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Matéria escura ultraleve

Ultralight Dark Matter

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"*Life is about the little things*"

—

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Projeto apresentado à Universidade de Aveiro para cumprimento dos requisitos necessários à obtenção do grau de Licenciado em Física, realizado sob a orientação científica do Doutor António de Aguiar e Pestana de Morais, Investigador do Departamento de Física da Universidade de Aveiro, e do Doutor Filipe Ferreira de Freitas, Investigador do Departamento de Física da Universidade de Aveiro.

o júri / the jury

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Palavras Chave

Fenomenologia, Matéria escura, Bosão pseudo-Nambu-Golstone, Mecanismo de desalinhamento, Taxa de interação

Resumo

Neste trabalho, foi estudado uma extensão do Modelo Padrão que permite o surgimento de um Bosão pseudo-Nambu-Golstone (pNGB) com uma massa na ordem dos $\mathcal{O}(10^{-20})\ \mathrm{eV}$, a partir de uma quebra de simetria espontânea e explicita. Além disso, foi mostrado como o pNGB se comporta termo-dinamicamente com o banho térmico do universo primordial e foi concluído que é um bom candidato a Matéria escura quando o ângulo de mistura, $\alpha \lesssim 10^{-7}$, e através do mecanismo de desalinhamento, foi possível obter uma densidade de relíquia coincidente com a ME, quando o valor esperado do vácuo, $v_\sigma \gtrsim 10^{18}\ \mathrm{GeV}$.

Keywords Phenomenology, Dark Matter, pseudo-Nambu-Goldstone Boson, Misalignment mechanism, Interaction rate

Abstract In this work we studied an extension of the standard model that permitted the appearance of a pseudo-Nambu-Goldstone Boson (pNGB) with a mass in the order of $\mathcal{O}(10^{-20})$ eV, from an explicit and spontaneous symmetry breaking. Furthermore, we show how the pNGB would behave thermodynamically with the thermal bath of the primordial universe and concluded it's a viable candidate for Dark Matter when the mixing angle, $\alpha \lesssim 10^{-7}$, and through the misalignment mechanism, we were able to obtain a relic density matching that of the DM, when the vacuum expected value, $v_\sigma \gtrsim 10^{18}$ GeV.

CHAPTER 1

Introduction

Dark Matter (DM) is one of the biggest mysteries in physics. There are various models that try to explain the abundance of DM in the universe, through extensions of the Standard Model (SM)[1][2][3]. The model we are focusing on has an ultralight pseudo-Nambu-Goldstone Boson (pNGB), as a DM candidate, which emerges from a Spontaneous Symmetry Breaking (SSB) [4]. In order to verify if the pNGB is a viable DM candidate we will do two analyses, first we will see if the pNGB will remain decoupled from the thermal bath, and second we will be using the Misalignment mechanism, to calculate its relic density in the early universe.

1.1 History of Dark Matter

In this section, we are going to talk about the historical evolution of DM, based on [5]. Perhaps the first person to talk about the existence of a specific unidentified astronomical object solely based on its gravitational influence was the mathematician Friederich Bessel. In a letter that was published in 1844 [6], he made the following claim:

If we were to think of Procyon and Sirius as double stars, their change in motion would not surprise us.

The motion of Sirius and Procyon could only be explained by the presence of faint companion stars that pull on the observed stars.

Lord Kelvin was one of the first to make an effort at a dynamical estimation of the Milky Way's dark matter abundance, he stated that the stars in the Milky Way can be thought of as gaseous particles moving under the force of gravity, and so we can determine the relation between the system's size and the star's velocity dispersion.

The most well-known and frequently referenced pioneer in the study of dark matter was the Swiss-American astronomer Fritz Zwicky. Hubble and Humason discovered a significant scatter in the apparent velocities of eight galaxies inside the Coma Cluster in 1933, with differences exceeding 2000 km/s, but Zwicky went one step further and used the virial theorem to calculate the cluster's mass [7].

He discovered that a sphere of 10^6 light-years should contain 800 galaxies with a mass of 10^9 solar masses and a velocity dispersion of 80 km/s. On the other hand, the average velocity dispersion along the line of sight was observed to be around 1000 km/s.

If this would be confirmed, we would get the surprising result that dark matter is present in a much greater amount than luminous matter

Assuming galaxies are in virial equilibrium, one would expect, $M(r) \propto v^2 r/G$, and be able to connect the mass of a galaxy at a given distance r from the galaxy's center.

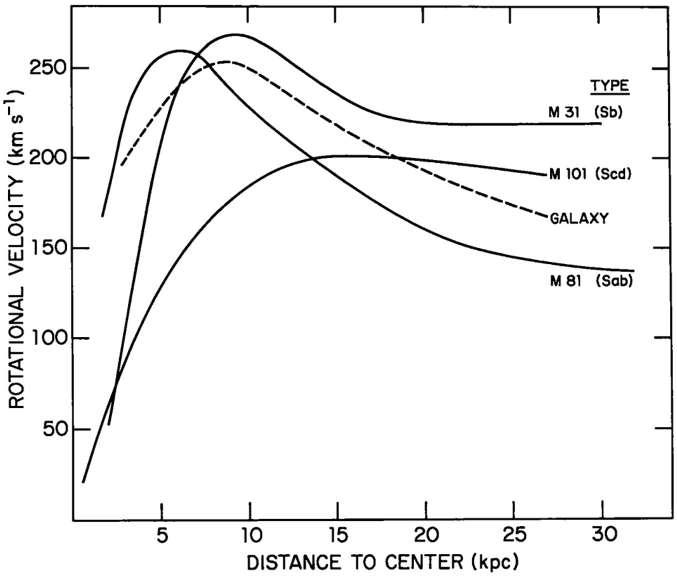

Figure 1.1: Rotation curves of various galaxies [8], where the continuous lines represents the rotational velocity measured experimentally and the dashed one represents the theoretical prediction.

As shown in Figure 1.1, this is not the case, as v is constant outside the galaxy's center, which indicates that $M \propto r$ outside the center. This is one of the most compelling arguments for the existence of dark matter.

1.2 Dark Matter as a particle

The term "dark matter" itself has undergone significant change during the previous few decades. Today, this phrase is most usually used to refer to the particles that make up the majority of the matter density in our universe. Via the Cosmic Microwave Background (CMB), we can say that the current matter density of the universe is, $\Omega^0 h^2 \simeq 0.13$, and the current baryonic matter density of the universe is, $\Omega^0 h^2 \simeq 0.0225 \pm 0.00023$, baryonic matter is matter that makes up protons and neutrons [9]. By a simple subtraction we can calculate the non-baryonic matter density to be, $\Omega^0 h^2 \simeq 0.11$, which allows us to see that most of the matter that makes up the universe aren't particles that constitute atoms, this non-baryonic matter is DM.

DM can be produced through: Freeze-in, Freeze-out, cosmic strings, domain walls and Misalignment mechanism [10][11][12][13].

When looking at the DM problem through the perspective of the SM of particle physics, we notice that DM has characteristics very similar to neutrinos. The main properties of DM are its stability, its neutrality, it only interacts with gravitational fields, and it's cold, after the structure formation. In contrast to every other known particle types, neutrinos are stable - or at least very long lived - and do not interact with electromagnetic or strong fields. These are some of the necessary properties for practically any viable candidate for DM [14]. And even though we now know that DM in the form of neutrinos from the SM would not be able to explain the observed large-scale structure of our Universe, these particles serve as a crucial model for an hypothetical class of particles, known as WIMPs, or weakly interacting massive particles. In this day and age, WIMPs are the strongest candidate for DM, but the lack of experimental evidence makes physicists lose interest in this candidate [15][16].

The main property of a given DM candidate, that numerical simulations can examine is whether it was relativistic (hot) or non-relativistic (cold) during the period of structure formation ($T \sim 10^5 K$).

Lately, there has been a lot of interest focused on ultralight particles for DM candidates, with masses, $m \in [10^{-10}, 10^{-20}]$ eV, however these particles cannot be detected through direct detection experiments, but there have been developments in the area of gravitational waves that can uncover the truth about this hypothesis, which will be the main topic in this work.

CHAPTER 2

Ultralight Bosons

2.1 Introduction to particle physics

Particle physics is an area of physics where it's studied the fundamental structure of the universe. In this work, we are going to use natural units where $c = \hbar = 1$, this is to simplify the calculations, and equations, as well use the term Lagrangian instead of Lagrangian density.

2.1.1 Symmetries

Particle physics revolves around the concept of symmetry and symmetry breaking, this concept is also important in mathematics in the area of Group theory. Symmetry is a property of an object such that under a certain transformation leaves the object invariant. Such operations can be mathematically described in the context of group theory.

Continuous symmetries

Certain symmetries are composed of continuous operations, for example, rotating a circle by an angle, it doesn't matter which angle we choose, the object suffers no change.

Noether's theorem says that [17], *The invariance of the Lagrangian $\mathcal{L}$ under a continuous symmetry implies the existence of a conserved quantity.*

Taking into account Noether's theorem, it's possible to verify:

- Space-time symmetry: They result in energy-momentum conservation and are characterized by the Poincaré group. Inferred from this law as conserved quantities are energy, linear momentum, and total angular momentum.
- Internal symmetry: Is a symmetry acting on fields, these can be global, independent of the space-time point, and local, dependant on the space-time point. The latter are known as gauge symmetries and result in the emergence of internal particle charges such as electric charge, hypercharge, weak isospin or colour charge. The conserved quantity is typically denoted as current-density.

2.1.2 U(1) symmetry

In this section, we are going to use the Klein-Gordon equation to prove the gauge symmetry, note that we can also use the Dirac equation,

$$(\Box + m^2)\phi = 0\,, \tag{2.1}$$

where ϕ is a scalar field [18] and $\Box = \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)$.

Let us consider the Klein-Gordon Lagrangian for a free complex scalar

$$\mathcal{L}_{KG}(\phi, \partial_\mu \phi) = K - V = g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi - m^2\phi^*\phi\,, \tag{2.2}$$

where K and V are the kinetic and potential term, respectively, $\partial_\mu = \frac{\partial}{\partial x_\mu} = (\frac{\partial}{\partial t}, -\vec{\nabla})$, and $g^{\mu\nu} = diag(+,-,-,-)$ is the Minkowski metric.

