Christian Parra
A16402854
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Shaft Design Code Writeup

Introduction

Shafts are commonly used in order to transfer rotational mechanical movement from one location to another. For example shafts are used to help transfer the rotational movement generated by a car engine to the wheels of the car. In order for shafts to function, many times they need to have differing diameters in order to fit equipment like gears and bearings in place. When a shaft has 2 different diameters, there are many new factors that must be taken into account in order to determine the minimum allowable shaft diameter for a specific material.

The task was to determine the minimum allowable diameter of a shaft with 2 different diameters and a filet connecting them. The shaft is rotating and experiencing both torsion and moment at varying degrees. The problem with finding the allowable diameter in this situation is that the equation that is used to find this diameter includes variables that depend on the diameter of the shaft in order to be determined. Therefore the diameter must be calculated using an iterative method, where with each iteration the allowable diameter found eventually converges to find the most accurate diameter.

The equation used to determine the minimum diameter is based off of the Modified Goodman line, which was developed in England in 1899. This equation was determined by using empirical data to estimate the mean stress effects on fatigue life cycle on a straight line. An 'infinite' life cycle can be achieved with alternating stresses when they remain on this line defined in the equation below.

$$\frac{\sigma_a}{\sigma'_e} + \frac{\sigma_m}{\sigma'_u} = \frac{1}{n}$$

In order to determine the diameter of the shaft, the alternating stress (σ_a) and the mean stress (σ_m) need to be decomposed into the variables that determine them. For a shaft with alternating moments and alternating torsion, normal and shear stresses are defined by the equations below.

$$\sigma_a = K_f \frac{M_a c}{I} \qquad \tau_a = K_{fs} \frac{T_a c}{J}$$

The equations for the moments of inertia and polar moments of inertia for a circular shaft are then substituted in, along with knowing that c is equal to half the diameter of a circular shaft. The solutions are shown below.

$$\sigma_a = K_f \frac{32 M_a}{\pi d^3} \qquad \tau_a = K_{fs} \frac{16 T_a}{\pi d^3}$$

A very similar procedure is used to find the mean stress and mean torsion that is produced by the mean moment and torque experienced by the shaft.

$$\sigma_m = K_f \frac{32M_m}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16T_m}{\pi d^3}$$

In order to determine the total alternating stress that the shaft experiences based on contributions from moments and torques, the 2D von mises equation below is used.

$$\sigma' = \sqrt{\sigma_x^2 + \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$$

Because only one stress and one torsion contribution exists, the new von mises equation for both alternating and mean becomes

$$\sigma_a = \sqrt{\sigma_a^2 + 3\tau_a^2} \quad \text{and} \quad \sigma_m = \sqrt{\sigma_m^2 + 3\tau_m^2}$$

Then the torsion and moment equations are substituted in resulting in

$$\sigma_a = \sqrt{(\frac{32M_a K_f}{\pi d^3})^2 + 3(\frac{16T_a K_{fs}}{\pi d^3})^2} \quad \text{and} \quad \sigma_m = \sqrt{(\frac{32M_m K_f}{\pi d^3})^2 + 3(\frac{16T_m K_{fs}}{\pi d^3})^2}$$

These resulting stresses can then be substituted into the modified goodman line twitch then becomes

$$\frac{1}{n} = \frac{16}{\pi d^3} \{\frac{1}{S_e} [\, 4(K_f M_a + 3K_{fs} T_a]^2 + \frac{1}{S_{ut}} [\, 4(K_f M_m + 3K_{fs} T_m]^2 \}$$

Which can then be rearranged in order to find d and have the factor of safety (n) as a known number.

$$d = (\frac{16n}{\pi} \{\frac{1}{S_e} [\, 4(K_f M_a + 3K_{fs} T_a]^2 + \frac{1}{S_{ut}} [\, 4(K_f M_m + 3K_{fs} T_m]^2 \})^{1/3}$$

In this equation, Se is dependent on the value of the ultimate stress that the chosen material can withstand and on different fatigue factors as seen below.

$$S_e = k_a\, k_b\, k_c\, k_d\, k_e\, k_f\, S'_e$$

This code takes into account the surface factor (k_a), size factor (k_b), and reliability factor (k_e). Loading factor (k_c) was not incorporated during this part because loading factor is automatically taken into account when using von mises when the total alternating and mean stresses experienced by the shaft are calculated. Temperature factor (k_d) was neglected because this code assumes that the shaft remains at room temperature.

The other factors that are incorporated are the fatigue stress concentration factors for moments and axial loads K_f and torsional loads K_{fs}. These factors are determined by the stress concentration factor K_t and notch sensitivity factor q.

The challenge in this project was to find the final shaft diameter which is dependent on stress concentration factor K_t and size factor k_b, which in themselves also depend on the final diameter of the shaft. In order to overcome this challenge, for the first iteration, typical values of K_f and k_b were used in order to obtain a first guess of the diameter. After this, the obtained diameter was then used to calculate new K_t and k_b values that would then be used to find the next diameter iteration. This process was repeated until the difference between the most recent diameter iterations was less than one tenth of a percent.

