

Mtl712

Assignment 2

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1. Euler:

Euler:

$$y' = \lambda y \quad y(0) = 1$$

$$y_{n+1} = y_n + hF(x_n, y_n)$$

$$y_{n+1} = y_n + h\lambda y_n \quad h\lambda = z$$

$$y_{n+1} = (1+z)y_n$$

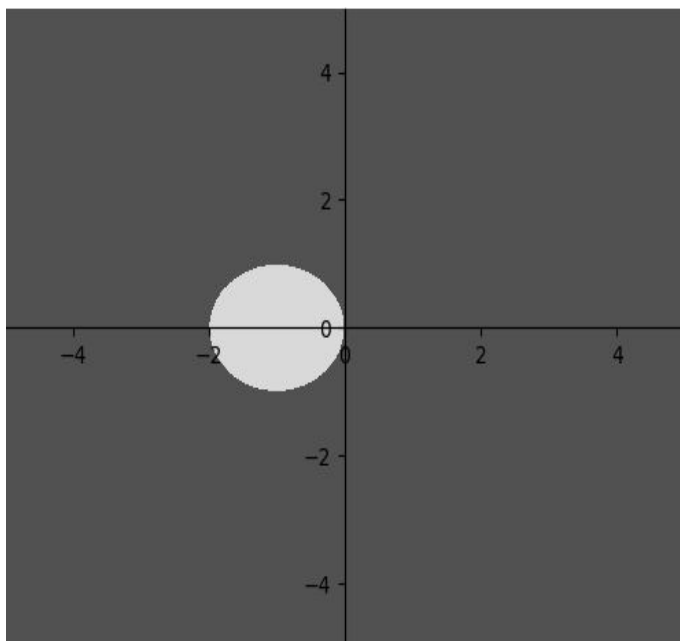
$$y_{n+1} = (1+z)^{n+1} y_0$$

$$|(1+z)| < 1$$

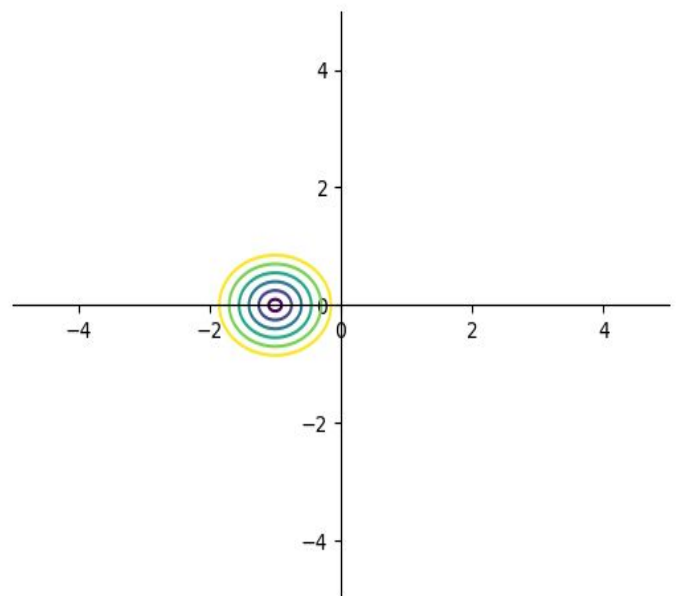
$$\Rightarrow |1+z| < 1$$

$$|1+i\lambda| < 1$$

$$1 - \lambda^2 < 1$$



Surface plot

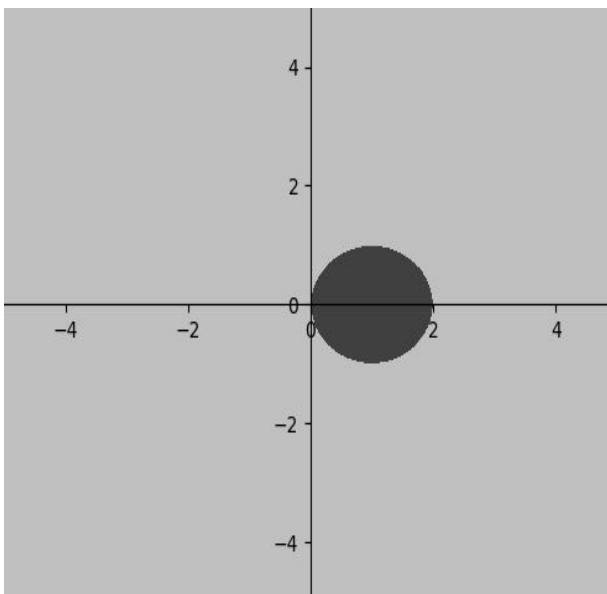


Contour

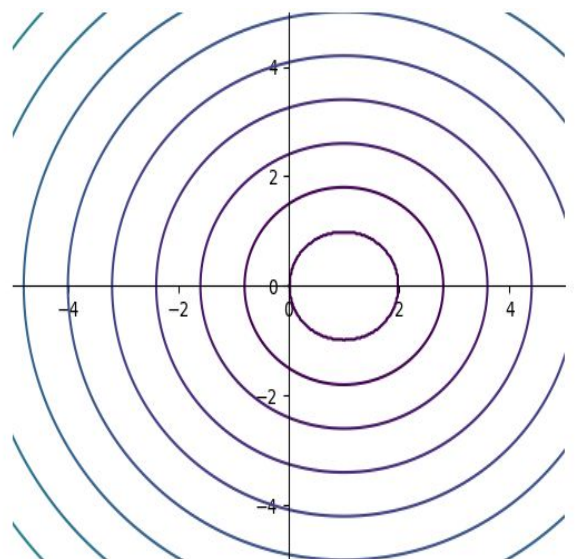
2. Backward Euler:

Backward Euler:

$$y_{n+1} = y_n + h f(x_{n+1}, y_{n+1})$$
$$y_{n+1} = y_n + h \lambda y_{n+1}$$
$$h\lambda = z$$
$$y_{n+1} = y_n + z y_{n+1}$$
$$y_{n+1} = y_n \left(\frac{1}{1-z} \right)$$
$$y_{n+1} = y_0 \left(\frac{1}{1-z} \right)^{n+1}$$
