Imperial College London

Relativity – Lecture 7

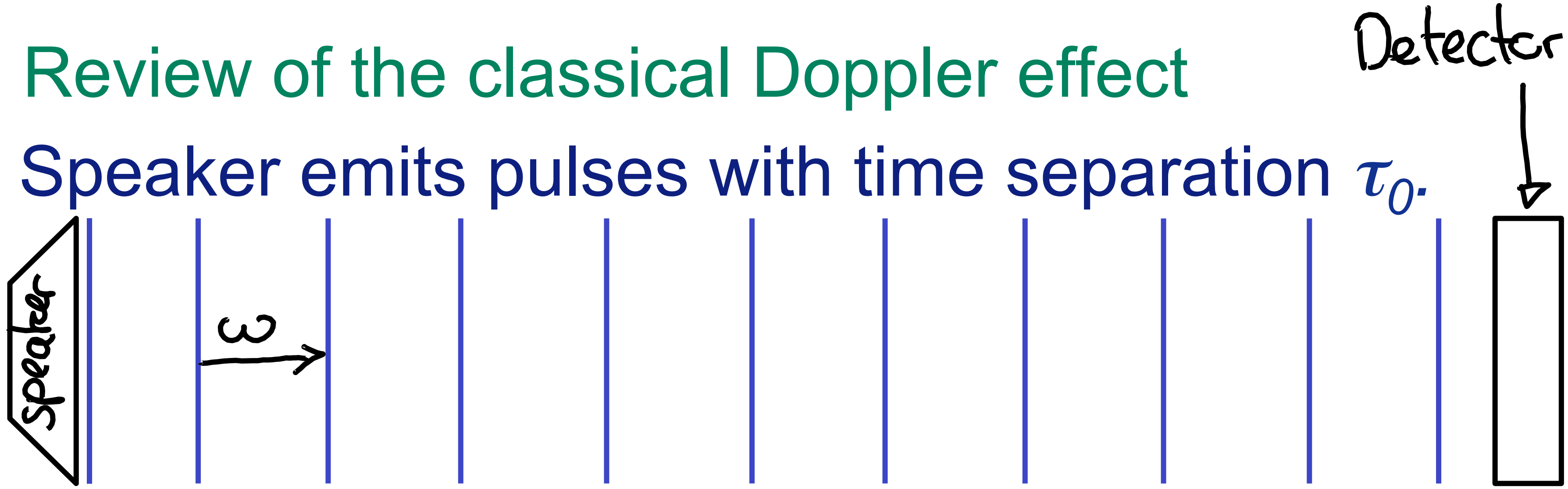
Dr Caroline Clewley

Key concepts of lecture 5 & 6

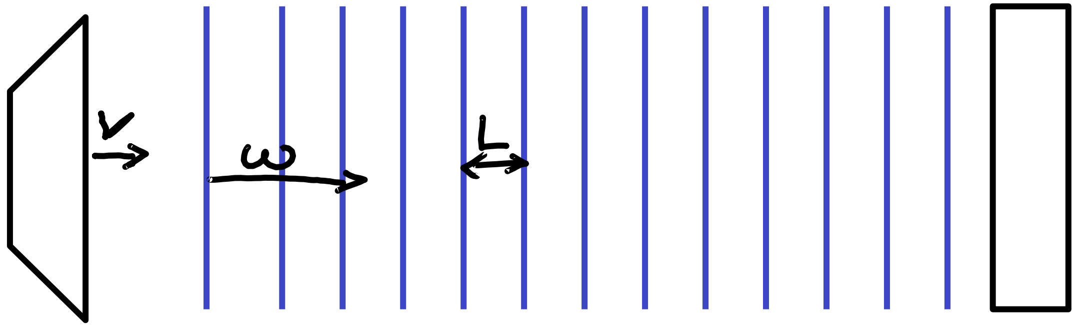
1. Events show up as points in a spacetime diagram. Moving objects have a worldline in this diagram.
2. The 4-position contains the four coordinates of an event in time and space.
3. The invariant interval $s^2 = c^2\Delta t^2 - \Delta \boldsymbol{r}^2$ denotes the separation between events.
4. $s^2 < 0$, spacelike separation,
   $s^2 > 0$, timelike separation,
   $s^2 = 0$, ~~time~~like separation.
   light

Review of the classical Doppler effect

Speaker emits pulses with time separation τ_0.



