Imperial College London

Relativity – Lecture 10

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Key concepts of lecture 9

Energy and momentum are conserved separately in any one frame.

However, when transforming frames, the energy and momentum change.

In other words, a Lorentz transformation changes energy into momentum, and vice versa.

However, the norm of the four-vector is invariant, so $E^2 = p^2c^2 + \left(mc^2\right)^2$ is always true.

Result of the frame transformation example

Frame in which target particle is at rest:

Particle	$\boldsymbol{P}_i = (E_i/c, \boldsymbol{p}_i)$	β_i	m_i
1	(17/c, 15/c)	15/17	$8/c^2$
2	(8/c, 0)	0	$8/c^2$
3	(25/c, 15/c)	3/5	$20/c^2$

Centre-of-momentum frame:

Particle	$\boldsymbol{P'}_i = (E'_i/c, \boldsymbol{p'}_i)$	β'_i	m'_i
1	(10/c, 6/c)	3/5	$8/c^2$
2	(10/c, -6/c)	-3/5	$8/c^2$
3	(20/c, 0)	0	$20/c^2$

Reminder: get the terminology right.

- **Conserved**: a quantity which is not changed by a physical process. This refers to one frame at a time, and a conserved quantity will typically have different numerical values in different frames.
- **Invariant**: a quantity which is not changed by a coordinate transformation. The term refers to more than one reference frame; an invariant quantity will not necessarily be conserved in a particular process.
- **Constant:** refers to a quantity which does not change in time, such as the mass of the Universe.

- The speed of light is conserved, invariant, and constant!

Tip: solving energy-momentum problems

Try to solve problems first by using energy conservation alone. Some problems require you to use both energy and momentum conservation.
You can eliminate one variable using

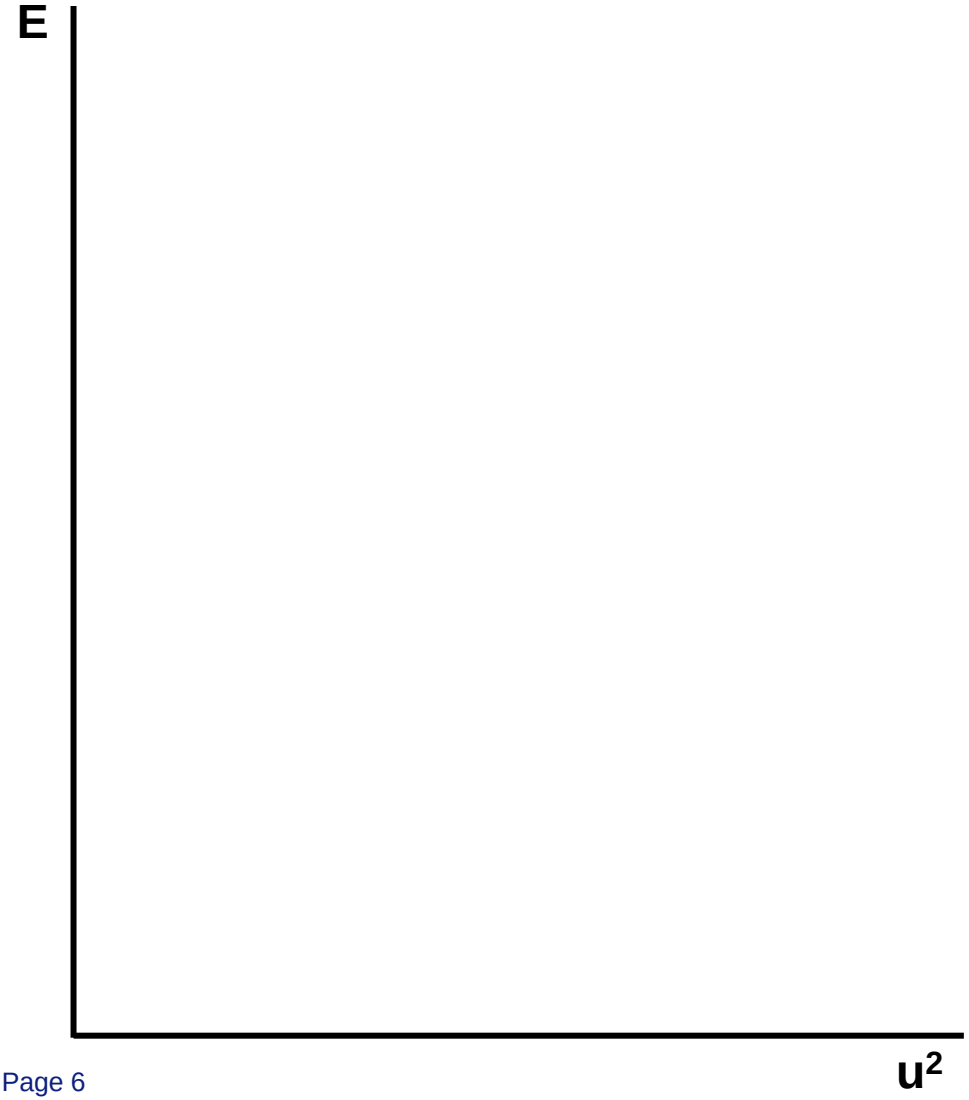
$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

