Lecture 5

Physical Realisation of Logic Gates

Boolean Algebra as a Model of Logic Circuits?

Boolean Algebra is a good framework for describing the behaviour of logic circuits, but it is an abstraction.

For a practical machine we need to use real voltages, (e.g. ~3.5v for one and ~0.5v for zero), and we need to consider time delays for the signals to propagate through the circuit.

Boolean Logic is only an approximation to the way in which a digital circuit operates.

Physical Models

All physical models are approximate.

For example Newtonian mechanics was thought to be exact until about 1900 when more accurate measurements showed that real planetary movements differed from the predicted ones.

However, Newtonian mechanics is enormously useful --you don't need quantum theory to design a car!

Time in Logic Circuits

We will see in this lecture that the most important deficiency of Boolean Logic is its inability to describe events happening at different moments in time.

Later in the course we will discuss ways in which we can cope with the problems caused by timing.

A more detailed model

We can introduce a more detailed model of the operation of logic circuits, and for this we need three components:

The Resistor

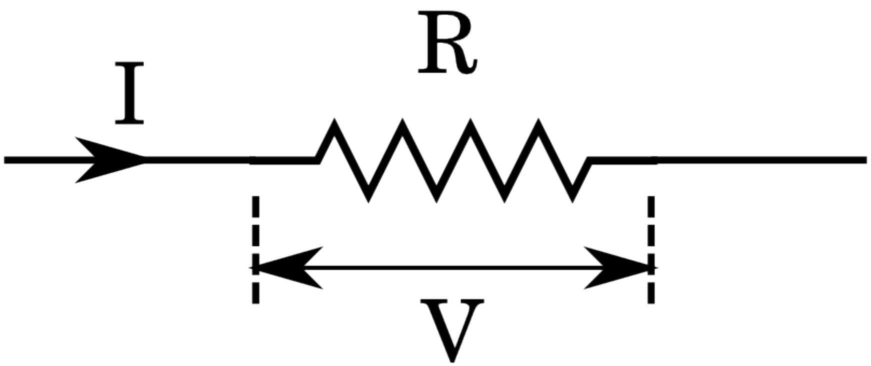
The Capacitor

The Transistor

The Resistor

This is a familiar device which is governed by Ohm's Law:

$V = I\,R$ (V=Voltage, I=Current, R=Resistance)



Procedural vs. Mathematical Models

Ohm's law is a simple mathematical model expressed by an equation.

For more complex devices, such as the transistor, it is possible to derive a mathematical equation, but it is much simpler to describe the behaviour of the device.

Such a description is called a procedural model.

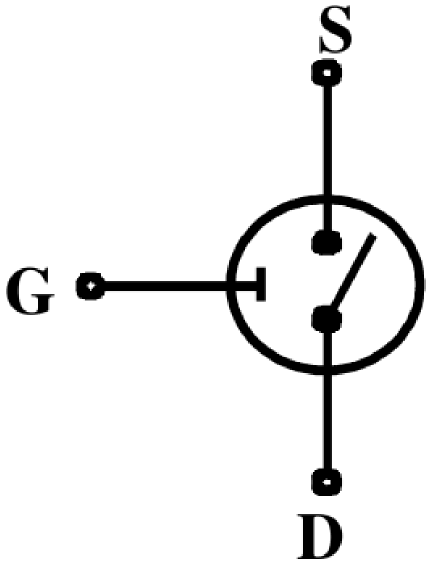
The transistor as a switch

The transistor may be thought of a a switch with the three terminals labelled:

S : Source

D : Drain

G : Gate

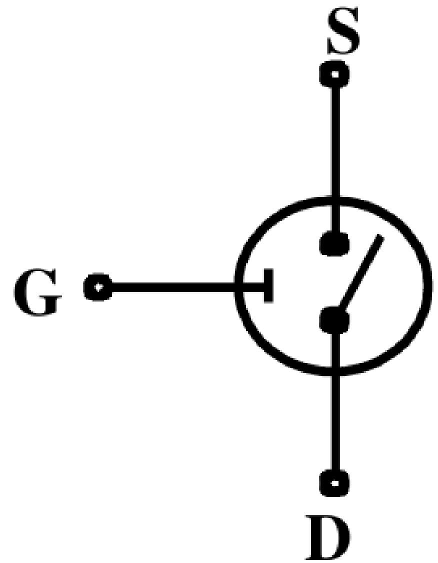


The rules

1. There is no connection between G and S or G and D

2. If the voltage between G and D (Vgd) is less than or equal to 0.5 volts there is no connection between S and D

3. If the voltage between G and D (Vgd) is greater than 2 volts S is connected directly to D



The Inverter Circuit

We can now build an inverter using a resistor and a transistor, but we need to define our Boolean States in terms of voltages:

For example:

V<=0.5volt is equivalent to Boolean 0

V>3volts is equivalent to Boolean 1

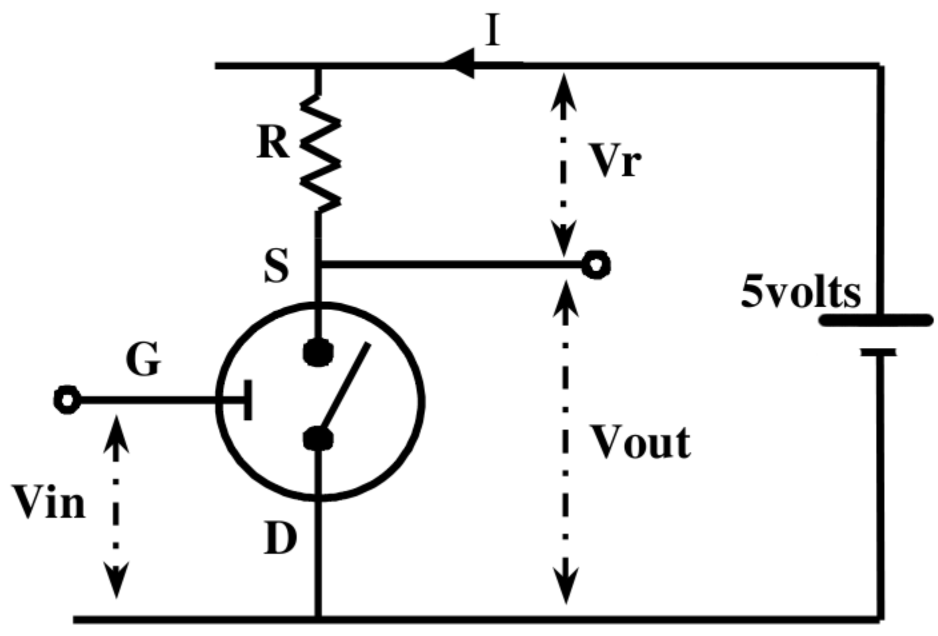
The Invertor Case 1: Vin = 0.5volt

Since Vin=0.5 the input is a Boolean 0, and by the procedural rules, the switch is open;

thus: I = 0

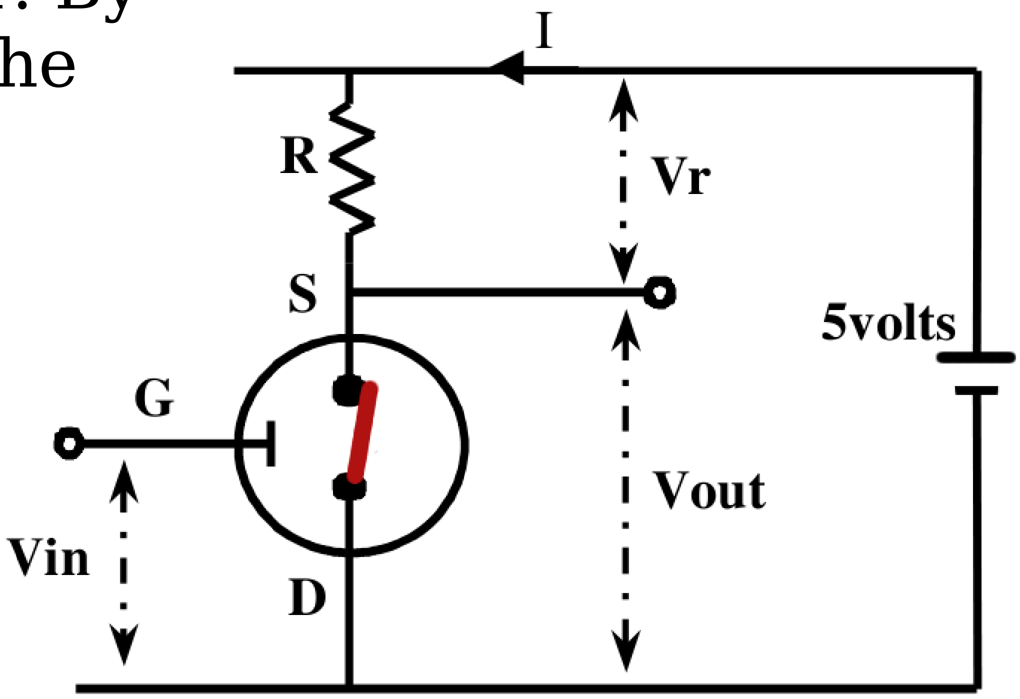
Vr = 0 (Ohm's law)

Vout = 5v = Boolean 1



The Invertor Case 2: Vin = 5volt

The input is 5v which we interpret as Boolean 1. By the procedural rules the switch is closed.



Thus:

Vout = 0

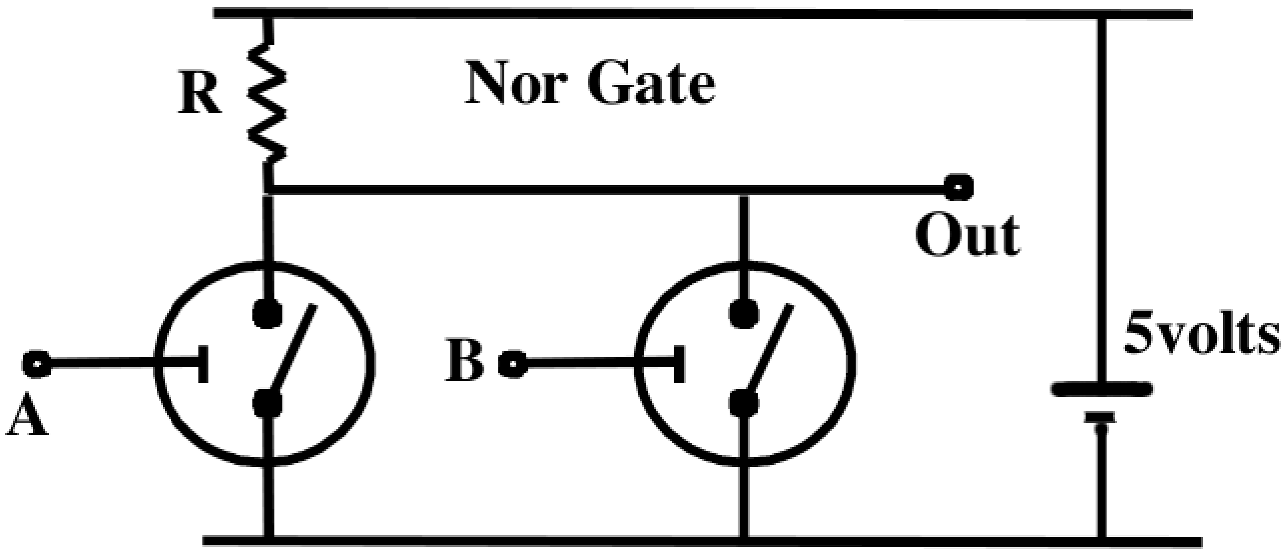
Vr = 5

I = 5/R (Ohm's law)

The *NOR* gate

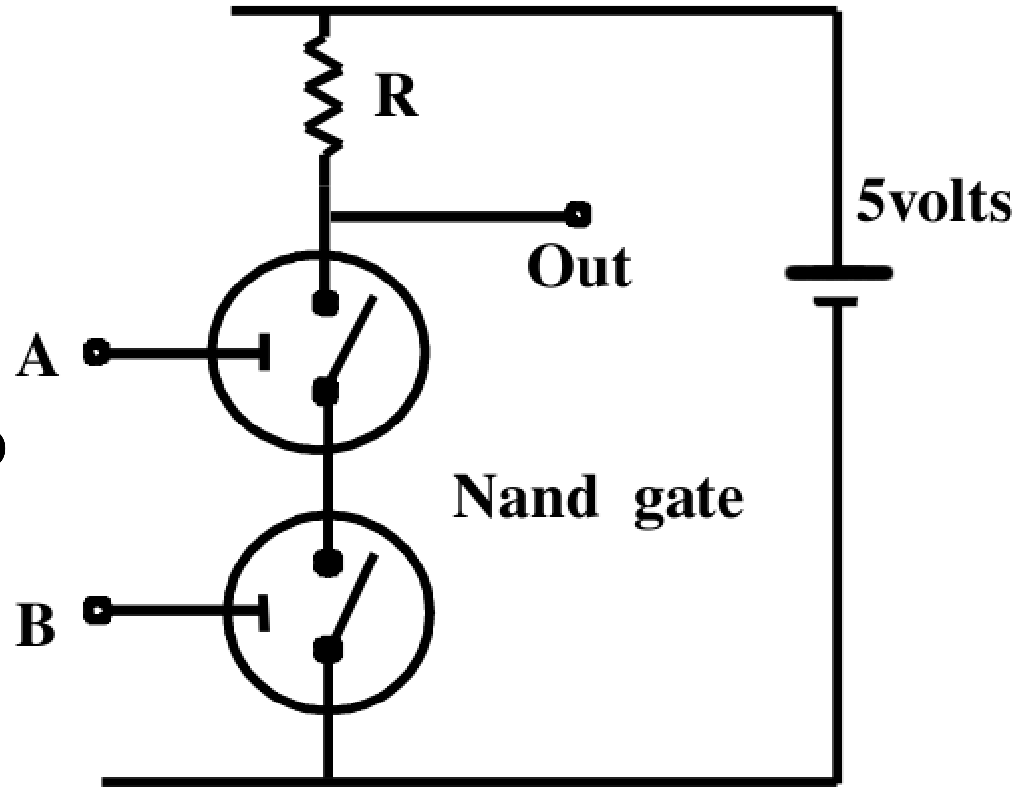
If both switches are open (input A and B both Boolean 0), the output is 5v (Boolean 1)

If either switch is closed (either, or both, ie A and/or B at Boolean 1 value), the output is 0v (Boolean 0)



The *NAND* gate

- The output falls to 0v (Boolean 0) only when both switches are closed.
- If either opens it rises to 5v (Boolean 1)



AND and *OR* gates

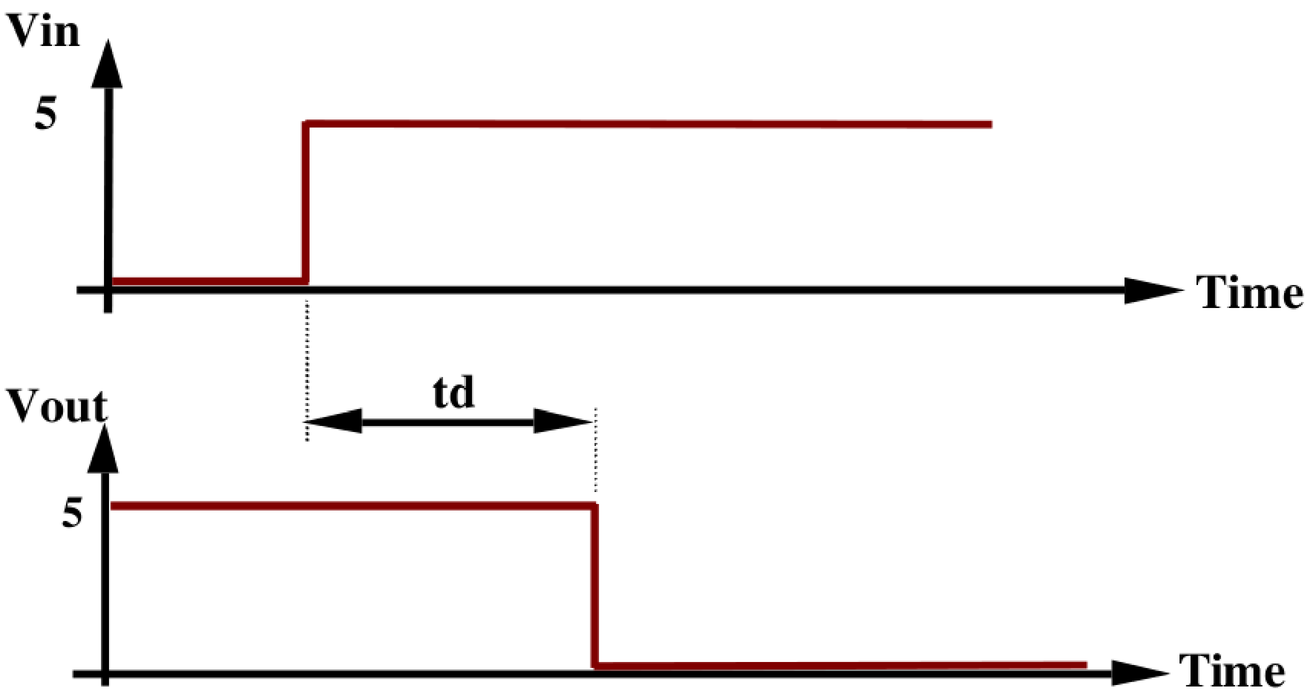
We can construct an *AND* gate by connecting a *NAND* gate and an invertor together.

