Impediments to Arbitrage in the NIFTY Index options Market - A Survival Analysis approach

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Abstract

This paper investigates the factors which affect the time it takes for an index options market to return from positive arbitrage profits to zero arbitrage profit level. We consider arbitrage profits to be zero when there is no deviation from the put-call parity. Using the inter day transaction data from the NIFTY index option market, we compute the time taken for arbitrage profits to go to zero. We employ survival analysis to characterize how limits to arbitrage influence this persistence of arbitrage opportunities.

1 Introduction

A market is said to be efficient when any deviation from no arbitrage values attracts the market participants to take advantage of the situation and make arbitrage profits. Thus, in an efficient market, there should be a rapid disappearance of mispricing. Hence, two assets providing identical future payoffs must have same price. However, as per the empirical evidences the market can remain away from no arbitrage value for a prolonged period of time, which goes to show that the arbitrageurs are not able to benefit out of the deviations, as suggested by the theory. Thus, certain factors must exist in the market which prevent the arbitrageurs from exploiting this arbitrage opportunity. In this paper we model the influence of various arbitrage prohibiting co-variates on the time taken for arbitrage profits to return to zero. The rest of the paper is divided into 5 sections. The first section highlights some relevant literature on this topic. The second section gives a brief theoretical background of the put call parity relation. The third section discusses the data and methodology used in the paper. In the fourth section we present and analyze the results. Concluding remarks are made in the fifth section.

2 Literature Review

Numerous studies about the put-call parity relations have been conducted in the American, European and Australian stock and stock index options markets. Investigations into apparent violations of put-call parity for the most part find that the violations can exist if the following market features are not taken into account such as dividend payments, the early exercise value of American options, short-sales restrictions, simultaneity problems in trading calls, puts, stocks and bonds at once, transaction costs, lending rates that do not equal borrowing rates, margin requirements, and taxes, to name a few.

Several recent papers argue that large deviations from put-call parity in options on individual stocks can arise in the presence of short sales constraints on the underlying stocks, e.g., Lamont and Thaler (2003), Ofek and Richardson (2003) and Ofek, Richardson and Whitelaw (2004). This is because if the price of a put option becomes sufficiently high relative to the price of the corresponding call and the underlying asset, then the standard arbitrage strategy involves selling the underlying asset short.

Transaction costs and holding costs may also be another reason for limits to arbitrage, as discussed in (Tuckman and Vila, 1992) and (Mitchell and Pulvino, 2001). Transaction cost are the cost which are incurred at the time of opening or closing of the trade. These costs are incurred per transaction, and include brokerage fees,market impact costs, and bid-ask spreads. In equilibrium, arbitrage trades should yield the same net return regardless of transaction costs.

Liquidity is a major determinant of arbitrageurs activity because illiquid markets complicate the completion of trades and makes arbitrage both risky and more costly. The fact that we focus on index derivatives makes liquidity all the more critical because the underlying asset is a basket of many stocks. On index futures markets, Roll, Schwartz and Subramanyam (2007) show that the basis mean reverts faster when the market is more liquid. On index options markets, Kamara and Miller (1995) provide evidence that deviations from put-call parity are related to proxies for liquidity risk.

3 Theoretical Background

Here we provide a brief introduction to the put-call parity relation.

The basic Put-call parity relation was given by Stoll. Stoll established a relationship between the prices of a put and a call option written on the same underlying asset, having the same exercise price and same expiration date. Stoll made the following assumptions in his derivation There are no transaction costs, options are to be exercised on their date of expiration itself (options are european), underlying assets do not pay dividend etc. Under these assumptions ,Stoll demonstrates that buying a put(p) is same as buying a call option (c)on the same stock, with the same expiration date (T) and strike price (K) combined with a short position in the underlying asset (-S) plus the loan of an amount of money equivalent to the exercise price discounted at the risk free interest rate (r):

$$-c_t = -S_t - p_t + Ke^{-rt} \tag{1}$$

In this study since our underlying asset is an index we replace S by index value I . We ignore the effect of dividends. Thus the put call parity equation is –

$$c_t + Ke^{-rt} = p_t + I_t \tag{2}$$

If equation (2) does not hold, the call option is either undervalued or overvalued with respect to the put option and an arbitrage portfolio can be constructed. In reality, we should consider the futures value on the NIFTY index instead if the index value, but, for the purpose of simplicity, we use the value of the index for our analysis. We can describe the opportunity for arbitrage profit in the following way. Suppose we have two portfolios.