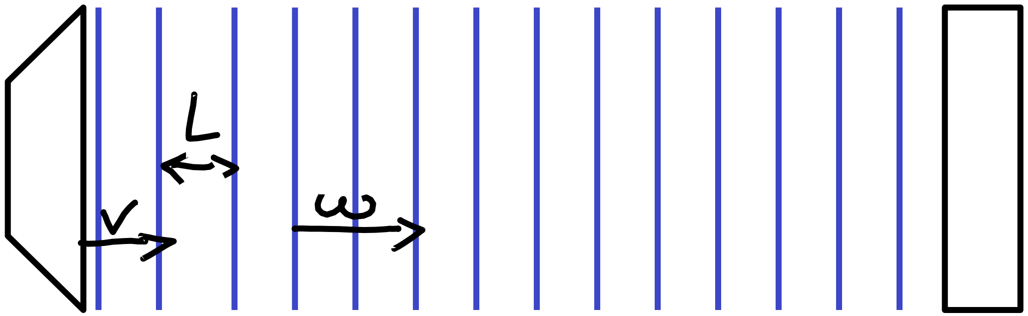
In time T pulse travels ωT (ω: wave speed)



Number of pulses that arrive at detector: $\dfrac{\omega T}{L}$

Per unit time: $\nu_D = \dfrac{\omega}{L}$ (ν_D: frequency of pulses at detector)

Review of the classical Doppler effect



What is *L*?

Pulse 1 emitted at $t = 0$.
Pulse 2 emitted at $t = \tau_0$.

Separation is $\omega\tau_0 - v\tau_0$

$$\Rightarrow L = (\omega - v)\tau_0 = \frac{\omega - v}{\nu_0}$$

original frequency

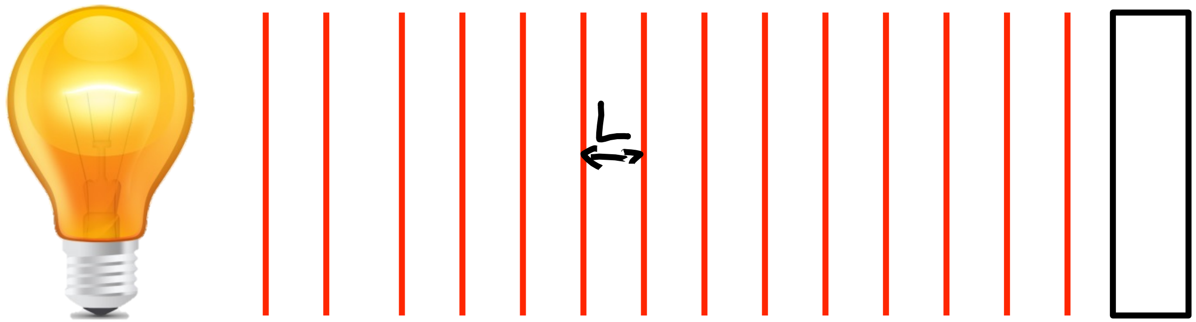
$$\Rightarrow \nu_D = \frac{\omega}{L} = \frac{\omega\nu_0}{\omega - v} = \nu_0 \frac{1}{1 - \frac{v}{\omega}}$$

(for moving source)

Source approaching: $\nu_D > \nu_0$

Source receding: $\nu_0 > \nu_D$

The Relativistic Doppler effect



Light flashes with period τ_0 in its rest frame.

Pulses leave bulb and arrive at detector with speed c.

Observer sees a longer emission period $\tau = \gamma\tau_0$

Frequency of pulses: $\nu_D = \frac{c}{L}$ with $L = (c-v)\tau$

So $\nu_D = \frac{c}{(c-v)\tau} = \frac{1}{(1-\frac{v}{c})\gamma\tau_0} = \nu_0 \frac{1}{\gamma}\frac{1}{1-\beta} = \nu_0\sqrt{\frac{1+\beta}{1-\beta}}$

Same result if observer's moving towards light.

