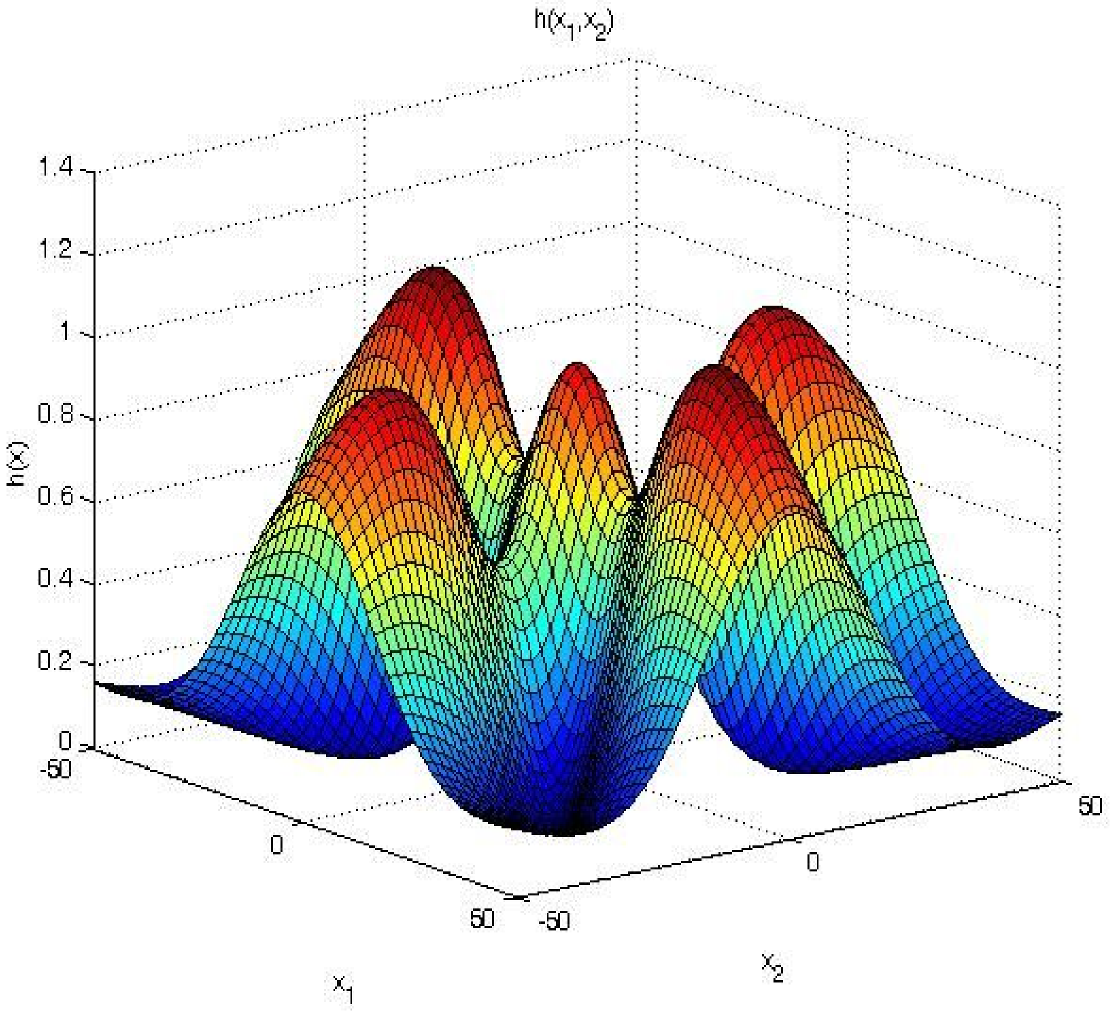
MTL704

ASSIGNMENT REPORT

NUMERICAL OPTIMIZATION



Submitted By :-

Parminder Singh (2016MT10630)

Naman Singh Kanaujia (2016MT10623)

CONTENTS :

Description of Code Files :

We are given with general quadratic problem :

$$f(x) = 0.5x^TQx-b^Tx$$

Q and b are input matrices. Q is nxn positive definite matrix and b is nX1 matrix. For a given n we have to generate 10 values of Q and b and then we have to solve the optimization problem by **exact steepest descent and backtrack line search(inexact) method**.