$$\Rightarrow \left| \frac{1}{1-z} \right| < 1$$
$$|1-z| > 1$$
$$|z-1| > 1$$



Surface plot(All area except circular region)



Contour plot

3. Modified Euler:

Improved Euler:

$$y_{n+1} = y_n + \frac{h}{2} (f(x_n, y_n) + f(x_{n+1}, y_{n+1}))$$

$$y_{n+1} = y_n + \frac{h}{2} [f(x_n, y_n) + f(x_n + h, y_n + hf(x_n, y_n))]$$

$$y_{n+1} = y_n + \frac{h}{2} [\lambda y_n + \lambda(y_n + h\lambda y_n)] \quad \downarrow \quad f(x_n, y_n) = \lambda y_n$$

$$y_{n+1} = y_n + \frac{1}{2} [\lambda h y_n + \lambda h(y_n + \lambda h y_n)]$$

$$\lambda h = z$$

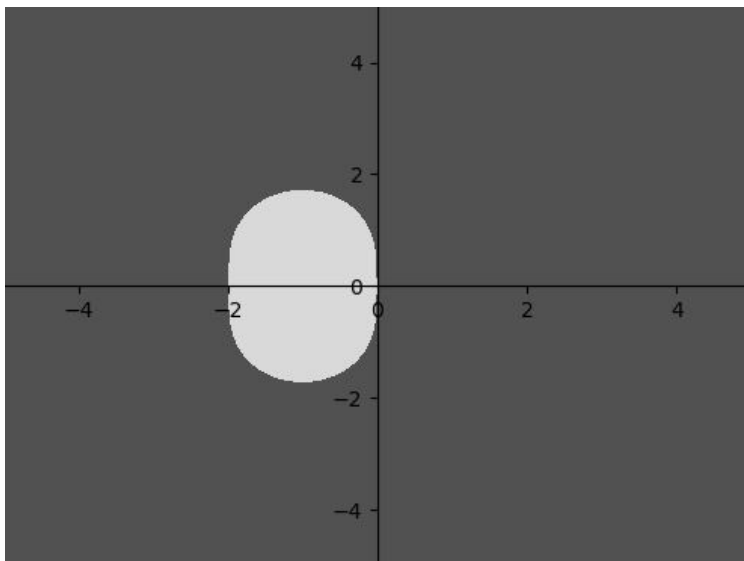
$$y_{n+1} = y_n \left[1 + \frac{1}{2} [z + z(1+z)] \right]$$

$$y_{n+1} = y_n \left[\frac{z^2 + z + 1}{2} \right]$$

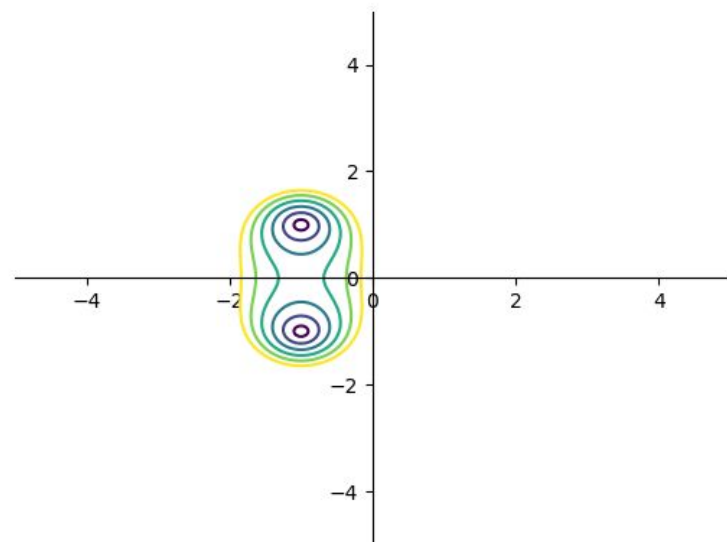
$$y_{n+1} = y_0 \left[\frac{z^2 + z + 1}{2} \right]^{n+1}$$