for example $$p = \sqrt{\left(E/c\right)^2 - \left(mc\right)^2}$$

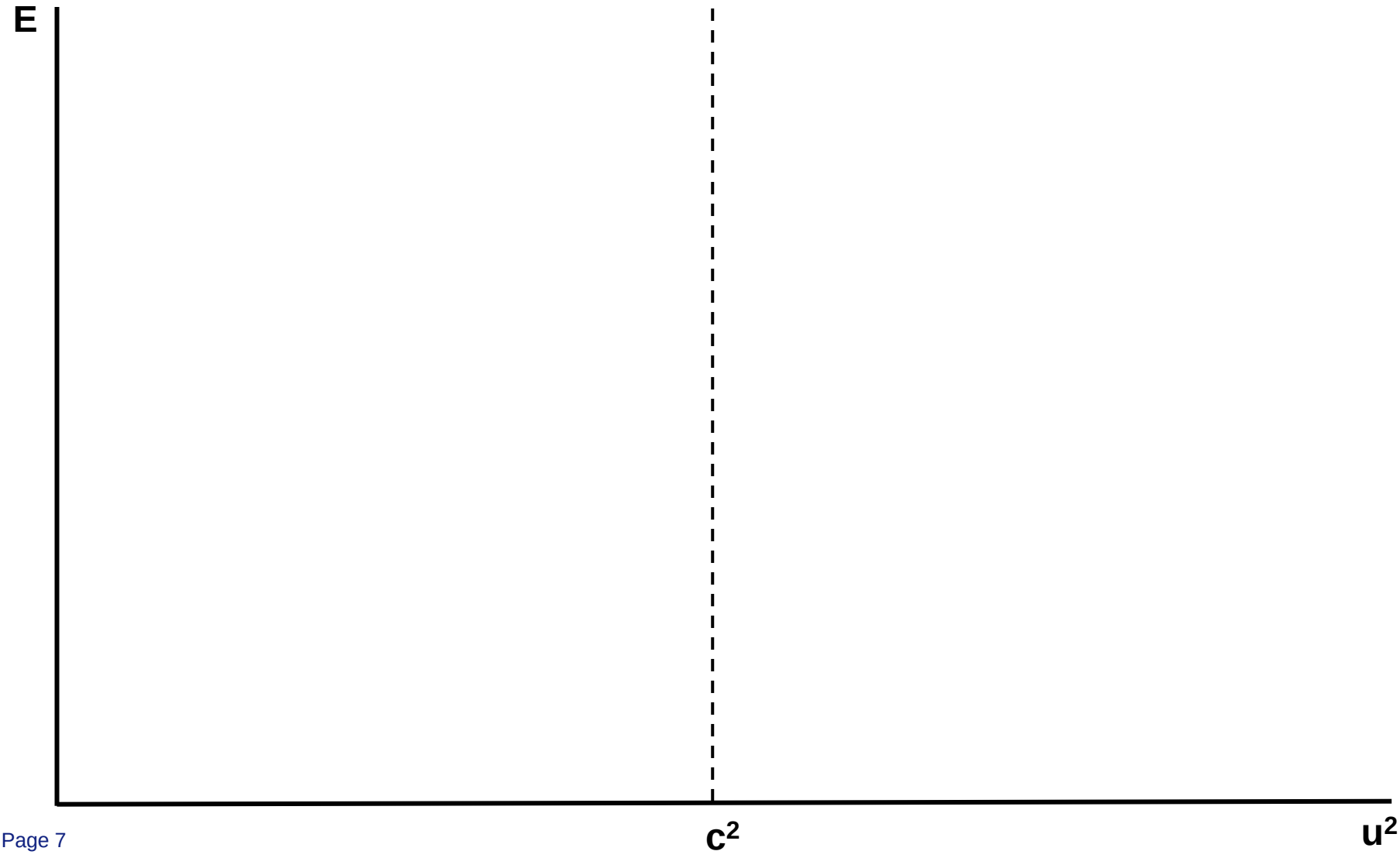
For a massless particle, $E = pc$.

