Imperial College London

Relativity – Lecture 10

Dr Caroline Clewley

Key concepts of lecture 9

Energy and momentum are conserved separately in any one frame.

However, when transforming frames, the energy and momentum change.

In other words, a Lorentz transformation changes energy into momentum, and vice versa.

However, the norm of the four-vector is invariant, so $E^2 = p^2c^2 + \left(mc^2\right)^2$ is always true.

Result of the frame transformation example

Frame in which target particle is at rest:

Particle	$P_i = (E_i/c, \boldsymbol{p}_i)$	β_i	m_i
1	(17/c, 15/c)	15/17	$8/c^2$
2	(8/c, 0)	0	$8/c^2$
3	(25/c, 15/c)	3/5	$20/c^2$

$\bar{P}_1 + \bar{P}_2 = \bar{P}_3$

$m_1 + m_2 \neq m_3$

Centre-of-momentum frame:

Particle	$P'_i = (E'_i/c, \boldsymbol{p}'_i)$	β'_i	m'_i
1	(10/c, 6/c)	3/5	$8/c^2$
2	(10/c, -6/c)	-3/5	$8/c^2$
3	(20/c, 0)	0	$20/c^2$

$\bar{P}'_1 + \bar{P}'_2 + \bar{P}'_3$

$\bar{P}_1 \neq P'_1$ $\qquad$ $m_1 = m'_1$

Reminder: get the terminology right.

- **Conserved**: a quantity which is not changed by a physical process. This refers to one frame at a time, and a conserved quantity will typically have different numerical values in different frames.
- **Invariant**: a quantity which is not changed by a coordinate transformation. The term refers to more than one reference frame; an invariant quantity will not necessarily be conserved in a particular process.
- **Constant:** refers to a quantity which does not change in time, such as the mass of the Universe.

- The speed of light is conserved, invariant, and constant!

Tip: solving energy-momentum problems

Try to solve problems first by using energy conservation alone. Some problems require you to use both energy and momentum conservation.