Similarly we can construct an *OR* gate by connecting a *NOR* together with an invertor.

These models, though simple are surprisingly close to the implementations used in practice.

Time rears its ugly head

Signal Propagation: it takes time for the transistor state to change. This is because electrons move through the transistor.



Goodbye to Boolean Algebra?

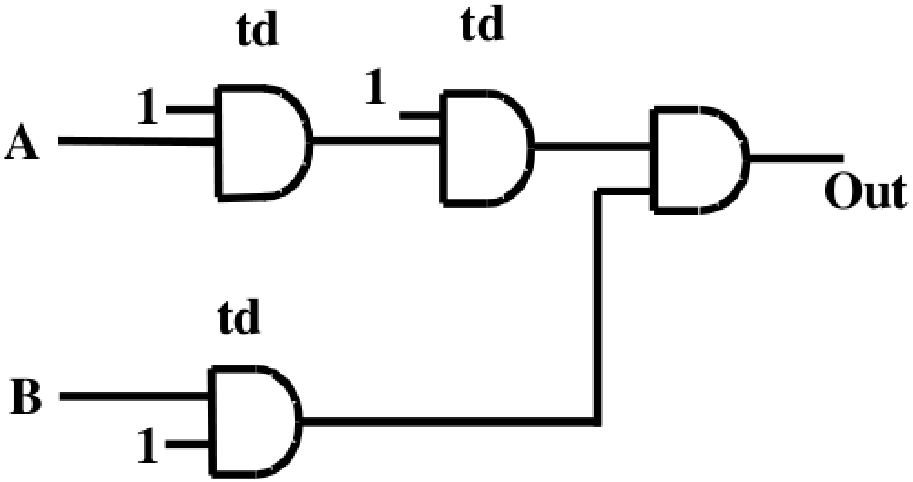
But Boolean Algebra does not incorporate a measure of “time”.

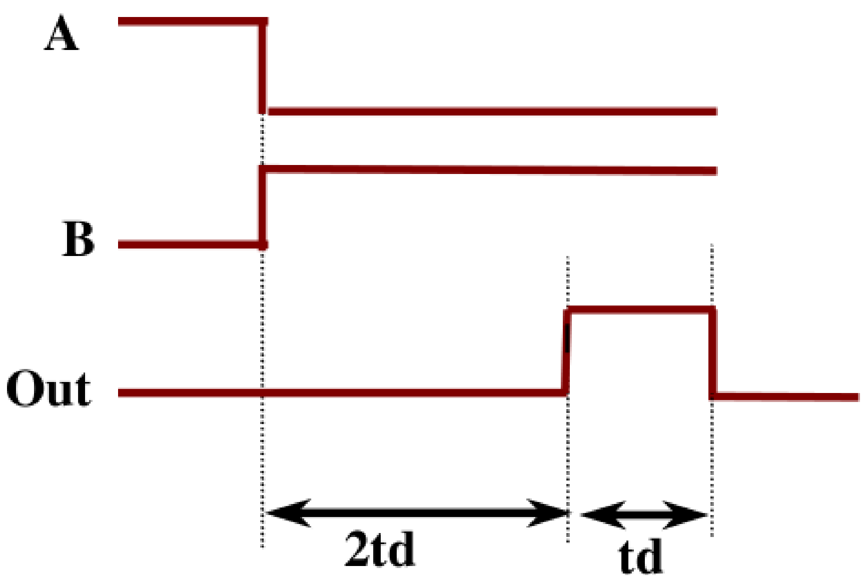
Although the time delay does not seem very important, in practice it complicates logic circuit design.

The larger the circuit and greater the difference in the number of gates in different paths the more reasoning about time becomes critical.