1. One call option (strike price K and premium C) and cash equal to K*exp(-rt).
2. One put option (strike price K and premium P) and the underlying index I.

If the value of the portfolio 1 is greater than portfolio 2, then we buy short the call and buy the put and asset(index) for making arbitrage profit. On the other hand, if portfolio 2 is overpriced relative to portfolio 1 then the correct arbitrage strategy is to buy the call and shorting both put and asset (index), generating a positive cash flow which can further be invested at risk free interest rate .

4 Data and Methodology

4.1 Methodology

4.1.1 The computation of the Time to no Arbitrage

The variable that we use to measure the arbitrage efficiency of the derivatives market is the time to no arbitrage (TTNA). TTNA is the number of days in which the market prices to revert back to no arbitrage values. To compute the TTNA, we first match pairs of call and put options having the same strike price and the same maturity. We then test the put call parity relationship on such matched put-call pairs. If the relationship does not hold, we compute the arbitrage profit by following appropriate strategies (as discussed in section 2). We compute the profits based on a lot size of 50 (1 lot contains 50 contracts) and factor in the appropriate transaction costs based on the ICICI direct brokerage structure. The profit, net of transaction costs, for each put-call pair is the realizable arbitrage profits for that pair.

The profits, obviously, can never be exactly equal to zero. Thus we define a zero profit window of [0,5] . If the absolute value of the net realized arbitrage profits from a put-call pair lies within the zero profit window, then we consider that the put call parity relationship holds for that particular put call pair for that particular day. In other words, if the absolute value of the net realizable profits for a put call pair for a particular day lie within [0,5], then we conclude that there are no arbitrage possibilities for that particular put call pair for that particular day. If the arbitrage profits are outside [0,5] then we conclude that arbitrage opportunities exist for that particular put call pair for that particular day. In such a case, we calculate the arbitrage profits for the same put call pair for the next trading day. If the newly calculated arbitrage profits are again outside [0,5], we again calculate the arbitrage profits for the same put call pair for the next trading day. We continue this process until a trading day comes when the calculated arbitrage profits lie within the zero profit window ([0,5]). The time to no arbitrage, TTNA, is then calculated as the number of days between the day when the arbitrage profits are zero and the day when the arbitrage opportunity existed for the first time.

Below, we provide an illustration of this process. We consider a put call pair in which both the put option and the call option expire at 25th October 2012 and both the put option and the call option have a strike price of 3800. We begin our calculation from the 14th of September, 2012.

1. At the 14th of September 2012, the net realizable arbitrage profits are 8.24. Since this lies outside the zero profit window of [0,5], we calculate the arbitrage profits for the next trading day.

2. At the 17th of September 2012 (the next trading day), the net realizable arbitrage profits are 18.78. This lies outside the zero profit window of [0,5]. Thus we calculate the arbitrage profits for the next trading day.

3. At the 18th of September 2012 (the next trading day), the net realizable arbitrage profits are 8.32. Again, this lies outside the zero profit window of [0,5]. Thus we calculate the arbitrage profits for the next trading day.

4. At the 20th of September 2012 (the next trading day), the net realizable arbitrage profits are -3.44. This lies inside the zero profit window of [0,5]. Thus arbitrage opportunities for this put call pair begins to exist at the 14th of September and ceases to exist at the 20th of September.

The time to no arbitrage (TTNA) is thus the number of days between the 20th and the 14th of September, i.e. 6 days.

4.1.2 The use of Survival Analysis

We use Survival Analysis to

1. - Contrast the TTNA of an ITM (In the money) call option with that of an OTM(Out of the money) put option
   - Contrast the TTNA of put call pairs with near, middle and far month maturities.
2. Model the influence of explanatory variables (arbitrage prohibiting co variates) on the TTNA.

We now provide a brief description of the survival analysis framework. Survival analysis models time to event data. Thus, in the present context, we will model the time to no arbitrage (TTNA) using survival analysis. The length of a spell for a subject (a matched put call pair in the present context) is a realization of the random variable T (the TTNA in the present context) with a cumulative distribution function (cdf) F(t), and a probability density function f(t). F(t) is the failure function, and S(t) = 1 F(t) is the survivor function. Here ‘t is the time elapsed since entry to the state at time 0.