There are two files attached with this report analysis and ass. ass1 file will generate some files and later those files will be used by analysis for analysis. It will also generate some csv files and plots.

File named ass1 contains implementation of both exact and inexact algorithms. In this file 10 different Q and b matrices are generated randomly and optimization problem is solved using both algorithms. This file forms dictionary of data and stores that dictionary in pickle file format. These files are generated for different values of n, gamma and beta for both algorithms. The format of the dictionary for both algorithms for given gamma and beta is as follows:

- d[i][0] = value of fxn at initial value of x
- d[i][1] = value of fxn at optimal value
- d[i][2] = number of steps taken by the algorithm
- d[i][3] = minimum value of eigenvalue of matrix Q
- d[i][4] = maximum value of eigenvalue of matrix Q

Here i varies from 0 to 9 for 10 different Q and b generated.

ass1 code files take initial guess on x randomly for a given n. The value of this initial guess **remains same** for all problems (for diff. Q,b,gamma,beta) for a given n. We also calculate the minimum and maximum value of eigenvalues of Q for calculation of **condition number** which is further used for analysis.

For all calculations value of epsilon is taken as 10^(-4)

Performance Comparison :

The following tables shows the results obtained from the code files. The first column indicates the problem number (Different Q and b), second column contains the number of steps taken by exact algo and the third column contains the number of steps taken by inexact algo and fourth column contains the value of condition number of matrix Q for the problem. These tables are generated for all values of n.

n=10,50,100,200,500 and the value of beta and gamma for inexact algorithm are 0.5 and 0.2 respectively.

For n=10 :

Total problems for n = 10	Number of steps taken in Exact line search	Number of steps taken in Inexact line search	Condition number (lambda(max) / lambda(min))
Problem 1	24	78	48.4064409988538
Problem 2	13	42	37.0294281610095
Problem 3	295	964	544.978945758886
Problem 4	159	478	294.202647980946
Problem 5	63	233	159.007644189273
Problem 6	77	396	205.255418972636
Problem 7	53	195	108.017338178014
Problem 8	60	137	97.7915415914019
Problem 9	101	192	194.718059301703
Problem 10	21	49	43.861601575811

For n = 50 :

Total problems for n = 50	Number of steps taken in Exact line search	Number of steps taken in Inexact line search	Condition number (lambda(max) / lambda(min))
Problem 1	84	742	3058.35982734382
Problem 2	308	1981	4282.97976389021
Problem 3	130	801	1359.69288136342
Problem 4	187	876	3818.64320202064
Problem 5	480	3218	18106.7368573576
Problem 6	137	1697	2338.46699046835
Problem 7	225	878	8399.32461122282
Problem 8	292	695	1865.19026029196
Problem 9	200	2164	3922.95802360801
Problem 10	411	945	5360.90913366614

For n = 100 :

Total problems for n = 100	Number of steps taken in Exact line search	Number of steps taken in Inexact line search	Condition number (lambda(max) / lambda(min))
Problem 1	162	667	8017.64930348481
Problem 2	415	2080	14663.9064117087
Problem 3	125	673	12912.866776339

Problem 4	61	692	3126.50711746642
Problem 5	308	2832	22923.4892746643
Problem 6	334	1300	23169.3933403049
Problem 7	401	1680	8737.97204244334
Problem 8	175	444	9041.33130547835
Problem 9	366	1477	5597.26634972261
Problem 10	165	3466	4081.37240110912

For n = 200 :

Total problems for n = 500	Number of steps taken in Exact line search	Number of steps taken in Inexact line search	Condition number (lambda(max) / lambda(min))
Problem 1	86	753	34620.5223982925
Problem 2	95	929	39954.9885102158
Problem 3	125	1569	17264.9669866651
Problem 4	284	817	50340.277308373
Problem 5	479	1533	69529.5577507345
Problem 6	2913	975	455144.43455956
Problem 7	168	1030	13762.1490085782
Problem 8	210	1341	85539.3947973127
Problem 9	399	3130	31028.447446784
Problem 10	179	1419	36045.040076047