The synchronisation problem

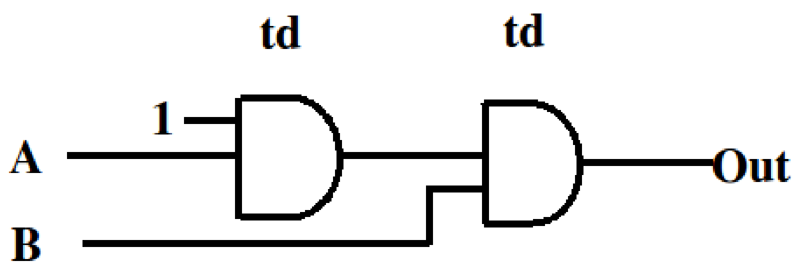
This example is artificial, but illustrates how a false result (sometimes called a spike) can be caused by time delays.

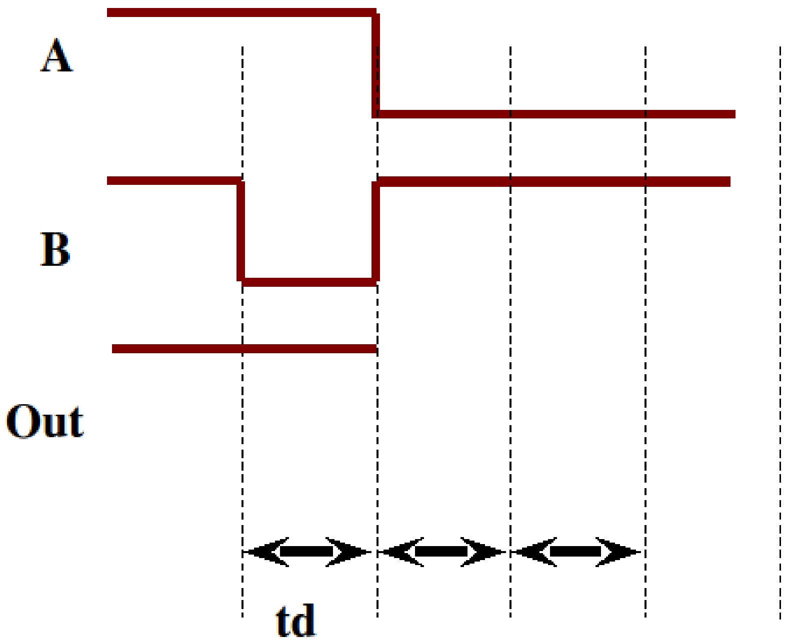




Problem Break

Given that A and B have had their starting values for some time what output would you expect to result from the timing diagram given?





Switch characteristics

Note that a transistor is not exactly a switch.

For a proper switch:

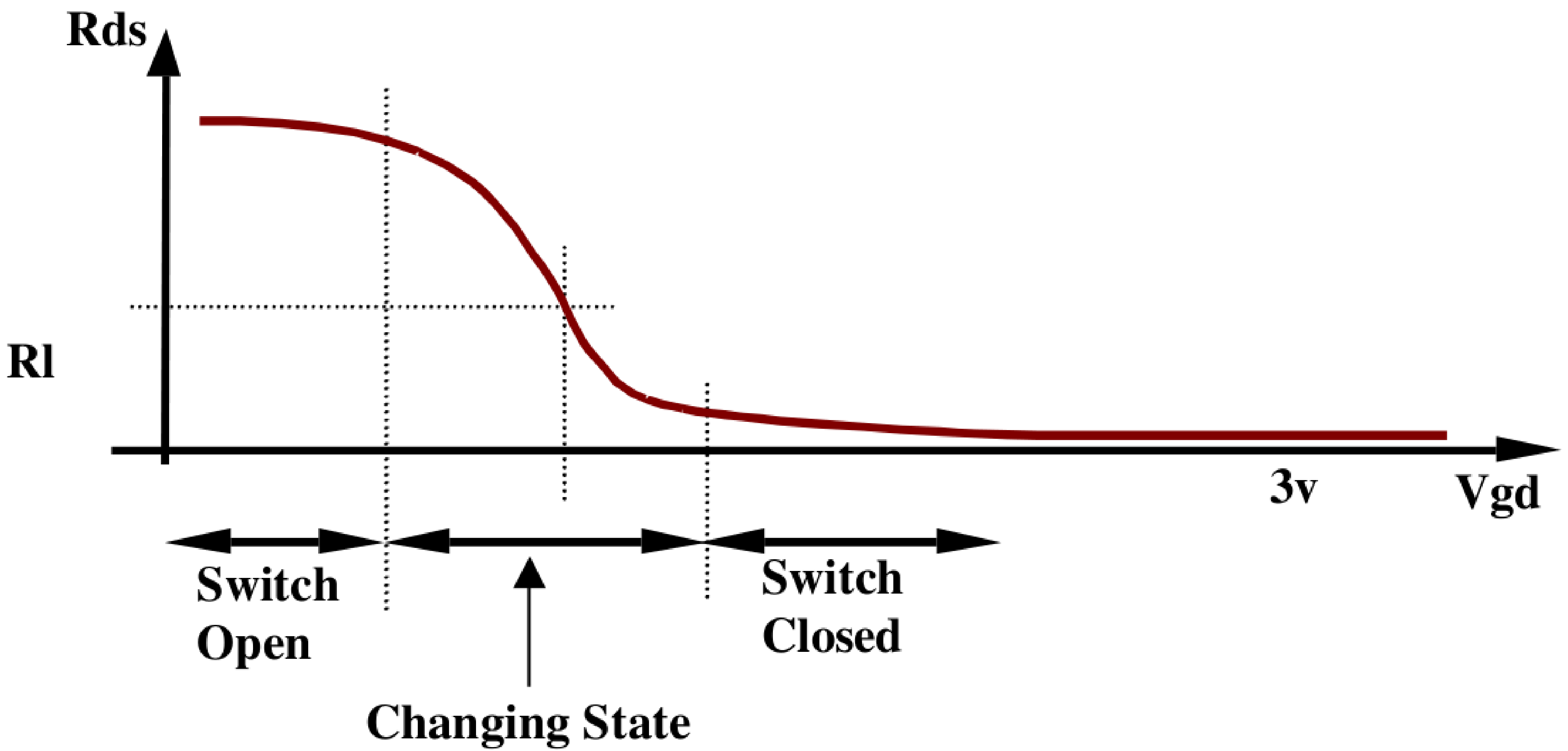
Switch Closed $\rightarrow$ 0 resistance

Switch Open $\rightarrow \infty$ resistance

But in practice neither of these extremes are reached.

