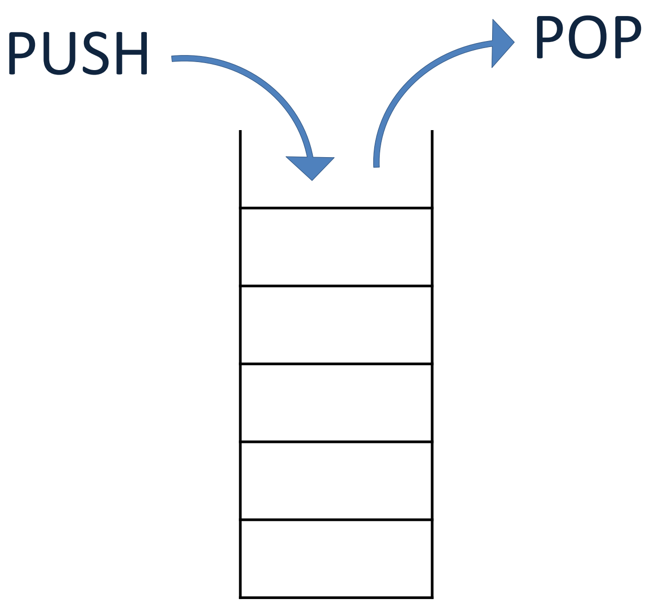
Unit-3
STACK and Queues

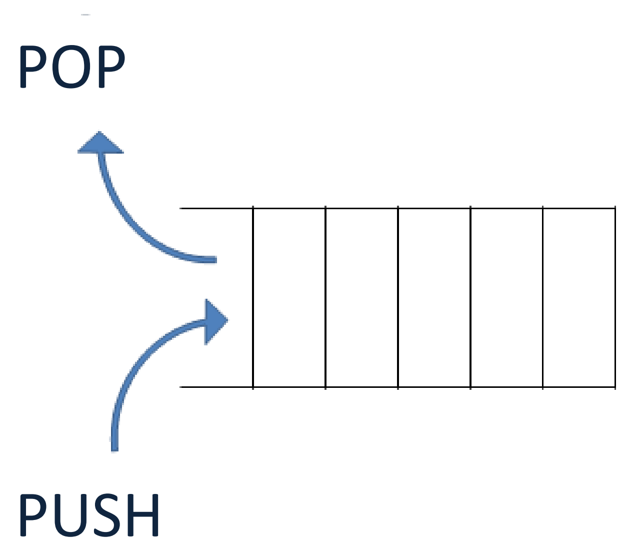
STACK

- Linear Data Structure
- Homogenous Data Structure
- ADT i.e. Abstract Data Type
- Elements are inserted and deleted only from one end, called top of stack.
- Stack is collection of elements that follows the LIFO order. **LIFO stands for Last In First Out**

STACK

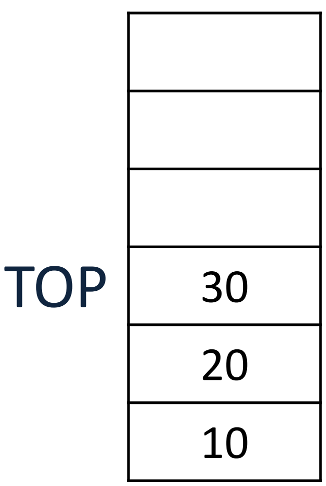
- In computer's memory stacks can be represented as a linear array.
- Insertion of element is called **PUSH** and deletion is called **POP**.
- These operations can be done from only one end of stack and we call that position as **top**.





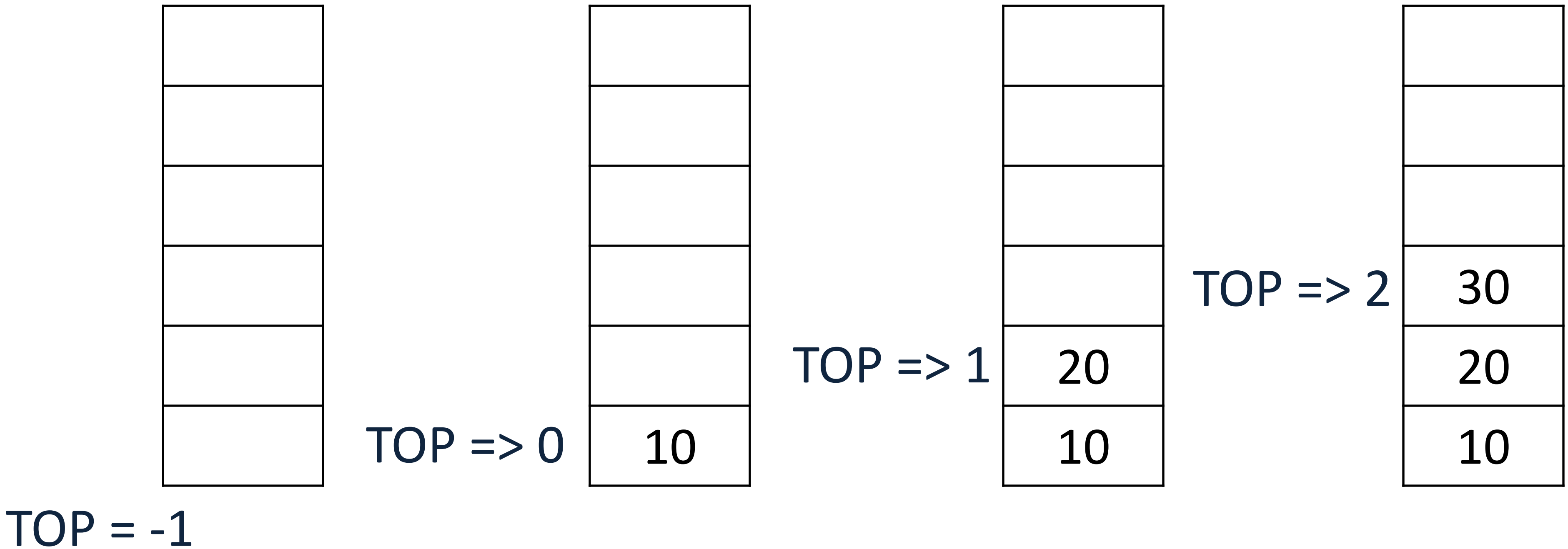
ARRAY REPRESENTATION OF STACK

- Every stack has a variable TOP associated with it.
- TOP is used to point to the top most element of the stack. It is the position from where the element will be added or deleted.
- Variable MAX will be used to store the maximum number of elements that the stack can hold.
- Initial value of TOP is -1 which indicates that stack is empty.
- TOP = MAX-1 indicates that stack is full.



PUSH OPERATION

- Used to insert an element into the stack.
- The new element is added at the topmost position of the stack.



TAKE CARE:

- **TOP = TOP + 1**
- **Check for TOP = MAX – 1 (Stack is full or not)**

ALGORITHM FOR PUSH OPERATION

Algorithm:

PUSH (VAL): This algorithm inserts an element to the top of the stack.

- S is stack which contains MAX elements.
- TOP is a pointer which points to the top element of the Stack.

Step 1) [Check for stack overflow]

    IF TOP = MAX - 1 then
    Write "OVERFLOW"
    Return

Step 2) [Increment TOP]

    TOP = TOP + 1

Step 3) [Insert Element]

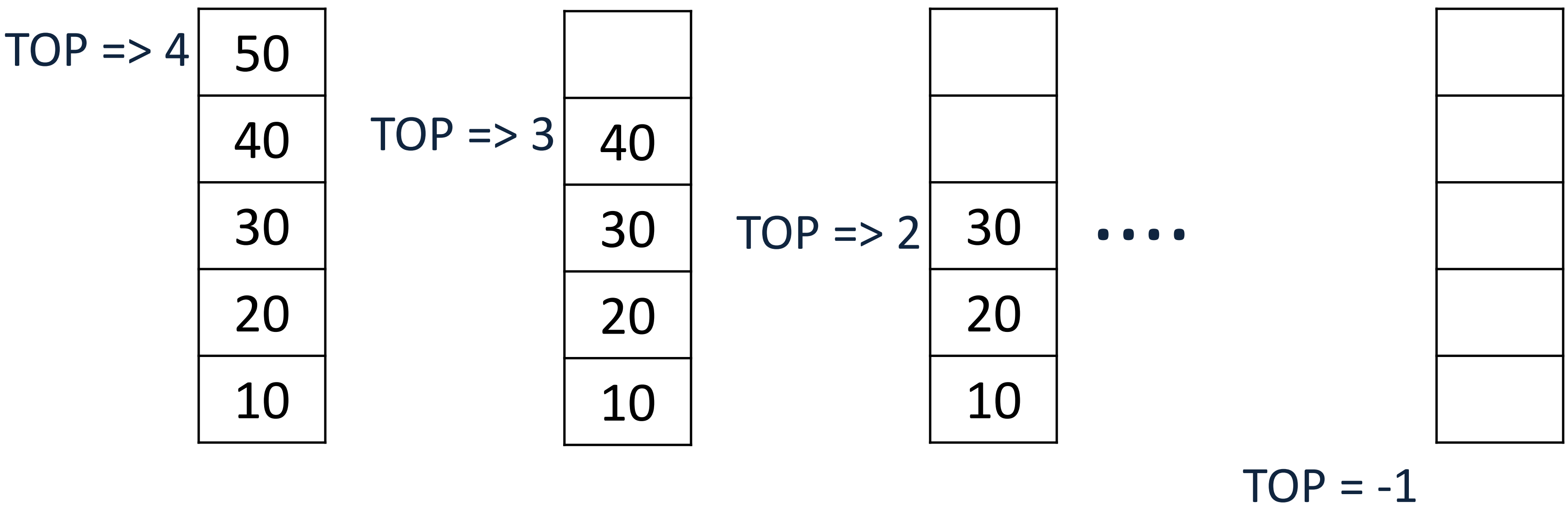
    S[TOP] = VALUE

Step 4) [Finished]

    Return

POP OPERATION

- Used to delete an element from the stack.
- The element is deleted from the topmost position of the stack.



