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MTH-371 SPA

ASSIGNMENT 1

QUESTION – 1

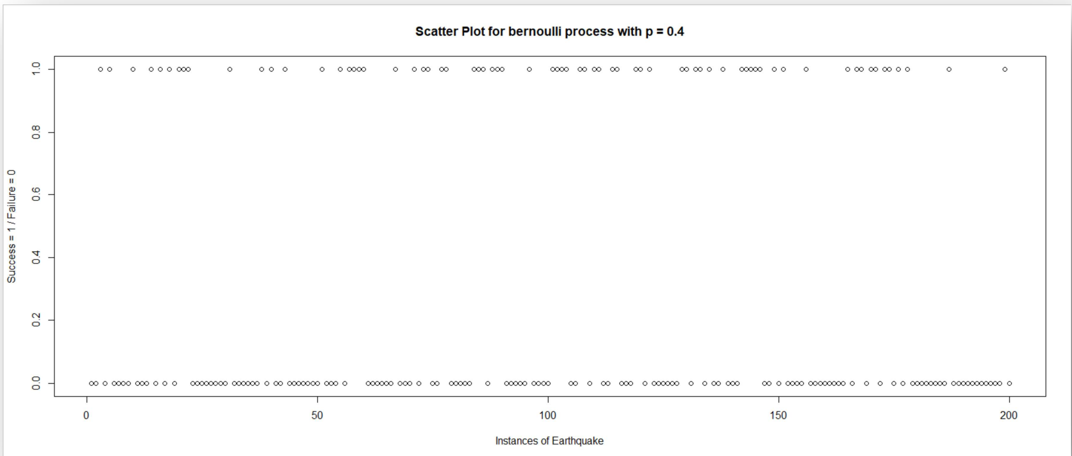
Magnitude of various earthquakes for a site.

The research conducted recorded the magnitude of various earthquakes for a fixed site from 2000 – 2022. To detect the presence of earthquakes of magnitude >= 6 can be modelled as a Bernoulli Process because:

- ✓ There are only 2 possible outcomes for each trial (trial represents instance of an earthquake happening); success or failure. Success indicates that the earthquake had magnitude >= 6, failure indicates that the earthquake had magnitude < 6.
- ✓ Each trial / occurrence of earthquake is independent of each other (given in the question itself).
- ✓ The probability p of success is same for all the trails / occurrence of earthquake (given in the question itself).
- ✓ The process is being studied over a discrete time (i.e., Number of instances of earthquakes are discrete).
- ✓ The process can start at time t = 0, but the first arrival / success (occurrence of earthquake of magnitude >= 6) can only occur at t >= 1 (Here time denotes instance of an earthquake).

- ➢ The first 3 points indicate that trials (instance of an earthquake) are IID (Independent and Identically Distributive) Bernoulli Random Variable.
- ➢ Therefore, detecting the presence of earthquakes of magnitude >= 6 on a fixed site can be modelled as a Bernoulli Process as it's a discrete state and discrete time random process.

PART – A



The scatter plot is shown for the given Bernoulli process with success probability p = 0.4, the x-axis denotes the instances of earthquakes and success/failure of an earthquake of magnitude >= 6 is taken on the y-axis.

PART B

As we have already established that this is a Bernoulli process where each trail is a Bernoulli RV.