Practical transistor characteristic

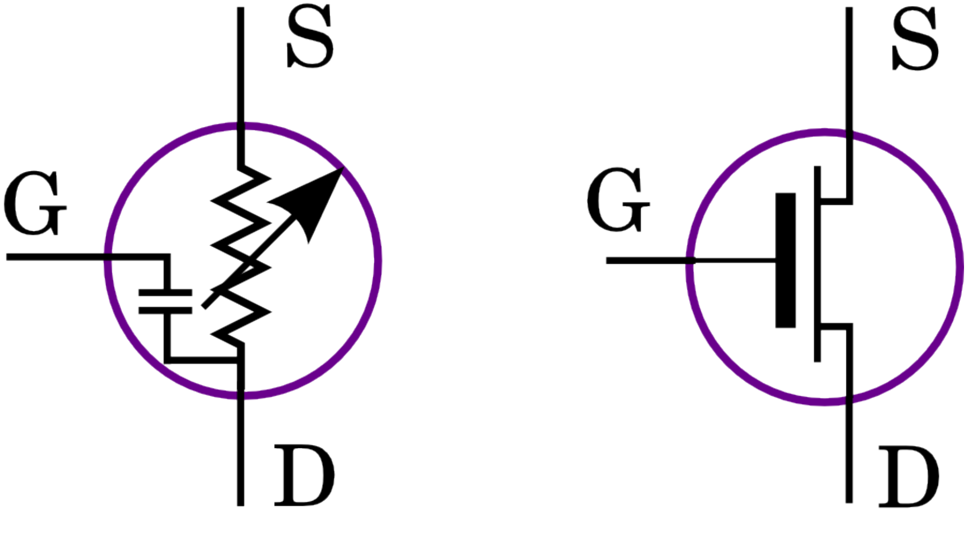
In fact a transistor behaves more like a variable resistor:



Input capacitance

Another feature of the real transistor is that it has a small capacitor connected between the gate and the drain.

We can represent it schematically thus:

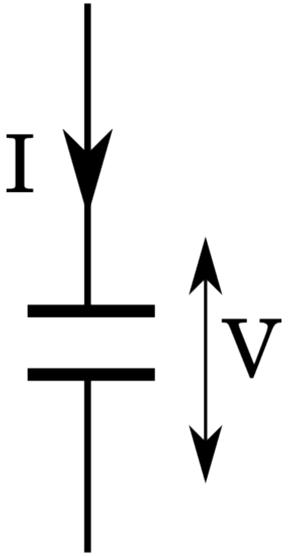


The effect of the capacitor

The capacitor has the effect of introducing a time delay. In fact, it is responsible for the time delay td that we talked about previously.

To see why we need to introduce a model of the capacitor:

$$I = C\,(dV/dt)$$



Calculating the effect of the capacitor

Assume A is 0V

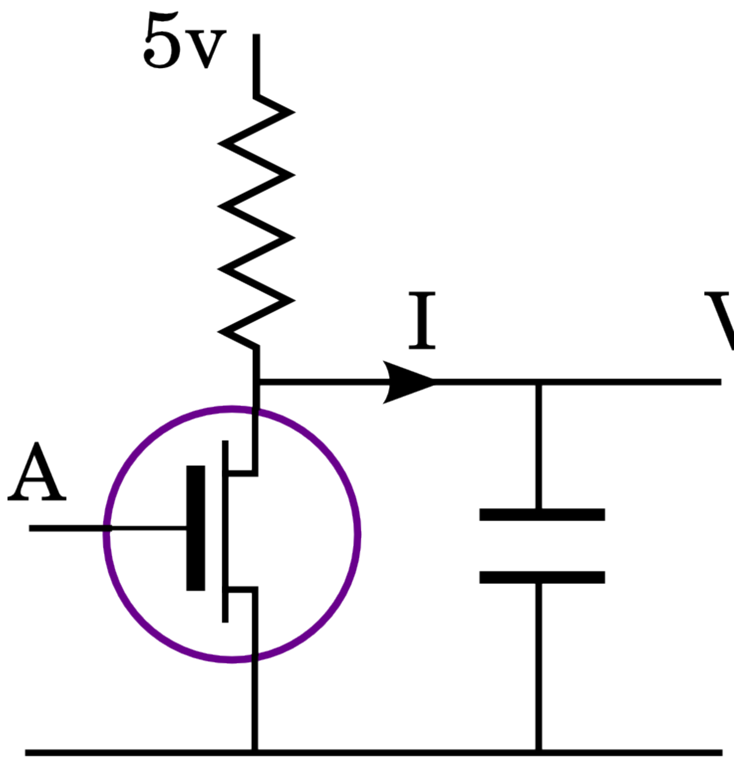
5 - V = IR   (Ohm's law)