TAKE CARE:

- **Access element at top of the stack**
- **TOP = TOP - 1**
- **Check for TOP = - 1 (Stack is empty or not)**

ALGORITHM FOR POP OPERATION

<u>Algorithm:</u>

POP (): This algorithm deletes an element at top of the stack.

- S is stack which contains MAX elements.
- TOP is a pointer which points to the top element of the Stack.

Step 1) [Check for stack underflow]

IF TOP = - 1 then

Write "UNDERFLOW"

Return

Step 2) [Access top most Element]

VALUE = S[TOP]

Step 3) [Decrement TOP]

TOP = TOP - 1

Step 4) [Finished]

Return VALUE

APPLICATIONS OF STACK

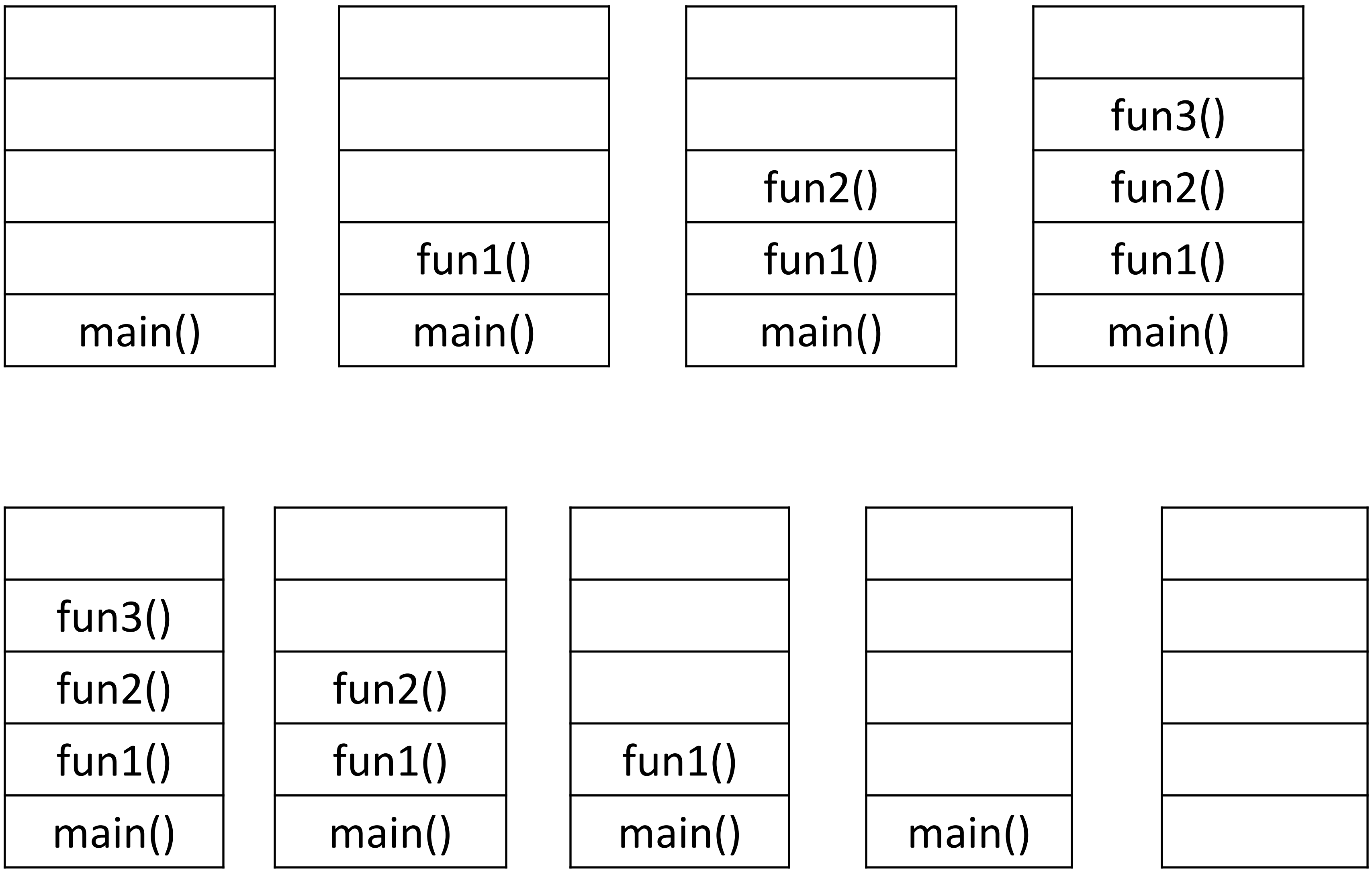
- Storing function calls
- Undo functionality
- Parentheses Checker
- Expression conversion
- Expression evaluation
- Recursion
- Tower of Hanoi

STORING FUNCTION CALLS

Example:

```
int main(){
    ....
    fun1();
    ....
}
void fun1(){
    ....
    fun2();
    ....
}
void fun2(){
    ....
    fun3();
    ....
}
void fun3(){
    ....
    fun4();
    ....
}
void fun4(){
    //code
}
```