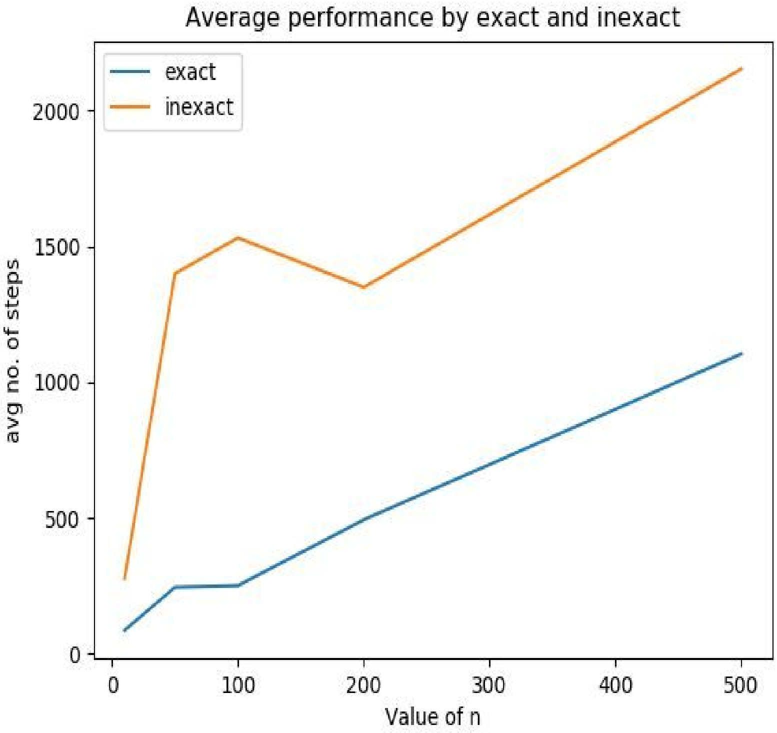
For n = 500 :

Total problems for n = 500	Number of steps taken in Exact line search	Number of steps taken in Inexact line search	Condition number (lambda(max) / lambda(min))
Problem 1	425	2160	221133.913003378
Problem 2	1156	1506	571311.339656517
Problem 3	3372	1533	2395806.96689454
Problem 4	1185	1736	401317.241354672
Problem 5	496	5493	994875.380763026
Problem 6	500	1444	222511.400046277
Problem 7	1390	1698	241371.114176569
Problem 8	1451	1777	4328556.23281082
Problem 9	712	2396	1828461.75091602
Problem 10	359	1790	365628.233804547

From the above tables we can observe two things. For a given value of n the number of steps taken by both algorithms are less in case of **matrix having small value of condition number** and steps taken by inexact algorithm are more as compared to exact algorithm. This could also be seen by the following table and plot.

Average Performance of Exact and Inexact Algorithms :

Value of n	Average Steps taken by Exact Algorithm	Average Steps taken by Inexact Algorithm
10	86.6	276.4
50	245.4	1399.7
100	251.2	1531.1
200	493.8	1349.6
500	1104.6	2153.3



These results are obtained after taking average for all 10 problems for a given n. We can clearly see from table and plot that number of steps taken in inexact algorithm are more.

Variation of Number of steps with gamma and beta:

Now we will analyze the variation of number of steps taken to solve the problem with inexact problem with beta and gamma. Different values of beta and gamma taken for these experiments are as follows:

gamma_values=[0.1,0.2,0.4,0.6,0.8]

