MATH96012 Project 1

Alexander John Pinches CID:01201653

November 1, 2019

Part 2

0.1 2.1

We see in figure 1 variability in α seems to increase as we increase A. This would make sense as increasing A increases the effect of the random component of the directions of the particles. α initially varies lots before becoming more stationary this seems to happen after the first 50 or so time steps. We can remove the initial transient and use a high Nt to reduce the effect of this. For $N = 32$ there seems to be less variation. We would expect this as more particles interacting with each other should force them into alignment faster.There also seems to be a point where the variability of the series changes rapidly with $A = 0.2$ being almost constant after the initial transient. The variability in the $N = 16$ series seems to increase more as we increase A this is most noticible when $A = 0.6$. If we use the variance and mean after the initial 50 time points in figure 2 the dotted lines represent the mean plus and minus two times the standard deviation. We see that mean α seems similar for both $N = 16, 32$ but the width of the dotted bars around the mean are smaller suggesting that the variability of α is smaller. We also see that the width increases from 0.2 to about 0.6 then reduces again as A reaches 0.8 suggesting there's a point around 0.6 where the variability of α is greatest. We can also see this in figure 3 that the variance has a maximum at $A = 0.6$ and that the variance of $N = 16$ is higher than that of $N = 32$. with both increasing rapidly from $A = 0.4$ to $A = 0.6$.

0.2 2.2

When trying to find A^* using the minimise function from scipy on the negative variance of A would find solutions outside the bounds or for methods we can bound they wouldn't converge or explore the space. Instead I opted to just perform a grid search of the parameter space as for $N = 32$ and $Nt = 1000$ this is still rather fast even when searching 1201 points. Allowing us to get of an estimate of A^* down to the 0.005. However, due to the stochastic nature of the system in general the result is similar between runs it can differ.In figure 4 we plot $\text{Var}[\alpha]$ against A and we mark A^* with a blue dot. We see that the variance of α increases slowly then rapidly up from 0.6 and down again after which is what we expect from the plot in figure 4. We could maybe apply some filter to remove the noise or fit a curve and take the max of that but whether this is actually anymore accurate I am unsure.

To see the dependence of α on $1 - \frac{A}{A^*}$ we plot them against each other figure 5. Showing the mean α as a red line and the plus and minus two times the standard deviation as blue dashed lines. We see as $1 - \frac{A}{A^*}$ increases the variation decreases and the mean increases which is as we would expect as alpha varies most when A is close to A^*. We see close to 0 ie when A is close to A^* the variation in alpha increases rapidly as we would expect. To see this better in figure 6 we plot $\mathrm{Var}[\alpha]$ against $1 - \frac{A}{A^*}$ and see the variance increases asymptotically as $1 - \frac{A}{A^*} \to 0$ and rapidly approaches 0 as it moves to the right.

Figures

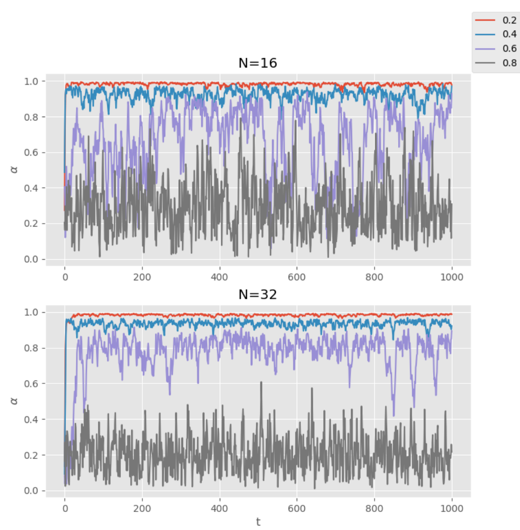

Figure 1: α over time for $N = 16, 32$

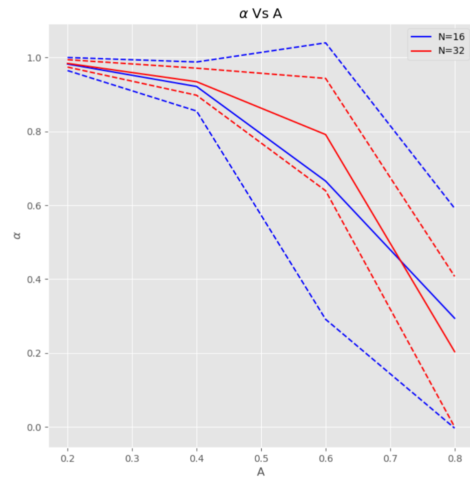

Figure 2: Mean α (red) for $N = 16, 32$ with first 50 steps omitted and $\pm 2\sigma[\alpha]$ dotted lines over A