Global symmetry

lets introduce the following global phase transformation

$$\phi \to \phi' = e^{-iq\alpha}\phi\,, \tag{2.3}$$

where α is a global parameter, global as in it's equal at every space-time point, and q represents the charge of the scalar field.

The Lagrangian in Equation 2.2 gets transformed to

$$\begin{aligned}\mathcal{L}_{KG}(\phi', \partial_\mu\phi') &= g^{\mu\nu}\partial_\mu(e^{iq\alpha}\phi^*)\partial_\nu(e^{-iq\alpha}\phi) - m^2(e^{iq\alpha}\phi^*)(e^{-iq\alpha}\phi) \\ &= g^{\mu\nu}\partial_\mu\phi^*\partial^\nu\phi - m^2\phi^*\phi \\ &= \mathcal{L}_{KG}(\phi, \partial_\mu\phi)\,,\end{aligned} \tag{2.4}$$

and so we conclude that the Klein-Gordon Lagrangian remains invariant under a global phase transformation. So we see that Equation 2.3 represents a unitary phase transformation of the field ϕ in one dimension such that the theory is said to be invariant under a $U(1)$ symmetry.

Local symmetry

Let us consider the same Lagrangian as in Equation 2.2, but now we replace the global parameter with a local one, $\alpha(x^\mu)$, which depends on the space-time point as

$$\phi \to \phi' = e^{-iq\alpha(x^\mu)}\phi\,. \tag{2.5}$$

The transformed Lagrangian becomes

$$\begin{aligned}\mathcal{L}_{KG}(\phi', \partial_\mu\phi') &= g^{\mu\nu}\partial_\mu(e^{iq\alpha(x^\mu)}\phi^*)\partial_\nu(e^{-iq\alpha(x^\mu)}\phi) - m^2(e^{iq\alpha(x^\mu)}\phi^*)(e^{-iq\alpha(x^\mu)}\phi) \\ &= g^{\mu\nu}(iq\partial_\mu\alpha(x^\mu)e^{iq\alpha(x^\mu)}\phi^* + e^{iq\alpha(x^\mu)}\partial_\mu\phi^*)\times \\ &(-iq\partial_\nu\alpha(x^\mu)e^{-iq\alpha(x^\mu)}\phi + e^{-iq\alpha(x^\mu)}\partial_\nu\phi) - m^2\phi^*\phi \\ &= \mathcal{L}_{KG}(\phi, \partial_\mu\phi) + q^2 g^{\mu\nu}\partial_\nu\alpha(x^\mu)\partial_\nu\alpha(x^\mu)\phi^*\phi \\ &-iqg^{\mu\nu}e^{iq\alpha(x^\mu)}\partial_\mu\phi^*\partial_\nu\alpha(x^\mu)\phi + iqg^{\mu\nu}e^{-iq\alpha(x^\mu)}\partial_\nu\phi\partial_\mu\alpha(x^\mu)\phi^*\,,\end{aligned} \tag{2.6}$$

therefore we see that the free Klein-Gordon theory is by no means local $U(1)$ invariant.

Assuming gauge invariance as a fundamental principle, the way we make our theory invariant is by redefining the partial derivative. A gauge transformation is not expected to modify any observable values as observable quantities should not be directly dependent on the field's derivative. Instead, we propose replacing the partial derivatives with the gauge covariant derivative as

$$\partial_\mu \to \mathcal{D}_\mu = \partial_\mu + V_\mu \,, \tag{2.7}$$

such that the condition that guarantees the theory to be invariant is given by

$$(\mathcal{D}_\mu \phi)' = e^{-iq\alpha(x)} \mathcal{D}_\mu \phi \,. \tag{2.8}$$

Using Equation 2.7 and Equation 2.8 we get

$$\begin{aligned} (\mathcal{D}_\mu \phi)' = (\partial_\mu + V'_\mu)\phi' \quad &= e^{-iq\alpha(x^\mu)} \left[\partial_\mu + V'_\mu - iq\partial_\mu \alpha(x^\mu)\right] \phi = e^{-iq\alpha(x^\mu)} \mathcal{D}_\mu \phi \\ &\Leftrightarrow \mathcal{D}_\mu = \partial_\mu + V'_\mu - iq\partial_\mu \alpha(x^\mu) \,, \end{aligned} \tag{2.9}$$

and choosing $V_\mu = iqA_\mu$, with A_μ the gauge field, we obtain by comparison with Equation 2.7 that

$$A'_\mu = A_\mu + \partial_\mu \alpha(x^\mu) \,, \tag{2.10}$$

for the gauge field transformations and consistently with

$$\mathcal{D}_\mu = \partial_\mu + iqA_\mu \,, \tag{2.11}$$

the definition of the covariant derivative.

Note that this gauge covariant derivative will remain unchanged whenever we apply a gauge transformation since the change in the partial derivative will now be balanced out by the change in the gauge field. The gauge covariant derivative can be explained intuitively as a means to alter the derivative operator so that the outcome is unaffected by a gauge transformation. This is crucial since a gauge transformation is meant to restore the system to its initial condition while merely changing the arrangement of unobservable degrees of freedom.

Another approach is using the "kinetic" term in Equation 2.2, it relates to the field propagator and it defines how the particle moves in empty space. The translation is generated by the derivative, and a finite translation is constructed by repeatedly applying infinitesimal translations. The consequence is to create an infinitesimal phase change that goes along with each infinitesimal translation when the partial derivative is replaced by the gauge covariant derivative. As a result, a phase shift is present along with a finite translation.

Spontaneous symmetry breaking

The mechanism of SSB is one of the most potent concepts in current theoretical physics. It serves as the foundation for the majority of recent advances in the statistical mechanics description of phase transitions as well as the description of collective events in solid state physics. Additionally, it has made it possible to combine electromagnetic, weak, and strong interactions in elementary particle physics. Philosophically, the concept is exceedingly complex and nuanced (which is partly why its successful application is a relatively recent accomplishment), and popular explanations of it fall short of doing it credit [19].

The usual, cheap explanation for this phenomenon is that it results from the presence of an asymmetrical absolute minimum, or "ground state", which is shown in the Figure 2.1, for the potential, $V(\phi) = \alpha|\phi|^2 + \beta|\phi|^4$

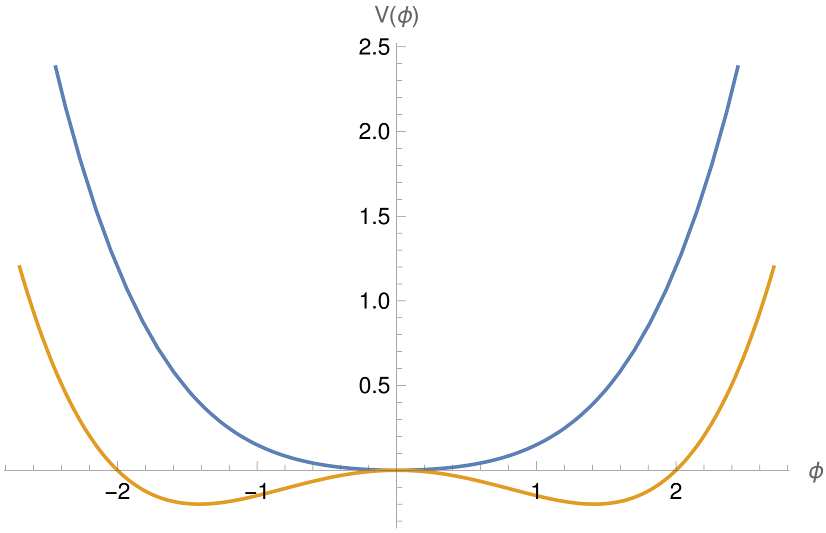

Figure 2.1: Example of a Spontaneous symmetry breaking, with the blue line representing the symmetrial ground state ($\alpha > 0$, $\beta > 0$), and the orange line representing the asymmetrical ground state ($\alpha < 0$, $\beta > 0$).

This symmetry breaking occurs when we change the sign of the coupling terms of the interactions.

The main advantage of this symmetry breaking is due to the Nambu-Goldstone theorem [18], which says

spontaneous breaking of a continuous global symmetry is always accompanied by the appearance of one or more massless scalar (spin-0) particles.

2.2 Bosonic sector

In this section, we are going to take a quick look at the theory invariant under the symmetry $\mathrm{SU(2)_W} \times \mathrm{U(1)_Y}$, denoted as Electroweak symmetry (EW), where W refers to the weak interactions and the Y refers to the weak hypercharge[18][4]. Let the Higgs potential scalar be given by

$$V_0(H) = \mu_H^2 H^\dagger H + \frac{1}{2}\lambda_H (H^\dagger H)^2\,, \tag{2.12}$$

where μ_H and λ_H are the self-interacting coupling terms, and H is the Higgs field, a $\mathrm{SU(2)_W}$ complex scalar doublet written as

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} w_1 + i w_2 \\ v_h + h + iz \end{pmatrix}\,, \tag{2.13}$$

where v_h is the vacuum expectation value VEV of the Higgs, which describes the classical ground state of the theory, $v_h \simeq 246\,\text{GeV}$, while h represents radial quantum fluctuations around the vacuum.

When a SSB occurs the vacuum configuration of the Higgs doublet can be cast as

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_h \end{pmatrix} \tag{2.14}$$

with the Nambu-Goldstone modes $w_{1,2}$ and z, that appear due to the Nambu-Goldstone Theorem 2.1.2, are absorbed by the longitudinal degrees of freedom of the $W^{\pm}$ and Z gauge bosons respectively.

The kinetic term of the Lagrangian, between the coupling of EW vector bosons with the Higgs doublet, is

$$\mathcal{L}_{kin} \supset (\mathcal{D}_\mu H^\dagger)(\mathcal{D}^\mu H)\,, \tag{2.15}$$

where $\mathcal{D}_\mu$ is the covariant derivative, which is defined by $\mathcal{D}_\mu = \partial_\mu + ig_1 \frac{\sigma_a}{2} A^a_\mu + ig_2 Y B_\mu$, $g_{1,2}$ are the $\mathrm{U(1)_Y}$ and $\mathrm{SU(2)_W}$ gauge couplings, σ_a $(a = x,y,z)$ are the Pauli matrices, and the B_μ, and A^a_μ are the massless EW gauge bosons[4][20].