$$\left| \frac{z^2 + z + 1}{2} \right| < 1 \quad \text{for } z = x + iy$$

$$\left(\frac{x^2 - y^2 + zx + z}{2} \right)^2 + (y(1+x))^2 < 1$$



Surface plot



Contour plot

4. 2nd Order Runge Kutta:

2nd order Runge Kutta:

$$K_1 = F(x_n, y_n) = \lambda y_n$$

$$K_2 = F\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} K_1\right) = \lambda \left(y_n + \frac{h}{2} K_1\right)$$

$$y_{n+1} = y_n + K_2 h$$

$$y_{n+1} = y_n + \lambda h \left(y_n + \frac{\lambda h}{2} y_n\right)$$

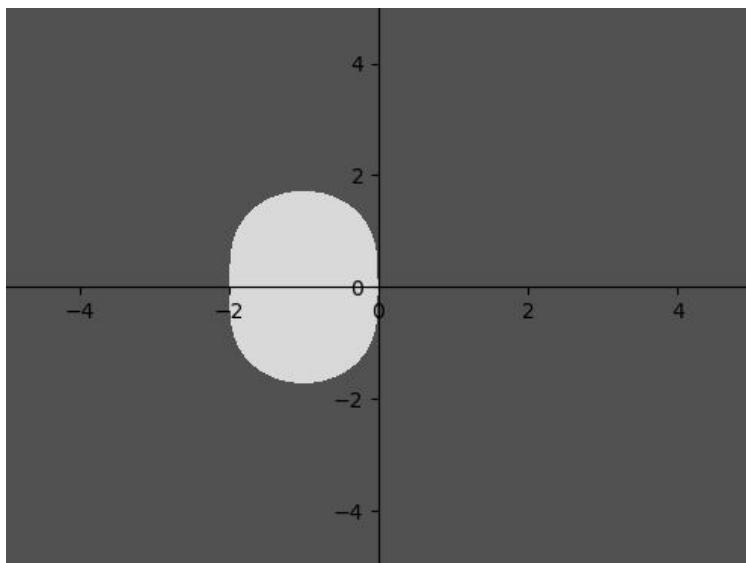
$$\lambda h = z$$

$$y_{n+1} = y_n + z \left(y_n + \frac{z}{2} y_n\right)$$

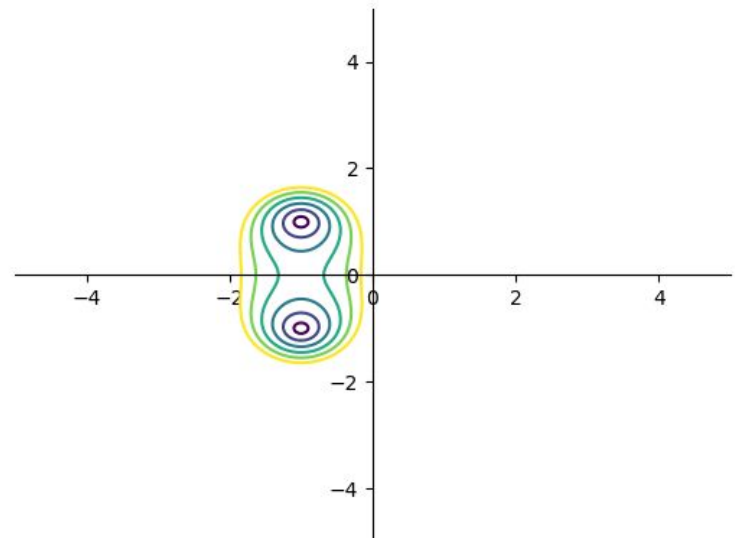
$$y_{n+1} = y_n \left(1 + z + \frac{z^2}{2}\right)$$

$$y_{n+1} = y_0 \left(1 + z + \frac{z^2}{2}\right)^{n+1}$$

$$\left|1 + z + \frac{z^2}{2}\right| < 1$$



Surface plot



Contour plot

Discussion:

1. Stability region of Euler is a circle centered at $(-1,0)$
2. Stability region of backward Euler is a region except for a circle whose center is $(1,0)$
3. Stability region of Modified Euler and 2nd order Runge Kutta is ellipse.