V = 5 - IR

I = C(dV/dt)

Now we eliminate I to get:

V = 5 - RC (dV/dt)



Calculating the effect of the capacitor

Re arrange and integrate

$\int dV/(5-V) = \int (1/RC)\, dt$

$- \log(5-V) = t/RC + K$

If V=0 at t=0 it follows that $K = -\log(5)$

$5-V = \exp(-t/RC + \log(5)) = \exp(-t/RC)\exp(\log(5))$

$= 5 \exp(-t/RC)$

$V = 5(1 - \exp(-t/RC))$

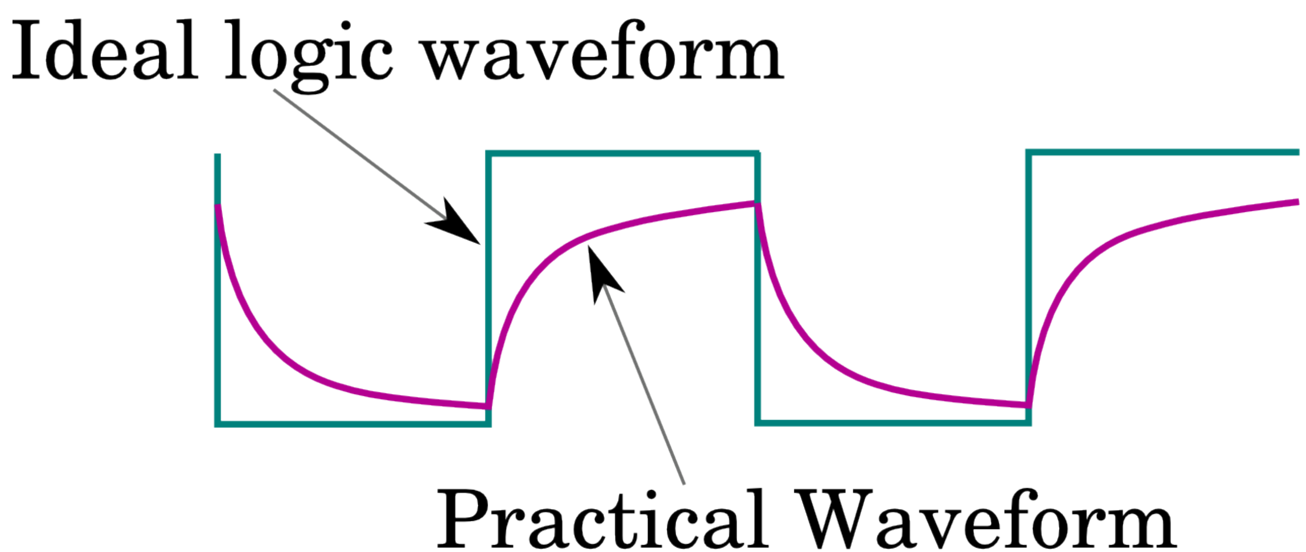
Plotting the effect of the capacitor

From the previous slide: $V = 5(1 - \exp(-t/RC))$

Practical representation of a square wave

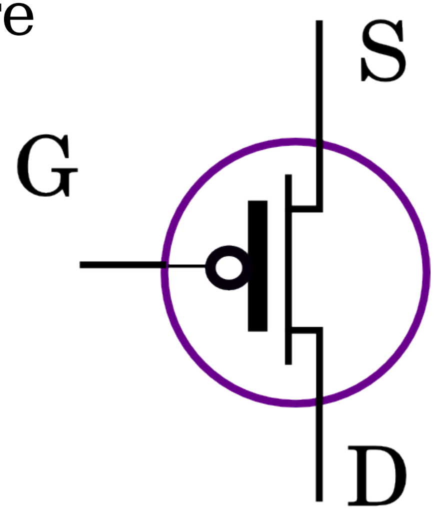
Notice that the voltage will never reach 1 or 0

There is a non-deterministic time interval which limits the speed that the computer can go



PMOS Transistors

The transistor we have discussed in this lecture is called the NMOS (Negative Metal Oxide Silicon) transistor. There is a dual type of transistor which operates in the opposite manner.



If the voltage between G and D (Vgd) is less than or equal to 0.5 volts there is low resistance between S and D

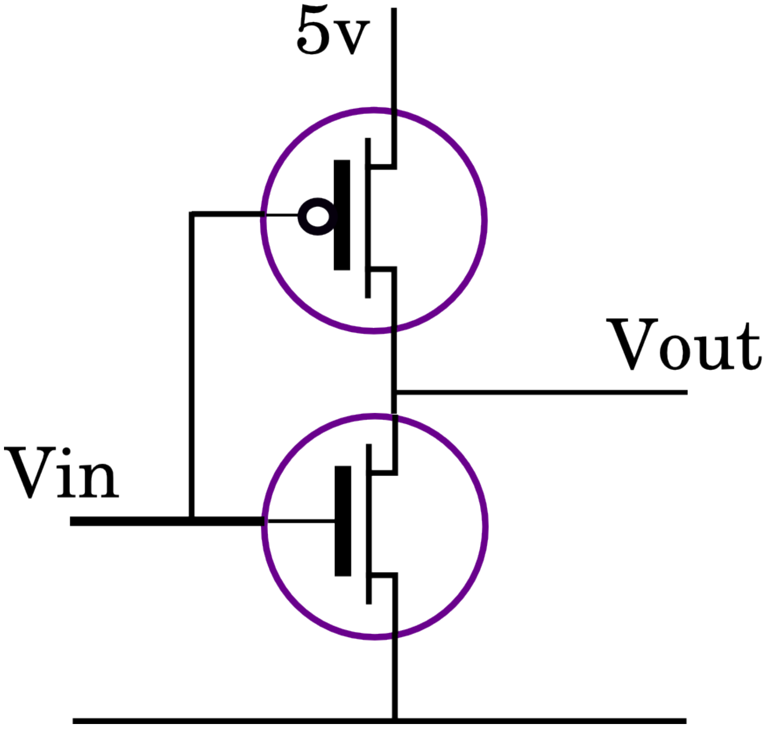
If the voltage between G and D (Vgd) is greater than 2 volts there is high resistance between S and D.