2.3 Ultralight complex scalar

This section serves as a simple example of how we can obtain an ultralight particle[4]. The model in question will contain an additional complex singlet ϕ charged under a global $\mathrm{U(1)_G}$ symmetry, such that it is invariant under the global phase transformation, α,

$$\phi \to e^{i\alpha}\phi\,. \tag{2.16}$$

The potential under this new complex singlet, before the electroweak symmetry breaking (EWSB), is defined as

$$V(H,\phi) = V_0(H) + \mu^2_\phi \phi^* \phi + \frac{1}{2}\lambda_\phi |\phi^*\phi|^2 + \lambda_{H\phi} H^\dagger H \phi^* \phi\,, \tag{2.17}$$

where $\lambda_{H\phi}$, is commonly known as the Higgs portal coupling, the coupling term between the boson scalar and the Higgs doublet.

The conditions that allow the potential to be bounded from below are

$$\lambda_H, \lambda_\phi > 0 \quad \text{and} \quad \lambda_H \lambda_\phi - \lambda^2_{H\phi} > 0\,, \tag{2.18}$$

while upon minimization one can write $\mu^2_H = -\lambda_H v^2_h$. Replacing μ^2_H in the Hessian matrix of Equation 2.17 yields [4]

$$M^2 = \begin{pmatrix} 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} \\ 0_{1\times3} & 2\lambda_H v^2_h & 0 & 0 \\ 0_{1\times3} & 0 & \mu^2_\phi + \frac{1}{2}\lambda_{H\phi} v^2_h & 0 \\ 0_{1\times3} & 0 & 0 & \mu^2_\phi + \frac{1}{2}\lambda_{H\phi} v^2_h \end{pmatrix}\,. \tag{2.19}$$

This matrix, already in its diagonal form, allows us to directly read the masses of the physical particles where we identify three Nambu-Goldstone bosons, the SM-Higgs boson and one new complex scalar with masses

$$m_h^2 = 2\lambda_H v_h^2 \qquad m_\phi^2 = \mu_\phi^2 + \frac{1}{2}\lambda_{H\phi} v_h^2\,. \tag{2.20}$$

The relevant interactions with the singlet ϕ are given by

$$\lambda_{h\phi\phi} = v_h \lambda_{H\phi}\,, \qquad \lambda_{hh\phi\phi} = \lambda_{H\phi}\,, \qquad \lambda_{\phi\phi\phi\phi} = \lambda_\phi\,. \tag{2.21}$$

This model can allow an ultralight complex scalar if μ_ϕ^2 is very small and in the limit of a feebly interacting scenario, which consists of making the portal coupling equally very small, i.e. $10^{-42} \gtrsim \lambda_{H\phi} \gtrsim 10^{-62}$ such that the mass of our complex scalar ϕ gets around, $10^{-10} \gtrsim m_\phi \gtrsim 10^{-20}$ eV.

2.4 pseudo-Nambu-Goldstone Boson

In this section we consider the same potential as in Equation 2.17 but now the ϕ scalar also develops a VEV, v_σ, such that the $\mathrm{U(1)_G}$ symmetry is broken. It follows from the Nambu-Goldstone theorem that a new massless real scalar emerges in the physical particle spectrum. If we further add a tiny soft symmetry breaking term [4]

$$V(H,\phi) \to V(H,\phi) + V_{\text{soft}}\,, \qquad \text{where} \qquad V_{\text{soft}} = \frac{\mu_s^2}{2}(\phi^2 + \phi^{*2})\,, \tag{2.22}$$

to also introduce an explicit $\mathrm{U(1)_G}$ breaking contribution in the theory, the Nambu-Goldstone boson will receive a small mass and is instead denoted as a pseudo-Nambu-Goldstone boson (pNGB).

The Nambu-Goldstone mode can be described as a phase, θ, and therefore the scalar ϕ can be expressed as

$$\phi = \frac{1}{\sqrt{2}}(\sigma + v_\sigma)e^{i\theta/v_\sigma}\,, \tag{2.23}$$

with σ denoting the quantum fluctuations in the radial directions around the VEV, v_σ. And so our V_{soft} scalar potential in Equation 2.22 can be written as

$$V_{\text{soft}} = \frac{\mu_s^2}{2}(\sigma + v_\sigma)^2 \cos\left(\frac{2\theta}{v_\sigma}\right)\,. \tag{2.24}$$

Lets determine the minimization conditions, knowing that our degrees of freedom are σ, h, and θ,

$$\frac{\partial V}{\partial \sigma} = \frac{\partial V}{\partial h} = \frac{\partial V}{\partial \theta} = 0 \Leftrightarrow \begin{cases} \mu_H^2 = -\frac{1}{2}(v_h^2\lambda_H + v_\sigma^2 \lambda_{H\phi}) \\ \mu_\phi^2 = -\frac{1}{2}(v_\sigma^2\lambda_\phi + v_h^2\lambda_{H\phi} + 2\mu_s^2) \end{cases}, \tag{2.25}$$

where $\theta = n\pi v_\sigma$, and $h = \sigma = 0$.

The corresponding Hessian matrix is defined by

$$M^2 = \begin{pmatrix} v_h^2\lambda_H & v_h v_\sigma \lambda_{H\phi} & 0 \\ v_h v_\sigma \lambda_{H\phi} & v_\sigma^2 \lambda_\phi & 0 \\ 0 & 0 & -2\mu_s^2 \end{pmatrix} \tag{2.26}$$

which is not yet diagonal and must be rotated to the physical basis, where we can more clearly see the physical particle spectrum. Doing this we obtain

$$m^2 = U^\dagger M^2 U = \begin{pmatrix} m_{h_1}^2 & 0 & 0 \\ 0 & m_{h_2}^2 & 0 \\ 0 & 0 & m_\theta^2 \end{pmatrix} , \tag{2.27}$$

where the eigenvalues are

$$\begin{aligned} m_{h_{1,2}}^2 &= \tfrac{1}{2}\left[v_h^2\lambda_H + v_\sigma^2\lambda_\phi \mp \sqrt{v_h^4\lambda_H^2 + v_\sigma^4\lambda_\phi^2 + 2v_h^2v_\sigma^2(2\lambda_{H\phi}^2 - \lambda_H\lambda_\phi)}\right] \\ m_\theta^2 &= -2\mu_s^2 , \end{aligned} \tag{2.28}$$

and where U is the matrix containing the eigenvectors, which can be expressed as a matrix rotation that reads as

$$U = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{2.29}$$

with α the scalar mixing angle. The physical basis vectors h_1 and h_2 can be represented in terms of the gauge eigenbasis ones h and σ, as follows

$$\begin{pmatrix} h_1 \\ h_2 \\ \theta \end{pmatrix} = U \begin{pmatrix} h \\ \sigma \\ \theta \end{pmatrix} \Leftrightarrow \begin{cases} h = \cos(\alpha)h_1 - \sin(\alpha)h_2 \\ \sigma = \sin(\alpha)h_1 + \cos(\alpha)h_2 \end{cases} . \tag{2.30}$$

When expanding the "kinetic" term of the scalar field, we get

$$\partial_\mu\phi^*\partial^\mu\phi = \frac{1}{2}\left[(\partial\sigma)^2 + (\partial\theta)^2\frac{(\sigma+v_\sigma)^2}{v_\sigma^2}\right] , \tag{2.31}$$

revealing a cubic interacting between $\sigma\theta\theta$ from $(\partial\theta)^2\sigma$ that, using Equation 2.30 and the equations of motion, reads as

$$(\partial\theta)^2\sigma = -\frac{1}{2}\left[\sin(\alpha)m_{h_1}^2 h_1 + \cos(\alpha)m_{h_2}^2 h_2\right]\theta^2 + \sigma m_\theta^2\theta^2 . \tag{2.32}$$

Notice that in Equation 2.24 expanding the cosine with a Taylor series, $\cos(x) \simeq 1 - \dfrac{x^2}{2}$, will get us

$$\cos\left(\frac{2\theta}{v_\sigma}\right) \simeq 1 - \frac{2\theta^2}{v_\sigma} , \tag{2.33}$$

which showns that the theory is invariant under a $Z_2 \subset U(1)_{\mathrm{G}}$ symmetry where the pNGB transforms as $\theta \to -\theta$. With this new expansion we can see that the soft breaking potential, in Equation 2.24, forms a cubic coupling term $\dfrac{m_\theta^2}{v_\sigma}\sigma\theta^2$, which will cancel out with the last term from Equation 2.32.

Taking a longer look at the Equation 2.32, there is a mixture of h and σ, that allows the decay of both Higgs like particles, h_1 or h_2, into a pair $\theta\theta$ with partial decay width $\Gamma(h_i \to \theta\theta) \propto m_\theta^3 U_{1i}^2/v_\sigma^2$. Current constraints on the mixing angle, backed up by experiments, imply that $|\sin(\alpha)| < 0.3$ [21], and as the value of $|\sin(\alpha)|$ decreases, the probability of h_1 decaying into $\theta\theta$ decreases as well, but the probability of h_2 decaying into $\theta\theta$ increases. Later in this work this will become relevant.

This approach provides three real scalars. First, one of the Higgs bosons, either h_1 or h_2, must be the SM-like Higgs, whereas the other can be heavier or lighter than 125 GeV. Second, the pNGB θ derives its mass from a soft-breaking parameter and is a candidate for a real scalar with an ultralight mass. Its mass comes from a marginal explicit violation of the global $U(1)_{\mathrm{G}}$ symmetry[4] as discussed above.

The expansion in Equation 2.33 allows us to see the appearance of interactions with θ, as mentioned before, and the cubic and quartic coupling terms of these interactions are [4]

$$\begin{aligned}
\lambda_{\theta\theta h_i} &= \frac{m_{h_1}^2}{v_\sigma} U_{2i} \,, \\
\lambda_{\theta\theta h_i h_j} &= \frac{1}{4}\left[\lambda_\phi U_{2i} U_{2j} + (-1)^{i+j} \lambda_{H\phi} U_{1i} U_{1j}\right] \,, \\
\lambda_{h_i SMSM} &= U_{1i} g_{SM} \,.
\end{aligned} \tag{2.34}$$

In terms of these parameters, the quartic terms can also be written as

$$\begin{aligned}
\lambda_{H\phi} &= \frac{\sin(2\alpha)(m_{h_1}^2 - m_{h_2}^2)}{2 v_\sigma v_h} \,, \\
\lambda_\phi &= \frac{\cos^2(\alpha) m_{h_2}^2 + \sin^2(\alpha) m_{h_1}^2}{v_\sigma^2} \,, \\
\lambda_H &= \frac{\cos^2(\alpha) m_{h_1}^2 + \sin^2(\alpha) m_{h_2}^2}{v_h^2} \,.
\end{aligned} \tag{2.35}$$

Last but not least, the quartic self-interaction of the pNGB reads as

$$\lambda_{\theta\theta\theta\theta} = -\frac{m_\theta^2}{6 v_\sigma^2} \,. \tag{2.36}$$

As a side comment, note that the pNGB is protected against radiative corrections no matter the values of the quartic couplings above. However this is not so relevant for this work and we refer to [4] for more information on the subject.