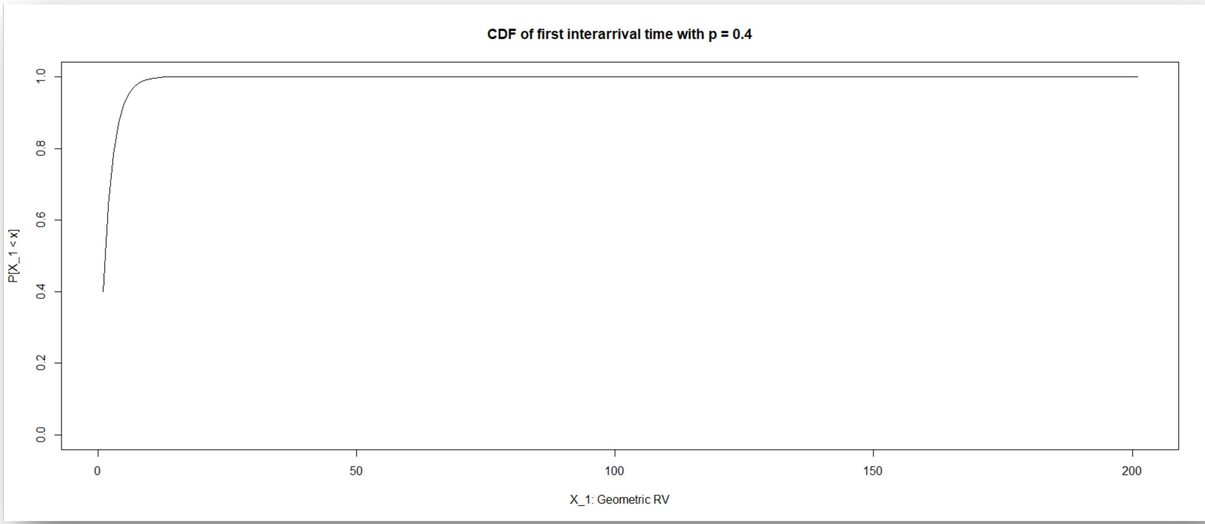
As derived in the class, the first interarrival time in a Bernoulli process follows geometric distribution.

Therefore, X1 the first interarrival time is a geometric RV,

$$P\,[X1 = t] = (1-p)^{t-1} * p$$

By fresh start property, it can be further shown that each interarrival time X2, X3, …, all of them follow geometric distribution.

The graph of the distribution CDF of the given geometric RV with success probability p = 0.4, x-axis denotes the values of first interarrival time and the CDF in on y-axis.



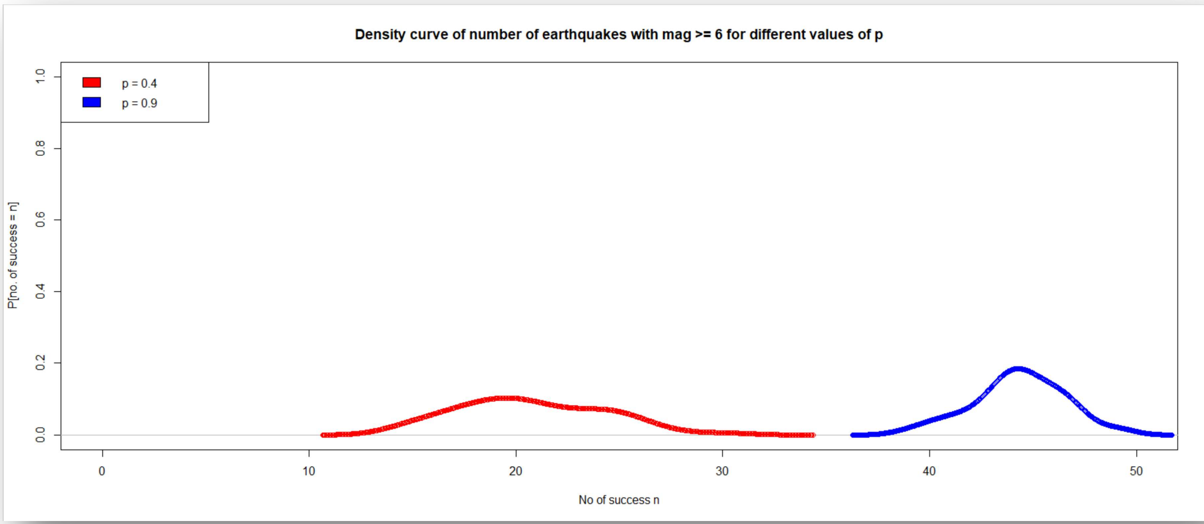
PART C

As we have already established that this is a Bernoulli process where each trail is a Bernoulli RV.

As derived in the class, the total number of arrivals/successes up to a certain point of time can be modelled as a Binomial RV.

