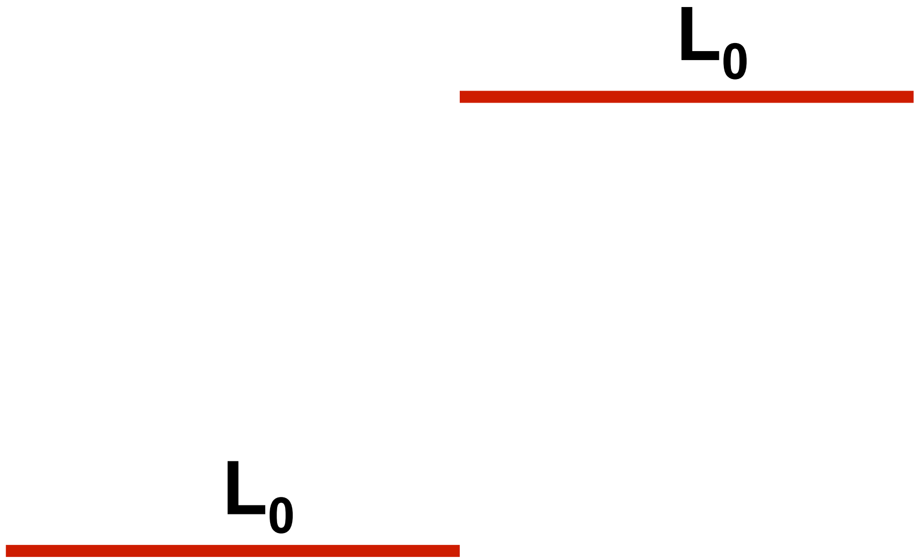
In other words:

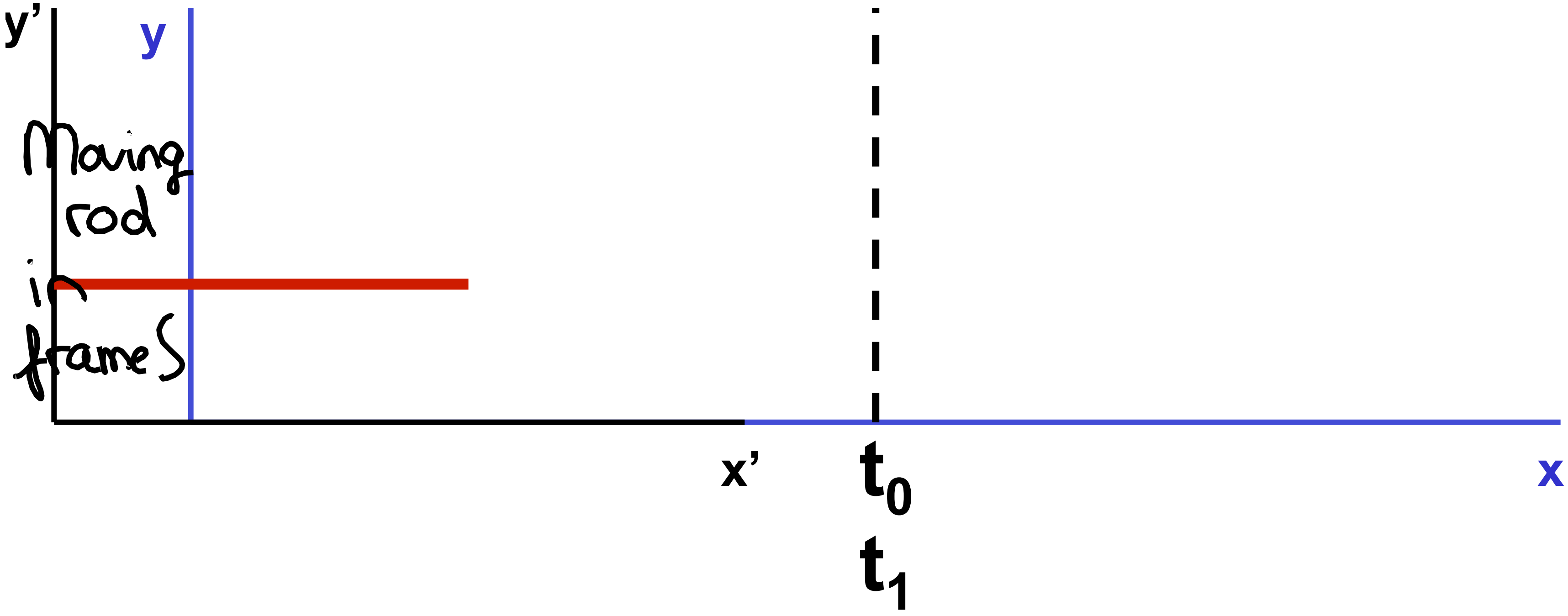
Stationary clocks measure the shortest time interval between two events.

Stationary clocks measure proper time.