CHAPTER 3

Thermodynamics of the expanding universe

In this chapter, our aim is to understand the thermal evolution of the early universe. In that regard we will make a brief review of the thermodynamics in the equilibrium, following [10].

Most of our knowledge about how the early universe behaves is thanks to the CMB being approximately equal to the spectra of a black body, in that way we can assume that in the primordial universe the particles from the SM are in an equilibrium with each other.

3.1 Thermodynamics in equilibrium

The primordial universe is a gaseous environment with most of its particles in equilibrium, and so we can use statistical mechanics to make a prediction of how it used to work in the past.

The number density, n, energy density, ρ, and the pressure, P of a gas of particles with a degree of freedom, g, distinguish the thermodynamic properties of the gas and are defined as a function of the distribution function $f(\vec{p}, t)$:

$$n(t) \equiv \frac{g}{(2\pi)^3} \int f(\vec{p}, t) d\vec{p}, \tag{3.1}$$

$$\rho(t) \equiv \frac{g}{(2\pi)^3} \int E(\vec{p}) f(\vec{p}, t) d\vec{p}, \tag{3.2}$$

$$P(t) \equiv \frac{g}{(2\pi)^3} \int \frac{p^2}{3E} f(\vec{p}, t) d\vec{p}, \tag{3.3}$$

where $\vec{p} = (p_x, p_y, p_z)$, $d\vec{p} \equiv dp_x dp_y dp_z$ and $E = \sqrt{p^2 + m^2}$. The phase space equilibrium distribution function for a particle i will be given by one of the three following distributions

$$f_i^{eq} = \begin{cases} \dfrac{1}{e^{(E_i-\mu_i)/T}+1}\,, & \text{Fermi-Dirac}\,, \\ \dfrac{1}{e^{(E_i-\mu_i)/T}-1}\,, & \text{Bose-Einstein}\,, \\ e^{-(E_i-\mu_i)/T}\,, & \text{Maxwell-Boltzmann}\,, \end{cases} \tag{3.4}$$

where E_i and μ_i is the energy and chemical potential of the particle i, respectively. The Fermi-Dirac (Bose-Einstein) statistic refers to particles with half-integer (integer) spin.

Using spherical symmetry we can simplify, $d\vec{p} \equiv 4\pi p^2 dp$ and rewrite the Equations (3.1), (3.2) and (3.3) in terms of the energy, E, and so they can be given as

$$n_R = \frac{g}{2\pi^2}\int_m^\infty \frac{(E^2-m^2)^{1/2}}{e^{(E-\mu)/T}\pm 1}dE\,, \tag{3.5}$$

$$\rho_R = \frac{g}{2\pi^2}\int_m^\infty \frac{(E^2-m^2)^{1/2}}{e^{(E-\mu)/T}\pm 1}E^2 dE\,, \tag{3.6}$$

$$P_R = \frac{g}{6\pi^2}\int_m^\infty \frac{(E^2-m^2)^{3/2}}{e^{(E-\mu)/T}\pm 1}dE\,, \tag{3.7}$$

where "R" refers to Radiation (relativistic limit), The inferior limit of the integral refers to the lowest energy a particle can have, which is its mass, m, where the + and - signs refer respectively to the Fermi-Dirac and Bose-Einstein distributions.

Lets take a look at a generalized solution for the Fermi-Dirac and Bose-Einstein distributions in the limit of small masses, $m \simeq 0$, where we can write

$$\int_m^\infty \frac{(E^2-m^2)^s}{e^{(E-\mu)/T}\pm 1}dE = \int_0^\infty \frac{E^s}{e^{\frac{E-\mu}{T}}\pm 1}dE\,.$$

Recasting $k = \frac{E}{T}$ and $\gamma = \frac{\mu}{T}$, we get [22][23]

$$\int_0^\infty \frac{E^s}{e^{\frac{E-\mu}{T}}\pm 1}dE = \int_0^\infty \frac{(Tk)^s}{e^{k-\gamma}\pm 1}Tdk = T^{s+1}\int_0^\infty \frac{k^s}{e^{k-\gamma}\pm 1}dk = \mp T^{s+1}\Gamma(s+1)Li_{1+s}(\mp e^\gamma)\,, \tag{3.8}$$

where the polylogarithm or Jonquière's function and the Gamma function, are respectively defined by [24]

$$Li_n(z) \equiv \sum_{k=1}^\infty \frac{z^k}{k^n} \quad \text{and} \quad \Gamma(s+1) = \int_0^\infty e^{-x}x^s dx = s!\,. \tag{3.9}$$

At the ultra-relativistic limit (radiation), $T \gg m, \mu$. We can simplify $\gamma = \mu/T \simeq 0$ and so the polylogarithm function in Equation 3.8 can be simplified into

$$Li_{s+1}(\mp 1) = \sum_{k=1}^\infty \frac{(\mp 1)^k}{k^{s+1}}\,. \tag{3.10}$$

Now, if we take a look at the solution of the following sum

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^s} = (1 - 2^{-s})\zeta(s+1)\,, \tag{3.11}$$

where $\zeta(s)$ is the Riemann zeta function defined as

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s}\,, \tag{3.12}$$

the generalized solution for ultra-relativistic particles, in Equation 3.8, gets reduced to

$$\int_0^{\infty} \frac{E^s}{e^{\frac{E-\mu_i}{T}} \pm 1} dE = \begin{cases} T^{s+1}(1-2^{-s})\zeta(s+1)s!\,, & \text{Fermi-Dirac}\,, \\ T^{s+1}\zeta(s+1)s!\,, & \text{Bose-Einstein}\,, \end{cases} . \tag{3.13}$$

Now we can easily integrate the Equations (3.5), (3.6) and (3.7), knowing that $\zeta(4) = \pi^4/90$ to

$$n_{\mathrm{R}} = \frac{g}{2\pi^2}\int_0^{\infty} \frac{E}{e^{E/T} \pm 1} dE = \frac{g}{\pi^2} T^3 \zeta(3) \begin{cases} 3/4\,, & \text{Fermi-Dirac}\,, \\ 1\,, & \text{Bose-Einstein}\,, \end{cases} \tag{3.14}$$

$$\rho_{\mathrm{R}} = \frac{g}{2\pi^2}\int_0^{\infty} \frac{E^3}{e^{E/T} \pm 1} dE = g\frac{\pi^2}{30} T^4 \begin{cases} 7/8\,, & \text{Fermi-Dirac}\,, \\ 1\,, & \text{Bose-Einstein}\,, \end{cases} \tag{3.15}$$

$$P_{\mathrm{R}} = \frac{g}{6\pi^2}\int_0^{\infty} \frac{E^3}{e^{E/T} \pm 1} dE = \frac{1}{3}\rho_{\mathrm{R}}\,. \tag{3.16}$$

In the non-relativistic limit (matter), $T \ll m, \mu$, which follows the Maxwell-Boltzmann distribution, we can rewrite Equations (3.1), (3.2) and (3.3) in terms of the momentum, p, given as

$$n_{\mathrm{M}} \equiv \frac{g}{(2\pi)^3}\int e^{-\frac{(E-\mu)}{T}} d\vec{p}, \tag{3.17}$$

$$\rho_{\mathrm{M}} \equiv \frac{g}{(2\pi)^3}\int E(\vec{p}) e^{-\frac{(E-\mu)}{T}} d\vec{p}, \tag{3.18}$$

$$P_{\mathrm{M}} \equiv \frac{g}{(2\pi)^3}\int \frac{p^2}{3E} e^{-\frac{(E-\mu)}{T}} d\vec{p}, \tag{3.19}$$

where the subscript "M" means Matter (non-relativistic limit).

Let's take a look at a generalized solution to a certain integral, where, in spherical coordinates $d\vec{p} \equiv 4\pi p^2 dp$, we get

$$\int p^k E^s e^{-\frac{(E-\mu)}{T}} d\vec{p} = 4\pi e^{\mu/T} T^s \int_0^{\infty} p^{k+2} \frac{(p^2+m^2)^{s/2}}{T^s} e^{-\frac{\sqrt{p^2+m^2}}{T}} dp\,. \tag{3.20}$$

Substituting $x = \frac{p}{T}$ and $\gamma = \frac{\mu}{T}$, in Equation 3.20 yields

$$4\pi e^{\mu/T} T^{s+k+3} \int_0^{\infty} x^{k+2}(x^2+\gamma^2)^{s/2} e^{-\sqrt{x^2+\gamma^2}} dx\,. \tag{3.21}$$

Taking into account that $T \ll m, \mu$, we can do the following approximation to simplify the integration

$$(x^2+\gamma^2)^{s/2} = \gamma^s\left[1 + \left(\frac{x}{\gamma}\right)\right]^{s/2} \approx \gamma^s + \frac{s}{2}\gamma^{s-2}x^2\,, \tag{3.22}$$

which, using Equation 3.22, we obtain

$$4\pi e^{\mu/T} T^{s+k+3} \gamma^s e^{-\gamma} \left[\int_0^\infty x^{k+2} e^{-\frac{x^2}{2\gamma}} dx + \frac{s}{2} \gamma^{-2} \int_0^\infty x^{k+4} e^{-\frac{x^2}{2\gamma}} dx \right] . \tag{3.23}$$

