

CSC 349a: Assignment #5

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augmented matrix

$$\begin{cases} 2x_1 + x_2 - x_3 = 1 \\ 5x_1 + 2x_2 + 2x_3 = -4 \\ 3x_1 + x_2 + x_3 = 5 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 5 & 2 & 2 & -4 \\ 3 & 1 & 1 & 5 \end{array} \right]$$

Forward Elimination

$$m_{21} = \frac{a_{21}}{a_{11}}, \quad m_{31} = \frac{a_{31}}{a_{11}} \quad \left\{ \begin{array}{l} E_2^* = E_2 - m_{21}E_1 \\ E_3^* = E_3 - m_{31}E_1 \end{array} \right.$$

$$= \frac{5}{2}, \quad = \frac{3}{2}$$

From this we obtain

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -\frac{1}{2} & \frac{9}{2} & -\frac{13}{2} \\ 0 & -\frac{1}{2} & \frac{5}{2} & \frac{7}{2} \end{array} \right]$$

$$m_{32} = \frac{a_{32}}{a_{22}} = 1 \quad \left\{ \begin{array}{l} E_3^{**} = E_3^* - m_{32}E_2^* \end{array} \right. \quad \text{we get} \quad \left[\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -\frac{1}{2} & \frac{9}{2} & -\frac{13}{2} \\ 0 & 0 & -2 & 10 \end{array} \right]$$

upper triangular form

back-substitution

starting from E_3^* , $-2x_3 = 10$
 $x_3 = -5$

(ii) $-\frac{1}{2}x_2 + \frac{9}{2}x_3 = -\frac{13}{2}$
 $x_2 - 9x_3 = 13$
 $x_2 = 13 + 9(-5)$
 $x_2 = -32$

(iii) $2x_1 + x_2 - x_3 = 1$
 $x_1 = \frac{1 + x_3 - x_2}{2}$
 $x_1 = 14$

$\therefore \hat{X} = \begin{bmatrix} 14 \\ -32 \\ -5 \end{bmatrix}$

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①

$$b) \begin{bmatrix} 2 & 1 & -1 & 1 \\ 5 & 2 & 2 & -4 \\ 3 & 1 & 1 & 5 \end{bmatrix}$$

Augmented Matrix

Using Gaussian Elimination With
Partial Pivoting:

Step 1

 $k=1$ the row index of the pivot is $p=2$, so
rows 1 and 2 are interchanged:

$$m_{21} = 2/5$$

$$m_{31} = 3/5$$

with elimination:

$$\begin{bmatrix} 5 & 2 & 2 & -4 \\ 2 & 1 & -1 & 1 \\ 3 & 1 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 & 2 & -4 \\ 0 & 1/5 & -9/5 & 13/5 \\ 0 & -1/5 & -1/5 & 37/5 \end{bmatrix}$$

Step 2

 $k=2$ the row index of the pivot is $p=2$, Since $p=k$
no interchange occurs between rows. Continuing
with elimination:

$$m_{32} = -1$$

$$\begin{bmatrix} 5 & 2 & 2 & -4 \\ 0 & 1/5 & -9/5 & 13/5 \\ 0 & 0 & -2 & 10 \end{bmatrix}$$

Using back-substitution:

$$i) -2x_3 = 10$$

$$x_3 = -5$$

$$ii) \frac{1}{5}x_2 - \frac{9}{5}x_3 = \frac{13}{5}$$

$$x_2 - 9x_3 = 13$$

$$x_2 = 13 + 9(-5)$$

$$x_2 = -32$$

$$iii) 5x_1 + 2x_2 + 2x_3 = -4$$

$$x_1 = \frac{-4 - 2x_2 - 2x_3}{5}$$

$$x_1 = 14$$

$$\therefore \mathbf{x} = \begin{bmatrix} 14 \\ -32 \\ -5 \end{bmatrix}$$

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- ① c) Using the upper triangular form from part a) and knowing that
- $$\det A = a_{11}^{(1)} a_{22}^{(2)} a_{33}^{(2)}$$
- we get
- $$\det A = (2) \cdot (-\frac{1}{2}) \cdot (-2) = 2$$