Redshift

For light: $c = \lambda \nu$ So $\frac{c}{\lambda_D} = \frac{c}{\lambda_0} \frac{1}{\gamma(1-\beta)}$

or $\lambda_D = \gamma \lambda_0 (1-\beta) = \lambda_0 \sqrt{\frac{1-\beta}{1+\beta}}$

For Hydrogen λ_0 = 656 nm, but in a distant galaxy this is observed at λ_D = 953 nm.

$\lambda_D > \lambda_0$: so $v < 0$ (galaxy is receding)

Solve: $\frac{953}{656} = \sqrt{\frac{1-\beta}{1+\beta}}$ $\therefore \beta = -0.36$

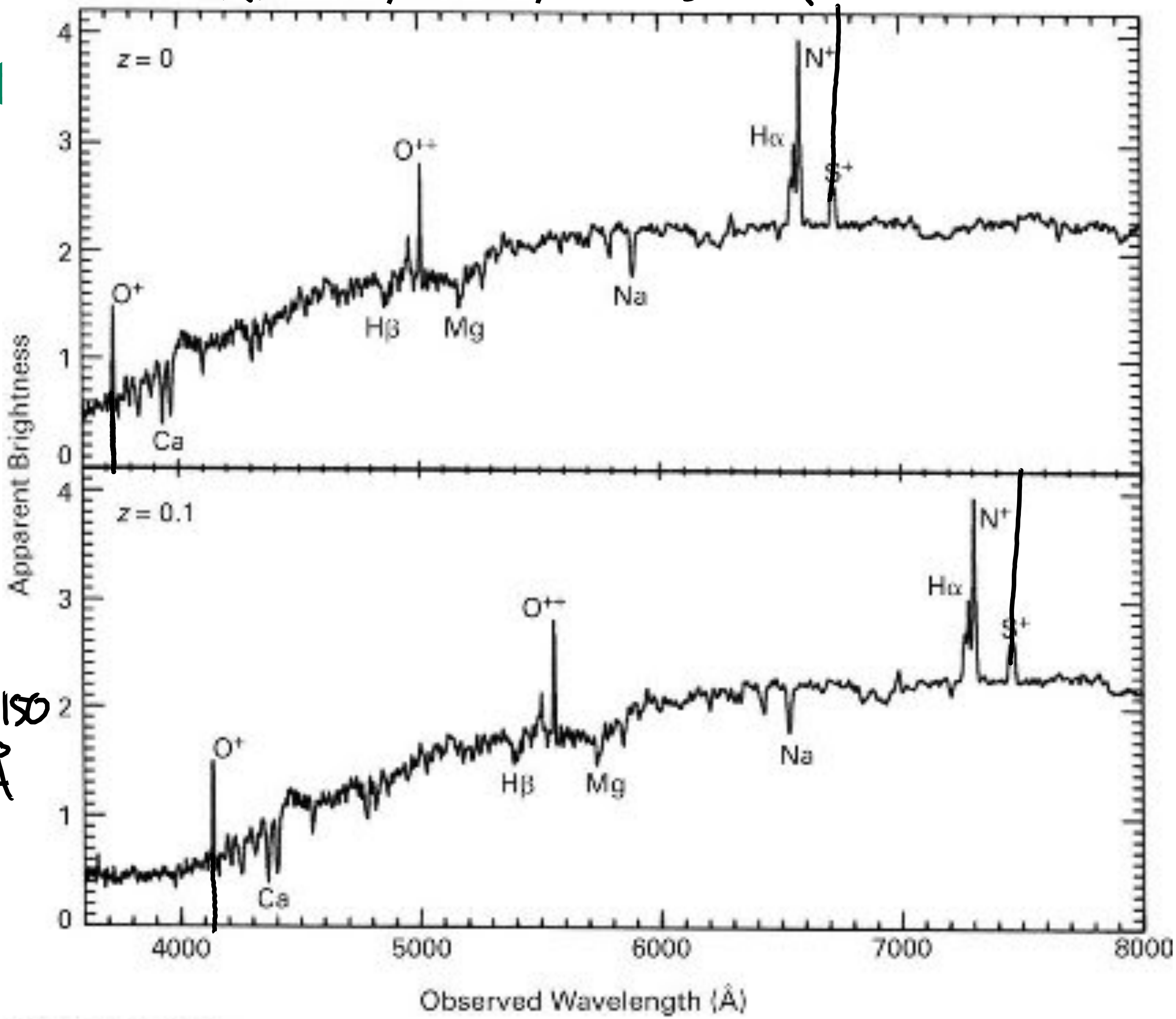
[In Astrophysics: $z + 1 = \frac{\lambda_D}{\lambda_0}$ => $z = 0.46$]