Thus we have Faliure function (cdf)

$$Pr(T <= t) = F(t) \tag{3}$$

Survivor function

$$Pr(T > t) = 1F(t) = S(t) \tag{4}$$

Probability density function (pdf)

$$f(t) = \delta F(t)/\delta t \tag{5}$$

We are working in a framework where the survival times are intrinsically discrete. The discrete time hazard at ‘t’, h(t), is the conditional probability of the event at ‘t’, (with conditioning on survival until completion of the cycle immediately before the cycle at which the event occurs). Thus, in the present context, the discrete time hazard at day ‘t’, h(t), is the conditional probability of the zero arbitrage profits at day ‘t’(for a particular put call pair), conditional on positive arbitrage profits from day ‘0’ (the starting day of arbitrage profits for that matched put call pair) till day ‘t’. Thus,

$$h(t) = Pr(T = t|T >= t) \tag{6}$$

$$h(t) = f(t)/S(t1) \tag{7}$$

There is a one-to-one relationship between a specification for the hazard rate and a specification for the survivor function. Thus, whatever functional form is chosen for h(t), one can derive S(t), F(t) and f(t) from it. To model the time to event data using survival analysis, we have to choose a specification for the hazard rate i.e. the shape of the relationship between the hazard rate and the survival time. In this case, the survival times are intrinsically discrete and we choose a proportional odds model to model the time to event data. Using a proportional odds model, we also account for potential differences in hazard rates between individuals. The characteristics of a given matched put call pair are summarized by a vector of variables X. For each individual, we observe k different variables and define a linear combination of characteristics β'X.

Thus the hazard and survivor functions are functions of both 't' and 'X'. Thus, we now refer to the hazard function by h(t,X) and the survivor function by S(t,X).

The proportional odds model assumes that the relative odds of making a transition in day 't', given survival up to end of the previous day, is summarized by an expression of the form:

$$h(t,X)/1 - h(t,X) = [h_0(t)/1 - h_0(t)] * e^{\beta X} \tag{8}$$

Here h(t,X) is the discrete time hazard rate for day t, and h_0(t,X) is the corresponding baseline hazard arising when X = 0. The relative odds of making a transition (i.e. the arbitrage profits go from positive to zero) at any given day is given by the product of two components: (a) a relative odds that is common to all matched put call pair, and (b) an put call pair specific scaling factor. It follows that

$$logit[h(t,X)] = log[h(t,X)/1 - h(t,X)] = \alpha_t + \beta^{'} X \tag{9}$$

where α_t = logit[h_0t]. Alternatively,

$$h(t,X) = 1/[1 + e^{-\alpha_t - \beta^{'} X}] \tag{10}$$

This is the logistic hazard model and it has a proportional odds interpretation. In our analysis, we take α_t to be equal to rlog(t). In this specification, the shape of the hazard monotonically increases if r > 0, decreases if r < 0, or is constant if r = 0 . Again, our premise is that if arbitrage opportunities exist and are not exercised immediately, then it implies that there are certain arbitrage prohibiting co-variates which prevent the traders from exercising this arbitrage opportunity. We consider the following explanatory variables as arbitrage prohibiting co-variates -

1. Volatility level of the underlying index As a proxy for the underlying volatility of the NIFTY index, we take the values of the India Volatility Index (India VIX)

2. Illiquidity as an impediment to arbitrage - Traders always face a trade-off between the costs of trading immediately and the risk of adverse price movements when delaying their trades. Illiquid markets make this tradeoff all the more critical and could thus deter traders from entering in arbitrage trades. Hence we hypothesize that TTNA is related to illiquidity. We proxy for liquidity by maturity and moneyness of matched put call pairs.

   - For maturity, we use dummy variables that take on the value one if the option belongs to the corresponding maturity and zero otherwise (Mat1 stands for options that expire by the end of the current month, Mat2 for options with maturity between two and three months and Mat3 for options with maturity more than three months)
   - We measure moneyness as the absolute value of the difference between the index value and strike price.
   - Liquidity in the underlying index should also be critical given that the index constitutes one leg of the arbitrage. We proxy for the liquidity in the index total index trading volume over the corresponding trading day.

We summarize these arbitrage prohibiting co-variates by the vector X. Thus our full model specification becomes

$$logit[h(t, X)] = rlog(t) + \beta_1 indexvol + \beta_2 maturitydumm + \beta_3 moneyness + \beta_4 tradingvolume \tag{11}$$

4.2 Data

We collect daily data on the expiry date, strike price, call premium and put premiums for PCP pairs from the Bhav-copy released daily by the NSE. We collect the data for three months from August 2012 to October 2012. We also collect daily data for these three months on the risk free interest rate from the ZCYC curve released daily by the NSE. Also, we collect data for these three months on the daily trading volume of the NIFTY index and the daily index closing value of the India Volatility Index from the CMIE PROWESS Database.

