CSC 349a Assignment #3

Parm Johal
V00787710

① $f(x) = 1 - \sin(x)$ (Note: $f'(x) = -\cos(x)$)

a) For values of x near $\pi/2$, we evaluate $\tilde{x} = 1.56$ with the condition number, which is defined as

$$\frac{\tilde{x} \cdot f'(\tilde{x})}{f(\tilde{x})} \rightarrow = \frac{(1.56)(-\cos(1.56))}{(1-\sin(1.56))} \cong -288.98.$$

Since $\left|\text{condition number}\right| \not< 1$, we conclude that $f(x)$ is ill-conditioned for values of x close to $\pi/2$.

For values of x near 0.5, we evaluate $\tilde{x} = 0.51$.

$$\frac{\tilde{x} \cdot f'(\tilde{x})}{f(\tilde{x})} = \frac{(0.51) \cdot (-\cos(0.51))}{(1-\sin(0.51))} \cong -0.8696.$$

Since $\left|\text{condition number}\right| < 1$, we conclude that $f(x)$ is well conditioned for values of x close to 0.5.

b) With relative error for both $\tilde{x} = 1.56$ & $\tilde{x} = 0.51$, we have

$$|\varepsilon_t| = \left|\frac{f(x) - f(\tilde{x})}{f(x)}\right| = \left|\frac{(1-\sin(\pi/2)) - (1-\sin(1.56))}{1-\sin(\pi/2)}\right| =$$

$=$ undefined

$$|\varepsilon_t| = \left|\frac{(1-\sin(0.5)) - (1-\sin(0.51))}{(1-\sin(0.50))}\right| = 0.016811635\ldots \approx 1.68\%$$

Both the relative error and condition number seem related in that the bigger the evaluated input to output, the more ill-conditioned it will be. In this case, if the condition number's value is less than 1, then $f(x)$ is well-conditioned, otherwise it is ill-conditioned.

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(2) $g(x) = \begin{cases} \frac{(1+\cos x)}{(x-\pi)^2} & x \neq \pi \\ 0.5 & x = \pi \end{cases}$

a) With $\cos x \approx -1 + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}$, we have
(when $x \neq \pi$)

$$g(x) = \frac{1 + \left(-1 + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}\right)}{(x-\pi)^2} \Leftrightarrow$$

$$\Leftrightarrow \boxed{\frac{1}{2} - \frac{(x-\pi)^2}{24}}$$

b) note: $g(x) = \frac{1}{2} - \frac{(x-\pi)^2}{24}$ when $x \neq \pi$.

and $fl(g(3.16)) = 0.5540$ w $|\varepsilon_t| \approx 10.8\%$.

Analyzing the stability of $fl(g(3.16))$:

given problem $\Longrightarrow$ 0.5540
$x = 3.16$

perturbed problem
$\tilde{x} = 3.16 + \varepsilon$ $\Longrightarrow$ $g(3.16+\varepsilon) = \frac{1}{2} - \frac{(3.16+\varepsilon-\pi)^2}{24}$
($\left|\frac{\varepsilon}{3.16}\right|$ is small)

if $\left|\frac{\varepsilon}{3.16}\right|$ is small, then

$$\frac{1}{2} - \frac{(3.16+\varepsilon-\pi)^2}{24} \approx \frac{1}{2} - \frac{(3.16-\pi)^2}{24}$$

≈ 0.499985 for all small values of ε.

$\therefore$ There are no small values for ε such that $g(3.16+\varepsilon)$ is close to 0.5540.
The computation is unstable.

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2 (continued)

c) note: the correct value of $g(1.41) = 0.386906095$

Analysis of stability

data

$x = 1.41 \implies fl(g(1.41)) = 0.3871$

perturbed data

$\tilde{x} = 1.41 + \varepsilon$, where $\left|\frac{\varepsilon}{1.41}\right|$ is 'small' $\implies g(1.41+\varepsilon) = \frac{1}{2} - \frac{(1.41+\varepsilon-\pi)^2}{24}$

with $|\varepsilon/1.41|$ 'small',

$\frac{1}{2} - \frac{(1.41+\varepsilon-\pi)^2}{24} \approx \frac{1}{2} - \frac{(1.41-\pi)^2}{24}$

≈ 0.3869 for all small values of ε.

$\therefore$ there exists a small value for ε such that $g(1.41+\varepsilon)$ is ≈ 0.3871.

3 $R(x) = x^4 - 6x^3 + 13.5x^2 - 13.5x + 5.06249999$
$= (x-1.5)^4 - 10^{-8}$

a) To find the zeros of $R(x)$, we equate $R(x)$ to zero.

$$R(x) = (x-1.5)^4 - 10^{-8} = 0$$
$$(x-1.5)^4 = 10^{-8}$$
$$(x-1.5) = \pm 0.01$$
$$x = 1.49, 1.51 \rightarrow \text{real roots}$$

$$\boxed{(x-1.5)^2 = -0.0001 = x^2 - 3x + 2.25} \rightarrow x = \frac{3 \pm 0.02i}{2}$$

$$x^2 - 3x + 2.2501 = 0$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(2.2501)}}{2(1)}$$

$$\boxed{x = 1.5 + 0.01i,\ 1.5 - 0.01i}$$

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3 b)

Relative change in $P(x) \to R(x)$:

$= (5.0625 - 5.062499999)/5.062499999 \cong 1.975 \times 10^{-9}$

Relative change in output (zeros) using $x = 1.51$ as a root:

$$\frac{1.51 - 1.50}{1.51} = 0.006622516\ldots$$

relative change in $P(x) \to R(x) \approx \underbrace{\left(\frac{\tilde{x} f'(\tilde{x})}{f(\tilde{x})}\right)}_{\text{condition number}} \cdot$ relative change in zeros

$$\therefore \text{condition } \# \approx \frac{0.006622516}{1.975 \times 10^{-9}} \approx 3.353 \times 10^{6}$$

Since condition number $>> 1$, we conclude that the problem of computing the zeros of $P(x)$ is ill-conditioned.