Results

This code starts off by determining the material and its properties that are going to be used for the shaft. The user first has the option of choosing to input properties of a custom material or choosing a material from the various ASTM steels that are either Hot-Rolled or Cold-Drawn that are embedded within the code. This selection will set the ultimate stress and the yield stress that are going to be used later on. The ultimate stress that was inputted is used in order to determine the endurance limit S'. The endurance limit is determined differently depending on the magnitude of the ultimate stress. If the ultimate stress is less than 200 kpsi, the the equation below for S' is valid.

$$S' = .5\ S_{ut}$$

However, if the material's ultimate stress is above 200 kpsi then a standard S' of 100 kpsi can be used.

Then the surface finish is chosen. If a custom material was selected, the user will select their desired surface finish. If a material from the saved steels is chosen, the user will have the option of choosing their own or choosing the finish that the processing style dictates. The selected surface finishes are then used with the defined ultimate stress to determine the surface factor k_a of the material. The equation that is used in all 4 cases is shown below.

$$k_a = a\ S_{ut}^{\ b}$$

The coefficients a and b that are used in the equation above are determined by the factors in table 1 below which are determined by the chosen surface finish.

Surface Finish	Factor a	Exponent b
Ground	1.34	-0.085
Machined or Cold-Drawn	2.70	-.0265
Hot-Rolled	14.4	-0.718
As-Forged	39.9	-.995

Table 1: coefficients to determine surface factor k_b for Ultimate stress in units of kpsi

Then the Moments and torques that the shaft experiences are imputed. The minimum and maximum Moments and torques are inputted where their magnitude does not exceed 60000 lb-in because at this point, the estimation of the shaft diameter becomes unreliable. With the imputed moments and torques, the Mean and alternating magnitudes can then be calculated using the equations below

$$T_{alternating} = \frac{T_{max} - T_{min}}{2} \qquad M_{alternating} = \frac{M_{max} - M_{min}}{2}$$

$$T_{mean} = \frac{T_{max} + T_{min}}{2} \qquad M_{mean} = \frac{M_{max} + M_{min}}{2}$$

Then the ratios of D/d and r/d are selected by the user. The user has the option of selecting D/d ratios from 1.1 to 1.3 in increments of .05. The range of D/d ratios that can be selected is limited to cases that are more likely to be occurring for a shaft of this type. Additionally, having D/d ratios in either extreme makes the shaft diameter prediction either unreliable or makes it unreasonable. The user has the option of selecting the r/d ratio of the filet as either .1 which is a well rounded shoulder or .2 which is a sharp shoulder. This is done because the first iteration of the fatigue stress concentration factors has commonly accepted first iteration estimations. These estimations are listed in table 2 below.

Shoulder Filet type	Bending (K_f)	Torsional (K_{fs})
Sharp (r/d =.02)	2.7	2.2
Well rounded (r/d) = .1	1.7	1.5

Table 2: first iteration estimations of K_f and K_{fs}

Then the factor of safety, n, is inputted by the user with no limitations. The last factor chosen by the user is the reliability factor k_e. The use is given 6 options ranging from 50% to 99.99% as defined in table 3 below.

Reliability [%]	50	90	95	99	99.9	99.99
Reliability factor [k_e]	1.000	0.897	0.868	0.814	0.753	0.702

Table 3: Reliability factors, k_e

At this point all of the inputs that are user selected have been determined. At this point, the final endurance limit S_e can be found using all of the corrective factors determined above, in addition to an estimation of .9 for the size factor, k_b due how this factor can only be determined with a known diameter which has not been calculated yet.

Once the diameter is calculated, the code enters an iteration stage. The size factor k_b and fatigue factors K_f and K_{fs} can now be calculated more accurately. The size factor is now calculated using 2 equations depending on the size of the shaft. If the shaft diameter is between .3 and 2 inches or equal to 2 inches, the equation below is used.

$$k_b = (\frac{d}{.3})^{-.107}$$

If the shaft diameter is between 2 and 10 inches or equal to 10 inches, the equation below is used.

$$k_b = .91d^{-.157}$$

At this point the fatigue stress concentration factors for moments (K_f) and torsional (K_{fs}) loads can be calculated. They are calculated using the following equations, there K_t and K_{ts} are the stress concentration factors for moment and torsional loads and where q and q_s are the notch sensitivity factors

$$K_f = 1+q(K_t-1);$$

$$K_{fs} = 1+q_s(K_{ts}-1);$$

The stress concentration factors K_t and K_{ts} are calculated using the provided D/d and r/d ratio and using the equation

$$K_{t,ts} = A\left(\frac{r}{d}\right)^b$$

Where the factors A and b are determined using the table below depending on if the concentration factor is applied or moment or torsional loads and then, the provided D/d. If a D/d that is provided does not fit a line directly, then its value is interpolated using the two D/d lines that surround that value.

