Weekly Exercises - Statistical Inference

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1 Week 7

1. Suppose $X_1, X_2, \ldots, X_n$ is an iid sample where $X_i \sim \text{Bernoulli}(p)$. What is the distribution of $Y = \sum_{i=1}^n X_i$?

2. [1, 5.3] Let $X_1, X_2, \ldots, X_n$ be iid random variables with continuous cumulative distribution function F_X, and suppose $E(X_i) = \mu$. Define the random variables $Y_1, \ldots, Y_n$ by

$$Y_i = \begin{cases} 1 & \text{if } X_i \geq \mu \\ 0 & \text{if } X_i \leq \mu. \end{cases}$$

   Find the distribution of $\sum_{i=1}^n Y_i$.

3. Read Section 5.3.1 in [1]. The goal is to understand the properties in the following theorem and the proof:

   Theorem 1.1. *Let $X_1, \ldots, X_n$ be a random sample from a $N(\mu, \sigma^2)$ distribution, and let $\overline{\mathbf{X}} = (1/n)\sum_{i=1}^n X_i$ and $S^2 = [1/(n-1)]\sum_{i=1}^n (X_i - \overline{X})^2$. Then*

   (a) $\overline{\mathbf{X}}$ and S^2 are independent random variables,

   (b) $\overline{\mathbf{X}}$ has a $N(\mu, \sigma^2/n)$ distribution,

   (c) $(n-1)S^2/\sigma^2$ has a chi-squared distribution with $n-1$ degrees of freedom.

4. [1, 6.6] Let $\mathbf{X} = \{X_1, \ldots, X_n\}$ be a random sample from a $\Gamma(\alpha, \beta)$ population. Find a two dimensional sufficient statistic for (α, β).

References

[1] G. Casella and R. L. Berger. *Statistical inference.* Cengage Learning, 2021.