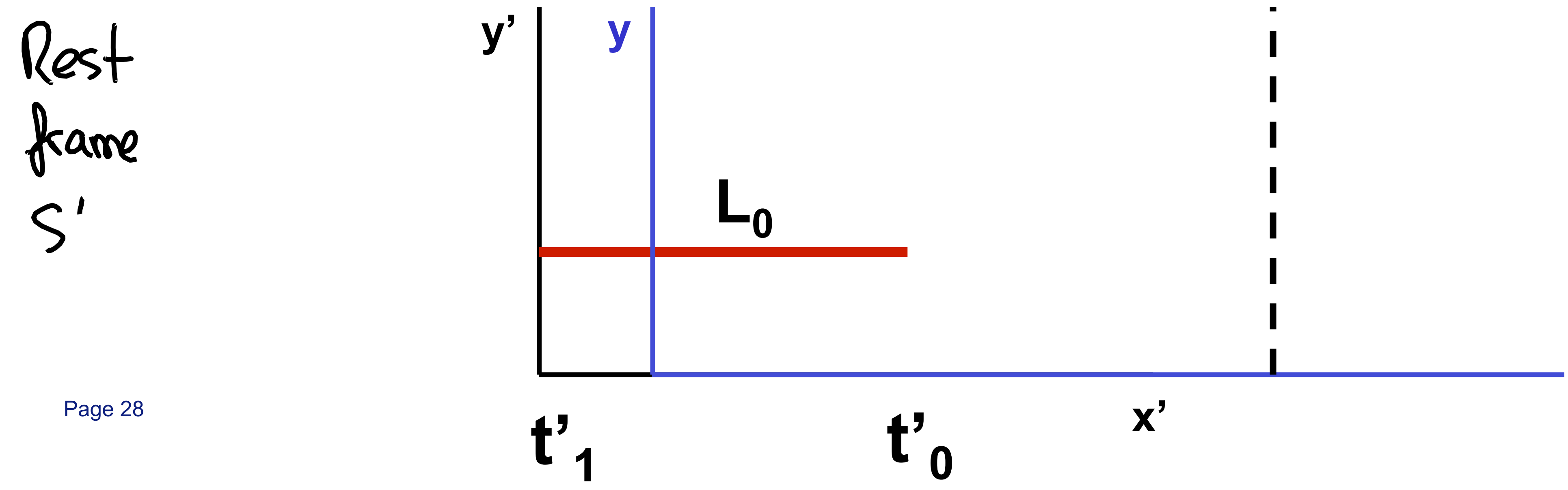
Proper time is the shortest time interval between 2 events.

Measuring the length of a rod





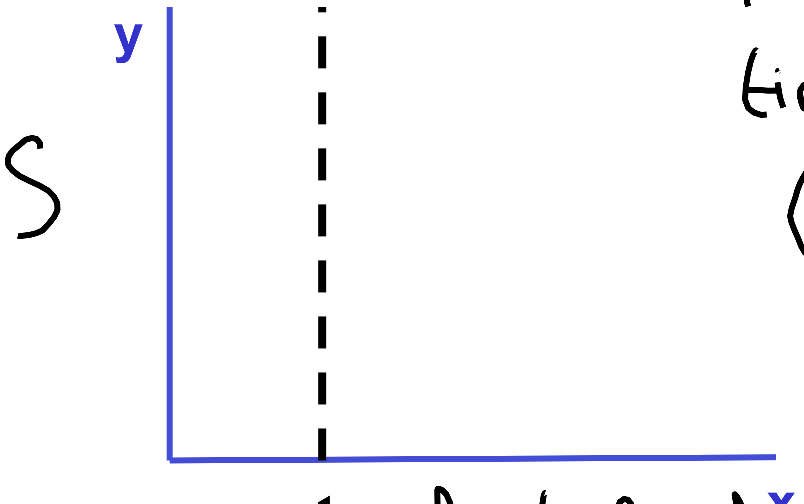
y'
y
Moving rod in frame S
x'
t_0
t_1
x



Rest frame S'
y'
y
L_0
t'_1
t'_0
x'

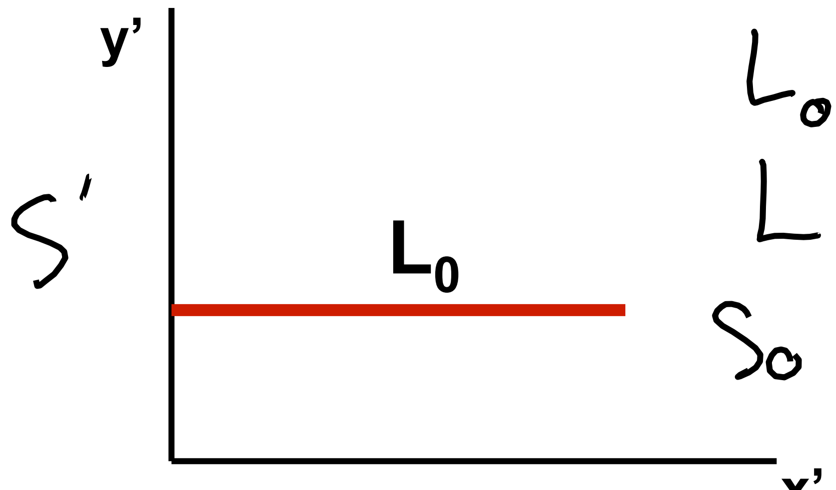
Frame S measures proper time between the 2 events.

$$\Delta t = t_1 - t_0 = \frac{\Delta t'}{\gamma}$$



t_0 : front of rod passes the marker

t_1 : back of rod passes the marker



t'_1 $\qquad$ $\longrightarrow t'_0$

marker passes front of rod

$$L_0 = v\Delta t' \; (= v\Delta t\,\gamma)$$

$$L = v\Delta t = v\frac{\Delta t'}{\gamma} = \frac{L_0}{\gamma}$$

So $L < L_0$!

$$\boxed{L = \frac{L_0}{\gamma}}$$

length contraction formula

Summary of concepts

- Events that are simultaneous in one inertial frame and spatially separated, are non-simultaneous in another inertial frame.

This causes:

- Time dilation: moving clocks run slow.

$\boxed{\Delta t = \gamma \tau}$ where τ is proper time

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

- Proper time: the time interval measured between 2 events by a stationary clock.

→ Always the shortest time interval

- Length contraction: moving objects are short.

$\boxed{L = \frac{L_0}{\gamma}}$, where L_0 is length of object in its own rest frame.