STORING FUNCTION CALLS

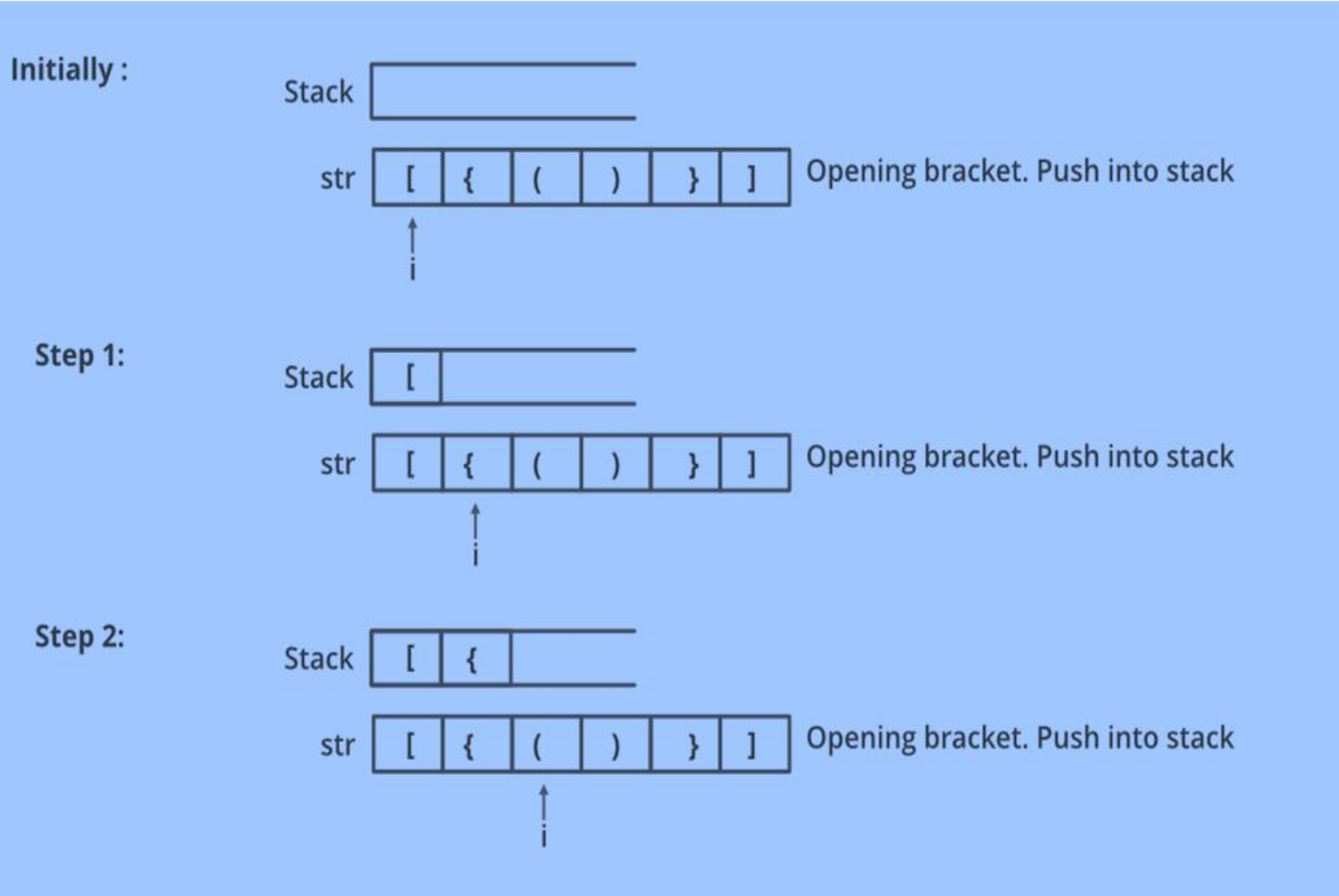


EXERCISE

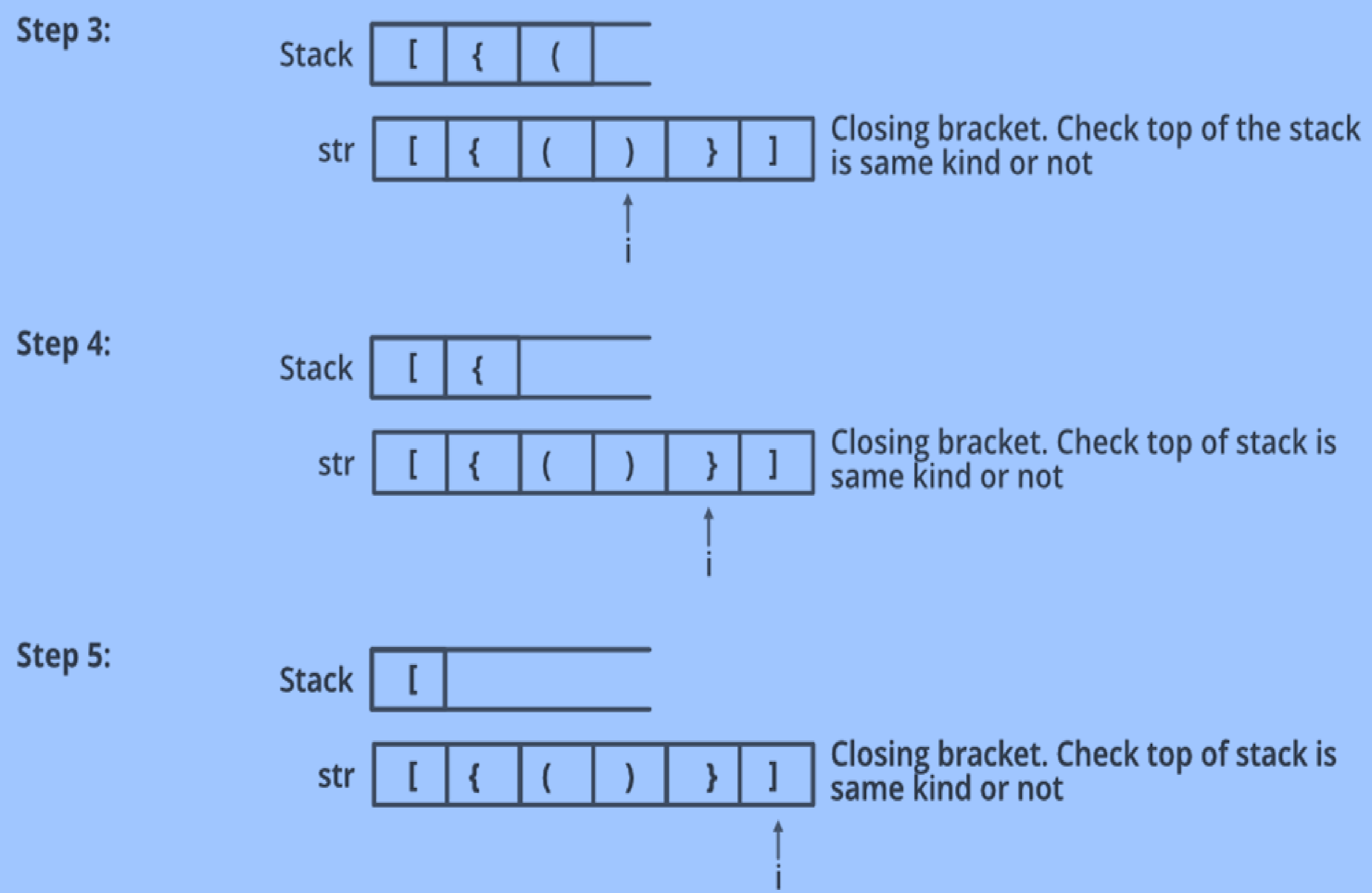
- Describe the output of the following series of stack operations:

  Push(8), Push(3), Pop(), Push(2), Push(5), Pop(), Pop(), Push(9), Push(1)

PARENTHESES CHECKER



PARENTHESES CHECKER



UNDO FUNCTIONALITY

- Maintain two stack say UNDO stack and REDO stack.
- Use UNDO stack to store all the operations that have been processed in the text editor.
- whenever an user encounter undo operation, pop the top of the element from UNDO stack and push it to REDO stack.
- Then, if the user want a redo operation pop the top of the element of REDO stack and push it to UNDO,
- If user performs a new operation then make the stack REDO empty.

EXPRESSION

- An expression is a collection of operators and operands that represents a specific value.
- Operator is a symbol which performs a particular task.
- Operands are the values on which the operators can perform the task. Here operand can be a direct value or variable.
- Based on the operator position, expressions are divided into THREE types. They are as follows...
  1. Infix Notation
  2. Postfix Notation
  3. Prefix Notation

INFIX NOTATION

- In expression using infix notation, the operator is placed in between the operands.
- **Structure: Operand1 Operator Operand2**
- **Example: A+B**

- For computers parsing of infix expression is difficult as lot of information is required to evaluate the expression.
- Information is needed about operator precedence and associativity rules, and brackets which override these rules.
- Computers work more efficiently with expressions written using prefix and postfix notations.

PREFIX NOTATION

- In prefix notation, the operator is placed before the operands.
- Also known as Polish Notation.
- **Structure: Operator Operand1 Operand2**
- **Example: +AB**

- **Evaluation of Prefix Expression:**

  Operator is applied to the operands that are present immediately on the right of the operator.

  Example: *+ABC

POSTFIX NOTATION

- In postfix notation, the operator is placed after the operands.
- Also known as Reverse Polish Notation(RPN).
- **Structure: Operand1 Operand2 Operator**
- **Example: AB+**

- **Evaluation of Prefix Expression:**

  Operator is applied to the operands that are present immediately on the left of the operator.

  Example: AB+C*

- Prefix and postfix expressions are evaluated from left to right.
- No need to follow operator precedence rules and associativity.
- Prefix and postfix notations are developed with aim to create parenthesis free expressions.

EXPRESSION CONVERSION

- Following conversions are possible:
  1. Infix to Postfix
  2. Infix to Prefix
  3. Prefix to Postfix
  4. Postfix to Prefix
  5. Prefix to Infix
  6. Postfix to Infix

- We consider five binary operators:

OPERATOR	PRECEDENCE	ASSOCIATIVITY
Exponentiation ($ or ↑ or ^)	Highest	Right to Left
* , /	Next Highest	Left to Right
+, -	Lowest	Left to Right

EXPRESSION CONVERSION

- Process for conversion:

1. Find all the operators in the given Infix Expression.
2. Find the order of operators evaluated according to their Operator precedence.
3. Convert each operator into required type of expression (Postfix or Prefix) in the same order.

- ***INFIX to POSTFIX:***

  A + B * C

  ✓ Here Operators are: **+ , ***

  ✓ Order of Operators according to their preference : *** , +**

  A + BC *

  ABC*+

EXPRESSION CONVERSION

- ***INFIX to PREFIX:***

  A + B * C

  ✓ Here Operators are: **+ , ***

  ✓ Order of Operators according to their preference : *** , +**

  A + *BC

  +A*BC

  Another Example:

  (A + B) * C

  What is the postfix and prefix expression???

  Postfix: AB+C*

  Prefix: *+ABC

MORE EXAMPLES

1. A + B * C + D
2. (A + B) * (C + D)
3. P * Q + R / S
4. (J – K / L) * (M / N – O)
5. (A – B) * (C + D)
6. (P + Q) / (R + S) – (T * U)
7. 14 / 7 * 3 – 4 + 9 / 2

INFIX TO POSTFIX USING STACK

1. Scan the infix expression from left to right.
2. If the scanned character is an **operand**, output it.
3. If the scanned character is an **operator**,

   3.1 If the **precedence of the scanned operator** is **greater** than the **precedence of the operator in the stack** (or the stack is empty or the stack contains a '('), **push** it.