- ② The algorithm for Forward Substitution:
- a)
- ```

1 $X_1 \leftarrow b_1/a_{11}$
2 for $i = 2, 3, \dots, n$
3 $\text{Sum} \leftarrow b_i$
4 for $j = i-1, i-2, \dots, 1$
5 $\text{Sum} \leftarrow \text{Sum} - a_{ij} * X_j$
6 end for
7 $X_i \leftarrow \text{Sum}/a_{ii}$
8 end for

```

- b)
- At line 1, we see 1 division occurring.
  - At line 7, this division occurs  $n-1$  times in the outer loop.
  - At line 5:
    - 1 subtraction & 1 division occur  $j$  times.
    - $j$  iterations depends on outer loop ( $i$ th) iteration.

So, looking at this closer:

$$i=2 \rightarrow j=1 \text{ iterations}$$

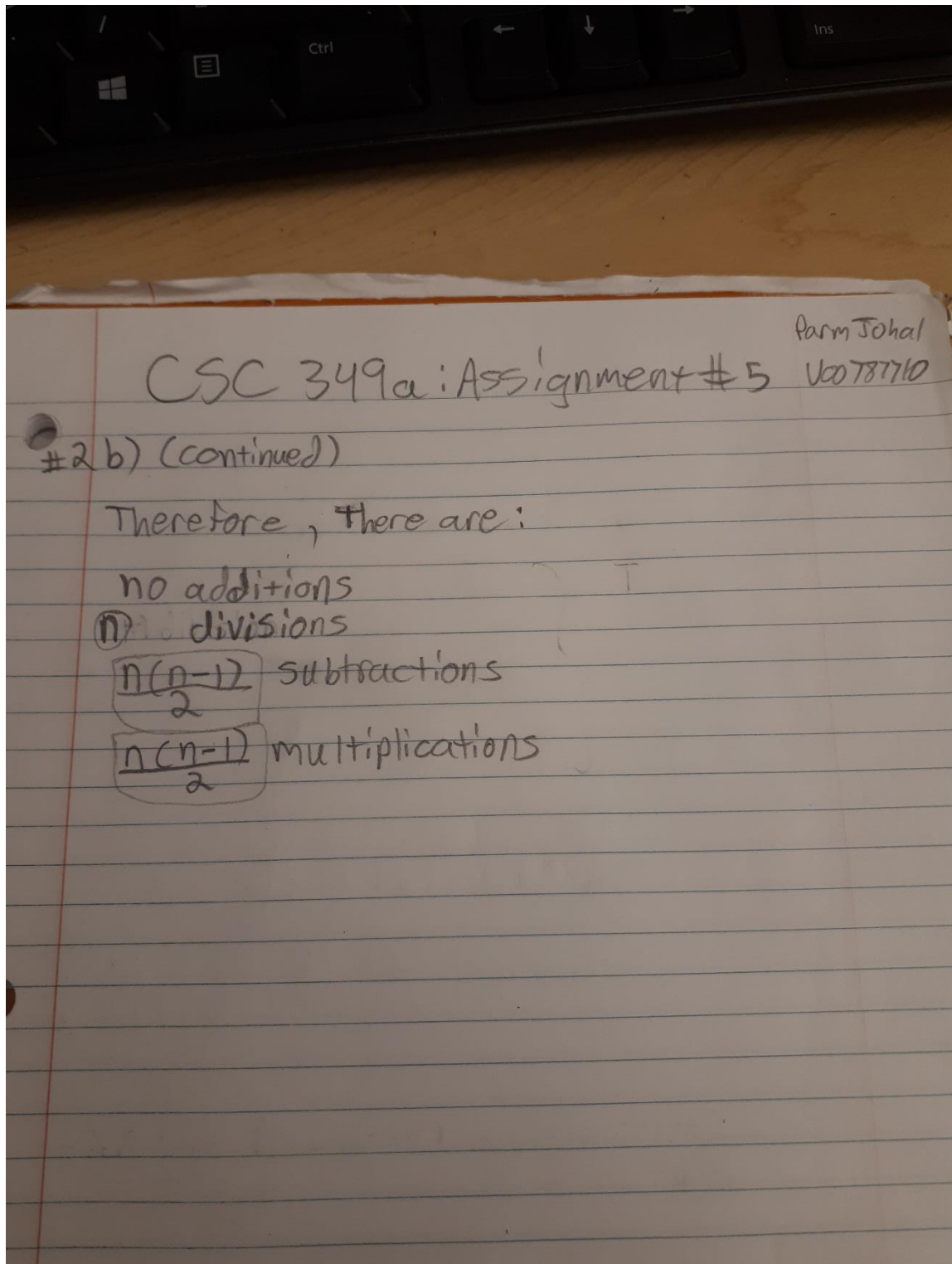
$$i=3 \rightarrow j=2 \text{ iterations}$$