From Equation 3.23 and performing the change of variable $u = \frac{x^2}{2\gamma}$, we obtain

$$4\pi e^{\mu/T} T^{s+k+3} \gamma^{s+1} e^{-\gamma} \left[(2\gamma)^{\frac{k+1}{2}} \int_0^\infty u^{(k+1)/2} e^{-u} du + \frac{s}{2} \gamma^{-2} (2\gamma)^{\frac{k+3}{2}} \int_0^\infty u^{(k+3)/2} e^{-u} du \right] , \tag{3.24}$$

such that, using the Gamma function, as shown in Equation 3.9, we get for Equation 3.20 the following result

$$4\pi T^{s+k+3} e^{-\frac{(m-\mu)}{T}} 2^{(k+1)/2} \left(\frac{m}{T} \right)^{\frac{k+3}{2}+s} \left[((k+1)/2)! + \frac{s}{2} \left(\frac{m}{T} \right)^{-1} ((k+3)/2)! \right] . \tag{3.25}$$

Using $(1/2)! = \sqrt{\pi}/2$ we can then obtain solutions to Equations (3.17), (3.18) and (3.19) in the form

$$n_{\mathrm{M}} = \frac{g}{(2\pi)^3} \int e^{-\frac{(E-\mu)}{T}} d\vec{p} = g \left(\frac{mT}{2\pi} \right)^{\frac{3}{2}} e^{-\frac{(m-\mu)}{T}} , \tag{3.26}$$

$$\rho_{\mathrm{M}} = \frac{g}{(2\pi)^3} \int E(\vec{p}) e^{-\frac{(E-\mu)}{T}} d\vec{p} = n_M m \left[1 + \frac{3}{2} \left(\frac{T}{m} \right) \right] , \tag{3.27}$$

$$P_{\mathrm{M}} = \frac{g}{(2\pi)^3} \int \frac{p^2}{3E} e^{-\frac{(E-\mu)}{T}} d\vec{p} = n_M T \left[1 - \frac{5}{6} \left(\frac{T}{m} \right) \right] . \tag{3.28}$$

Assuming a period when $T \sim 300\,\mathrm{GeV}$ such that $T \gg m, \mu$, we will be in the relativistic limit where the total energy density is equal to the sum of the energy density for each relativistic species

$$\rho_{\mathrm{Total}} = \sum_i \rho_{\mathrm{R}}^{(i)} , \tag{3.29}$$

with i representing SM particles.

As the universe expands, its temperature decreases and so some particles become non-relativistic. When $T \sim m$ such particles will start behaving accordingly to the Maxwell Boltzmann distribution increasing the matter energy density component in the total energy density of the universe defined as

$$\rho_{\mathrm{Total}} = \sum_i \rho_{\mathrm{R}}^{(i)} + \sum_j \rho_{\mathrm{M}}^{(j)} , \tag{3.30}$$

with i and j representing relativistic and non-relativistic particles, respectively.

Although some particles become non-relativistic as the temperature decreases, their contribution becomes negligible because their energy density, ρ_{M}, decreases exponentially as shown in Equation 3.27, and so we can approximate the energy density in the primordial universe to

$$\rho_{\mathrm{Total}} \simeq \sum_i \rho_{\mathrm{R}}^{(i)} . \tag{3.31}$$

Taking into account that species of particles can evolve with different temperatures, even in equilibrium, we can write the ρ_{R} in terms of the temperature of photons, T, as follows

$$\rho_{\mathrm{R}} = \sum_b \rho_b + \sum_f \rho_f = \frac{\pi^2}{30} T^4 g_*(T)\,, \tag{3.32}$$

where "b" stands for bosons, "f" for fermions and $g_*(T)$ is

$$g_*(T) = \sum_b g_b \left(\frac{T_b}{T}\right)^4 + \frac{7}{8} \sum_f g_f \left(\frac{T_f}{T}\right)^4 . \tag{3.33}$$

Factorizing with respect to photons γ and neutrinos ν, we can write

$$\rho_{\mathrm{R}} = \frac{\pi^2}{30} T^4 \left[g_\gamma + \frac{7}{8} \sum_\nu g_\nu \left(\frac{T_\nu}{T}\right)^4 + \sum_b g_b \left(\frac{T_b}{T}\right)^4 + \frac{7}{8} \sum_f g_f \left(\frac{T_f}{T}\right)^4 \right] =$$

$$= \rho_\gamma \left[1 + \frac{1}{g_\gamma} \frac{7}{8} \left(\frac{T_\nu}{T}\right)^4 \left(\sum_\nu g_\nu + \sum_b g_b \left(\frac{T_b}{T_\nu}\right)^4 + \sum_f g_f \left(\frac{T_f}{T_\nu}\right)^4 \right) \right] ,$$

where ρ_γ is

$$\rho_\gamma = \frac{\pi^2}{30} T^4 g_\gamma\,. \tag{3.34}$$

Finally we get

$$\rho_{\mathrm{R}} = \rho_\gamma \left[1 + \frac{7}{8} \left(\frac{T_\nu}{T}\right)^4 N_{\mathrm{eff}} \right] , \tag{3.35}$$

where

$$N_{\mathrm{eff}} = \frac{1}{g_\gamma} \left(\sum_\nu g_\nu + \sum_b g_b \left(\frac{T_b}{T_\nu}\right)^4 + \sum_f g_f \left(\frac{T_f}{T_\nu}\right)^4 \right) , \tag{3.36}$$

is the effective number of neutrino species ignoring the photon and $g_\gamma = 2$. Experiments using CMB data determine $N_{\mathrm{eff}} = 2.99 \pm 0.14$[9]. The number N_{eff} is important has it will tell us the quantity of radiation in the universe.

3.2 Collision Operator

In the early universe, its contents were for the most part in a thermal equilibrium, which allows us to say the universe was in an equilibrium, following [10]. Lets assume that in this equilibrium occurred interactions between the particles. The collision operator allows us to determine quite a number of things. In particular it is useful in solving the Boltzmann Equation, which describes the microscopic evolution of a particle's phase space distribution function $f(p^\mu, x^\mu)$, and is given by

$$\hat{L}[f] = C[f]\,, \tag{3.37}$$

where $\hat{L}$ is the Liouville operator and C is the collision operator. The collision operator also helps describing processes of annihilation and decay.

In this work we will only be concerned with annihilation processes where for a process $(1+2 \leftrightarrow 3+4)$ we have

$$\begin{aligned}\frac{g}{(2\pi)^3}\int C[f_1]\frac{d^3p_1}{E_1} &= \pm\int (2\pi)^4\delta^4(\mathcal{P}_1+\mathcal{P}_2-\mathcal{P}_3-\mathcal{P}_4) \\ &\times[|\overline{M}|^2_{1+2\to 3+4} f_1 f_2(1\pm f_3)(1\pm f_4) \\ &-|\overline{M}|^2_{3+4\to 1+2} f_3 f_4(1\pm f_1)(1\pm f_2)]d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4\,,\end{aligned} \tag{3.38}$$

with $\mathcal{P}_i = (E_i, \vec{p}_i)$ and $d\Pi_i = \dfrac{g_i d\vec{p}_i}{2(2\pi)^3 E_i}$. The $\pm$ refers to which direction the process takes, where + indicates to the production of particle 1 ($3{+}4 \to 1{+}2$) and - refers to it's consumption $(1+2 \to 3+4)$. In the term $(1 \pm f_i)$, the + appears when i is a boson and - when it's a fermion and $|\overline{M}|^2$ is the mean of initial and final spins squared [25]. In order to simplify we are going to assume

$$|\overline{M}|^2 \equiv |\overline{M}|^2_{1+2\to 3+4} \equiv |\overline{M}|^2_{3+4\to 1+2}\,. \tag{3.39}$$

For further details about this assumption see [10].

The mean amplitude squared is defined as

$$|\overline{M}|^2 = \frac{S_{12}S_{34}}{g_1g_2g_3g_4}|M|^2\,, \tag{3.40}$$

where $|M|^2$ is the amplitude squared of the processes and the symmetrization factor $S_{i,j} = 1/n_{i,j}!$ which accounts for identical particles. In particular, if the particles i and j are the same, $S_{i,j} = 0.5$ and if they are different $S_{i,j} = 1$ (note that particles and anti-particles are different). If we assume that a particle i is in thermal equilibrium, we have that $E_i > T$ so we can approximate the term $1 \pm f_i \simeq 1$, as the particle i will behave according to the Maxwell-Boltzmann distribution, as shown in Figure 3.1.

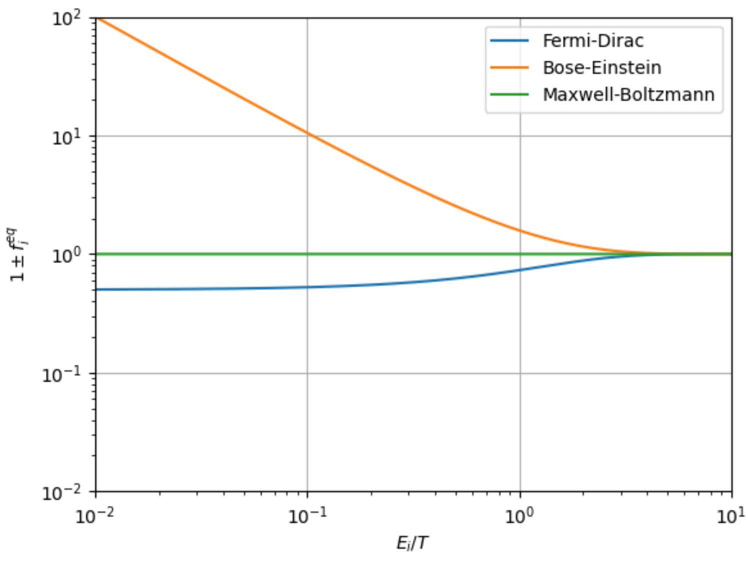

Figure 3.1: Dependence of $1 \pm f_i$ with E_i/T.