5 Results

5.1 Graphical Analysis of moneyness and maturity

The bar charts are shown in the appendix. In both the ITM and the OTM cases, we observe that the arbitrage window of one day has the highest frequency. However, the interesting thing to note in both cases is that a considerable number put call pairs demonstrate arbitrage opportunities in excess of 1 day. In case of the OTM call, as the arbitrage window increases, the number of put call pairs promising the arbitrage opportunity decreases at first, but later shows an increasing trend from day 5 onward. This is an interesting observation and should be analyzed further.The initial decrease is probably because the OTM option is expected to have higher liquidity which leads to lesser time in obtaining no arbitrage prices. The greater liquidity is probably because out-of-the-money options represent a cheaper way to speculate on or hedge against changes in future volatility in the presence of leverage constraints. The bar chart of the ITM call, with increase of the arbitrage window, shows a decrease in the number of put call pairs giving the opportunity of arbitrage. This result is expected, but, on the whole, the prevalence of arbitrage opportunities for a sizable number of put call pairs is surprising.

The bar charts in the cases of options having near, middle and far month maturities show the expected trend. The options with near month maturities are considered to be most liquid. Thus illiquidity factors should have the least impact on options with near month maturities. This fact is backed up by the bar chart which shows that as the arbitrage window increases, the number of arbitrage opportunities decrease. In the case of middle and far month maturities, the number of arbitrage opportunities do not decrease as the arbitrage window increases. In fact, for options having far month maturities, the number of arbitrage opportunities actually increase as the arbitrage window increases. However, the huge number of arbitrage opportunities for options of any maturity are indeed surprising.

5.2 Analysis of Maximum Likelihood Estimates

The result shown in the appendix gives the value of the coefficients of the independent variables. The model used here is a Logit model of where the dependent variable is the log of odds of arbitrage profit being zero. Hence the results are to be interpreted as the increase in the log of the odds ratio due to one unit increase in independent variable. For example, mat1 is a dummy variable which takes the value of 1 if the corresponding pair is in the near month category. Thus the intercept gives the mean log hazard odds and the coefficient of mat1, .9131, gives the difference in mean log hazard odds of near month maturity pairs as opposed to middle and far month maturity pairs. The other significant parameter is log(t). We had specified the hazard rate to be a product of two components – a component that is common to all put call pairs, and a component that is specific to a particular put call pair. The parameter log(t) is the component which is common to all put call pairs. Thus we conclude that a significant portion of the total hazard can be explained by a component which is common to all the put call pairs. The rest of the parameters, accounting for the volatility level and liquidity of the underlying nifty index, and the moneyness of the options, turn out to be insignificant. This result is not consistent with the posited theory and should be analyzed further.

6 Conclusion

On options markets, deviations with respect to arbitrage relationships are temporary. We show that, eventually, prices typically revert to no arbitrage levels. But the data shows that a substantial amount of time elapses before prices return to no arbitrage levels. Using survival analysis, we model the time to no arbitrage for NIFTY index options market and relate it to the constraints investors confront in their arbitrage activity. We find that the persistence time to arbitrage opportunities is impacted by some of the determinants of liquidity constraint like the effect of time to maturity. However, the other factors considered like moneyness, trading volume and volatility are found to be insignificant.Thus, to conclude, we can say that a substantial amount of research needs to be done in this area to accurately justify the market behavior.

7 References

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- Jenkins survival analysis
- Tests of the put-call parity relation using options on futures on the S&P 500 index., Urbi Garay, Maria Celina Ordonez and Maxmilliano Gonzalez