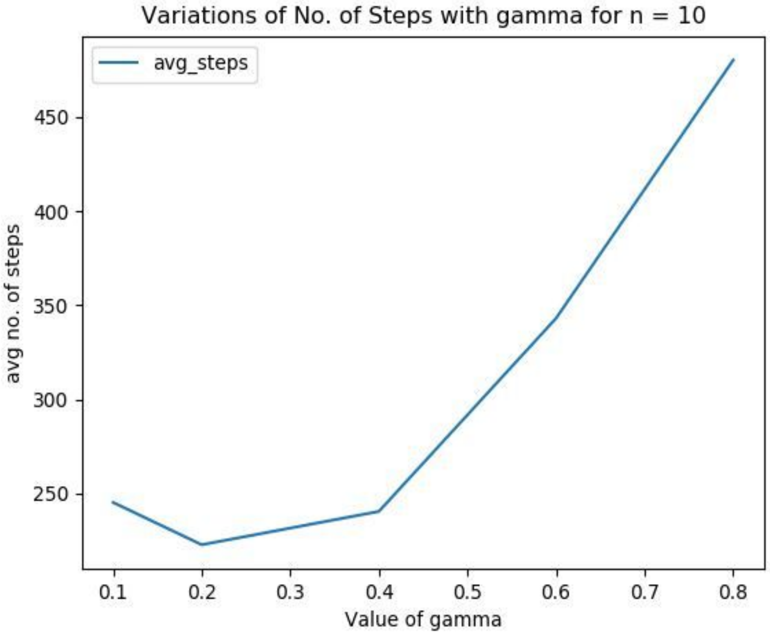
beta_values=[0.2,0.3,0.5,0.7,0.9]

1. Variation with gamma:

In the following experiments for a given value of n. The value corresponding to one gamma value is obtained by fixing gamma and then taking the average of number of steps taken over all values for beta. Plot for these obtained values are also generated.

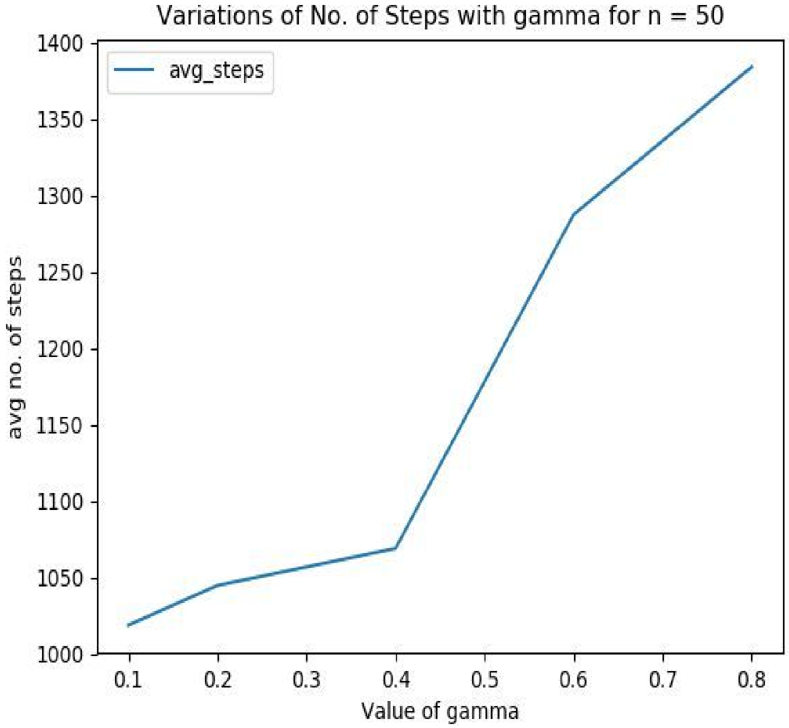
For n = 10 :

Value of Gamma	Average No. of Steps
0.1	245.3
0.2	222.8
0.4	240.5
0.6	343.12
0.8	480.32