It is called the PMOS transistor

CMOS Circuits

In practice most integrated circuits do not use resistors but use a combination of NMOS and PMOS transistors. The technology is called CMOS (Complementary MOS).

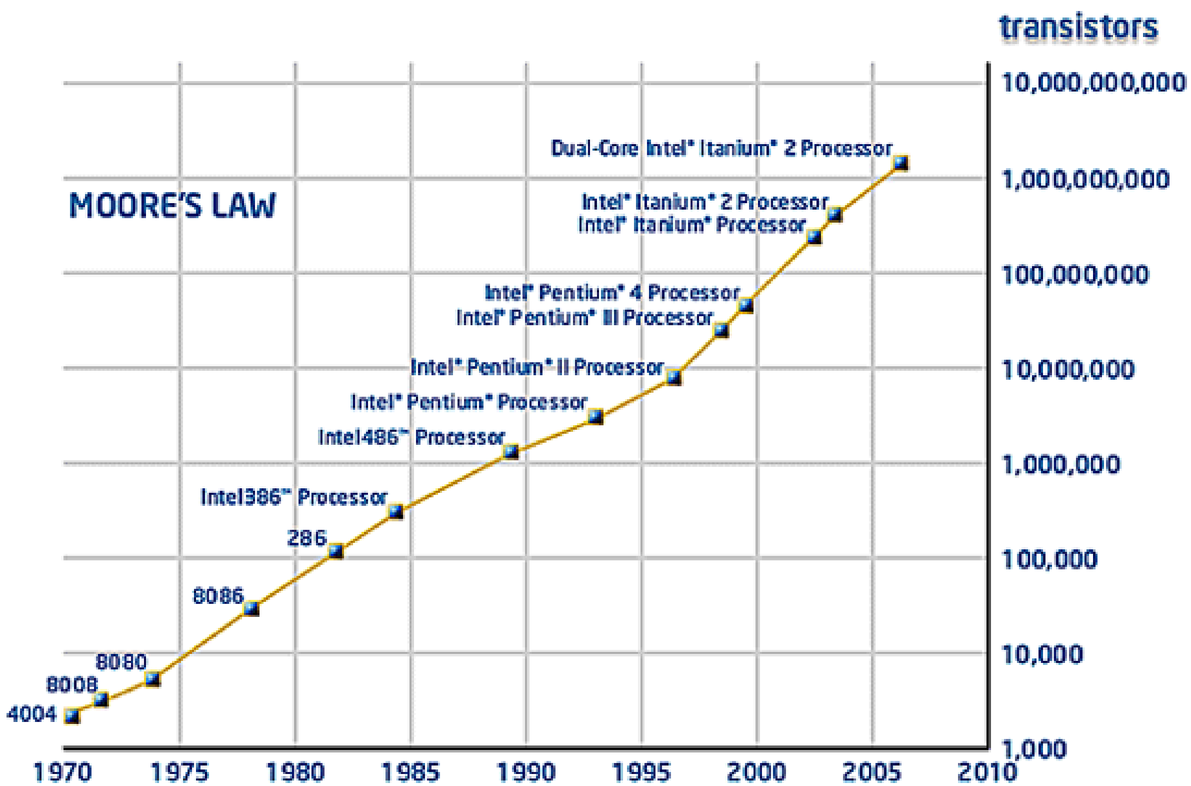
A CMOS inverter circuit has a PMOS transistor in place of the resistor. This means lower power consumption and faster switching.

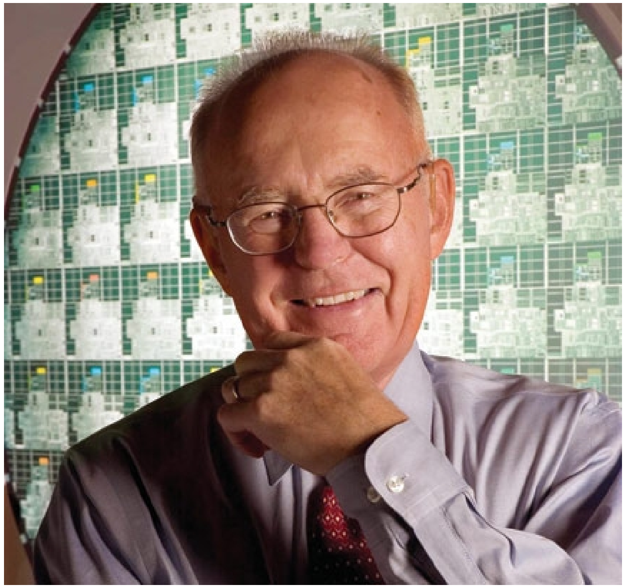


Transistor economics

Moore's Law (1965)

"Transistors per square inch doubles every 18 months"





Software Efficiency

May's Law (1985)

Software efficiency halves every 18 months, compensating Moore's Law.