$\approx -\frac{v}{c}$ for small v.

(Classical approximation ok if $\frac{v}{c} < 0.1$)

Redshift

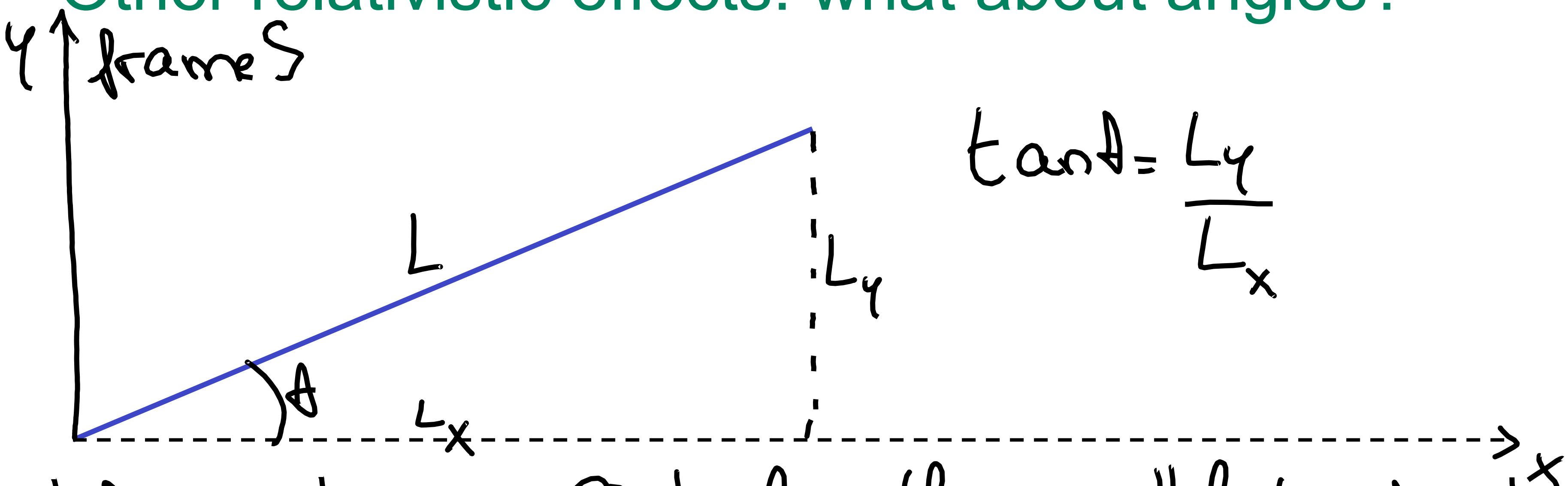
$\Delta\lambda = 6750 - 3750 = 3000\,\text{Å}$



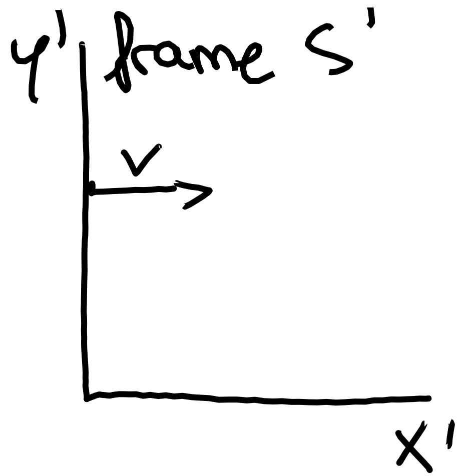
$\Delta\lambda = 7500 - 4150$
$= 3350\,\text{Å}$

A. Filippenko and R. J. Foley

Other relativistic effects: what about angles?