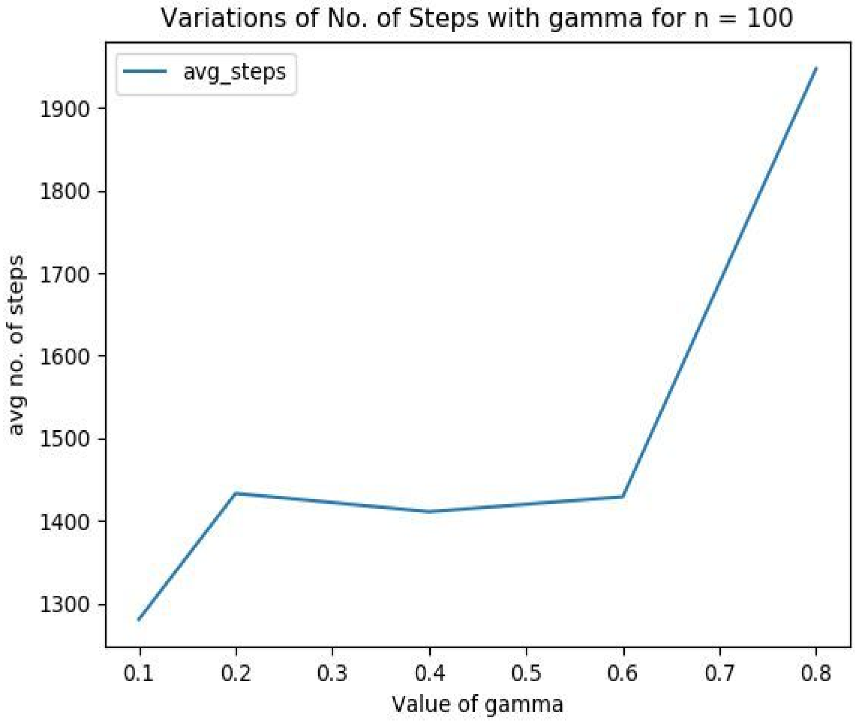
For n=50 :

Value of Gamma	Average No. of Steps
0.1	1018.76
0.2	1044.76
0.4	1068.9
0.6	1287.32
0.8	1383.88



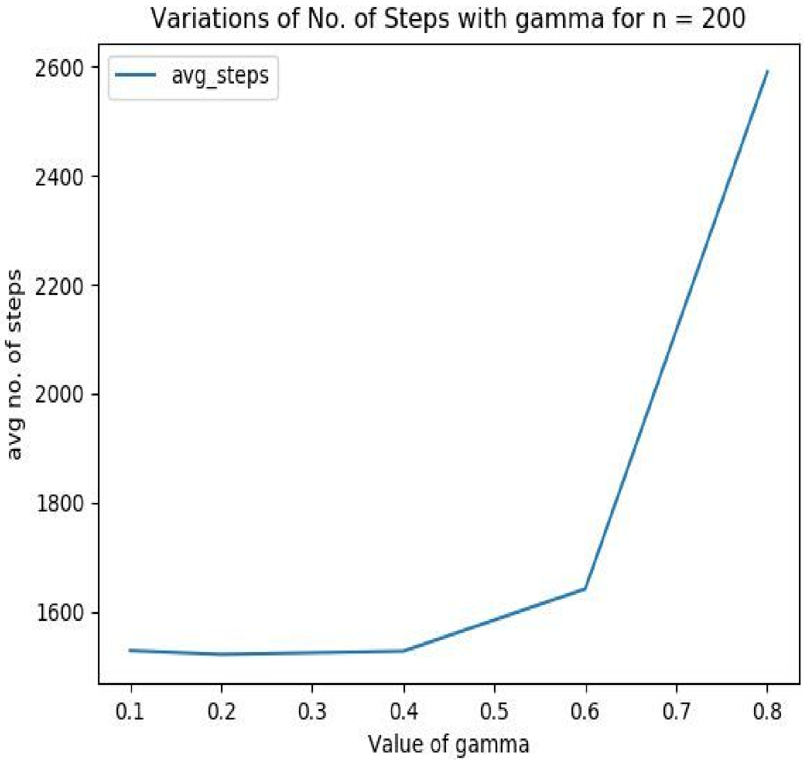
For n = 100 :

Value of Gamma	Average No. of Steps
0.1	1281.08
0.2	1433.42
0.4	1411.34
0.6	1429.32
0.8	1947.06



