Weekly Exercises - Statistical Inference

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1 Week 9

1. [1, 7.9] Let $X_1, \ldots, X_n$ be an iid sample with the same pdf

$$f(x|\theta) = \frac{1}{\theta}, \quad 0 \le x \le \theta, \quad \theta > 0.$$

Estimate θ using both the method of moments and maximum likelihood. Calculate the means and variances of the two estimators: Which one should be preferred and why?

2. [1, 7.19](This problem might be a little bit longer, but it is very relevant). Suppose that the random variables $Y_1, \ldots, Y_n$ satisfy

$$Y_i = \beta x_i + \epsilon_i, \quad i = 1, \ldots, n$$

where $x_1, \ldots, x_n$ are fixed constants, and $\epsilon_1, \ldots, \epsilon_n$ are iid $n(0, \sigma^2)$, where σ is unknown. (Hint: Check the Theorem 5.4.4 in [1] to find the distribution of the ordered statistics)

   (a) Find a two-dimensional sufficient statistic for (β, σ^2).

   (b) Find the MLE of β, and show that it is an unbiased estimator of β.

   (c) Find the distribution of the MLE of β.

3. Let $\mathbf{X} = \{X_1, \ldots, X_n\}$ be an i.i.d. sample from an exponential distribution with scale parameter $\theta > 0$,

$$f(x|\theta) = \begin{cases} \frac{1}{\theta} e^{\frac{-x}{\theta}} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the MLE $\hat{\theta}_{\text{MLE}}$ for θ.

   (b) Compute the MSE (mean squared error) for $\hat{\theta}_{\text{MLE}}$, is this a biased estimator?

   (c) Find a UMVUE for θ. You may use the fact that the family of exponential distributions is an exponential family.

References

[1] G. Casella and R. L. Berger. *Statistical inference*. Cengage Learning, 2021.