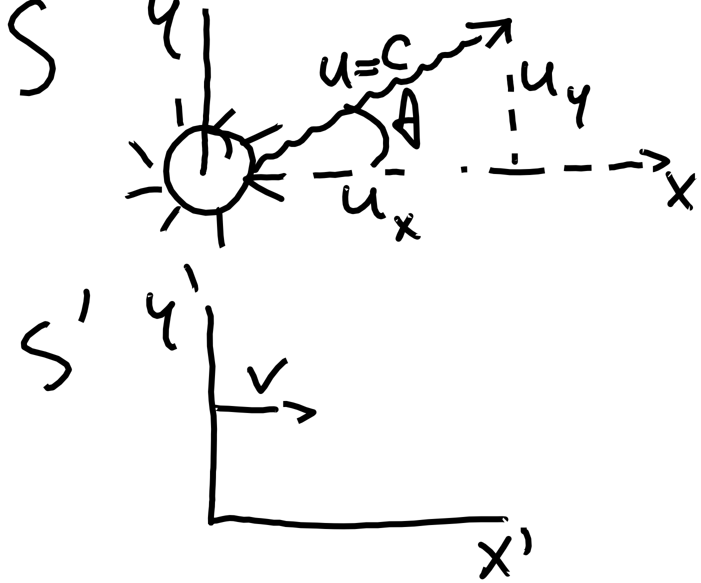
$$\tan\theta = \frac{L_y}{L_x}$$



Only length parallel to direction of motion is length contracted

$$\Rightarrow \tan\theta' = \frac{L'_y}{L'_x} = \frac{L_y}{\frac{L_x}{\gamma}} = \gamma\tan\theta$$

Relativistic aberration (non-examinable)