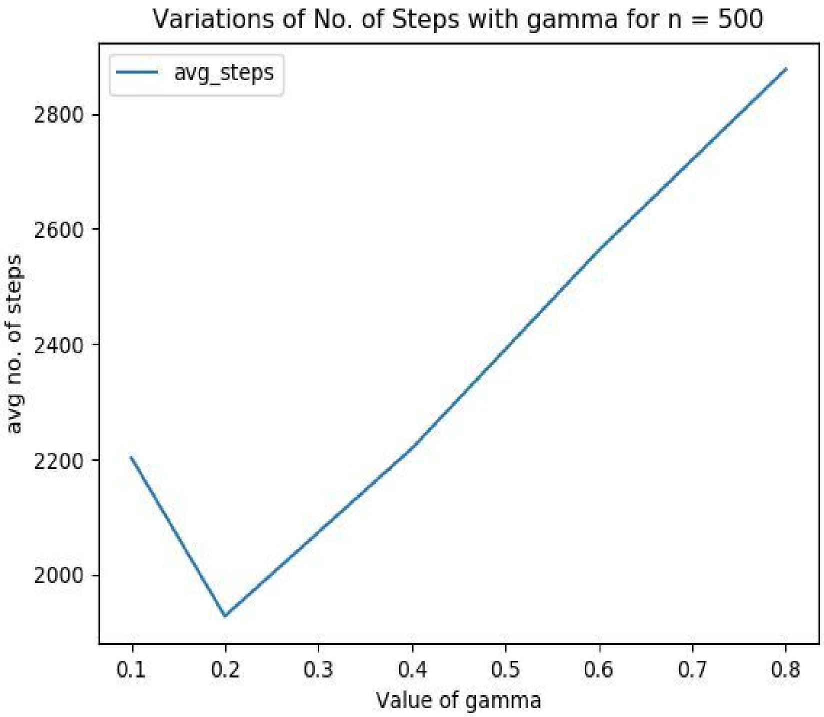
For n = 200 :

Value of Gamma	Average No. of Steps
0.1	1529.4
0.2	1522.36
0.4	1528.32
0.6	1642.46
0.8	2589.32



For n = 500 :

Value of Gamma	Average No. of Steps
0.1	2202.66
0.2	1927.96
0.4	2218.84
0.6	2562.78
0.8	2877.06

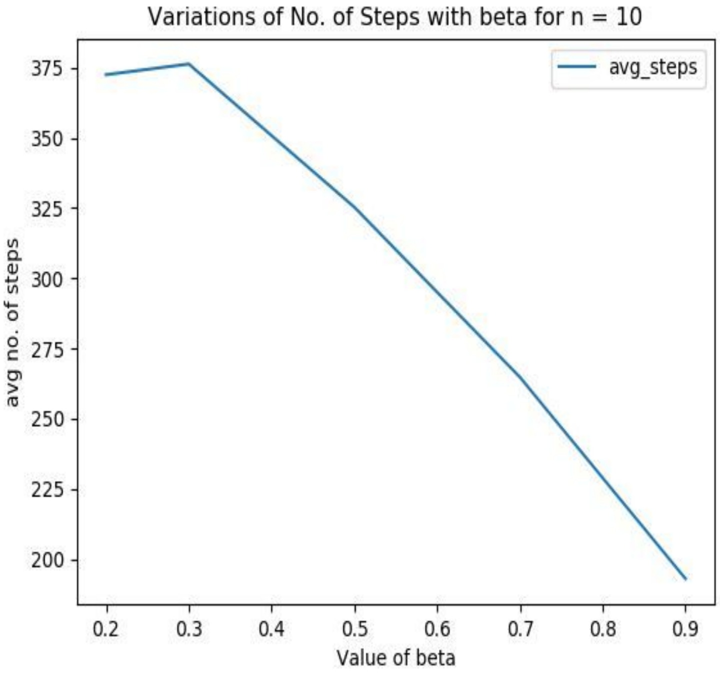


2. Variation with beta:

In the following experiments for a given value of n. The value corresponding to one beta value is obtained by fixing beta and then taking the average of number of steps taken over all values for gamma. Plot for these obtained values are also generated.