Therefore, N_k denotes the total number of arrivals/successes/earthquakes with magnitude >= 6 till time k,

$$P\,[\,N_k = n] = {}^{k}C_n * (1-p)^{k-n} * (p)^n$$



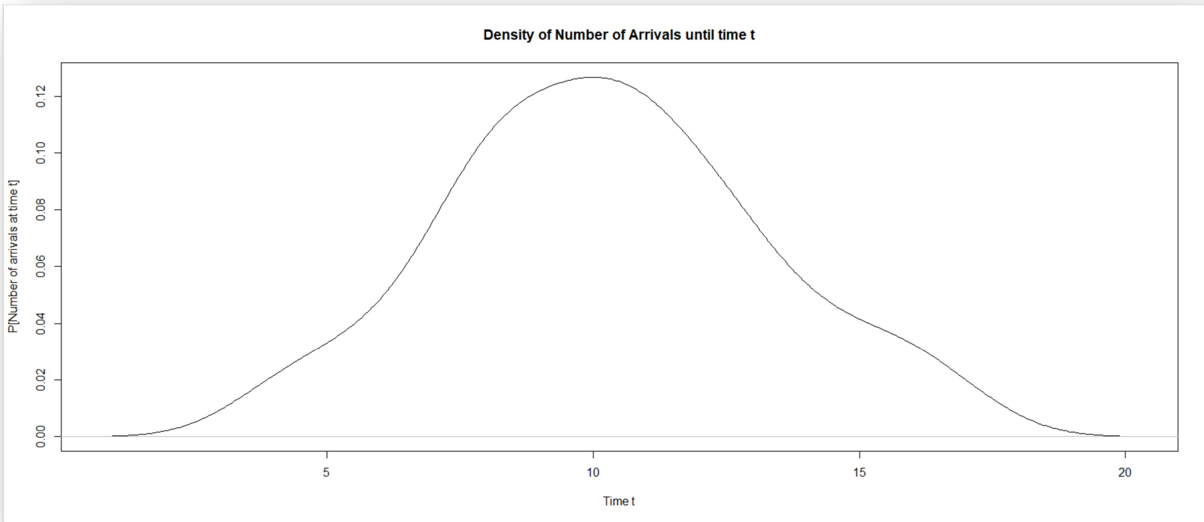
As the probability of success increases, the distribution shifts towards right i.e., the expectation increases.

QUESTION – 2

Requests received by a website.

Aim is to study the number of visitors at a website in the time interval (0, t] where t is continuous and the average rate of visitors is 10 per hour. This can be modelled as a Poisson process as given in the question.

PART – A

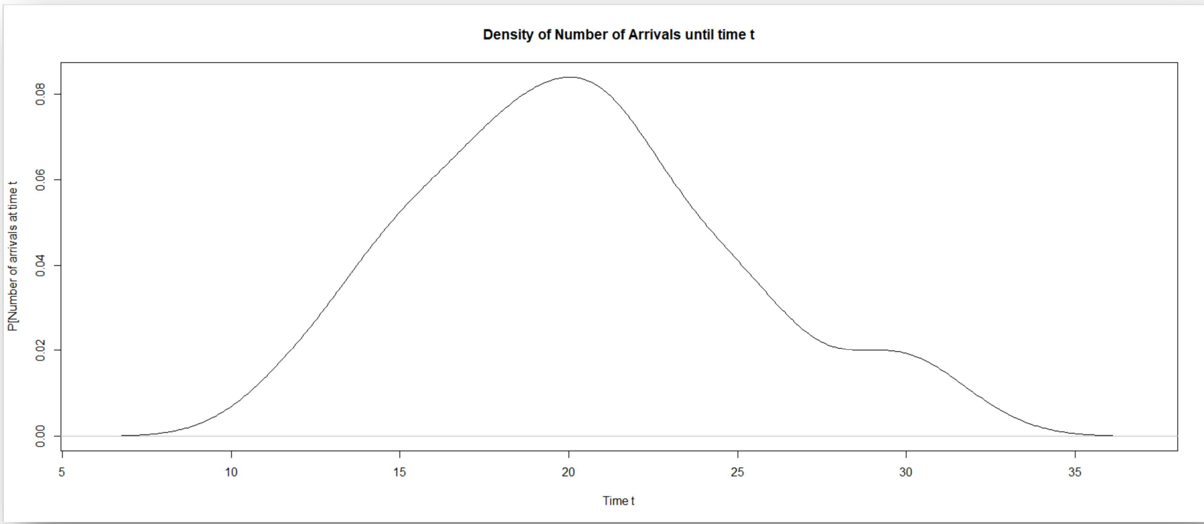


The above graph is shown for density of number of arrivals until time t (t = 100 in this graph) with lambda = 10. We can see that denser part of the graph is around 10 which is the density of the distribution.