You can eliminate one variable using

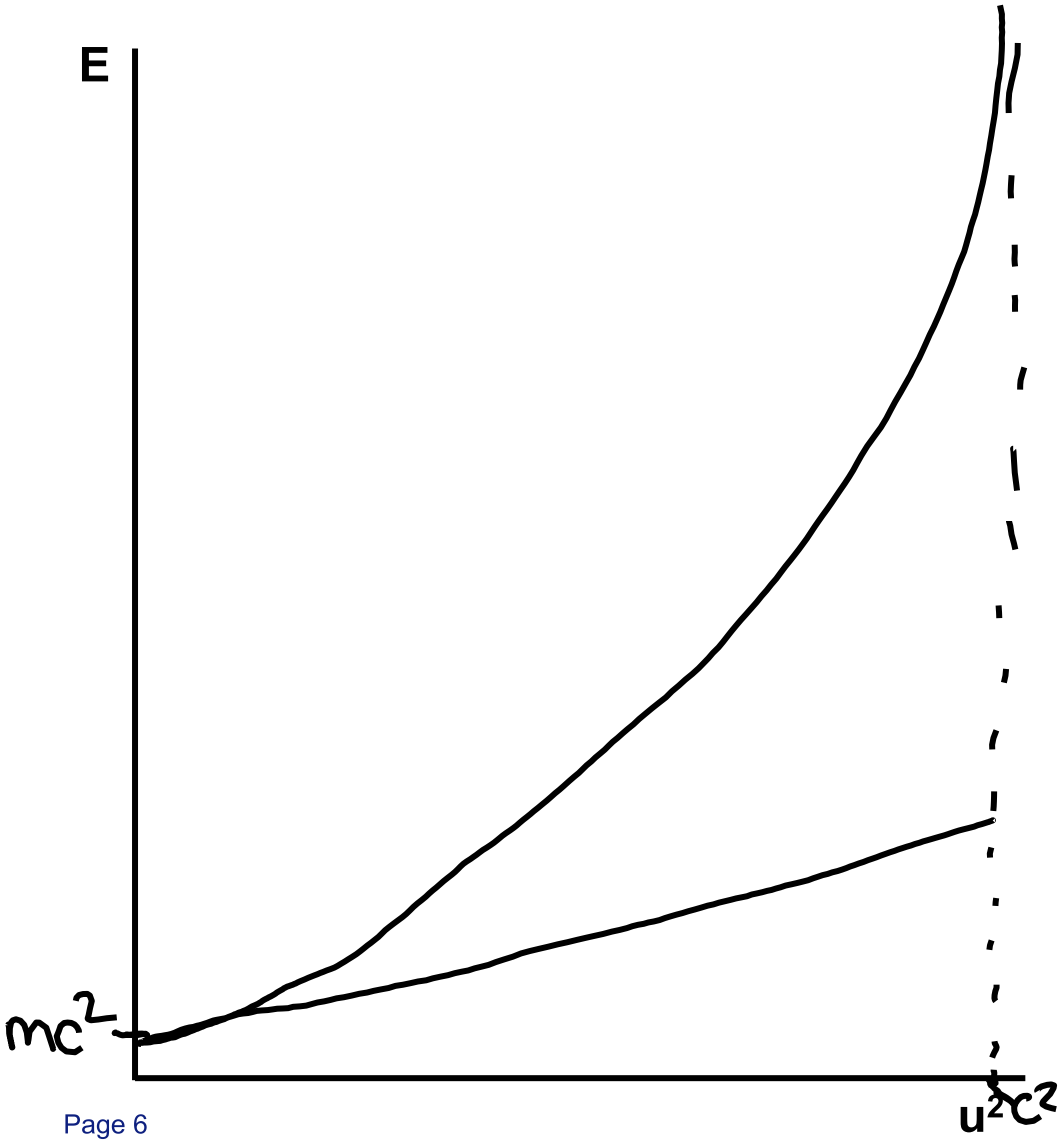
$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

for example $$p = \sqrt{\left(E/c\right)^2 - \left(mc\right)^2}$$

For a massless particle, $E = pc$.

You can also leave out all of the c's and put them in at the end using dimensional analysis.

Total energy as *u* approaches *c*



$$E = \gamma_u mc^2 = \frac{mc^2}{\sqrt{1-\left(\frac{u}{c}\right)^2}} \quad (1)$$

