Statistics & Probability

Week 2 Assignment

Instructions

1. Read the problem statements carefully.
2. Identify the concept in which this problem fits into and state the concept.
3. Solve the problem.
4. Detail out the steps you have taken to solve this. Do not give only answers.

Problem 1

A random sample of 225 First Year statistics tutorials was selected from the past 5 years and the number of students absent from each one recorded. The sample mean and standard deviation are 11.6 and 4.1 respectively. Estimate the mean number of absences per tutorial over the past 5 years with 90% confidence.

Solution:

n = 225, X = 11.6, SD = 4.1,
Confidence Limit = 90% so the Z = 1.645

Confidence Interval = [X + (Z * SD/ sqrt(n)) , X - (Z * SD/ sqrt(n))]

= [11.6 + 1.645 * 4.1/sqrt(225), 11.6 - 1.645 * 4.1/sqrt(225)]
= [11.6 + 1.645 * 0.2733, 11.6 - 1.645 * 0.2733]
= [12.05, 11.15]

So the confidence interval for the mean is given as [12.05, 11.15]

Problem 2

Sample of 15 test-tubes were tested for number of times they can be heated on Bunsen burner before they cracked. The sample and standard deviation were observed to be 1230 and 270 respectively. Construct a 99% confidence interval for the population mean.

SOLUTION:

n = 15, x = 1230, SD = 270,
Confidence Limit = 99% so the Z = 2.575

Confidence Interval = [X + (Z * SD/ sqrt(n)) , X - (Z * SD/ sqrt(n))]

= [1230 + 2.575 * 270/sqrt(15), 1230 - 2.575 * 270/sqrt(15)]
= [1409.512, 1050.488]

So the confidence interval for the mean is given as [1409.512, 1050.488]

Problem 3

A quality control engineer finds that a sample of 100 light bulbs had an average life-time of 470 hours. Assuming a population standard deviation of $\sigma = 25$ hours, test whether the population mean is 480 hours at a significance level of $\alpha = 0.05$

SOLUTION:

n = 100, SD = 25

The hypothesis statements will be H0 : X >= 470 , H1 : X < 470

Test Statistic Z = mean - X / SD/ sqrt (n)
= 480 - 470 / 25/ sqrt(100)

= 10/ 2.5
= 4
Therefore, the p-value = .00032

Comparing this with 0.05 , p-value(.00032) <= type 1 error value(0.05)
which means it is at significant level so we can reject the null hypothesis.

Problem 4

A batch 100 resistors have an average of 102 Ohms. Assuming a population standard deviation of 8 Ohms, test whether the population mean is 100 Ohms at a significance level of $\alpha = 0.05$

SOLUTION:

n = 100, SD = 8

The hypothesis statements will be H0 : X >= 102 , H1 : X < 102

Test Statistic Z = mean - X / SD/ sqrt (n)
= 100 - 102 / 8/ sqrt(100)
= -2/ 0.8
z score = -2.5

Therefore, the p-value = .00621

Comparing this with 0.05 , p-value(.00621) <= type 1 error value(0.05)
which means it is at significant level so we can reject the null hypothesis.

Problem 5

A batch 100 resistors have an average of 101.5 Ohms. The standard deviation of the resistors sampled is 5 Ohms:

(a) Test whether the population mean is 100 Ohms at a significance level of $\alpha = 0.05$
(b) Compute the p-value

SOLUTION:

n = 100, SD = 5

The hypothesis statements will be H0 : X >= 101.5 , H1 : X < 101.5

Test Statistic Z = mean - X / SD/ sqrt (n)
= 100 - 101.5 / 5/ sqrt(100)
= -1.5/ 0.5
z score. = -3

Therefore, the p-value = .00135

Comparing this with 0.05 , p-value(.00135) <= type 1 error value(0.05)
which means it is at significant level so we can reject the null hypothesis.