$$\vdots$$

$$i=n \rightarrow j=n-1 \text{ iterations}$$

we then get  $\sum_{i=2}^n (i-1) = 1+2+3+\dots+(n-2)+(n-1) = \frac{n(n-1)}{2}$

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#2 c)

```

function [x] = ForwardSub(A, b)
%Algorithm for forward substitution, A is a lower
%triangular matrix and b is a column vector.
 %First calculate the number of columns n in b
 n = length(b);
 %Empty vector x
 x = [];
 %Start by solving trivial first step and
 %storing it into x(1)
 x(1) = b(1)/A(1,1);
 fprintf ('i-val: %2.0f \t x(i): %2.8f \n\n', 1, x(1));
 %Outer loop iterates through each row
 for i = 2 : n
 fprintf ('i-val: %2.0f \n', i);
 %Make sum equal to the row's corresponding
 %b value
 sum = b(i);
 %Inner loop iterates through each nonzero
 %value of the ith row and multiplies it
 %with the solved x value from previous
 %row
 for j = i-1 :-1: 1
 sum = sum - A(i,j) * x(j);
 fprintf ('j = %2.0f \t sum = %2.8f \n', j, sum);
 end
 x(i) = sum/A(i,i);
 fprintf ('sum: %2.8f \t A(i,i): %2.8f \t x(i): %2.8f \n', sum,
A(i,i), x(i));
 fprintf('\n');
 end
 fprintf('Vector x =\n')
 fprintf('%i\n', x)
end

```



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output:

```
>> ForwardSub(A,b);
```

```
i-val: 1 x(i): 1.00000000
```

```
i-val: 2
```

```
j = 1 sum = 3.00000000
```

```
sum: 3.00000000 A(i,i): 3.00000000 x(i): 1.00000000
```

```
i-val: 3
```

```
j = 2 sum = 10.00000000
```

```
j = 1 sum = 6.00000000
```

```
sum: 6.00000000 A(i,i): 6.00000000 x(i): 1.00000000
```

```
i-val: 4
```

```
j = 3 sum = 25.00000000
```

```
j = 2 sum = 17.00000000
```

```
j = 1 sum = 10.00000000
```

```
sum: 10.00000000 A(i,i): 10.00000000 x(i): 1.00000000
```

Vector x =

1

1

1

1

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V00787710(3) a)  $f(x) = \cos(x)$  w. Interval  $[0, 8]$  &

points:

|       | x | f(x)    |
|-------|---|---------|
| $x_0$ | 1 | 0.5403  |
| $x_1$ | 2 | -0.4161 |
| $x_2$ | 6 | 0.9602  |

Using  $P(x) = L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2)$ ;

$$L_0(x) = \prod_{j=0, j \neq 0}^2 \frac{x-x_j}{x_0-x_j} = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-2)(x-6)}{(-1)(-5)}$$

$$L_1(x) = \prod_{j=0, j \neq 1}^2 \frac{x-x_j}{x_1-x_j} = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-1)(x-6)}{(1)(-4)}$$

$$L_2(x) = \prod_{j=0, j \neq 2}^2 \frac{x-x_j}{x_2-x_j} = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-1)(x-2)}{(5)(4)}$$