$$\lim_{u \to c} (E) = \infty$$

$$(1) \quad mc^2\left(1 + \frac{1}{2}\left(\frac{u}{c}\right)^2 + \dots\right)$$

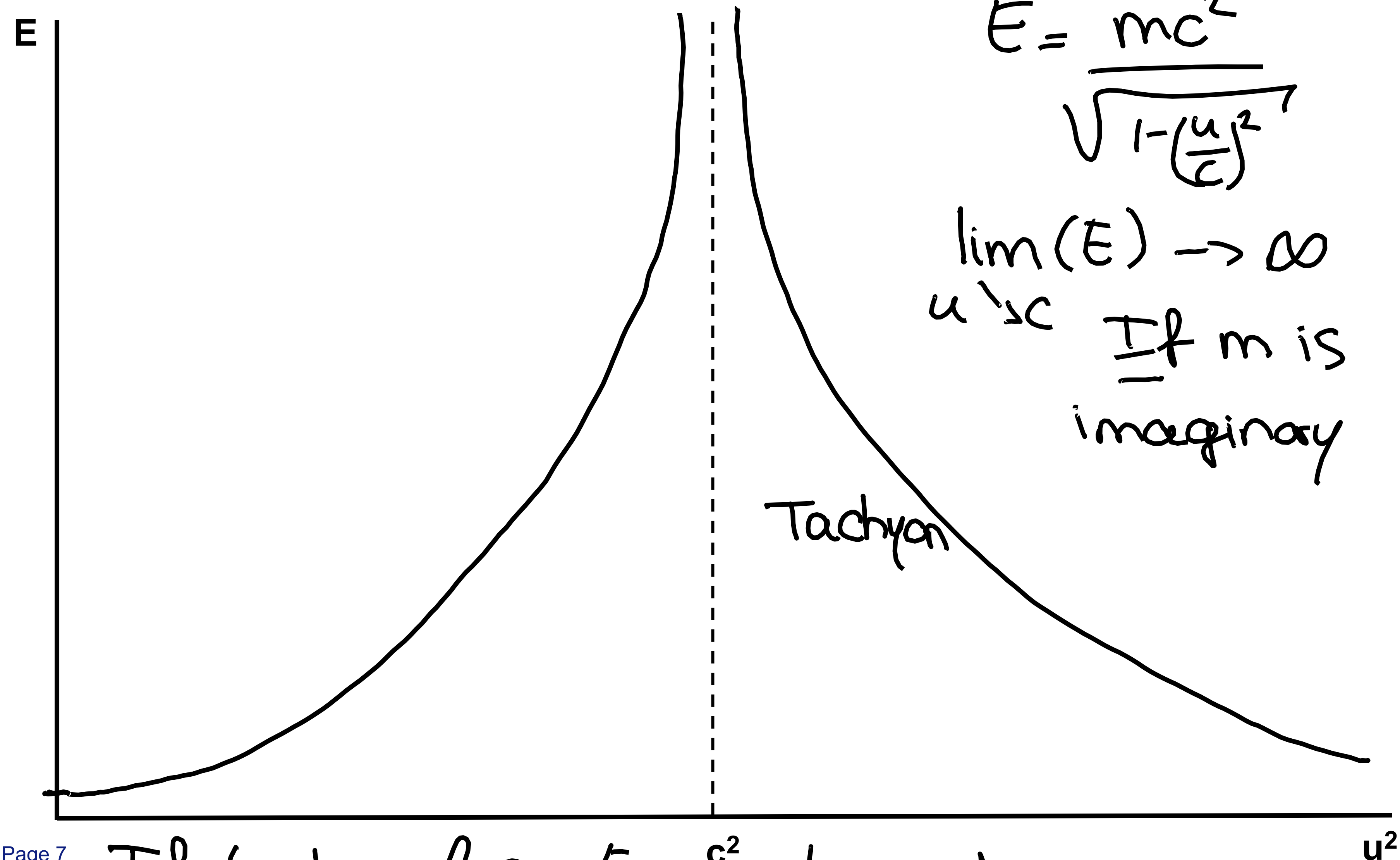
$$= \underbrace{mc^2 + \frac{1}{2}mu^2}_{\text{low-velocity limit}} + \dots$$