K_t			K_{ts}		
D/d	A	b	D/d	A	b
1.50	0.93836	-0.25159	1.33	0.84897	-0.23161
1.20	0.97098	-0.21796	1.20	0.83425	-.021649
1.10	.095120	-0.23757	1.09	0.90337	-0.12692

Table 4: coefficients to determine stress concentration factors K_t and K_{ts}

Finally the notch sensitivity factors q and q_s are calculated using the following equations

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}}$$

The two equations below that determine $\sqrt{a}$ are only valid if the ultimate allowable stress is between 50 and 250 kpsi and S_{ut} values must be in kpsi.

$$\sqrt{a}_{\text{Bending moment}} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S^2_{ut} - 2.67(10^{-8})S^3_{ut}$$

$$\sqrt{a}_{\text{Torsion}} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S^2_{ut} - 2.67(10^{-8})S^3_{ut}$$

At this point all of the necessary values for the second iteration of the diameter calculation have been determined. The factors k_b, q, and q_s will continue to be calculated until the two most recent diameter calculations are within .01% of each other. At this point an accurate value for d has been determined and any additional iterations will not yield any significant increase in accuracy.

At this point the code then checks if the shaft will yield using the equation below.

$$n_y = \frac{S_y}{\sigma'_{max}}$$

where

$$\sigma'_{max} = \sqrt{(\frac{32M_aK_f}{\pi d^3})^2 + 3(\frac{16T_aK_{fs}}{\pi d^3})^2} + \sqrt{(\frac{32M_mK_f}{\pi d^3})^2 + 3(\frac{16T_mK_{fs}}{\pi d^3})^2}$$

If the shaft diameter that is found leads to a yielding safety factor (n_y) that is less than one, the code will tell the user to run the code again using smaller forces or a different material with a higher yielding stress in order to find a valid minimum diameter.

Some example inputs to this code are displayed in table 5 below.

Test #	Material	S_u [kpsi]	S_y [kpsi]	Surf finish	M_{min} [lb-in]	M_{max} [lb-in]	T_{min} [lb-in]	T_{max} [lb-in]	reliability	D/d	r/d	n
1	custom	75	50	machined	1000	5000	0	1800	99.99%	1.2	.1	1.5
2	1035 CD	80	67	CD	5000	10000	2000	4000	99.99%	1.3	.02	1.5
3	1015 HR	50	27.5	HR	-6000	15000	-50	5000	99%	1.1	.1	2
4	1050 CD	100	84	CD	-60000	60000	-60000	60000	99.9	1.3	.02	2

Table 5: Examples of input values into shaft code

The outputs for the examples shown in table 5 are shown in table 6

Test #	d first iteration [in]	# d iterations [in]	D final [in]
1	1.5553	3	1.5137
2	2.0827	4	1.9137
3	2.9865	3	3.0045
4	5.3085	4	5.4001

Table 5: shaft code sample outputs

Discussion

For all of the varying inputs that were entered into the code, even with torques and moment combinations being set to the maximum amount that the code is set to allow, the final shaft diameters were reasonable and the diameters converged quickly. After 4 tests with varying parameters, the code only performed a maximum of 4 iterations in order to converge to a final minimum shaft diameter, with the exit condition of the code only allowing for a .1% difference in diameters between the last two iterations.The shaft diameters that were found also incorporated several options of the code, including several D/d and r/d ratios. The largest diameter of 5.4001 inches was reached by setting both the torque and moments to have minimums and maximums of 60000 lb-in. Considering the large magnitude of these stresses, the code managed to converge quickly and still give a reasonable shaft diameter that is within the accurate calculation limits of between .3 in and 10 in. In a

Conclusion

In order to design a shaft, there are many factors that must be chosen in order to create an effective design that is also known to be safe. The code that I developed had many advantages in terms of both the wide options that it allows the user to suggest while also adding some limitations so that the user inputs options that will not invalidate the code. For example, the user can enter 23 different steel options that are already embedded into the code, in addition to custom material options if the user so desires. The users can also input Moment and Torques that range from -60000 lb-in to 60000 lb-in, while also checking the inputs to ensure that the minimum values are not swapped and allowing the user to re-input these values incase there was a mistake. The code also allows for 6 reliability options for the user to choose from. And if the diameter that was chosen leads to a yielding factor of safety that is less than one, the code shows recommendations on what inputs to possibly change in order to obtain a valid output of a minimum diameter.

Some additional suggestions that this code can make for the future is to include more material options that are already built into the code. Another suggestion is incorporating a way to only change one factor while keeping the rest the same in order for the user to perform trial and error of several factors more quickly. Another suggestion to this code is also incorporating more types of additions that can be included in shaft design such as keyholes. Designing a code to calculate one aspect that is ready for user customization is an intensive process where the engineer not only needs to input the necessary formulas correctly, but where the engineer also needs to think of how to help guide a user who may not know the intricacies of shaft design to include valid inputs, and provide recommendations if the initial inputs are not valid.