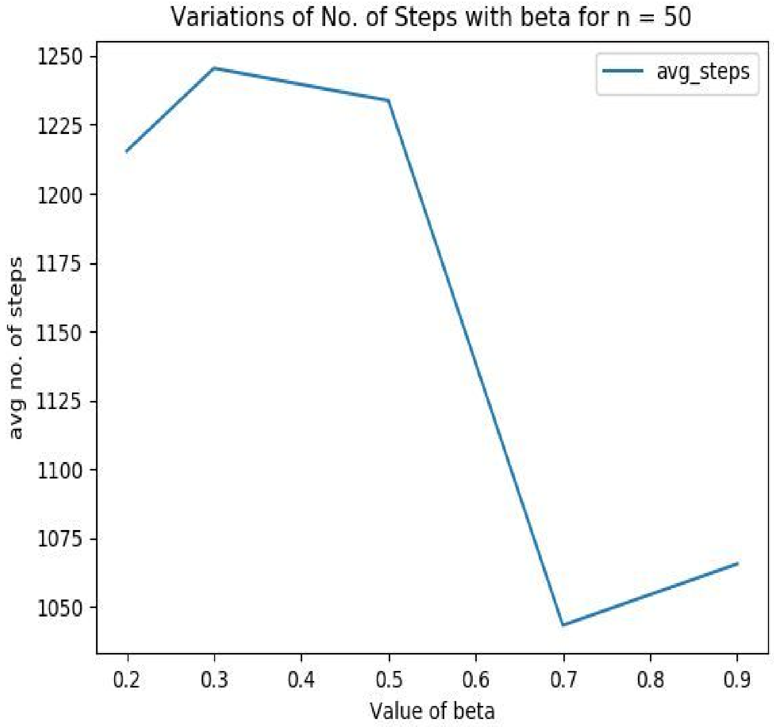
For n = 10 :

Value of beta	AverageNo. of Steps
0.2	372.52
0.3	376.32
0.5	325.36
0.7	264.8
0.9	193.04



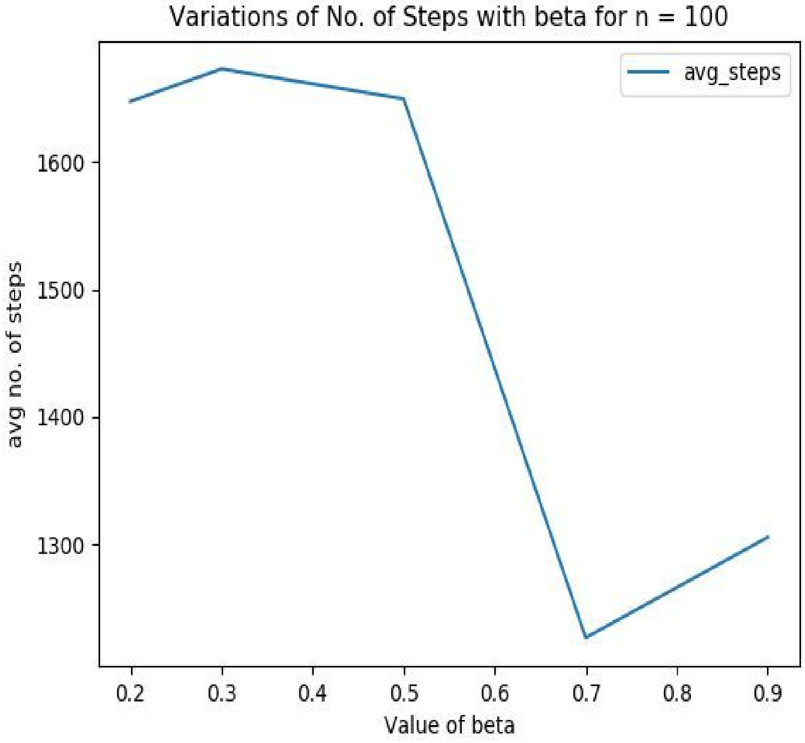
For n = 50 :

Value of beta	Average No. of Steps
0.2	1215.5
0.3	1245.52
0.5	1233.78
0.7	1043.28
0.9	1065.54