   3.2 Else, **Pop all the operators** from the stack which are **greater than or equal to in precedence** than that of the scanned operator. After doing that **Push the scanned operator** to the stack. (If you encounter parenthesis while popping then stop there and push the scanned operator in the stack.)
4. If the scanned character is an **'('**, **push** it to the stack.
5. If the scanned character is an **')'**, **pop** the stack and output it **until a '(' is encountered**, and discard both the parenthesis.

INFIX TO POSTFIX USING STACK

6. Repeat steps 2-5 until infix expression is scanned.
7. **Pop** from the stack **and output it** until stack is not empty.
8. Print the output.

EXAMPLE

1. A * B + C

SCANNED CHARACTER	OPERATOR STACK	POSTFIX STRING (OUTPUT)
A		A
*	*	A
B	*	A B
+	+	A B * **{POP * before pushing the +}**
C	+	A B * C
		A B * C +

EXAMPLE

2. A * (B + C)

SCANNED CHARACTER	OPERATOR STACK	POSTFIX STRING (OUTPUT)
A		A
*	*	A
(	* (	A
B	* (	A B
+	* (+	A B **{PUSH +}**
C	* (+	A B C
)	*	A B C + **{POP FROM STACK UNTIL) ENCOUNTERED}**
		A B C + *

EXAMPLE

3. A * (B + C * D) + E

SCANNED CHARACTER	OPERATOR STACK	POSTFIX STRING (OUTPUT)
A		A
*	*	A
(	* (	A
B	* (	A B
+	* (+	A B
C	* (+	A B C
*	* (+ *	A B C
D	* (+ *	A B C D
)	*	A B C D * +
+	+	A B C D * + *
E	+	A B C D * + * E
		A B C D * + * E +

MORE EXAMPLES

1. 3 + 4 * 5 / 6
2. ((A + B) — C * (D / E)) + F
3. (300 + 23) * (43 – 21) / (84 + 7)
4. x + y * z / w - v
5. (4 + 8) * (6 – 5) / ((3 – 2) * (2 + 2))

EXPRESSION EVALUATION

- EVALUATION OF INFIX EXPRESSION:

  a + b / c –d * e   where a=10, b=6, c=2, d=8, e=13

  10 + 6 / 2 – 8 * 13

  10 + 3 – 8 * 13

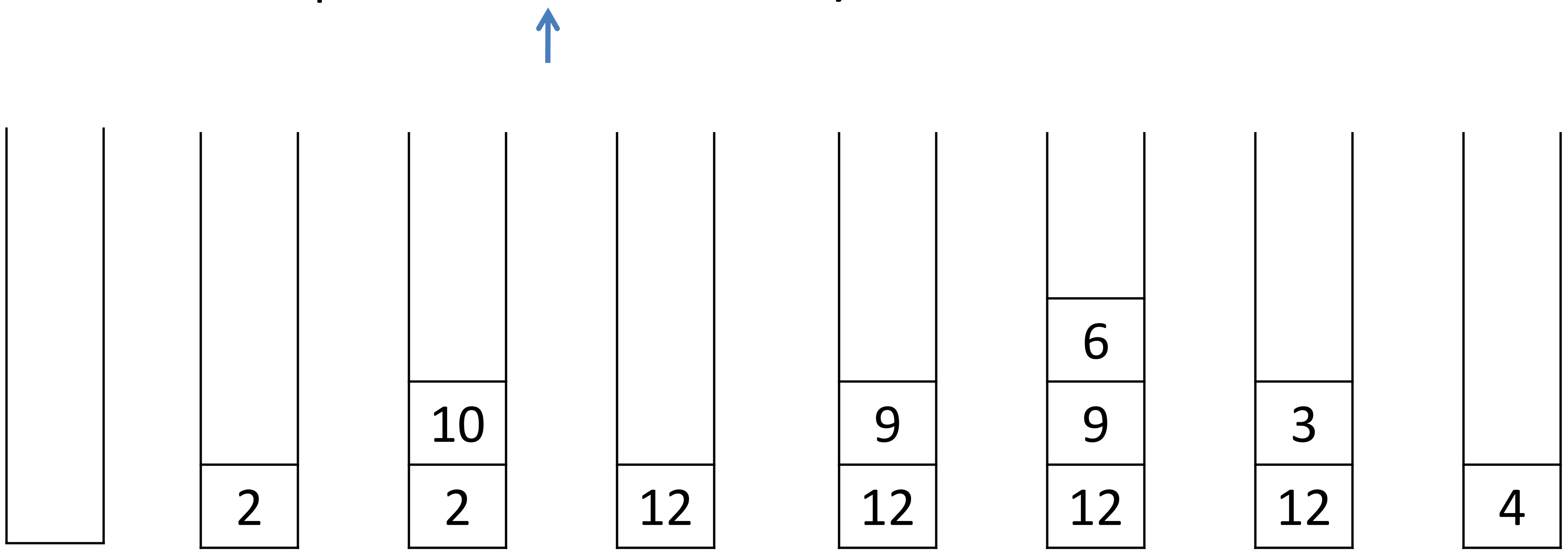
  10 + 3 - 104

  13 - 104

  - 91

POSTFIX EXPRESSION EVALUATION

- Postfix expression: 2 10 + 9 6 - /



POSTFIX EXPRESSION EVALUATION

1. Create a stack to store operands (or values) and scan the postfix expression from left to right.
2. If the scanned element is an operand, push it into the stack.
3. If the scanned element is an operator(O),
   a) Pop two operands from stack as A and B.
   b) Evaluate B O A where A is top most element and B is element below the A
   c) Push the result back to the stack.
4. When the expression is ended, the number in the stack is the final answer.

POSTFIX EXPRESSION EVALUATION

EVALUATE POSTFIX EXPRESSION: 2 7 * 18 - 6 +

SCANNED ELEMENT	STACK	CALCULATION
2	2	
7	2, 7	
*	14	2 * 7
18	14, 18	
-	-4	14 - 18
6	-4, 6	
+	2	-4 + 6

POSTFIX EXPRESSION EVALUATION

EVALUATE POSTFIX EXPRESSION: 30, 23, +, 43, 21, -, *, 6, 5, +, /

SCANNED ELEMENT	STACK	CALCULATION
30	30	
23	30, 23	
+	53	30 + 23
43	53, 43	
21	53, 43, 21	
-	53, 22	43 - 21
*	1166	53 * 22
6	1166, 6	
5	1166, 6, 5	
+	1166, 11	6 + 5
/	106	1166 / 11

RECURSION

- A recursive function is defined as a function that calls itself.
- Final call does not require to call itself.
- Recursive function makes use of stack to temporarily store the return address and local variables of the calling function.
- Every recursive solution has two major cases.

1. Base case: no further calls to function itself.

2. Recursive case: Problem is divided into subparts, function calls itself and result is obtained by combining the solutions of subparts.

RECURSION

- Example: Factorial of number (n)
- n! = n * (n–1)!
- Let us say we need to find the value of 5!

PROBLEM	SOLUTION
$5!$	$5 \times 4 \times 3 \times 2 \times 1!$
$= 5 \times 4!$	$= 5 \times 4 \times 3 \times 2 \times 1$
$= 5 \times 4 \times 3!$	$= 5 \times 4 \times 3 \times 2$
$= 5 \times 4 \times 3 \times 2!$	$= 5 \times 4 \times 6$
$= 5 \times 4 \times 3 \times 2 \times 1!$	$= 5 \times 24$
	$= 120$

RECURSION

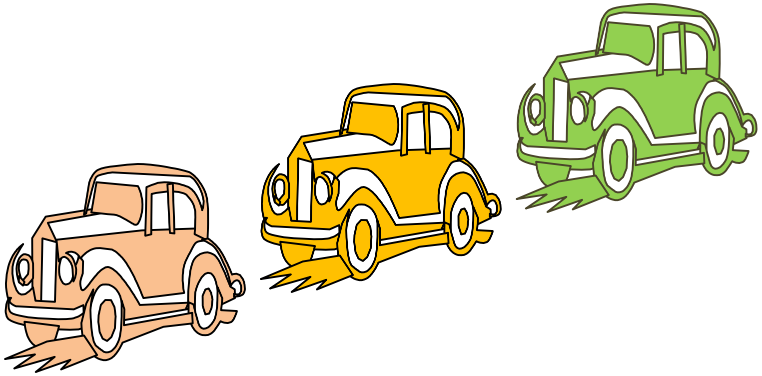
- **Base case: n = 1**, because if n = 1, the result will be 1 as 1! = 1.
- **Recursive case:** factorial function will call itself but with a smaller value of n, i.e.

  factorial(n) = n × factorial (n–1)

QUEUE

QUEUE

- Linear Data Structure
- ADT i.e. Abstract Data Type
- Unlike stack, open at both end.
- One end is always used to insert data and another end is used to delete data.
- Queue is collection of elements that follows the FIFO order. **FIFO stands for First In First Out**