Total energy as *u* approaches *c*

$$E = \frac{mc^2}{\sqrt{1-\left(\frac{u}{c}\right)^2}}$$

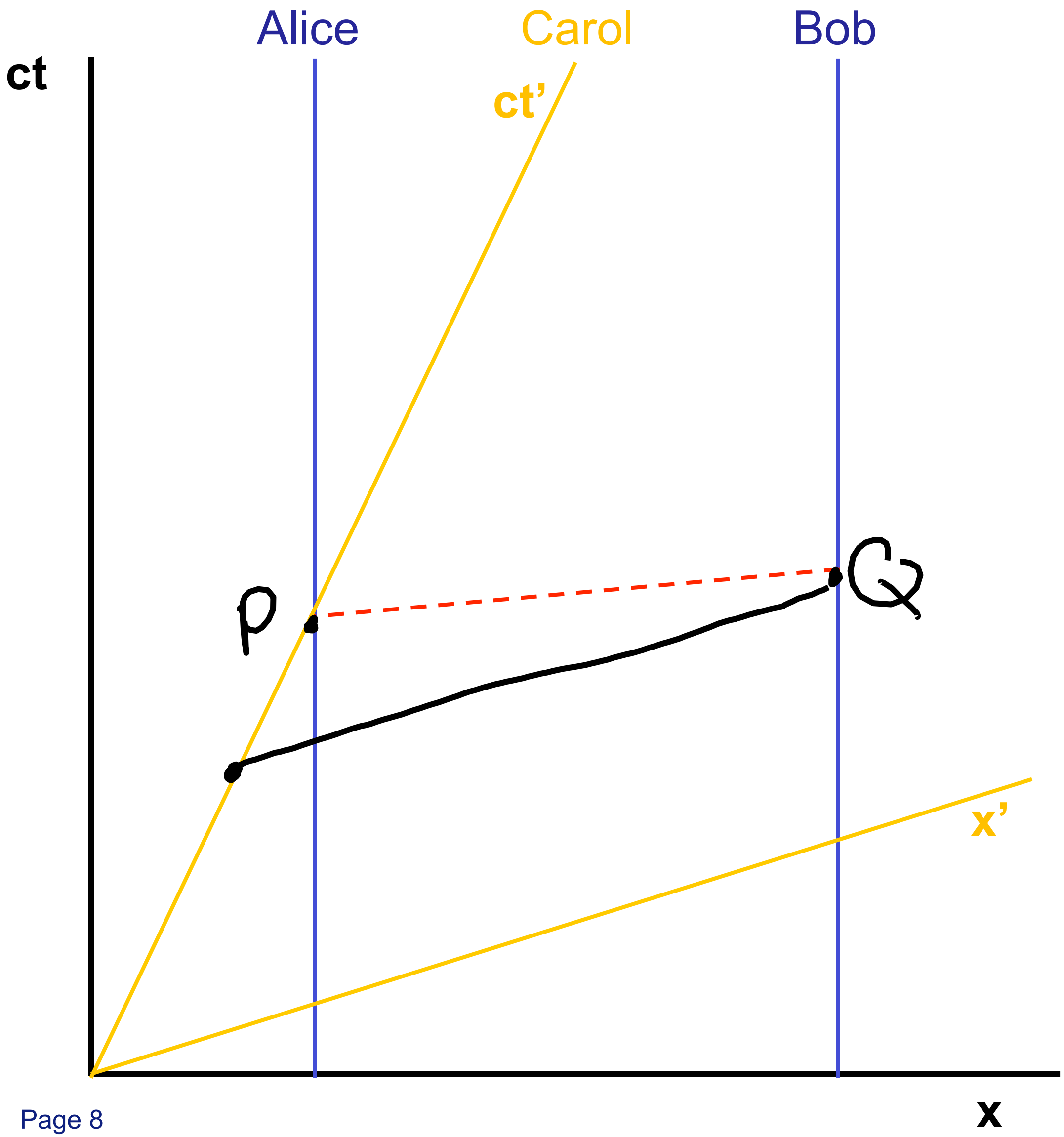
$$\lim_{u \to c} (E) \to \infty$$

If m is imaginary



If tachyon loses Energy, it speeds up.