So then we have:

$$P(x) = \frac{1}{5}(x-2)(x-6)(0.5403) - \frac{1}{4}(x-1)(x-6)(-0.4161) \oplus$$

$$\oplus \frac{1}{20}(x-1)(x-2)(0.9602)$$

$$= 0.10806 \cdot (x^2 - 8x + 12) + 0.104025(x^2 - 7x + 6) \oplus$$

$$\oplus 0.04801(x^2 - 3x + 2)$$

$$\star P(x) = 0.260095x^2 - 1.736685x + 2.01689 \star$$

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#3 b)

&gt;&gt; x = 0:0.1:8;

&gt;&gt; y = cos(x);

&gt;&gt; plot(x, y);

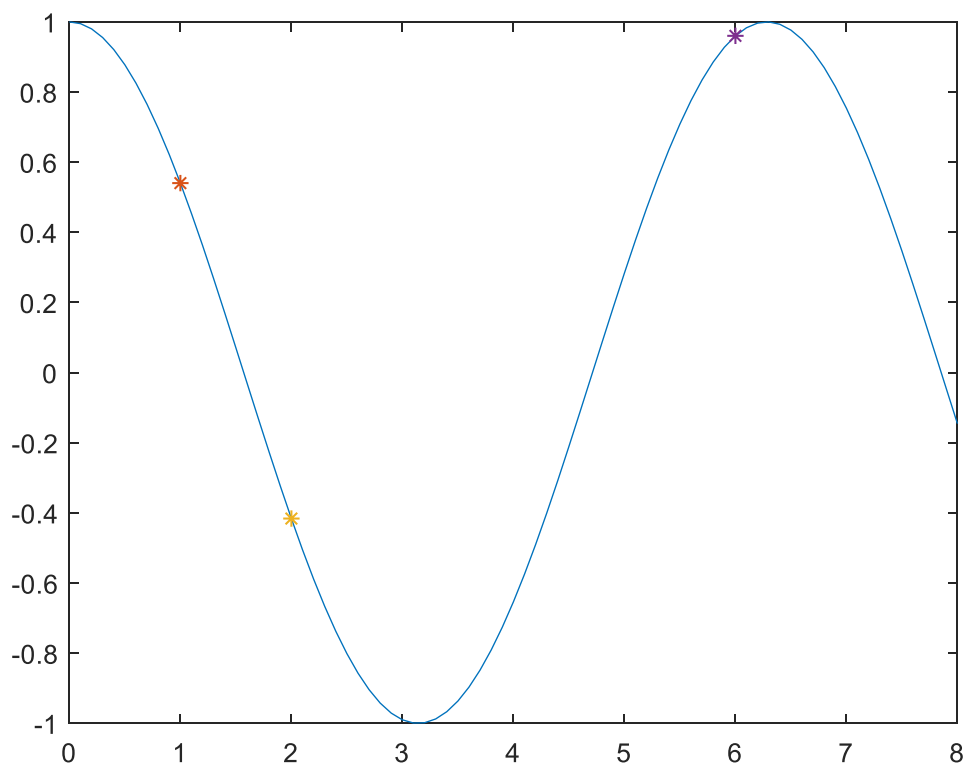
&gt;&gt; hold on

&gt;&gt; plot(1, 0.5403, '\*')

&gt;&gt; plot(2, -0.4161, '\*')

&gt;&gt; plot(6, 0.9602, '\*')