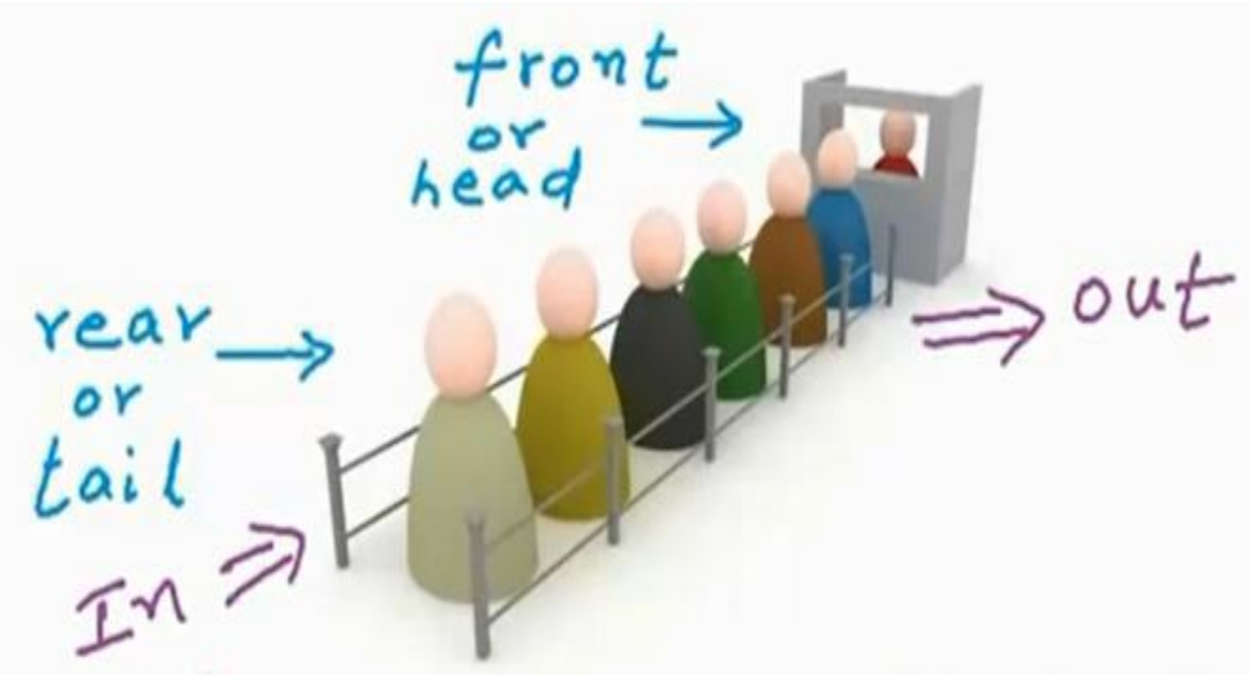
QUEUE

- Examples:
  1. Customers’ line at a grocery store
  2. The cars at a stop light
  3. People moving on an escalator
  4. People waiting for a bus
  5. Queue of people at ticket window
  6. Luggage kept on conveyor belts
  7. Patients waiting outside the doctor’s clinic

QUEUE

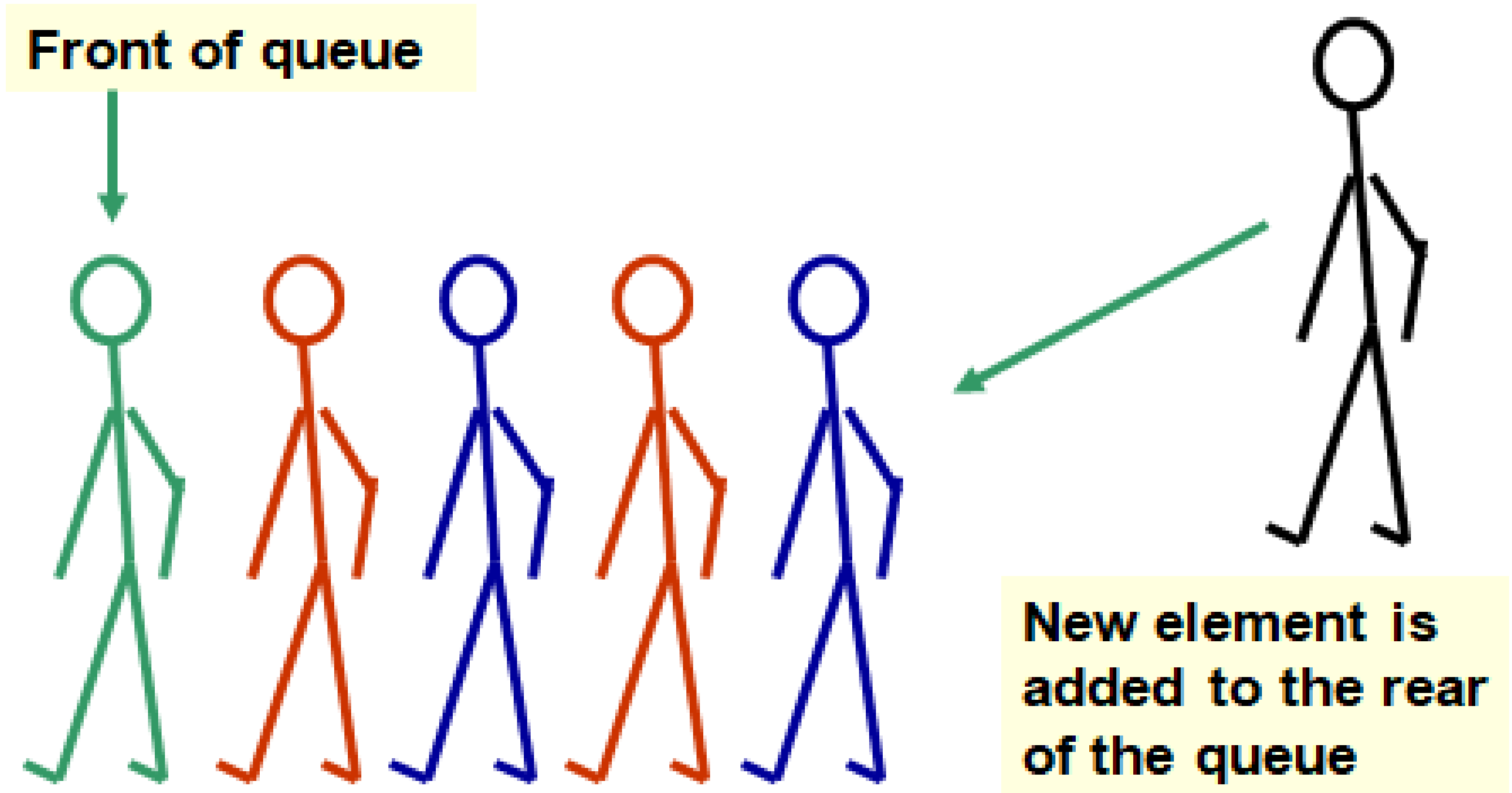
- The end from where element is inserted is called **rear**.
- The end from where element is deleted is called **front**.