Tachyons in a spacetime diagram

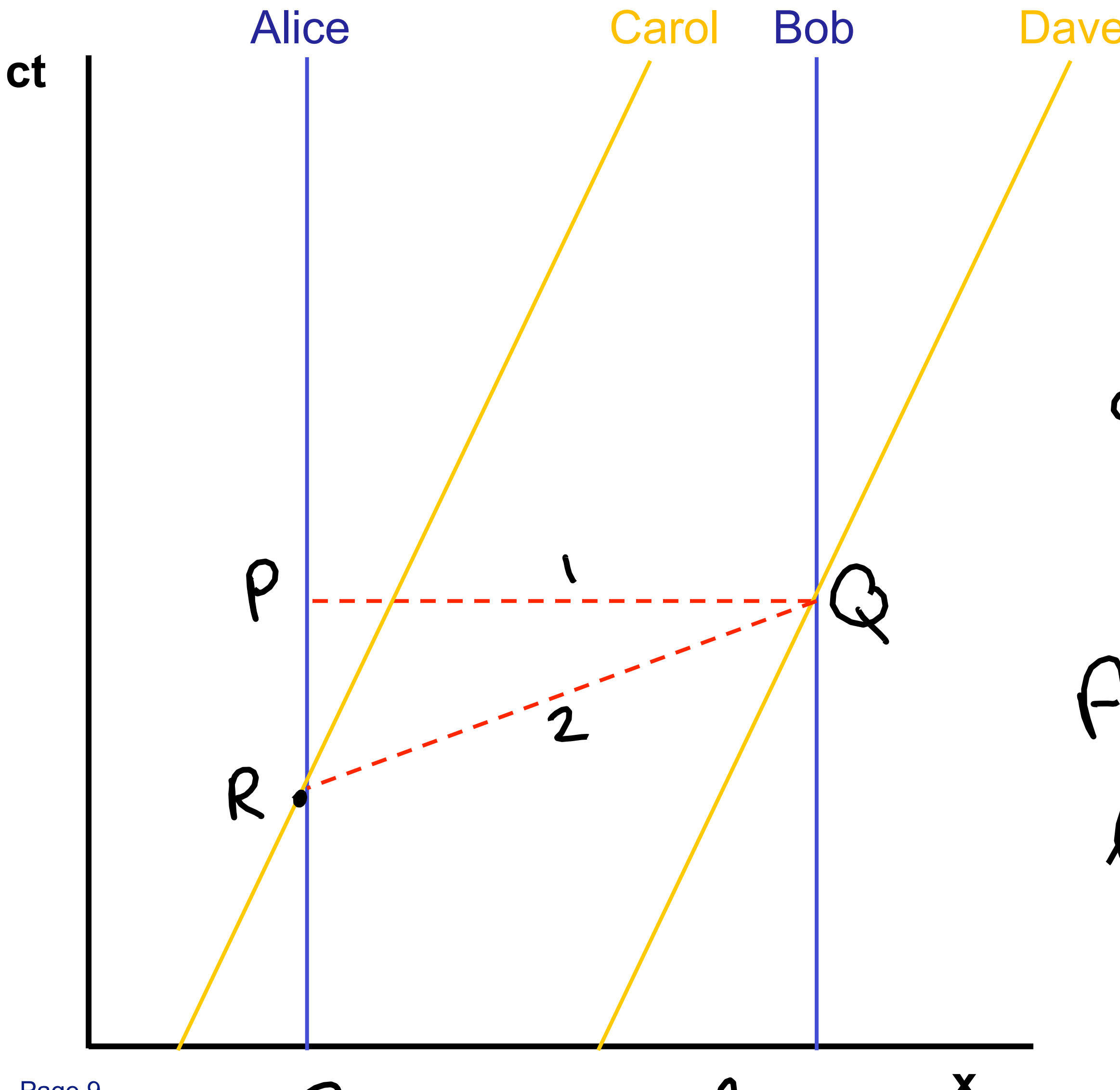


P: Alice sends tachyon message ($u > c$)

Q: Bob receives message.

In Carol's frame message is received before it was sent.

Tachyons and causality



Here $u_{\text{Tachyon}} \to \infty$

R: Carol receives 2nd message and passes it to Alice

Alice receives reply <u>before</u> message <u>1</u> was sent.

→ Causality breaks down.

$E_{tachyon}$ in Carol & Dave's frame

A-B-frame: $E = \dfrac{mc^2}{\sqrt{1-\left(\frac{u}{c}\right)^2}}$

$u > c$

m imaginary

E real & positive

LT: $\dfrac{E'}{c} = \gamma\left(\dfrac{E}{c} - \beta p\right)$

$$\Rightarrow E' = \gamma(E - up) = \gamma(\gamma_u mc^2 - \gamma_u m u^2)$$

$$= -\underbrace{\gamma\gamma_u m}_{\text{real, +ive, finite}} \underbrace{(c^2 - u^2)}_{\text{real, finite, negative}}$$

So E' is real, but negative...