PART – B

The below graph is shown for density of number of arrivals until time t (t = 100 in this graph) with lambda = 20. We can see that denser part of the graph is around 20 which is the density of the distribution.

The difference between this graph with lambda = 20 and previous with lambda = 10 for the same time t = 100 is the increase in the expected value as in case of Poisson RV, the mean and the variance are equal to the rate of arrival which is lambda. So, we can clearly see the graph below is more spread than the above because as the rate increases, more arrivals are bound to happen (20 in previous graph, 35 in this graph), which in turn decreases the maximum probability (0.12 in previous graph, 0.08 in this graph).



PART – C

In this part, we have 2 independent Poisson processes. Let $N_1(t)$ and $N_2(t)$ be the 2 Poisson processes with corresponding rate of arrivals as lambda_1 and lambda_2.

According to the question, we are interested in studying the total number of arrivals/visitors on both the processes combine.

Let N(t) be the combined Poisson process:

$$N(t) = N_1(t) + N_2(t)$$

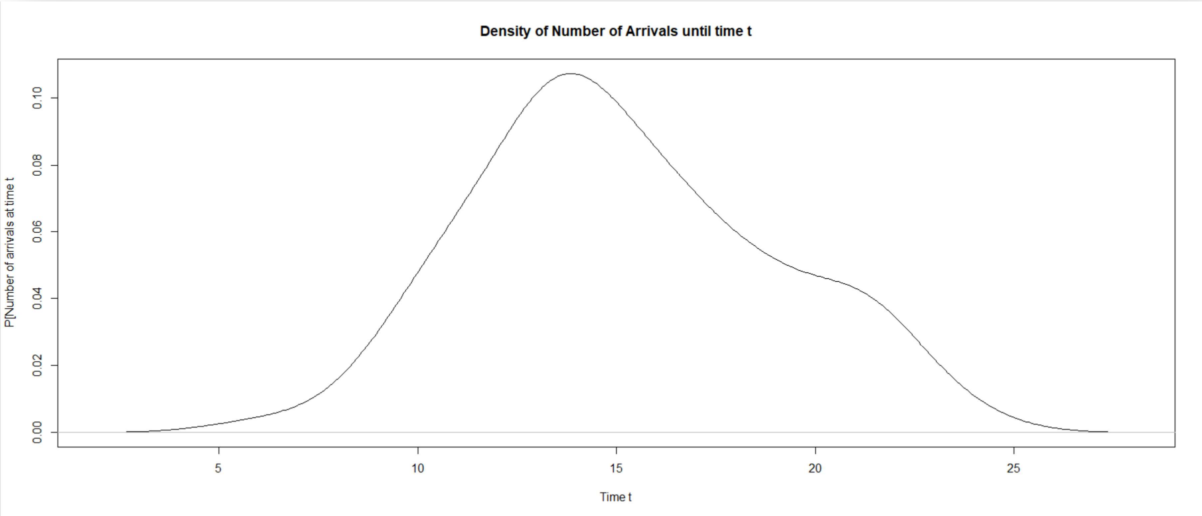
$$\Rightarrow \quad M_{N(t)}(s) = E[e^{N(t)s}]$$

$$= E[e^{[N1(t) + N2(t)]\, s}]$$

$= M_{N1(t)}(s) * M_{N2(t)}(s)$

$= \exp [(\text{lambda\_1} + \text{lambda\_2}) (e^s - 1)]$

- ⇨ lambda = lambda_1 + lambda_2
- ⇨ lambda = 10 + 5
- ⇨ lambda = 15



The above graph is shown for density of number of arrivals until time t (t = 100 in this graph) with lambda = 15 for the combined processes. We can see that denser part of the graph is around 15 which is the density of the distribution.

Now, as already derived in class, the first interarrival time in a Poisson process follows the exponential distribution. This curve shows first arrival around 0.4 unit of time.