- Insert-> Enqueue
- Delete -> Dequeue



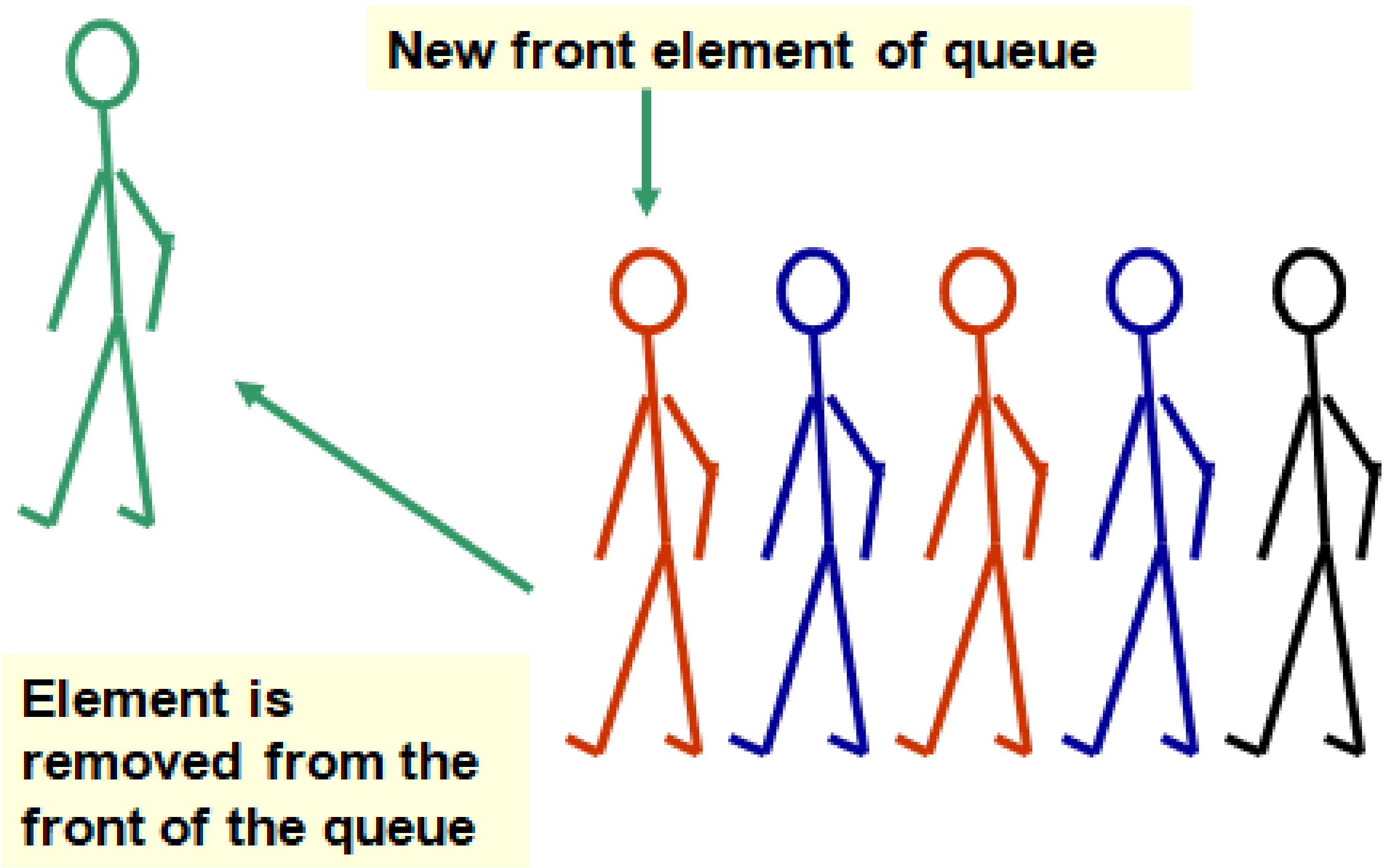
ENQUEUE OPERATION

Adding an element



DEQUEUE OPERATION

Removing an element



OPERATIONS ON QUEUE

FRONT = -1

REAR = -1

ENQUEUE(10)

10				

FRONT = 0
REAR = 0

ENQUEUE(20)

10	20			

FRONT = 0 REAR = 1

ENQUEUE(30)

10	20	30		

FRONT = 0 **REAR = 2**

OPERATIONS ON QUEUE

ENQUEUE(40)

10	20	30	40	

FRONT = 0 REAR = 3

ENQUEUE(50)

10	20	30	40	50

FRONT = 0 REAR = 4

DEQUEUE

	20	30	40	50

FRONT = 1 REAR = 4

DEQUEUE

		30	40	50

FRONT = 2 REAR = 4

OPERATIONS ON QUEUE

DEQUEUE

			40	50

FRONT = 3 REAR = 4

DEQUEUE

				50

FRONT = 4

REAR = 4

DEQUEUE

FRONT = -1

REAR = -1

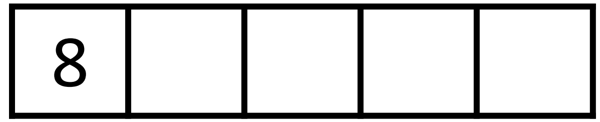
EXAMPLE

Describe the output of the following series of queue operations

- enqueue(8)
- enqueue(3)
- dequeue()
- enqueue(2)
- enqueue(5)
- dequeue()
- dequeue()
- enqueue(9)
- enqueue(1)

EXAMPLE

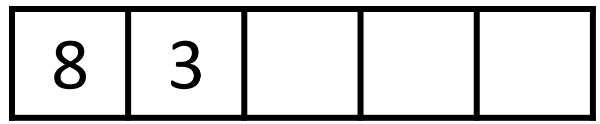
- enqueue(8)



FRONT = 0

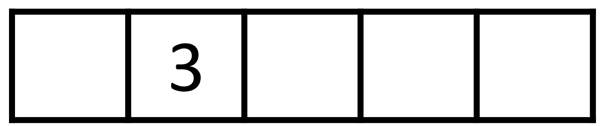
REAR = 0

- enqueue(3)



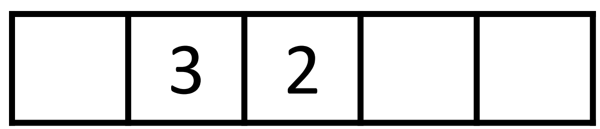
FRONT = 0 REAR = 1

- dequeue()



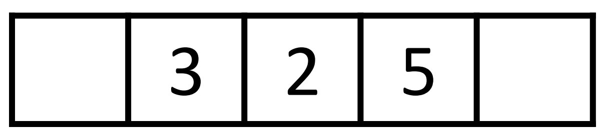
REAR = 1
FRONT = 1

- enqueue(2)



FRONT = 1 REAR = 2

- enqueue(5)



FRONT = 1 REAR = 3

EXAMPLE

- dequeue()

		2	5	

FRONT = 2 REAR = 3

- dequeue()

			5	

REAR = 3
FRONT = 3

- enqueue(9)

			5	9

FRONT = 3 REAR = 4

- enqueue(1)

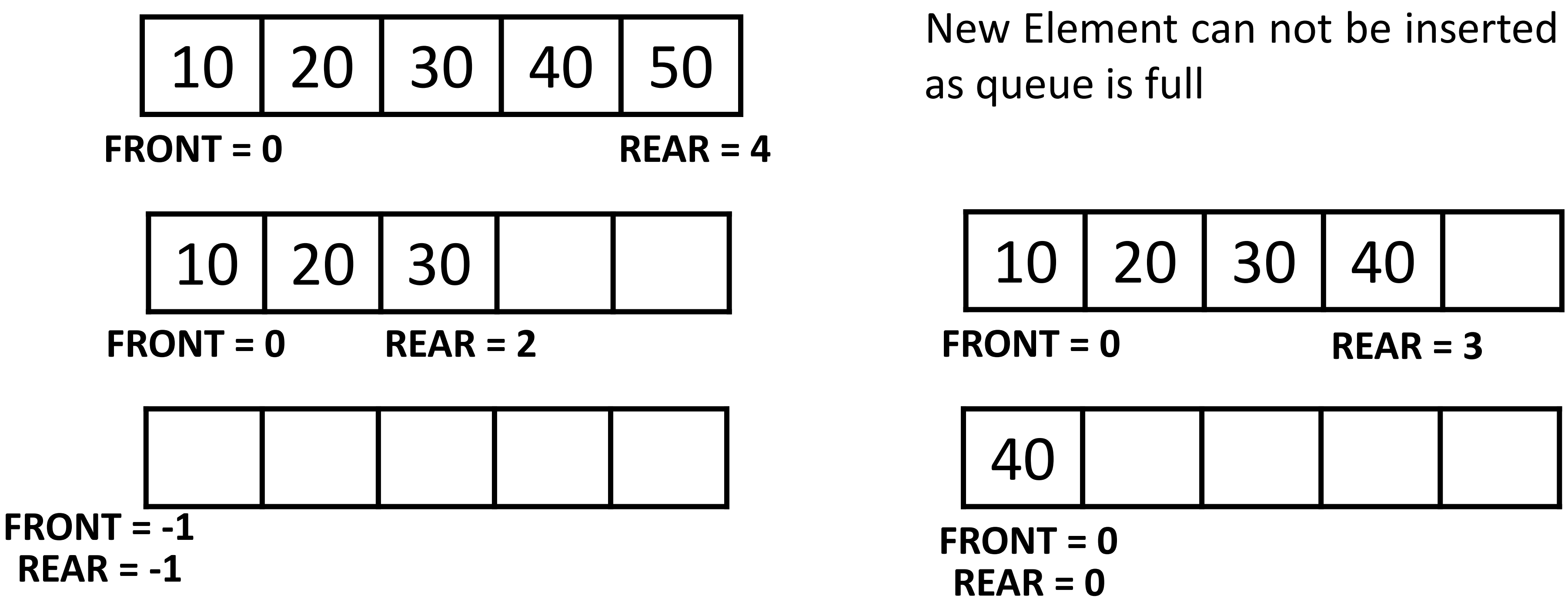
1 CAN NOT BE INSERTED.

ARRAY REPRESENTATION OF QUEUE

- Requirements:
- ✓ Array Q[MAX], where MAX is the size of the array.
- ✓ Two variables FRONT and REAR are used.
- ✓ REAR is used to point to the rear end of the queue. FRONT is used to point to the front end of the queue.
- ✓ Initial value of FRONT and REAR is -1 which indicates that queue is empty.
- ✓ REAR = MAX-1 indicates that queue is full.

ENQUEUE OPERATION

- ENQUEUE(40)



✓ Check for Queue Overflow (i.e. REAR=MAX-1).

✓ Increment value of REAR by 1.

✓ Store value at the position pointed by REAR.

✓ Increment value of front if front is set to -1.

ALGORITHM FOR ENQUEUE OPERATION

Algorithm:

ENQUEUE(VAL): This algorithm inserts an element to rear end of the queue.

- Q is queue which contains MAX elements.
- VAL is the element to be inserted in queue.
- REAR and FRONT are pointers which point to rear and front elements of the queue respectively.

Step-1) [Overflow?]