8 Appendix – Charts & Tables

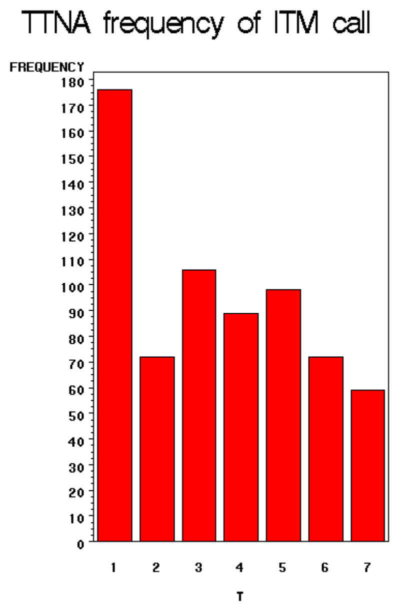

Figure 1: Bar Chart for In−The−Money call options

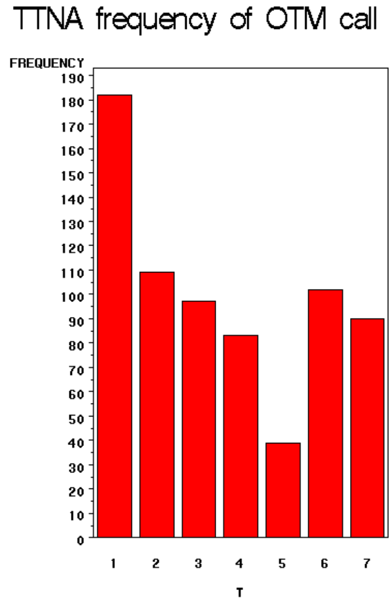

Figure 2: Bar Chart for Out−of−The−Money call options

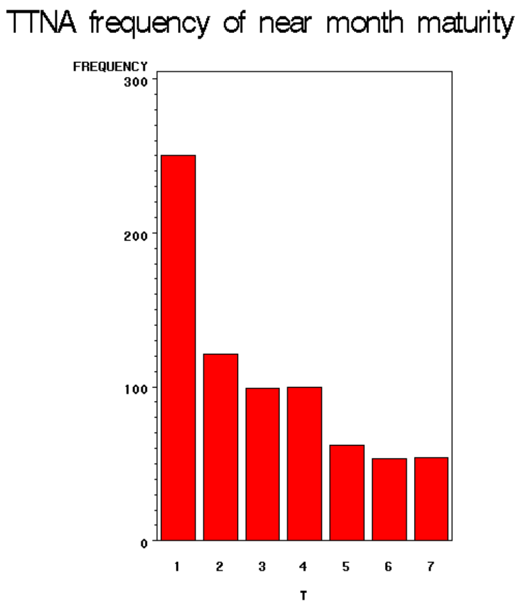

Figure 3: Bar Chart for near month maturity options

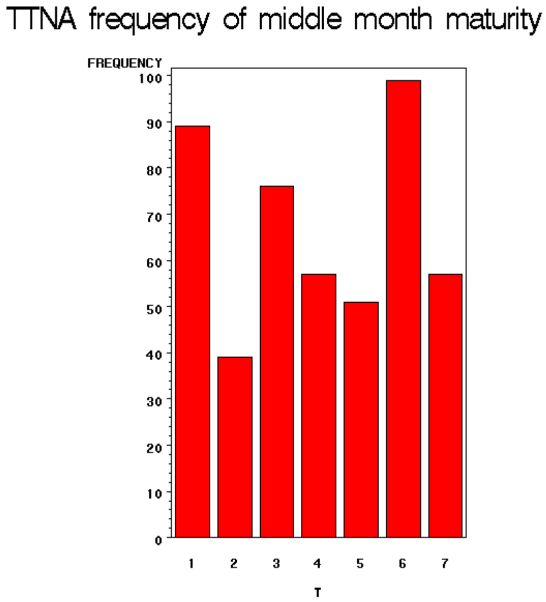

Figure 4: Bar Chart for middle month maturity options

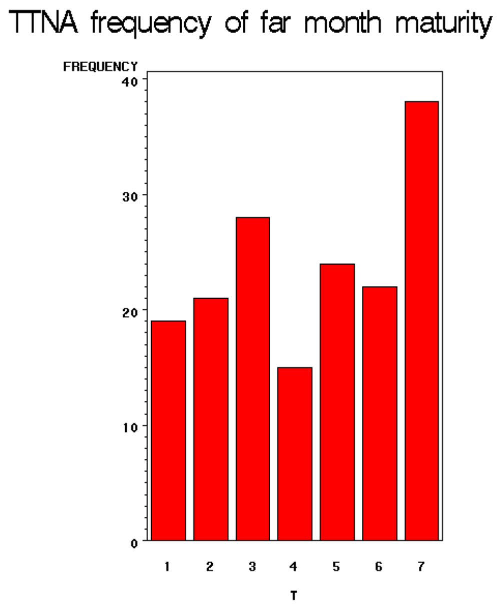

Figure 5: Bar Chart for far month maturity options

Table 1: Survival Analysis Results

Parameter	Maximum Likelihood estimates
Intercept	-1.4661 -0.1494
ln(t)	2.407 <.0001
index_volatility	-0.0762 0.247
mat1	0.9131 <.0001
mat2	0.1959 0.4027
moneyness	0.0001 0.4982
trading volume	-0.0058 0.3046