For the process of annihilation ($1+2 \leftrightarrow 3+4$), where the particles 3 and 4 are in thermal equilibrium, and considering the conservation of energy ($E_1 + E_2 = E_3 + E_4$) we check that

$$f_3^{eq} f_4^{eq} = e^{-(E_3+E_4)/T} = e^{-(E_1+E_2)/T} = f_1^{eq} f_2^{eq} \,. \tag{3.41}$$

The cross section for a process $1+2 \to 3+4$ is [10]

$$\sigma = \frac{S_{12} S_{34}}{4\sqrt{(\mathcal{P}_1 \cdot \mathcal{P}_2)^2 - (m_1 m_2)^2}} \int (2\pi)^4 \delta^4(\mathcal{P}_1 + \mathcal{P}_2 - \mathcal{P}_3 - \mathcal{P}_4) \frac{|M|^2}{g_1 g_2} \frac{d\Pi_3 d\Pi_4}{g_3 g_4} \,. \tag{3.42}$$

Using Equation 3.1 for the number density as [10]

$$dn_i = \frac{g_i}{(2\pi)^3} \frac{d\vec{p}_i}{2E_i} \,, \tag{3.43}$$

and Equation 3.42, we can simplify Equation 3.38 to

$$\int C[f_1] d\Pi_1 = -\int \sigma v_{\text{Møller}} \left[dn_1 dn_2 - dn_1^{eq} dn_2^{eq} \right] \,, \tag{3.44}$$

where $v_{\text{Møller}}$ is the Møller velocity defined as

$$v_{\text{Møller}} = \frac{\sqrt{(\mathcal{P}_1 \cdot \mathcal{P}_2)^2 - (m_1 m_2)^2}}{E_1 E_2} \,, \tag{3.45}$$

with $v_{\text{Møller}} n_1 n_2$ a Lorentz invariant [12].

Therefore we can write our Equation 3.44, as

$$\frac{g_1}{(2\pi)^3} \int \frac{d\vec{p}_1}{2E_1} C[f_1] = -\langle \sigma v_{\text{Møller}} \rangle [n_1 n_2 - n_1^{eq} n_2^{eq}] \,, \tag{3.46}$$

where the thermal averaged cross section is defined as

$$\langle \sigma v_{\text{Møller}} \rangle = \frac{\int (\sigma v_{\text{Møller}}) dn_1^{eq} dn_2^{eq}}{\int dn_1^{eq} dn_2^{eq}} \,, \tag{3.47}$$

where for simplicity we are going to denote the thermal averaged cross-section as $\langle \sigma v \rangle$.

Lets take a look at the number density of the particles coupled with the thermal bath. Knowing these particles will follow the Maxwell-Boltzmann distribution, with $\mu = 0$, we can write Equation 3.17 as

$$n = \frac{g}{(2\pi)^3} 4\pi \int_m^\infty E\sqrt{E^2 - m^2} e^{-E/T} dE \,, \tag{3.48}$$

where we use $E^2 = |\vec{p}|^2 + m^2$ and spherical symmetry of the momentum, $d\vec{p} = 4\pi E\sqrt{E^2 - m^2} dE$. The inferior limit comes from the minimum energy possible of the particle, which is its rest mass.

Integrating by parts Equation 3.48, we get

$$n = \frac{g}{(2\pi)^3} 4\pi \left(\frac{1}{3} \left[(E^2 - m^2)^{3/2} e^{-E/T} \right]_m^\infty + \frac{1}{3T} \int_m^\infty (E^2 - m^2)^{3/2} e^{-E/T} dE \right) \,, \tag{3.49}$$

where we can easily see that the left side gives zero, and the right side can be related to the Bessel function defined as

$$K_\alpha(z) = \frac{\sqrt{\pi}}{\Gamma(\alpha+1/2)}\left(\frac{z}{2}\right)^\alpha \int_1^\infty (x^2-1)^{\alpha-1/2} e^{-zx} dx\,. \tag{3.50}$$

Making $x = E/m$ and $z = m/T$ and knowing that $\Gamma(5/2) = 3\sqrt{\pi}/4$, we get

$$n = \frac{g}{2\pi^2} m^2 T K_2(m/T)\,. \tag{3.51}$$

From Equations (3.47) and (3.43) we can see that

$$\langle \sigma v_{\text{Møller}} \rangle = \frac{\displaystyle\int (\sigma v_{\text{Møller}}) e^{-E_1/T} e^{-E_2/T} d^3p_1 d^3p_2}{\displaystyle\int e^{-E_1/T} e^{-E_2/T} d^3p_1 d^3p_2}\,. \tag{3.52}$$

Let's now consider that particle 1 is equal to particle 2, as it will be the main scope of this project. Following the Equation 3.51, it's easy to see that the denominator of Equation 3.52 will be

$$\int e^{-E_1/T} e^{-E_2/T} d\vec{p}_1 d\vec{p}_2 = (4\pi m^2 T K_2(m/T))^2\,, \tag{3.53}$$

where $m = m_1 = m_2$.

Looking now at the numerator of Equation 3.52, using spherical symmetry ($d\vec{p}_i = 4\pi E_i|\vec{p}_i|dE_i$) we can see that

$$d^3p_1 d^3p_2 = 4\pi E_1 |\vec{p}_1| 4\pi E_2 |\vec{p}_2| \frac{1}{2} dE_1 dE_2 d\cos(\theta)\,, \tag{3.54}$$

where θ is the angle between $\vec{p}_1$ and $\vec{p}_2$. Notice that the term $d\cos(\theta)/2$ is invariant under integration. For simplicity we are going to make the following change of variables

$$\begin{cases} E_+ = E_1 + E_2 \\ E_- = E_1 - E_2 \\ s = 2m^2 + 2E_1E_2 - 2|\vec{p}_1||\vec{p}_2|\cos(\theta) \end{cases} \Leftrightarrow \begin{cases} E_1 = \frac{E_+ + E_-}{2} \\ E_2 = \frac{E_+ - E_-}{2} \\ \cos(\theta) = \dfrac{-s + 2m^2 + 2(E_+^2 - E_-^2)}{2|\vec{p}_1||\vec{p}_2|} \end{cases}, \tag{3.55}$$

where $s = (\mathcal{P}_1 + \mathcal{P}_2)^2$ is a Mandelstam variable [26].

Using the Jacobian matrix we can see that

$$\begin{aligned} dE_1 dE_2 d\cos(\theta) = & \begin{vmatrix} \frac{\partial E_1}{\partial E_+} & \frac{\partial E_1}{\partial E_-} & \frac{\partial E_1}{\partial s} \\ \frac{\partial E_2}{\partial E_+} & \frac{\partial E_2}{\partial E_-} & \frac{\partial E_2}{\partial s} \\ \frac{\partial \cos(\theta)}{\partial E_+} & \frac{\partial \cos(\theta)}{\partial E_-} & \frac{\partial \cos(\theta)}{\partial s} \end{vmatrix} dE_+ dE_- ds \\ & = (2|\vec{p}_1||\vec{p}_2|)^{-1} dE_+ dE_- ds\,. \end{aligned} \tag{3.56}$$

The volume element, in Equation 3.54 becomes

$$d^3p_1 d^3p_2 = 2\pi^2 E_1 E_2 dE_+ dE_- ds\,, \tag{3.57}$$

and the integration limits, from $E_1 \geq m_1$, $E_2 \geq m_2$, $|\cos(\theta)| \leq 1$, become

$$\begin{cases} s \geq 4m^2 \\ E_+ \geq \sqrt{s} \\ |E_-| \leq \sqrt{1-\frac{4m^2}{s}}\sqrt{E_+^2 - s} \end{cases} . \tag{3.58}$$

Therefore, the numerator in Equation 3.52 will be

$$\int (\sigma v_{\text{Møller}}) e^{-E_1/T} e^{-E_2/T} d^3p_1 d^3p_2 = 2\pi^2 \int dE_- \int dE_+ \int \sigma v_{\text{Møller}} E_1 E_2 e^{-E_+/T} ds\,, \tag{3.59}$$

such that from the definition of $v_{\text{Møller}}$ in Equation 3.45,

$$\begin{aligned} \int (\sigma v_{\text{Møller}}) e^{-E_1/T} e^{-E_2/T} d^3p_1 d^3p_2 &= 4\pi^2 \int \sigma G \sqrt{1-\tfrac{4m^2}{s}} ds \int e^{E_+/T} \sqrt{E_+^2 - s} dE_+ \\ &= 2\pi^2 T \int \sigma (s-4m^2) \sqrt{s} K_1(\sqrt{s}/T) ds\,, \end{aligned} \tag{3.60}$$

where we define $G \equiv v_{\text{Møller}} E_1 E_2 = \frac{1}{2}\sqrt{s(s-4m^2)}$ [26] and where we have used the Bessel function in Equation 3.50 with $\alpha = 1$. We can finally use Equations (3.53) and (3.60) to simplify the thermal averaged cross section, in Equation 3.47, to

$$\langle \sigma v_{\text{Møller}} \rangle = \frac{1}{8m^4 T K_2^2(m/T)} \int_{4m^2}^{\infty} \sigma (s-4m^2) \sqrt{s} K_1(\sqrt{s}/T) ds\,. \tag{3.61}$$

CHAPTER 4

Dark Matter candidate

As discussed in chapter 1, the key to find if a particle is a viable candidate for DM or not, is to check whether a particle is coupled or decoupled from the thermal bath and calculate its relic density.

The interactions of the particles occur within a homogeneous and isotropic gaseous equilibrium denominated as thermal bath. If the interactions are rapid enough to adjust to the changing of the temperature of the expanding universe, we can maintain our universe in a nearly thermal equilibrium. The particles that remain in equilibrium with the thermal bath are denominated as being coupled with the thermal bath, and the ones that leave the equilibrium, are decoupled from the thermal bath. We can say that a particle is either coupled or decoupled with the thermal bath, by the relation between the rate of interaction of said particle Γ, and the rate of expansion of the universe H as

$$\begin{aligned} &\Gamma > H \quad (coupled) \\ &\Gamma < H \quad (decoupled)\,. \end{aligned} \tag{4.1}$$

4.1 Interaction rate

In this section, we are going to check if the pNGB studied in chapter 2 is a viable candidate for DM.

The rate of interaction is given by [10]

$$\Gamma = n\langle\sigma v\rangle\,, \tag{4.2}$$

where n is the number density of our particle.

For the rate of interaction of pNGB, we are only interested in the processes of annihilation $\theta + \theta \to SM + SM$, which were our main focus in chapter 3. And so the rate of interaction can be written as

$$\Gamma = \frac{g}{16m^2\pi^2 K_2(m/T)} \int_{4m^2}^{\infty} \sigma(s-4m^2)\sqrt{s}K_1\left(\frac{\sqrt{s}}{T}\right) ds\,. \tag{4.3}$$

To calculate the rate of interaction, first we need the amplitude square of the processes of annihilation being studied, $|\mathcal{M}|^2$ in Equation 3.42, in order to calculate it we used CalcHEP. CalcHEP is a package for the calculation of Feynman diagrams [27], and implementing the model studied in the chapter 2, while considering only the s-channels with mediators like the SM-Higgs and h_2, where the t and u channels were not considered as they would have near to no impact on the amplitude of the process of annihilation of pNGB. For the calculation of the integrals, we used Cuba, a library for multidimensional numerical integration [28], in C++ language.

