Relativity – Lecture 8

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Key concepts of lecture 7

The relativistic Doppler effect is caused by:

1. The source 'catching up' to the emitted waves (classical Doppler effect).
2. Time dilation.

Compare formulae:

$$\nu_D^{Rel} = \frac{\nu_O}{\gamma}\frac{1}{1 - v/c} = \nu_0\sqrt{\frac{1+\beta}{1-\beta}}$$

$$\nu_D^{Cl} = \nu_O\frac{1}{1 - v/c}$$

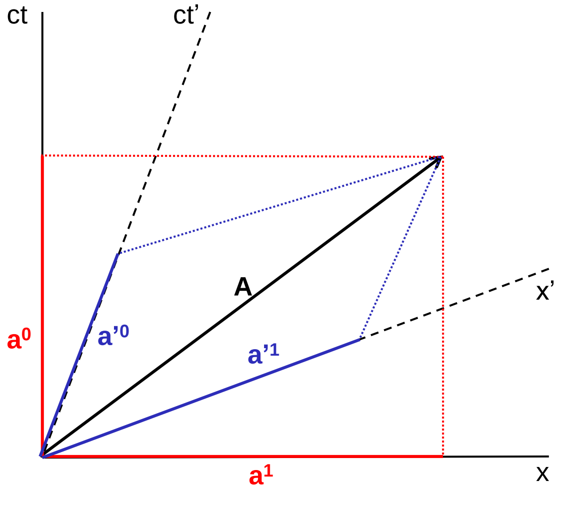
Four-vectors

A four-vector is a vector with four elements, and transforms under the Lorentz transformations between inertial reference frames.

Notation: $\boldsymbol{F} = (f^0, f^1, f^2, f^3) = (f^0, \boldsymbol{f})$.

Example: four-position $\boldsymbol{X} = (ct, x, y, z)$.

Four-vectors in spacetime



Four-vector algebraic rules

The sum of two four-vectors is a four-vector:

The inner product of two four-vectors is invariant:

So the norm of two four-vectors is invariant:

The four-velocity

$\boldsymbol{U} = \cancel{\mathrm{d}\boldsymbol{X} / \mathrm{d}t}$? $\boldsymbol{U} = \mathrm{d}\boldsymbol{X} / \mathrm{d}\tau$!

The norm of the four-velocity

What is the norm of the four-velocity?

$\Rightarrow$ Invariant, as expected.

The energy-momentum four-vector

What about momentum?

Try $\boldsymbol{P} = m\boldsymbol{U}$:

$\boldsymbol{P}$ is the energy-momentum four-vector.

The norm of the energy-momentum four-vector

What is the norm of ***P***?

So *E* and ***p*** are frame-dependent, but they combine into a frame-independent quantity ***P***, whose invariant length is the mass.

A note on mass

The invariant mass *m* we use here is also called the rest-mass, or invariant mass, m_0.

Some people use $m = \gamma m_0$, and call *m* the relativistic mass. The relativistic mass of an object increases as its velocity increases.

Momentum of light

Light has no mass, so we cannot derive momentum in the usual way.

Instead, use $E^2 = (mc^2)^2 + (pc)^2$.

Kinetic energy

Kinetic energy is the difference between the total energy and the rest energy.

Summary - 1

- A physical vector quantity is represented by a four-vector in Special Relativity.
- A four-vector transforms between inertial frames under the Lorentz transformations.
- The norm of a four-vector is invariant.
- The four-velocity is $\boldsymbol{U} = \gamma_u(c, \boldsymbol{u})$.

Summary - 2

- The energy-momentum four-vector is $\boldsymbol{P} = (E/c, \mathbf{p})$.
- Here $\mathbf{p} = \gamma_u mu$, and $E = \gamma_u mc^2$ is the total energy of the particle.
- The norm of $\boldsymbol{P}$ is m^2c^2. So $(mc^2)^2 = E^2 - (pc)^2$.
- The (rest-) mass is therefore invariant.
- The kinetic energy is $T = E – mc^2$.