&gt;&gt; hold off





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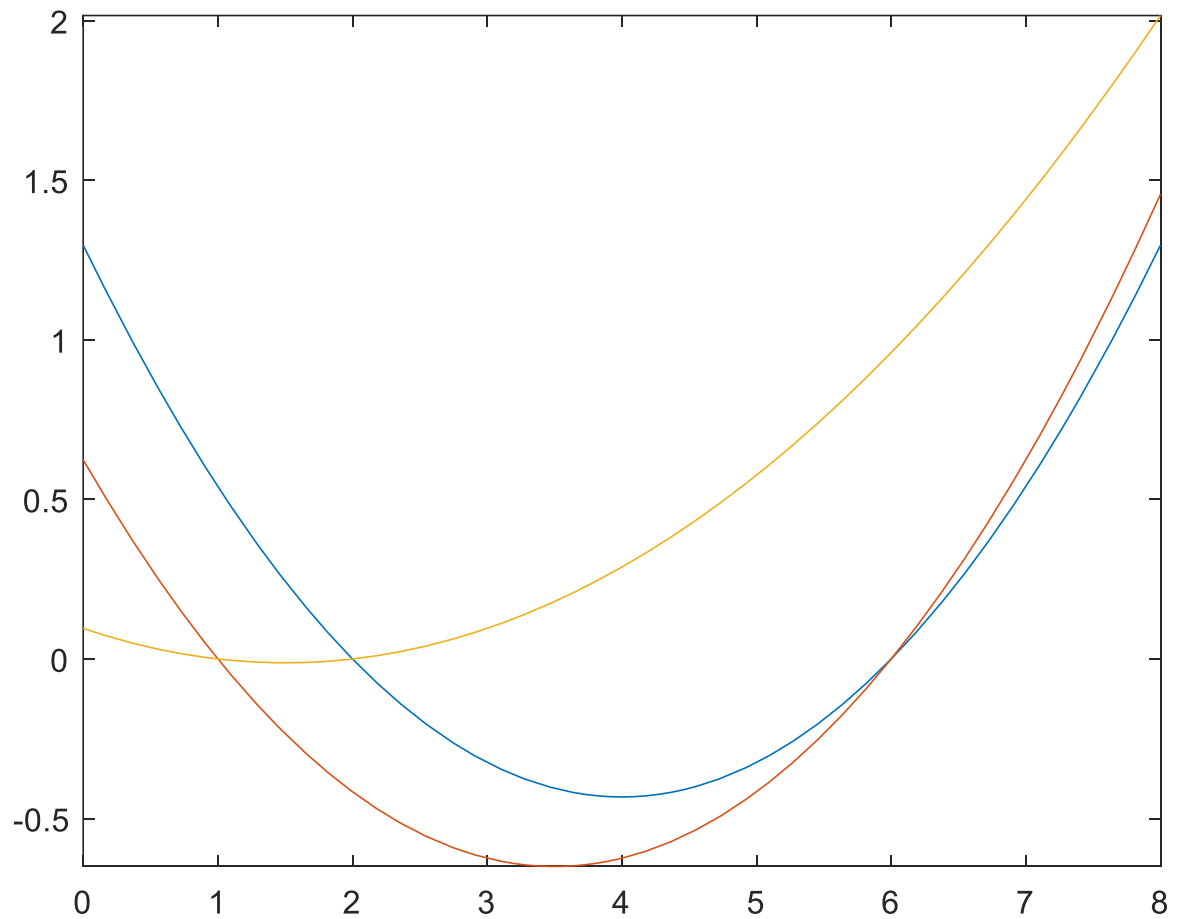
c)

```
>> fplot(@(x) 0.10806*(x*x-8*x+12), [0,8])
```

```
>> hold on
```

```
>> fplot(@(x) 0.104025*(x*x-7*x+6), [0,8])
```

```
>> fplot(@(x) 0.04801*(x*x-3*x+2), [0,8])
```



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d)

```
>> fplot(@(x) 0.260095*x*x - 1.736685*x + 2.01689, [0,8])
```

```
>> hold on
```

```
>> fplot(@(x) cos(x), [0,8])
```

```
>> plot(1, 0.5403, '*')
```

```
>> plot(2, -0.4161, '*')
```

```
>> plot(6, 0.9602, '*')
```