For simplification reasons, we are going to make all the coupling terms, λ_H, $\lambda_{H\phi}$ and λ_ϕ constant, remember that they are bounded by the conditions in Equation 2.18. Notice that the VEV, v_σ, is directly related to the mass of the h_2. If we recall Equation 2.28, we can manipulate both h_1 and h_2 masses, in order for one of them to equal the mass of the SM-Higgs and the other one to have any values depending on the VEV v_σ.

So we obtain the following rates of interaction

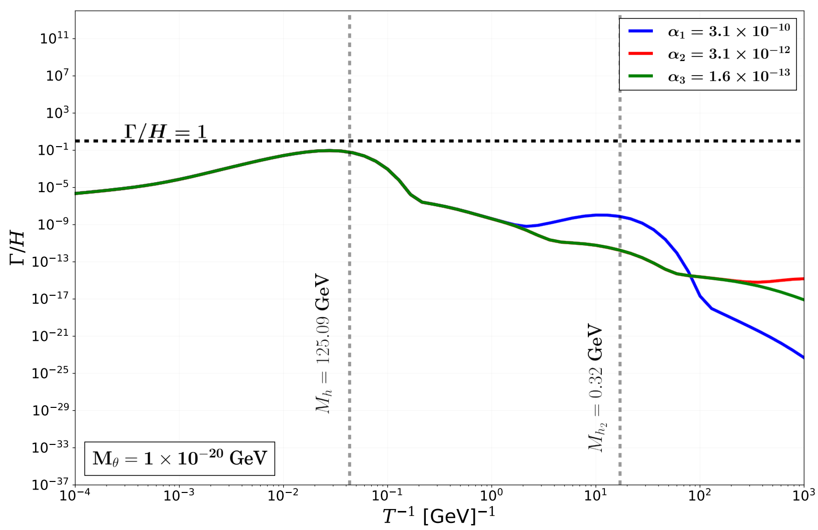

Figure 4.1: Rate of interaction divided by the Hubble constant assuming $m_h \gg m_{h_2}$, for $\nu_\sigma = 1$ GeV ($M_{h_2} = 3.2 \times 10^{-1}$ GeV), $\nu_\sigma = 0.01$ GeV ($M_{h_2} = 3.2 \times 10^{-3}$ GeV) and $\nu_\sigma = 5 \times 10^{-4}$ GeV ($M_{h_2} = 1.6 \times 10^{-4}$ GeV), for blue, red and green curves, respectively. Additionally, we have used $\lambda_H \sim 0.26$, $\lambda_{H\phi} = 2 \times 10^{-8}$ and $\lambda_\phi = 0.1$.

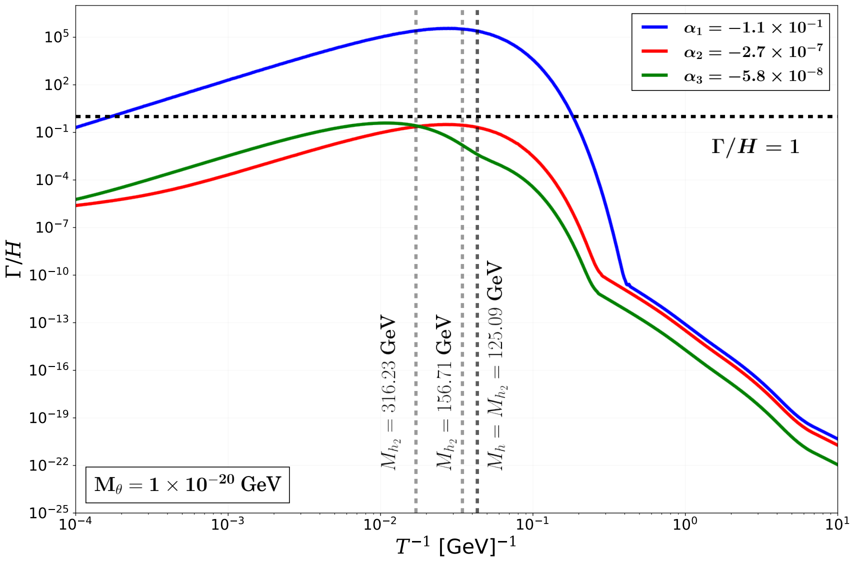

Figure 4.2: Rate of interaction divided by the Hubble constant assuming $m_h = m_{h_2}$, for $\nu_\sigma = 395.57$ GeV ($M_{h_2} = 125.09$ GeV), $\nu_\sigma = 495.57$ GeV ($M_{h_2} = 156.71$ GeV) and $\nu_\sigma = 10^3$ GeV ($M_{h_2} = 316.23$ GeV), for blue, red and green curves, respectively. Additionally, we have used $\lambda_H \sim 0.26$, $\lambda_{H\phi} = 2 \times 10^{-8}$ and $\lambda_\phi = 0.1$.

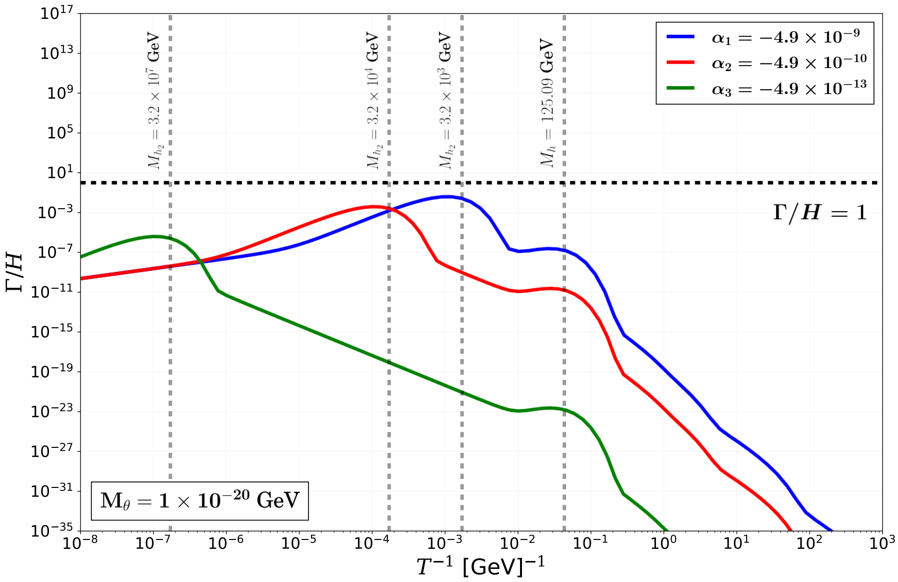

Figure 4.3: Rate of interaction divided by the Hubble constant assuming $m_h \ll m_{h_2}$, for $\nu_\sigma = 10^4$ GeV ($M_{h_2} = 3.2 \times 10^3$ GeV), $\nu_\sigma = 10^5$ GeV ($M_{h_2} = 3.2 \times 10^4$ GeV) and $\nu_\sigma = 10^8$ GeV ($M_{h_2} = 3.2 \times 10^7$ GeV), for blue, red and green curves, respectively. Additionally, we have used $\lambda_H \sim 0.26$, $\lambda_{H\phi} = 2 \times 10^{-8}$ and $\lambda_\phi = 0.1$.

We can prove that for as long as the mixing angle $\alpha \lesssim 10^{-7}$, our pNGB doesn't couple with the thermal bath, and so doesn't become relativistic, it remains cold. Notice that this constraint in the mixing angle allows our VEV v_σ to become a free parameter, as long as the quartic coupling terms remain the same. As mentioned in chapter 2, the mixing angle being small only allows decays of the h_2 into $\theta\theta$, and seeing as v_σ becomes a free parameter, for large values of v_σ, this decay also becomes rather suppressed.

Note that there is a resonance whenever the temperature is approximately equal to the mass of the mediator, SM-Higgs or h_2, by a factor of $T \sim m_{h_{1,2}}/5.4$, this comes from

$$\sigma \propto |\overline{M}|^2 \propto \frac{1}{(s - m_{h_{1,2}}^2)^2 + \gamma(m_{h_{1,2}}^2)^2}\,, \tag{4.4}$$

when the term $(s - m_{h_{1,2}}^2)^2 = 0$, knowing that for bosons we have $\langle E \rangle \simeq 2.7T$ [10][25].

4.2 Misalignment Mechanism

In this section, we are going to consider that the soft-symmetry breaking of the $\mathrm{U(1)_G}$ symmetry, which produced the pNGB, occurred before the end of inflation. Inflation is a theory that explains the exponential rate of expansion of the early universe.

Using Equation 2.24, and introducing the Misalignment angle as $\Theta = \dfrac{2\theta}{v_\sigma}$

$$V_{\text{soft}}(\Theta) \simeq \frac{m_\theta^2}{2}\left(\frac{v_\sigma}{2}\right)^2 \Theta^2\,. \tag{4.5}$$

From the energy-momentum tensor T^μ_ν of θ [10][11]

$$T^\mu_\nu = g^{\mu\alpha}(\partial_\alpha\theta)(\partial_\nu\theta) - \frac{\delta^\mu_\nu}{2}[g^{\alpha\beta}(\partial_\alpha\theta)(\partial_\beta\theta) + 2V(\theta)]\,, \tag{4.6}$$

where $g^{\mu\nu} = diag(-1,1,1,1)$ and the energy density reads as

$$\begin{aligned}\rho_\theta = -T^0_0 = -g^{0\alpha}(\partial_\alpha\theta)(\partial_0\theta) + \frac{1}{2}[g^{\alpha\beta}(\partial_\alpha\theta)(\partial_\beta\theta) + 2V(\theta)] = \frac{1}{2}\left(\frac{\partial\theta}{\partial t}\right)^2 - \frac{1}{2}\frac{\vec{\nabla}^2\theta}{a^2} + V(\theta) = \\ = \frac{1}{2}\left(\frac{\partial\theta}{\partial t}\right)^2 + V(\theta) = \left(\frac{v_\sigma}{2}\right)^2\left[\frac{\dot{\Theta}^2}{2} + \frac{m_\theta^2}{2}\Theta^2\right]\,,\end{aligned} \tag{4.7}$$

where θ is homogeneous throughout the universe, $\theta = \theta(t)$, and so only varies with time, due to the symmetry breaking only occurring before the end of inflation.