If REAR >= MAX-1

Then Write "Queue Overflow"

Exit

Step-2) [Increment rear pointer]

REAR = REAR + 1

Step-3) [Insert element]

Q[REAR] = VAL

Step-4) [Is front pointer properly set?]

If FRONT = -1

Then FRONT = 0

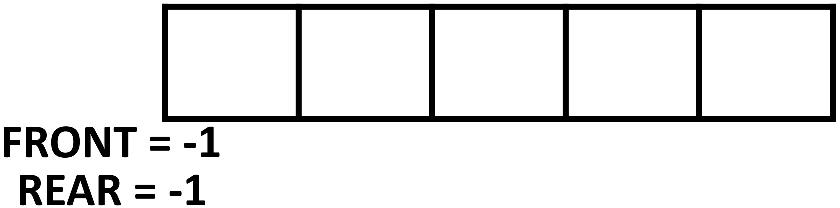
Return

Step-5) [Finished]

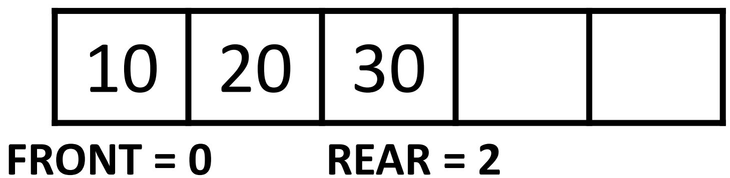
Exit

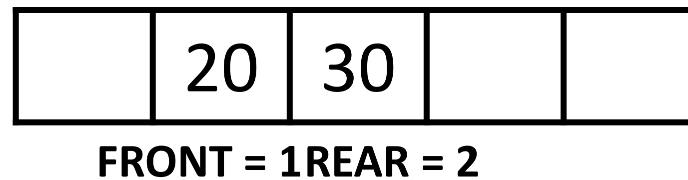
DEQUEUE OPERATION

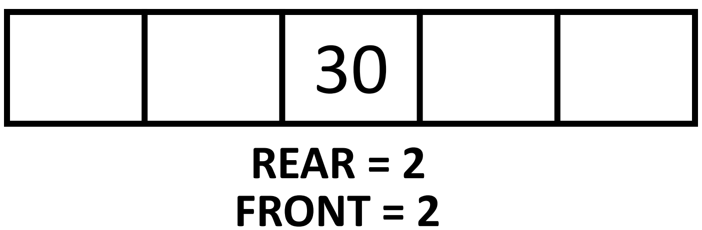
- DEQUEUE():

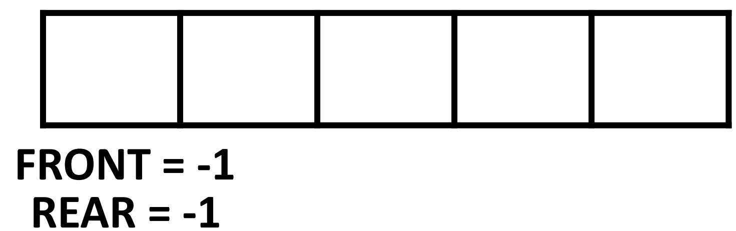


Element can not be deleted as queue is empty.









- ✓ Check for Queue Underflow (i.e. FRONT=REAR=-1).
- ✓ Access the front element that is to be deleted.
- ✓ Increment value of FRONT by 1.
- ✓ Set FRONT and REAR to -1, if queue contains single element.

ALGORITHM FOR DEQUEUE OPERATION

Algorithm:

DEQUEUE(): This algorithm deletes an element from front end of the queue.

- Q is queue which contains MAX elements.
- REAR and FRONT are pointers which point to rear and front elements of the queue respectively.

Step-1) [Check Queue Underflow]
If FRONT = -1
Then Write ('Queue Underflow')
Exit

Step-2) [Delete element]
VAL = Q [FRONT]

Step-3) [Check For Queue empty]
If FRONT = REAR
Then FRONT = REAR = -1
Else FRONT = FRONT + 1

Step-4) [Return element]
return (VAL)

LIMITATION OF SIMPLE QUEUE

- Consider following queue.

21	4	98	60	19

FRONT = 0 **REAR = 4**

- Now, enqueue operation can not be performed as queue is full.
- Performing dequeue operation 2 times to make some empty space in queue.

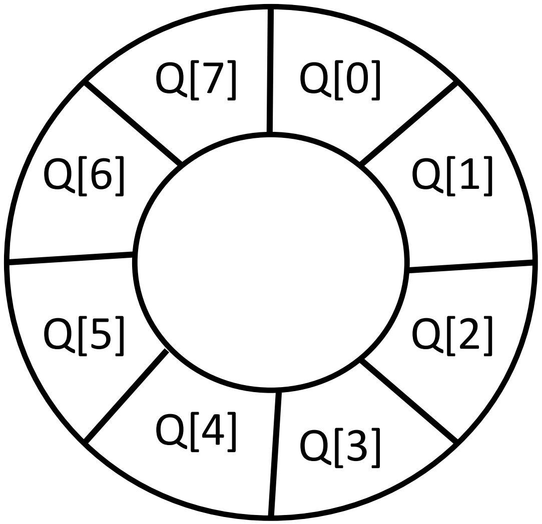
		98	60	19

FRONT = 0 **REAR = 4**

- Suppose we want to enqueue new element to queue. But it is not possible as overflow condition(i.e. REAR = MAX-1) still holds true.
- So even if empty space available, we can not insert new element.

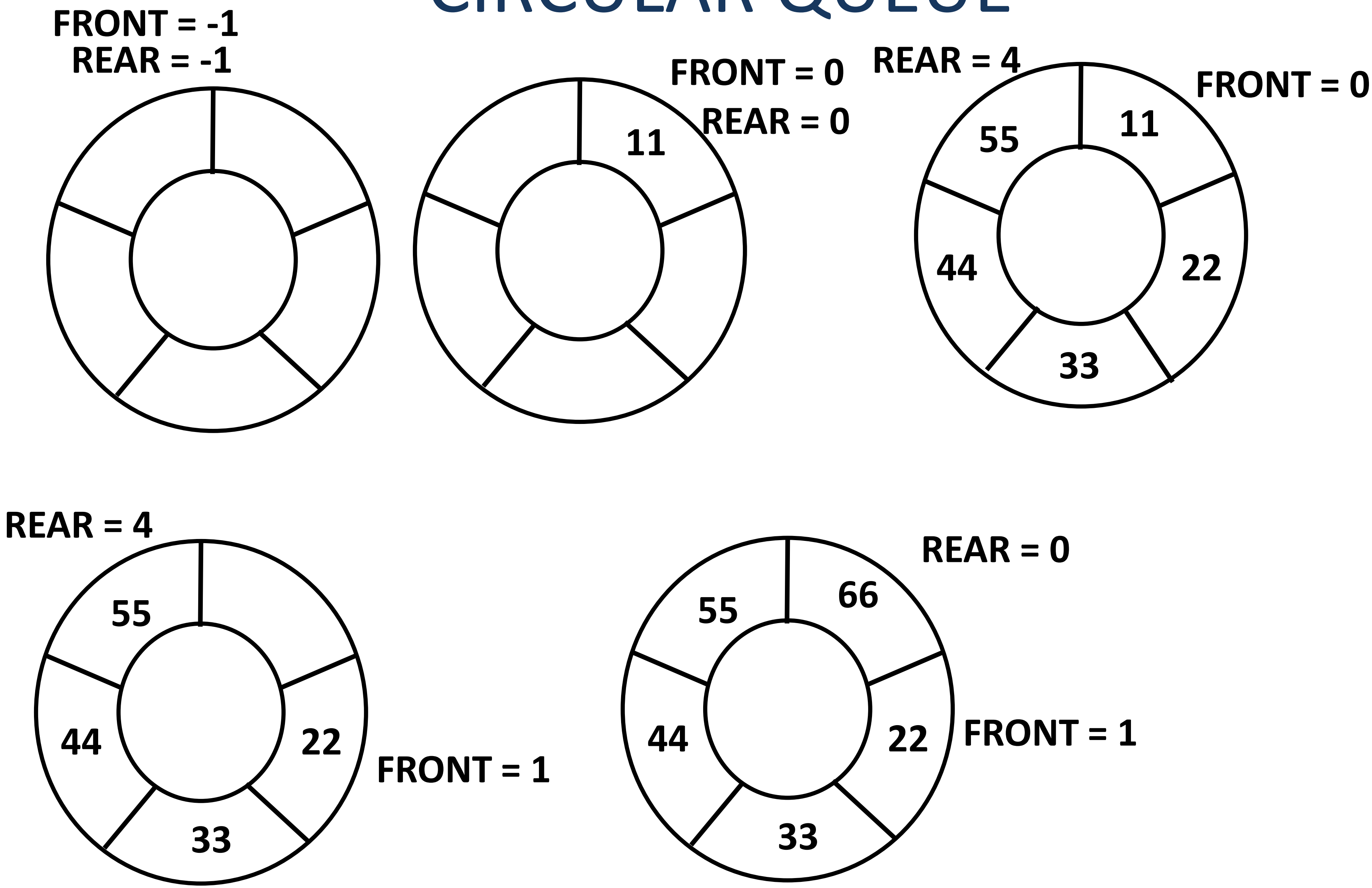
CIRCULAR QUEUE

- Solution for problem occurred in linear queue is to use circular queue.
- In the circular queue, the first index comes right after the last index. Conceptually, circular queue looks like



- The circular queue will be full only when FRONT = 0 and REAR = MAX – 1.