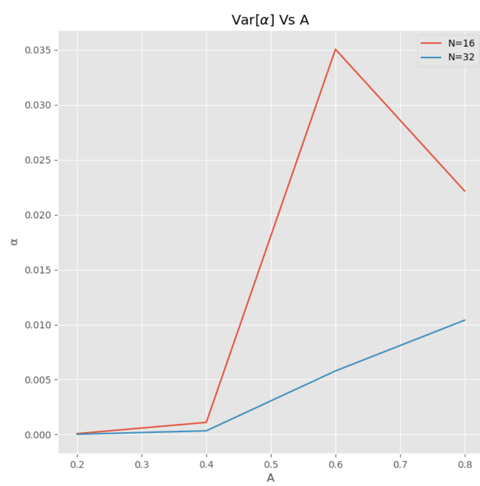

Figure 3: Variance of α against A for $N = 16, 32$

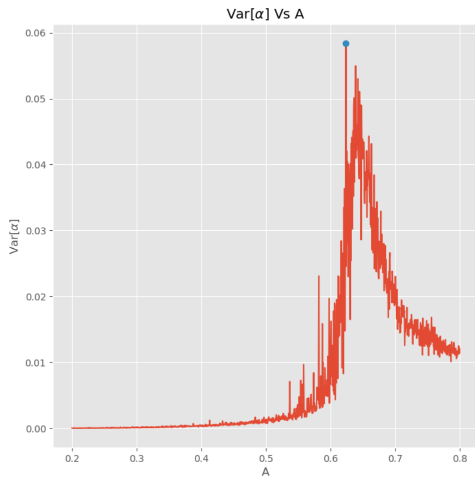

Figure 4: Var[α] against A with A^* represented by a blue dot

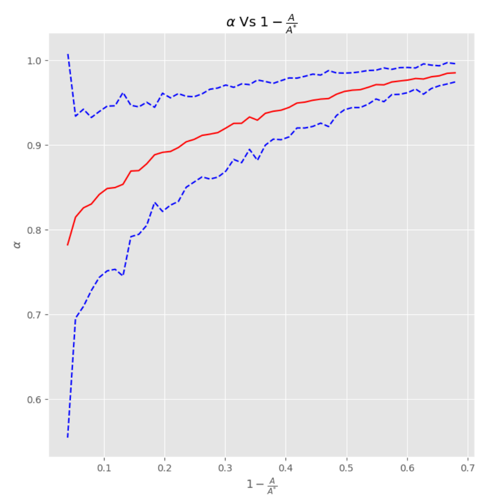

Figure 5: Mean α with against $1 - \frac{A}{A^*}$

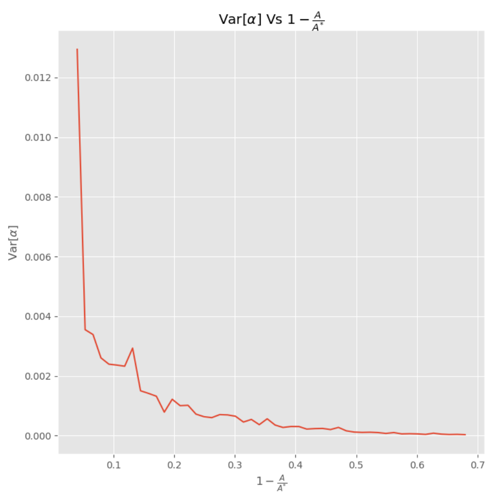

Figure 6: Var[α] with $\pm 2\sigma[\alpha]$ dotted lines against $1 - \frac{A}{A^*}$