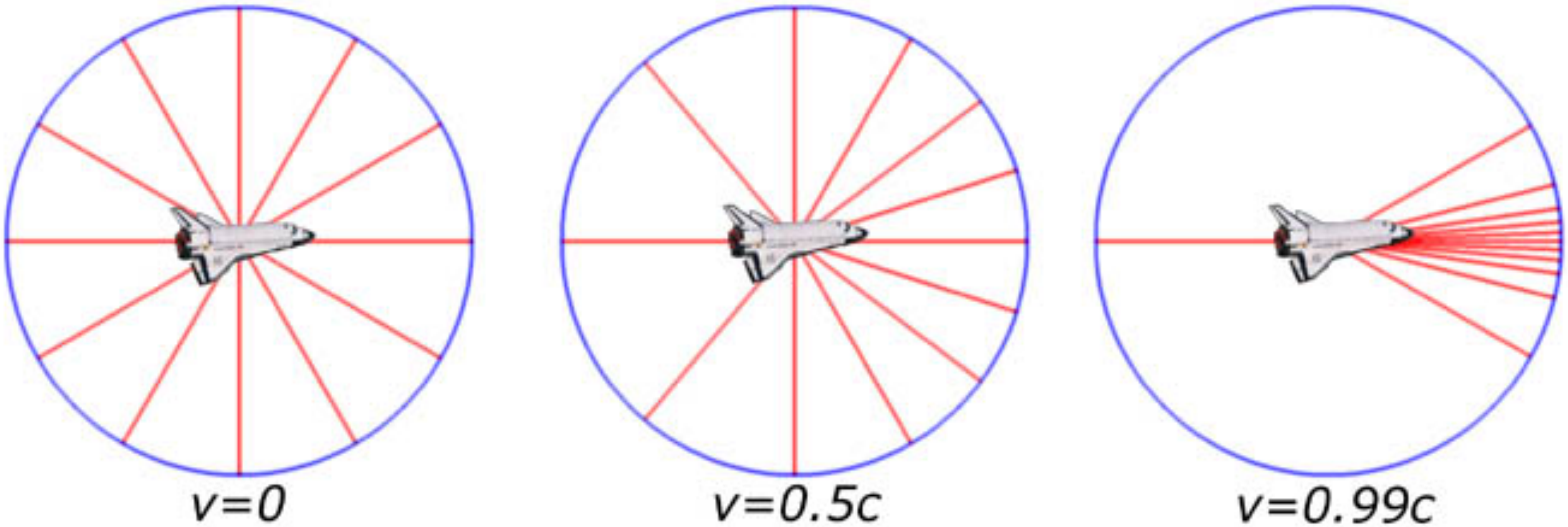
$$\cos\theta = \frac{u_x}{c}$$

$$\cos\theta' = \frac{u'_x}{c} = \frac{u_x - v}{1 - \frac{v u_x}{c^2}} \cdot \frac{1}{c} =$$

$$\frac{\frac{u_x}{c} - \frac{v}{c}}{1 - \frac{v}{c}\frac{u_x}{c}} = \frac{\cos\theta - \frac{v}{c}}{1 - \frac{v}{c}\cos\theta}$$

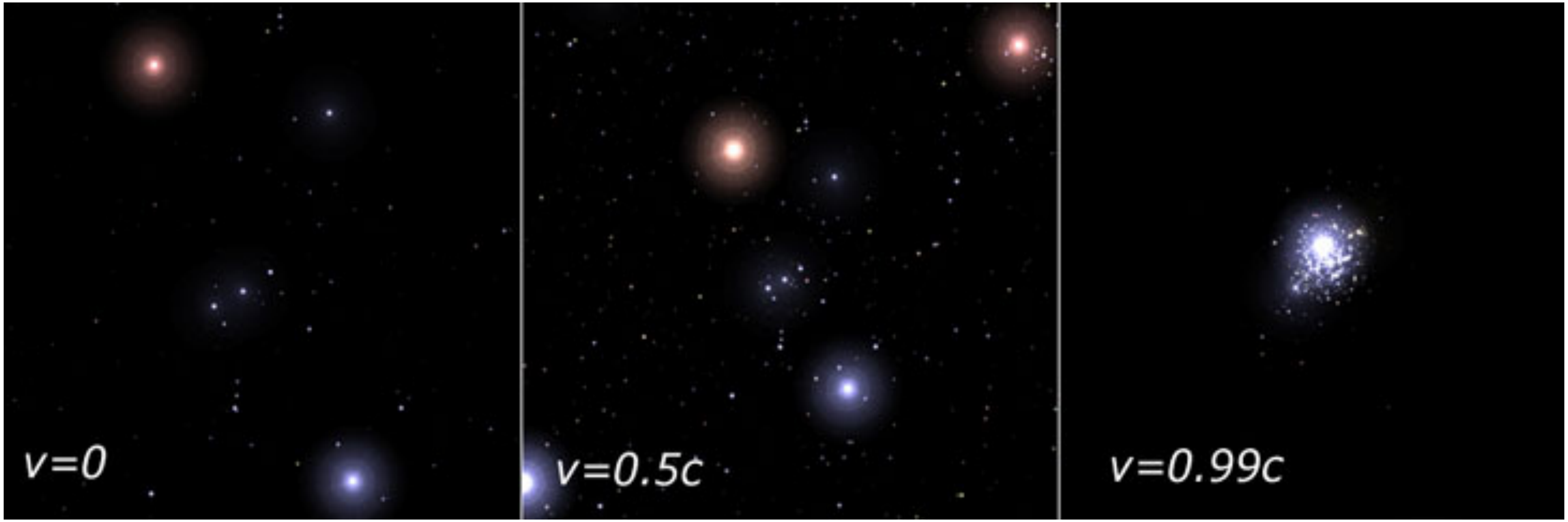
Combined effect of finite speed of light (light travel time) and relativistic velocity addition.