You can also leave out all of the c's and put them in at the end using dimensional analysis.

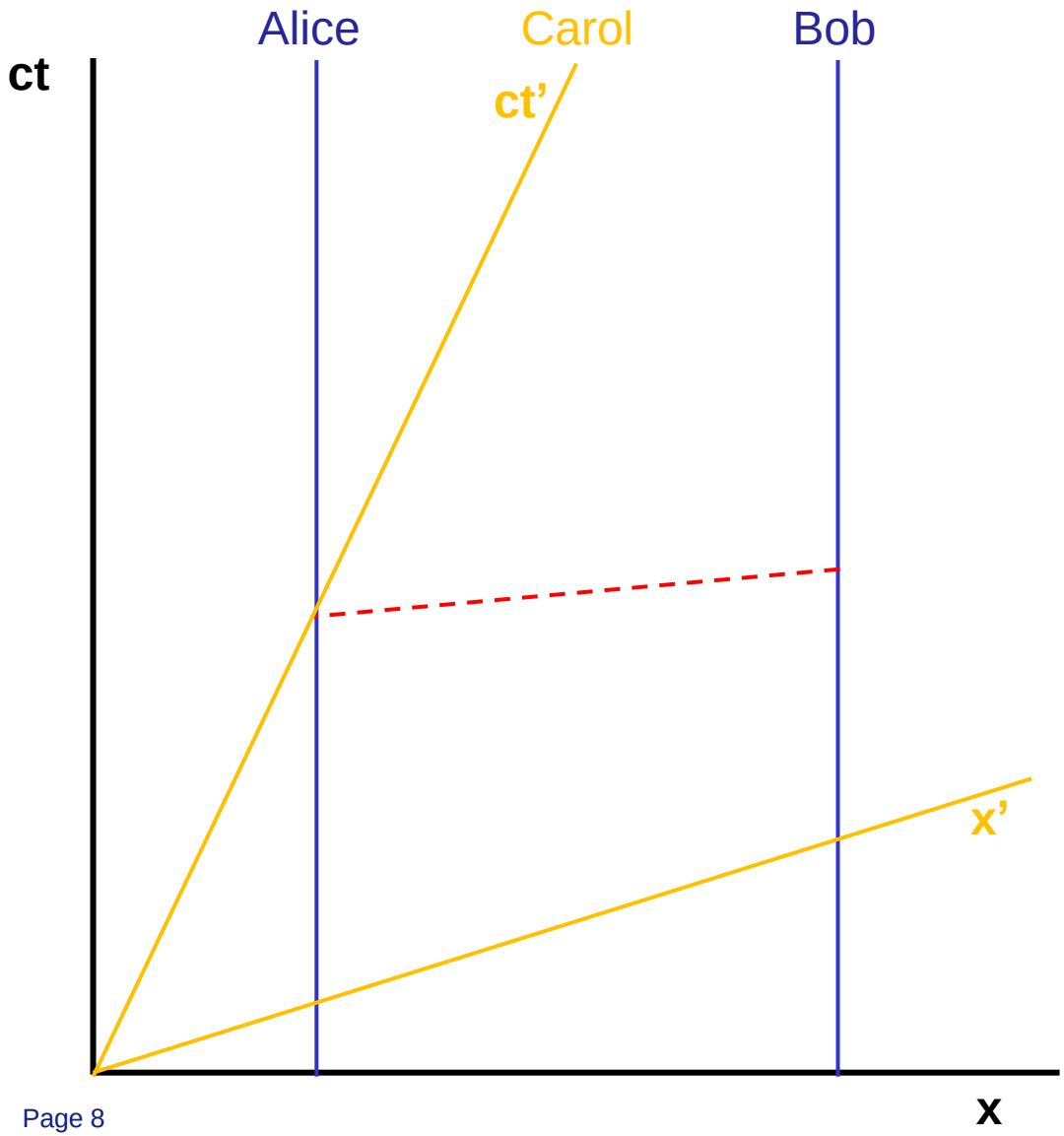
Total energy as *u* approaches *c*