From the Continuity equation, we have that for matter ($p = 0$), we obtain that the energy density of θ reads as

$$\rho_\theta(a) = \rho_\theta(a_i)\left(\frac{a_i}{a}\right)^3\,, \tag{4.8}$$

where a is the scale factor, and the a_i relates do any value of the scale factor.

The equation of motion for θ can be obtained by varying the action, $S = \int \mathcal{L}a^3 d^4x$, and computing the D'Alembertian for the Friedmann–Robertson–Walker (FRW) metric we obtain[11]

$$\ddot{\Theta}^2 + 3H\dot{\Theta} + m_\theta^2\Theta = 0\,. \tag{4.9}$$

During the radiation era we have that $a \propto \sqrt{t}$, where t is the age of the universe, so we have that the rate of expansion $H = 1/t$.

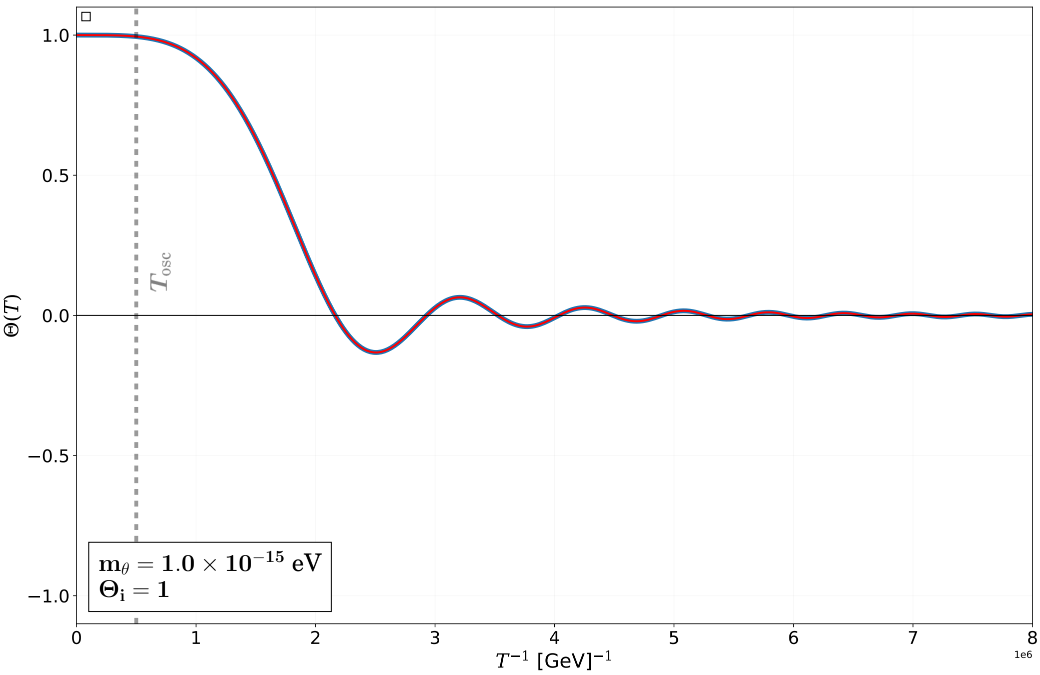

Figure 4.4: Solution of the equation of motion in Equation 4.9.

Figure 4.4, allows us to see that the Misalignment angle, Θ, is constant for $T^{-1} \in [0, T_{\text{osc}}^{-1}]$ and from Equation 4.7 we can see that

$$\rho_\theta(a_{\text{osc}}) = \left(\frac{v_\sigma}{2}\right)^2 \frac{m_\theta^2}{2} \Theta_{\text{osc}}^2 \,. \tag{4.10}$$

Using Equation 4.8, for an $a_i = a_{\text{osc}}$, we have

$$\rho_\theta(a) = \left(\frac{v_\sigma}{2}\right)^2 \frac{m_\theta^2}{2} \Theta_{\text{osc}}^2 \left(\frac{a_{\text{osc}}}{a}\right)^3 \,. \tag{4.11}$$

Let us take a look at the entropy in the primordial universe, it can be written in terms of the pressure P, and energy density ρ, as

$$S = sa^3 = \frac{\rho + P}{T} a^3 \,, \tag{4.12}$$

where s is the entropy density. Recalling Equations (3.32) and (3.16) we have that the entropy can be written as

$$S = \frac{2\pi^2}{45} g_{*s}(T) T^3 a^3 \,, \tag{4.13}$$

where, $g_{*s}(T)$ is

$$g_{*s}(T) = \sum_b g_b \left(\frac{T_b}{T}\right)^3 + \frac{7}{8} \sum_f g_f \left(\frac{T_f}{T}\right)^3 \,. \tag{4.14}$$

Taking the following ratio,

$$\frac{S_{\text{osc}}}{S} = \frac{g_{*s}(T_{\text{osc}})}{g_{*s}(T)} \left(\frac{T_{\text{osc}} a_{\text{osc}}}{Ta}\right)^3 \,, \tag{4.15}$$

knowing that in the primordial universe the entropy is constant, so $S_{\text{osc}} = S$, Equation 4.15 can be recast as

$$\left(\frac{a_{\text{osc}}}{a}\right)^3 = \frac{g_{*s}(T)}{g_{*s}(T_{\text{osc}})}\left(\frac{T}{T_{\text{osc}}}\right)^3 . \tag{4.16}$$

Using Equation 4.16, the current energy density of θ, in Equation 4.11, becomes

$$\rho_\theta(a_0) = \left(\frac{v_\sigma}{2}\right)^2 \frac{m_\theta^2}{2}\Theta_{\text{osc}}^2 \frac{g_{*s}(T_0)}{g_{*s}(T_{\text{osc}})}\left(\frac{T_0}{T_{\text{osc}}}\right)^3 . \tag{4.17}$$

The current relic density of any matter is given by

$$\Omega^0 = \frac{\rho(a_0)}{\rho_{\text{crit}}} , \tag{4.18}$$

and so using Equation 4.17, the current relic density of θ reads as

$$\Omega_\theta^0 = \frac{\rho_\theta(a_0)}{\rho_{\text{crit}}} = \frac{1}{\rho_{\text{crit}}}\left(\frac{v_\sigma}{2}\right)^2 \frac{m_\theta^2}{2}\Theta_{\text{osc}}^2 \frac{g_{*s}(T_0)}{g_{*s}(T_{\text{osc}})}\left(\frac{T_0}{T_{\text{osc}}}\right)^3 . \tag{4.19}$$

This can be further simplified as we know that $\rho_{\text{crit}} = 1.88 \times 10^{-29} h^2 \text{gcm}^{-3} = 8.1 \times 10^{-47} h^2$ GeV^4. Furthermore, using Equation 2.28 for $\mu_s^2 = -m_\theta^2/2$, $g_{*s}(T_0) = 3.91$, $g_*(T_0) = 3.36$ and $T_0 = 2.3 \times 10^{-4}$ eV, we derive the formula needed to calculate the relic abundance of the ultralight boson θ as

$$\Omega_\theta^0 h^2 = 0.11 \left(\frac{m_\theta}{10^{-14}\text{eV}}\right)^{1/2} \left(\frac{v_\sigma}{\sqrt{50}\times 10^{17}\text{GeV}}\right)^2 \left(\frac{\Theta_{\text{osc}}}{10^{-3}}\right)^2 \left(\frac{3.91}{g_{*s}(T_{\text{osc}})}\right)\left(\frac{g_*(T_{\text{osc}})}{3.36}\right)^{3/4} . \tag{4.20}$$

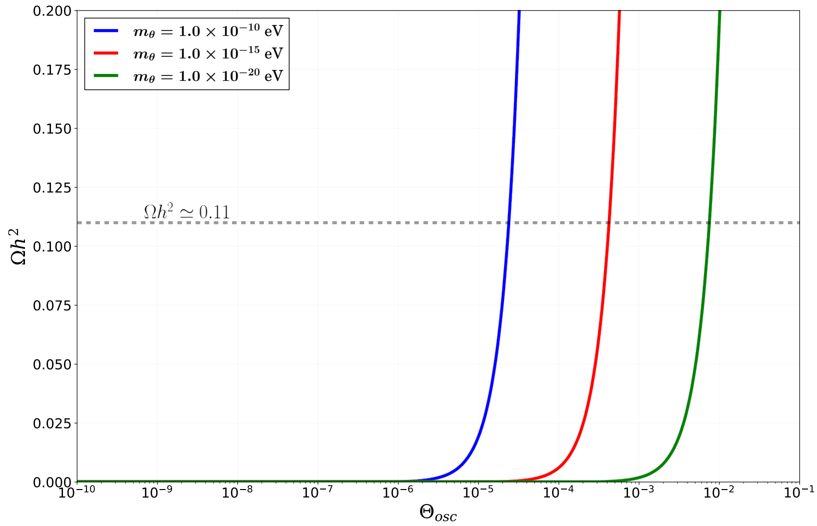

Figure 4.5: Relic density change with the initial Misalignment angle Θ_{osc}, with $v_\sigma = 3 \times 10^{18}$, for various values of m_θ .

We then reach our final result and verify that for a certain mass and v_σ, it is possible to produce the observed DM relic density for an ultralight real scalar via the Misalignment mechanism. In Figure 4.5 we further find the required misalignment angle to be $\Theta_{\text{osc}} \sim 2.5 \times 10^{-5}$, 4.5×10^{-4}, 8.3×10^{-2} for the pNGB masses 10^{-10} GeV, 10^{-15} GeV and 10^{-20} GeV, respectively.

CHAPTER 5

Conclusion

In this work we took a look at an extension of the SM that allows a pNGB to emerge in the physical basis, and then did a brief review of the thermodynamics of the primordial universe. We were able to conclude that for a mixing angle, $\alpha \lesssim 10^{-7}$ our dark matter candidate doesn't couple with the thermal bath and therefore doesn't become relativistic. We also confirmed that any scenario where h_2 is heavier, equal or lighter, than the SM-Higgs, is valid. We were also able to use the Misalignment mechanism in order to derive a formulae for the relic density of this candidate, and say that for a value of v_σ in the order of $\mathcal{O}(10^{18})$ is a viable option for a DM candidate. The culmination of this work allows us to see that an ultralight pNGB is a viable candidate for DM.

CHAPTER 6

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