Relativistic aberration: result



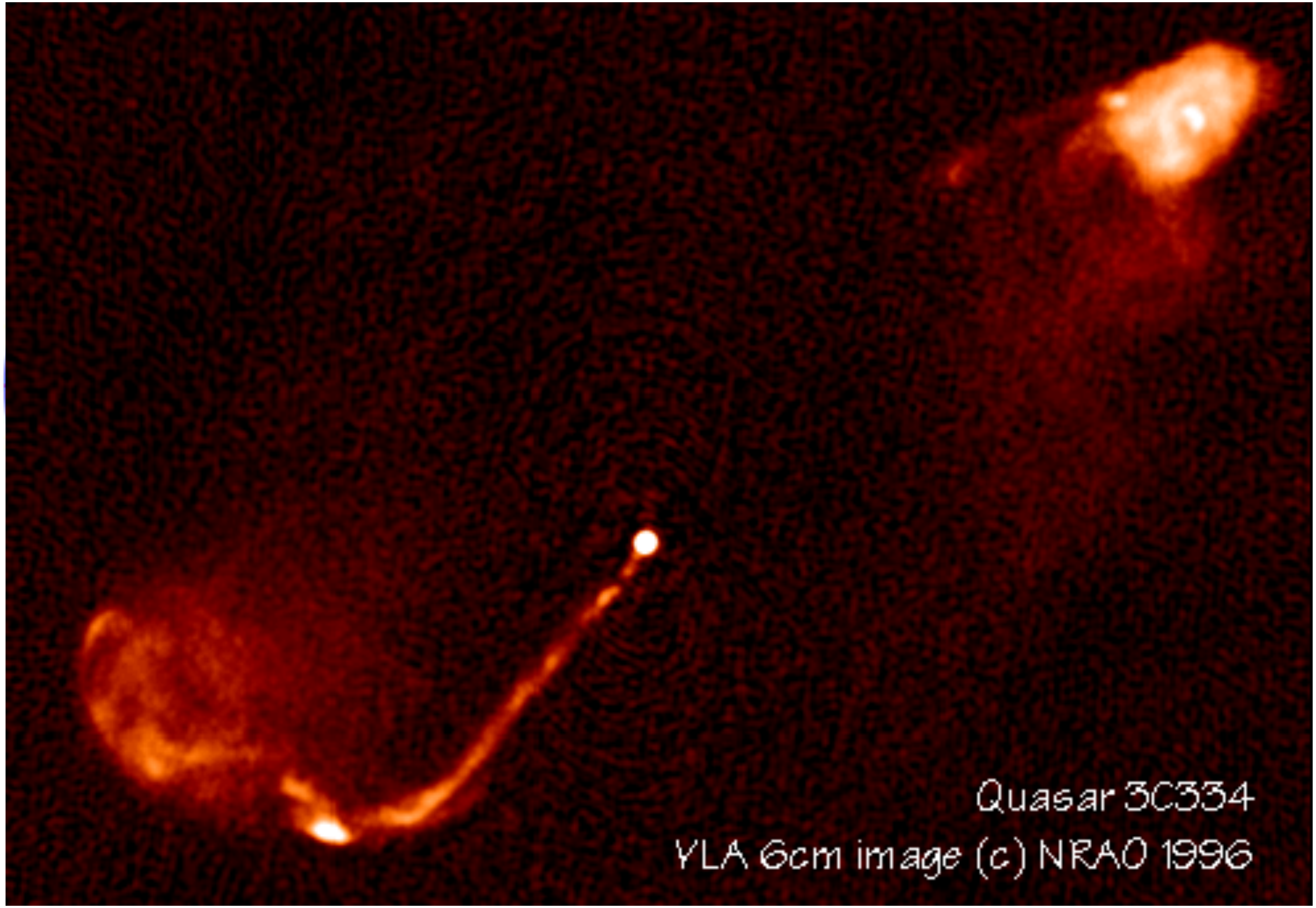
Moving observer sees light originating from a cone in front of them.

Relativistic aberration: example



Emitted light is concentrated in forwards cone. Luminosity is boosted.

Relativistic beaming.



Summary

The relativistic Doppler effect is caused by:

1. The source ‘catching up’ to the emitted waves (classical Doppler effect).
2. Time dilation.