CIRCULAR QUEUE



ENQUEUE OPERATION IN CIRCULAR QUEUE

- ENQUEUE(50)

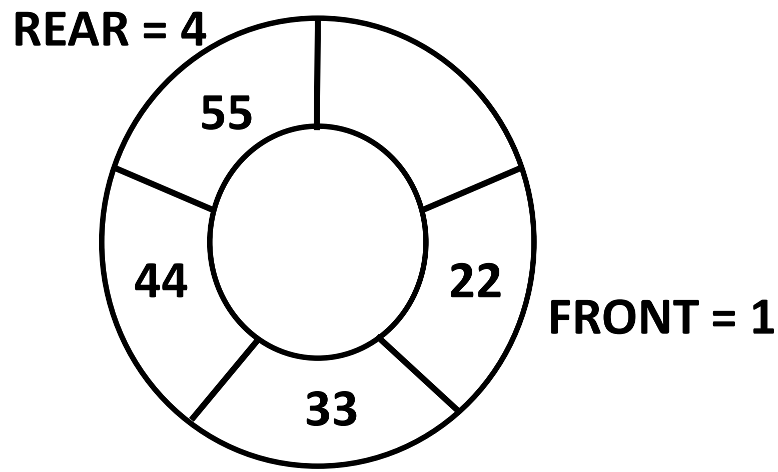
1. Queue is full.

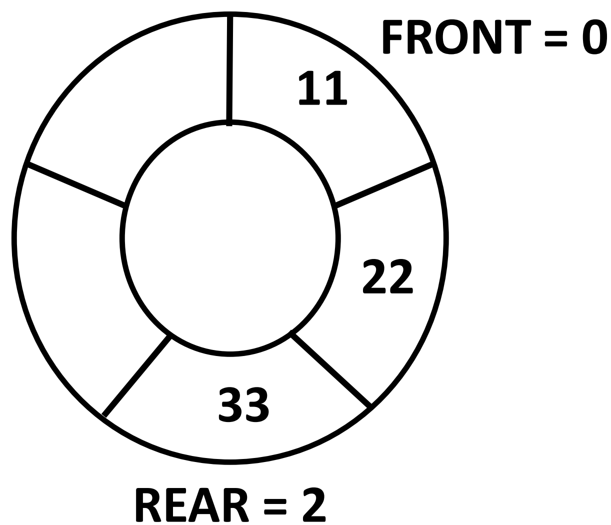
✓ Condition: FRONT = 0 & REAR = MAX-1
OR FRONT = REAR + 1

2. If Queue is not full then do circular increment of rear pointer
   REAR = (REAR + 1) % MAX

   Let MAX = 5
   if REAR = 4 , REAR = (4+1)%5 = 5%5 = 0
   Suppose REAR = 2, REAR = (2+1)%5 = 3%5 =3

3. Add new element in circular queue
4. For first element, set front pointer to 0.





ALGORITHM FOR ENQUEUE OPERATION

Algorithm:

ENQUEUE(VAL): This algorithm inserts an element in to circular queue.

- Q is queue which contains MAX elements.
- REAR and FRONT are pointers which point to rear and front elements of the queue respectively.
- VAL is the value to be inserted.

Step-1) [Check Queue Overflow]
If (FRONT = 0 and REAR =MAX-1) or FRONT = REAR + 1
Then Write ('Queue Overflow')
Exit

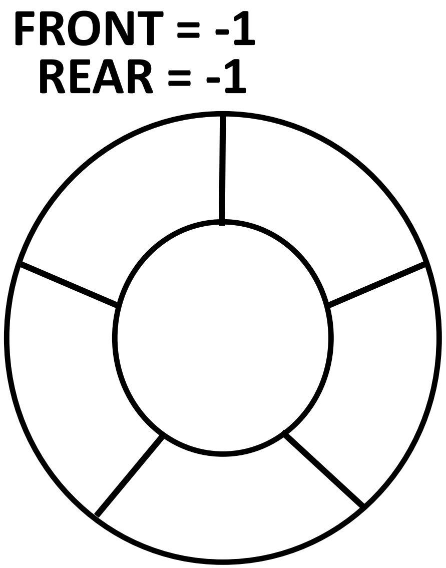
Step-2) [Reset rear pointer]
REAR = (REAR + 1) % MAX

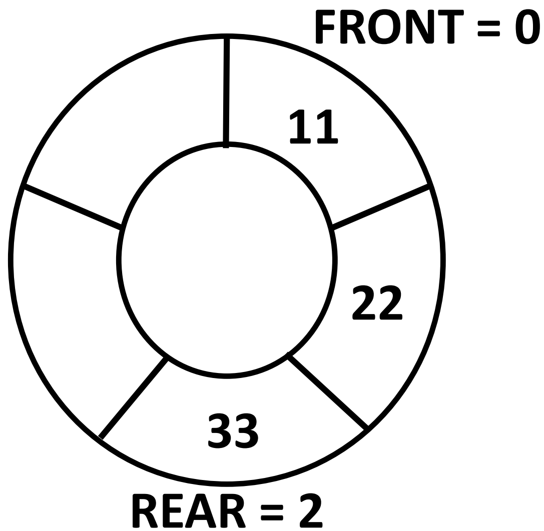
Step-3) [Insert new element]
Q[REAR] = VAL

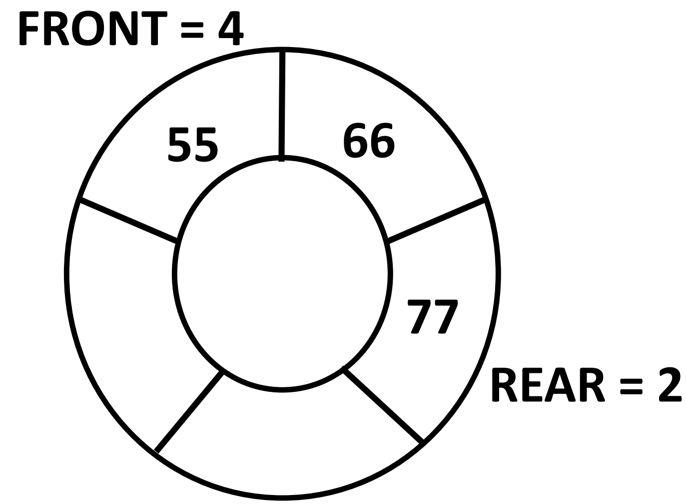
Step-4) [Is front pointer properly set?]
if FRONT = -1
Then FRONT = 0

Step-5) [Finished]
Exit

DEQUEUE OPERATION ON CIRCULAR QUEUE





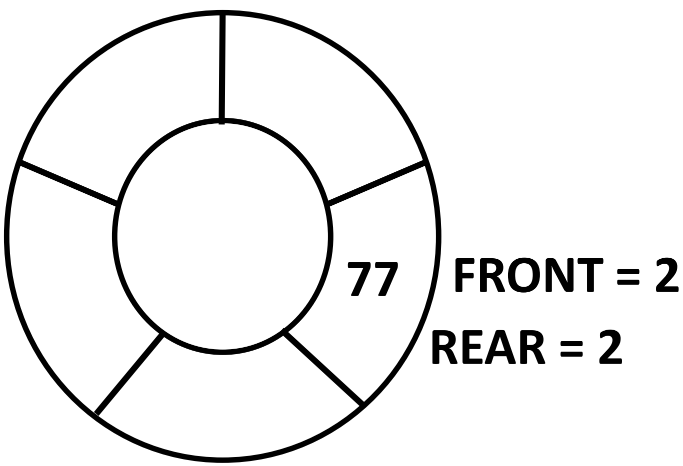


1. Underflow

✓ Condition: If FRONT = -1

2. Access the value that is to be deleted
3. Circular increment front pointer

✓ (FRONT = (FRONT +1) % MAX)



4. For last element, set FRONT and REAR to -1

✓ Condition: REAR = FRONT

ALGORITHM FOR DEQUEUE OPERATION

Algorithm:

DEQUEUE(): This algorithm deletes an element from circular queue.

- Q is queue which contains MAX elements.
- REAR and FRONT are pointers which point to rear and front elements of the queue respectively.

Step-1) [Check Queue Underflow]
If (FRONT = -1)
Then Write ('Queue Underflow')
Return -1

Step-2) [Delete element]
VAL = Q[FRONT]

Step-3) [Check For Queue empty]
If FRONT = REAR
Then FRONT = REAR = -1
Else FRONT = (FRONT + 1) % MAX

Step-4) [Return element]
return (VAL)

THANK YOU