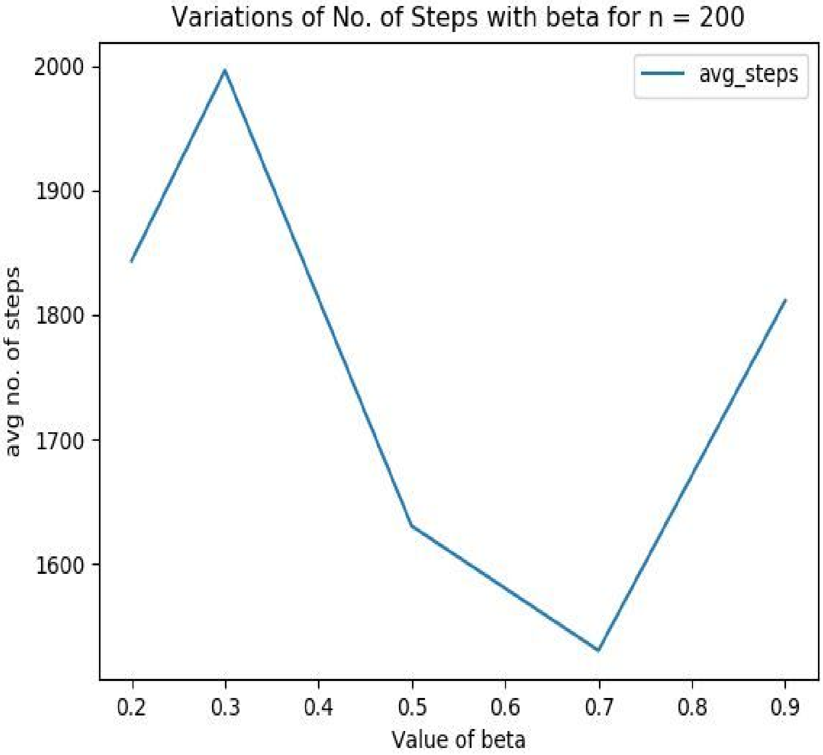
For n = 100 :

Value of beta	Average No. of Steps
0.2	1647.62
0.3	1673.06
0.5	1649.64
0.7	1226.5
0.9	1305.4



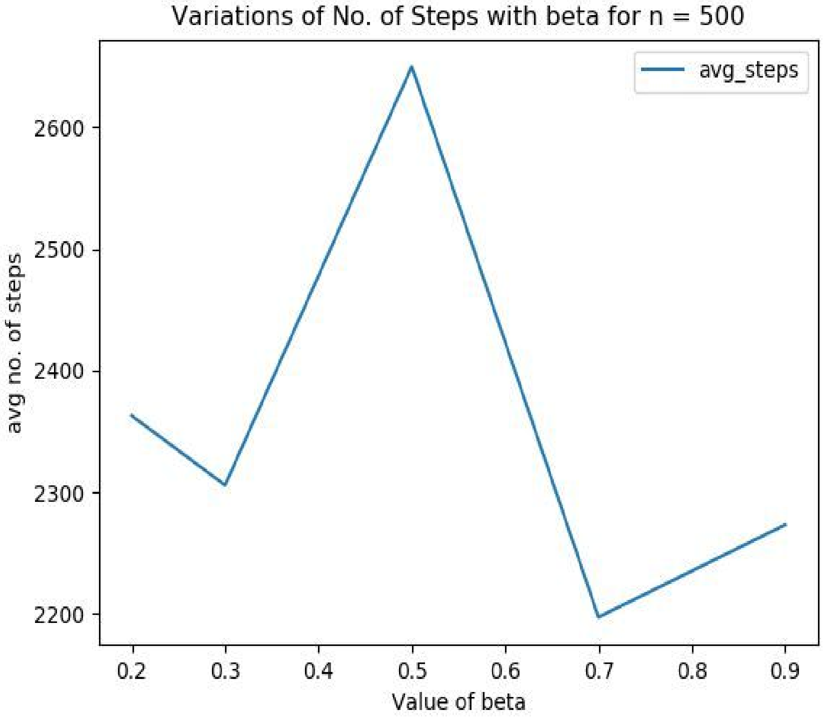
For n = 200 :

Value of beta	Average No. of Steps
0.2	1843.4
0.3	1996.32
0.5	1630.36
0.7	1530.42
0.9	1811.36



For n = 500 :

Value of beta	AverageNo. Of Steps
0.2	2362.98
0.3	2305.84
0.5	2649.9
0.7	2197.34
0.9	2273.24



Observations:

1. Both exact and inexact steepest descent algorithms converge faster for a matrix having low condition number as compared to a matrix having high condition number.
2. For a given values of n,Q and b the number of steps taken by inexact algorithm are more as compared to number of steps taken by exact algorithm(beta=0.5, gamma=0.2)
3. For inexact algorithm number of steps increase as we increase the value of gamma.
4. For inexact algorithm number of steps taken first increases slightly and then decreases (till beta=0.7) and then increases slightly as we increase the value of beta.