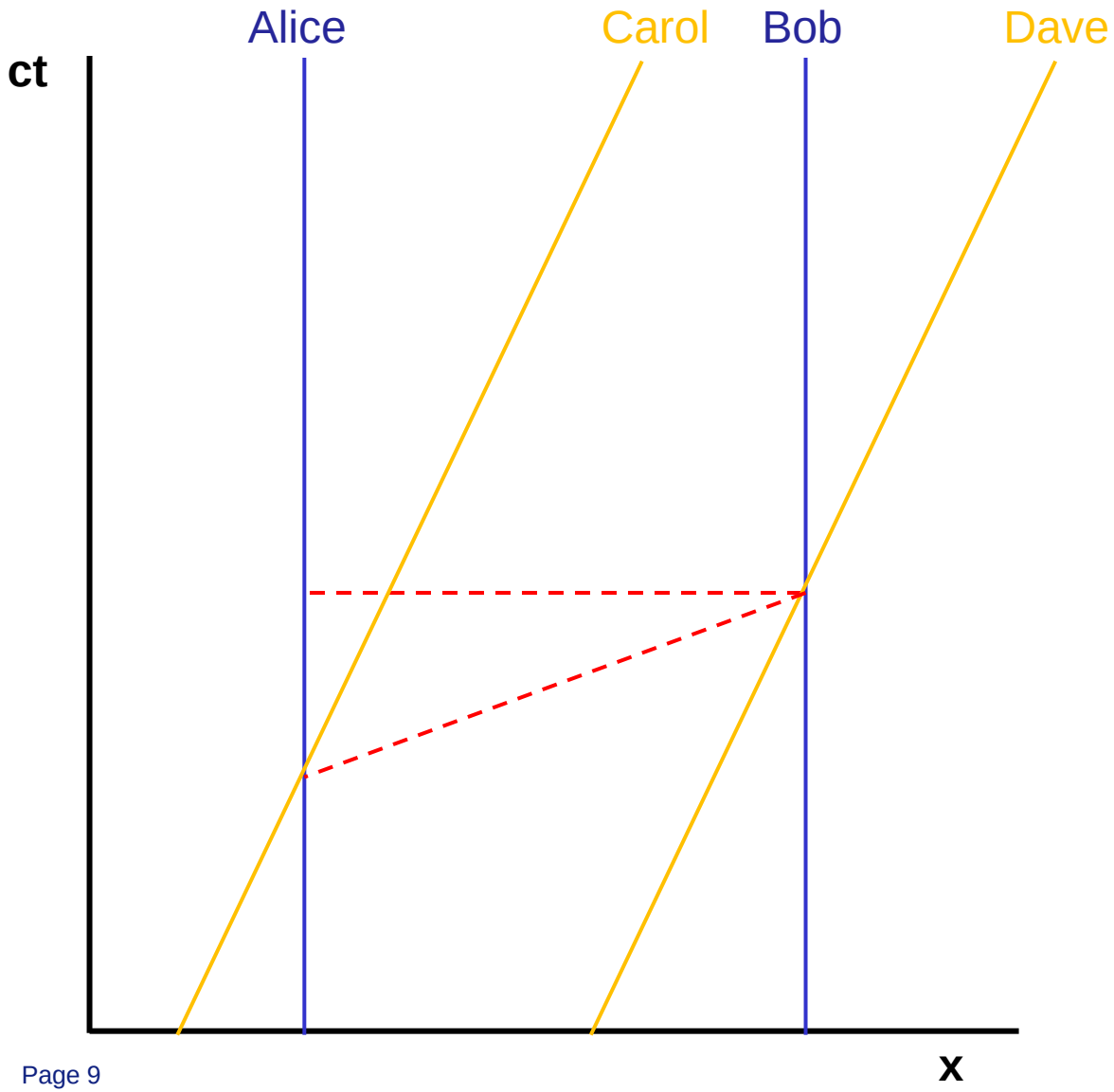
Total energy as *u* approaches *c*



Tachyons in a spacetime diagram



Tachyons and causality



E_{tachyon} in Carol & Dave's frame