Generate random numbers from beta distribution using the Acceptance-Rejection Method

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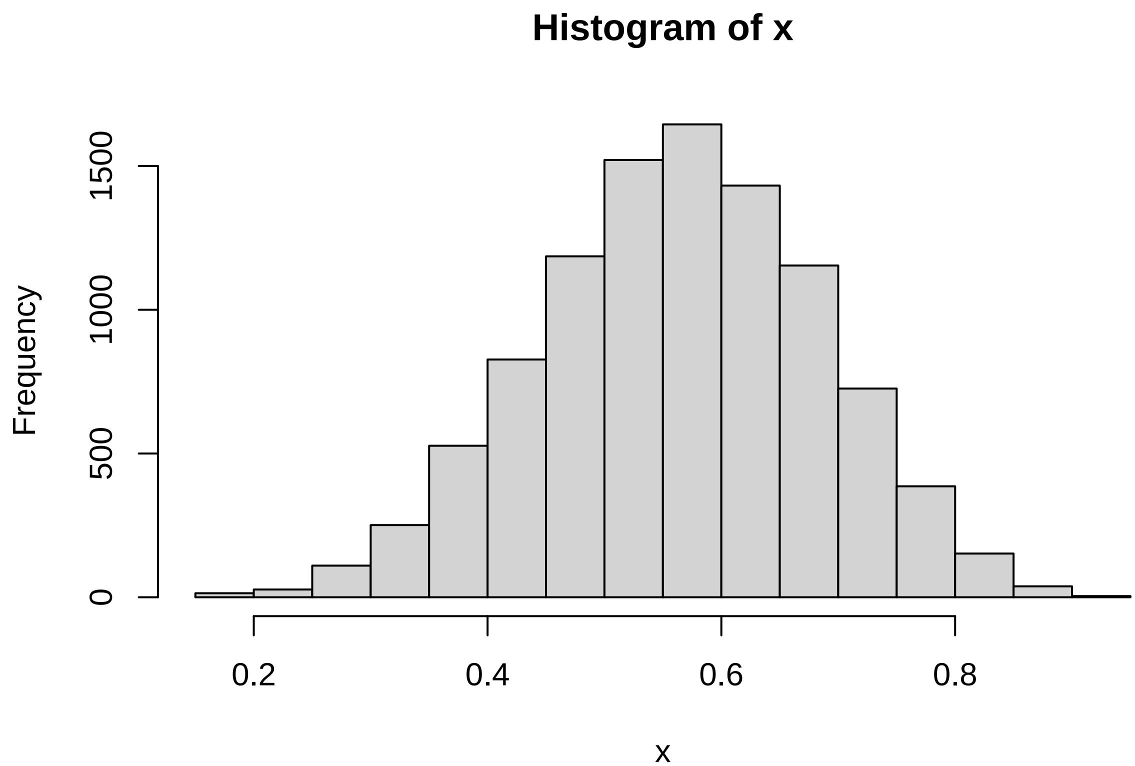
$\alpha = 9$
$\beta = 7$

Now we will compute the optimal `c`.We know that `Beta` distribution has the followingb density function:
$f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}$, $x \in (0,1)$
where:
$B(\alpha,\beta) = \left(\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\, dx\right)$

so we will have:
$B(9,7) = \frac{1}{45045}$
and $f(x)$ will be maximum when $x = \frac{4}{7}$:
$max f(x) = 45045 * x^8(1-x)^6 \approx 3.17... < 3.18$

```
set.seed(666)
n <- 10000
c <- 3.18
x <- rep(0,n)
for(i in 1:n){
  repeat{
    y <- runif(1)
    u <- runif(1)
    if(u < (dbeta(y,9,7)/(c * dunif(y)))) break
  }
  x[i] <- y
}
```

```
hist(x)
```

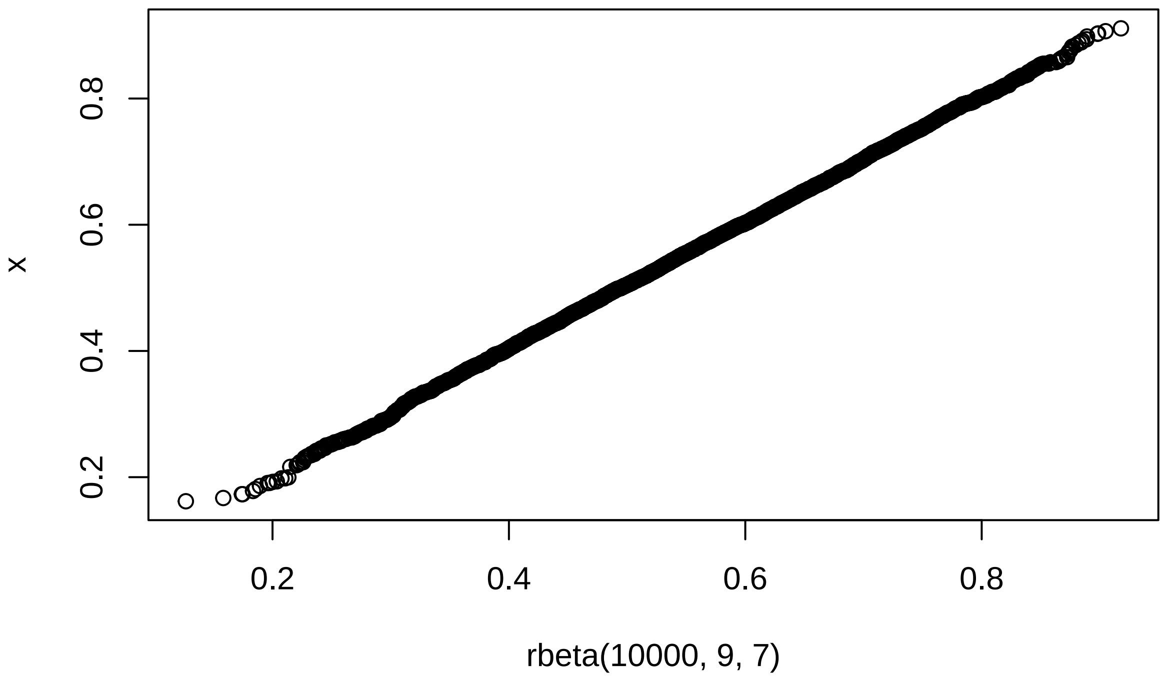



```
summary(x)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.1619  0.4832  0.5653  0.5638  0.6484  0.9112
```

We will use `qqplot` to see if our generated numbers have beta distribution:

```
set.seed(666)
qqplot(rbeta(10000,9,7),x)
```



based on the qqplot above,we can say that our generated numbers have beta distribution.