# Linear Optimization Assignment

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**Instructions**

- The complete submission should include: (a) the report (in PDF format), (b) the source code(s), compressed into one zip file (YourFullName_StudentID.zip).

- You can use either Python or MATLAB for the coding part.

- You are strictly prohibited from copying solutions from another person. Violation of this would be considered an act of plagiarism.

- Note that there will be a demo where you will be called individually to discuss your codes and solutions. The schedule will be released shortly.

## Question 1.

Write a code for the Revised Simplex method from scratch. **(10 marks)**

## Question 2.

Given a graph $G = (V; E)$, a matching $M \subset E$ of $G$ is a collection of vertex disjoint edges. The size of the matching $M$ is the number of edges present in $M$ and is denoted by $\|M\|$. Maximum matching of $G$ is a matching of maximum size. Given a graph G with $\|V\|$ vertices and $\|E\|$ edges, formulate an ILP(integer linear program) to solve the maximum matching problem. For the given adjacency matrix of network 1, solve the problem using the Simplex code you have written. **(4 marks)**

## Question 3.

Given a graph $G = (V; E)$, a vertex-cover $V^1 \subset V$ of $G$ is a collection of vertices such that each edge in $E$ is incident to at least one of the vertex in $V'$. The size of a vertex cover is the number of vertices present in the cover. Write an ILP to find a minimum size vertex cover for $G$. For the given adjacency matrix of network 1, solve the problem using the Simplex code you have written. **(4 marks)**

## Question 4.

We are given an adjacency matrix representation of the flow network 2. Each edge $(i, j)$ is a directed edge from $i$ to $j$. The capacity of each edge is given in the adjacency matrices. Node $S$ represents the source, and node $T$ represents the sink in both networks. Note that each capacity is an integer. Both the flow networks satisfy the following properties

- Conservation of flow at each node except Source (i.e., Total incoming flow at a node must equal total outgoing flow at the node).

- For each edge, any flow respects the capacity constraint of the edge.

Do the following

- Write a code for the Interior point method (IPOPT, based on newton iteration) from scratch. **(15 marks)**

- Formulate LPs for flow networks 1 and 2 to find out the maximum flow from $S$ to $T$ and solve it using the IPOPT code you have written. **(3 marks)**

- Take the Duals of each LP which corresponds to $S - T$ Min-Cut in the respective network and solve it using the IPOPT code you have written. **(4 marks)**

| Network 1 | 1 | 2 | 3 | 4 |
|-----------|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 |
| 2 | 0 | 1 | 1 | 0 |
| 3 | 0 | 0 | 1 | 1 |
| 4 | 0 | 1 | 0 | 1 |

| Network 2 | S | A | B | C | D | E | F | G | H | I | J | T |
|-----------|---|---|---|---|---|---|---|---|---|---|---|---|
| S | 0 | 11 | 15 | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | 0 | 0 | 0 | 0 | 0 | 18 | 4 | 0 | 0 | 0 | 0 | 0 |
| B | 0 | 3 | 8 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 0 | 0 | 6 | 0 | 0 | 3 | 11 | 0 | 0 | 0 |
| D | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 17 | 6 | 0 | 0 | 0 |
| E | 0 | 0 | 0 | 0 | 3 | 16 | 0 | 0 | 0 | 13 | 0 | 0 |
| F | 0 | 12 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 21 |
| G | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 9 | 4 | 3 |
| H | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 5 | 4 |
| I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 9 |
| J | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 15 |
| T | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

## Question 5.

Write the codes in AMPL to solve all LPs (you formed). Show that min-cut is equal to max-flow for question 4(i.e., an LP and its dual have the same optimal). **(10 marks)**