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Ultralight Scalar Dark Matter

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Abstract:

Contents

1 Introduction

There are several evidences about the existence of dark matter [1], which compose 27% of the total energy density.

2 The essence of the model

Following [2], we consider a simple Standard Model extensions with additional real singlet ϕ charged under a global $\mathrm{U}(1)_G$ such that the theory is invariant under transformation

$$\phi \to e^{i\alpha}\phi\,. \tag{2.1}$$

In this case the potential is

$$V\left(H,\phi\right) = V_0\left(H\right) + \mu_\phi^2\phi\phi^* + \frac{1}{2}\lambda_\phi|\phi\phi^*|^2 + \lambda_{H\phi}H^\dagger H\phi\phi^* + V_{soft}\,, \tag{2.2}$$

where

$$V_0(H) = \mu_H^2 H^\dagger H + \frac{1}{2}\lambda_H\left(H^\dagger H\right)^2\,, \tag{2.3}$$

and V_{soft} is the term that softly breaks the the $\mathrm{U}(1)_G$ global symmetry

$$V_{soft} = \frac{1}{2}\mu_s^2\left(\phi^2 + \phi^{*2}\right)\,. \tag{2.4}$$

The mass matrix for the states (h,σ,θ) is

$$\boldsymbol{M}^2 = \begin{pmatrix} v_h^2\lambda_H & v_h v_\sigma\lambda_{H\phi} & 0 \\ v_h v_\sigma & v_\sigma^2\lambda_\phi & 0 \\ 0 & 0 & -2\mu_s^2 \end{pmatrix}\,, \tag{2.5}$$

$\boldsymbol{M}^2$ can be diagonalised by the orthogonal transformation

$$\boldsymbol{m}^2 = \boldsymbol{O}^\dagger \boldsymbol{M}^2 \boldsymbol{O} = \begin{pmatrix} m_{h_1}^2 & 0 & 0 \\ 0 & m_{h_2}^2 & 0 \\ 0 & 0 & m_\theta^2 \end{pmatrix} \tag{2.6}$$

where

$$\boldsymbol{O} = \begin{pmatrix} cos\alpha & sin\alpha & 0 \\ -sin\alpha & cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2.7}$$

where the eigenvalues are

$$m_{h_{1,2}^2} = \frac{1}{2}\left[v_h^2\lambda_H + v_\sigma^2\lambda_\phi \mp \sqrt{v_h^4\lambda_H^2 + v_\sigma^4\lambda_\phi^2 + 2v_h^2v_\sigma^2\left(2\lambda_{H\phi}^2 - \lambda_H\lambda_\phi\right)} \right], \tag{2.8}$$

$$m_\theta^2 = -2\mu_s^2\,. \tag{2.9}$$

And the angle α satisfies

$$tan\,(2\alpha) = \left(\frac{2\lambda_{H\phi}\nu_h\nu_\sigma}{\lambda_H\nu_H^2 - \lambda_\phi\nu_\sigma^2} \right) \tag{2.10}$$

.

Here we will assume that h_1 is the SM Higgs boson, with mass of 125 GeV. As mentioned in [2], the mass of the second scalar (h_2) is a free parameter of the model. The coupling between the h_1, h_2 and θ with SM particles are shown in [2]. Additionally, we will have a quartic coupling

$$\lambda_{\theta\theta\theta\theta} = -\frac{m_\theta^2}{6\nu_\sigma^2}, \tag{2.11}$$

as $\lambda_{\theta\theta\theta\theta} < 0$ we have a attractive ultralight dark matter scenario, instead the repulsive ones [3].

3 Dark Matter

An ultralight scalar dark matter can be produced in the early universe through four principle mechanism: misalignment mechanism [4–8], decay of thermal relics [9], thermally (freeze-out) [10–12] and topological defects [13]. θ emerges as a Nambu-Goldstone boson when the global $U(1)_{\mathrm{G}}$ symmetry is spontaneously broken, this occur when $T \sim \nu_\sigma$ [11, 12, 14], which may happens before the end of inflation (scenario I) or after that (scenario II). We will address both scenarios in this work. Additionally, it acquires mass due the soft-breaking mass term, V_{soft}, becoming a pseudo Nambu-Goldstone Boson (pNGB).

We will show that we need $\nu_\sigma \sim \mathscr{O}\left(10^8 - 10^{19}\right)$ GeV in order to obtain θ as DM. This range of value of ν_σ suppress the coupling between the θ with SM particles [2], this implies that θ is not thermally produced, additionally the production via Higgs decays ($h \to \theta\theta$) becomes negligible. A thermally produced ultralight DM behaves as hot DM and it can jeopardize the period of structure formation [15]. Finally, θ will be only produced via misalignment mechanism and trough topological defects.

Here we are interested in the case where 10^{-22} eV $< m_\theta <$ 1 eV, mass below 10^{-22} eV is not excluded, however it candidate needs to compose a small fraction of dark matter [16, 17]. Additionally, when $m_\theta \lesssim 10^{-33}$ eV, that is approximately the value of Hubble constant today, θ behaves like dark energy [17].

In order to understand how θ will be produced, we need to clarify the pNGB cosmology in the early universe. As we mentioned above the symmetry breaking occurs at high scale, when $T \sim \nu_\sigma$, and establishes θ as a pNGB. However, spontaneous breaking of a global symmetry leads to the formation of global strings [11, 18]. As we will discuss, theses strings will decay and produce dark matter. The soft-breaking term in the broken phase of ϕ is

$$V_{\rm soft} = \frac{\mu_s^2}{2}\left(\sigma + \nu_\sigma\right)^2 \cos\left(2\frac{\theta}{\nu_\sigma}\right) , \tag{3.1}$$

and it is clear that from the global $U(1)_{\rm G}$ symmetry breaking emerges a Z_2 shift symmetry $\theta \to \theta + \pi\nu_\sigma k$, where $k = 0, 1$.

After the Spontaneous Symmetry Breaking (SSB) of $U(1)_G$ the pNGB field assumes some initial non-zero value, $\theta_{\rm ini}$. Then, as the initial value of θ is not aligned with the minimum of potential, the pNGB starts to oscillate coherently when $m_\theta \simeq 3H$, at some temperature $T_{\rm osc}$ and θ is produced non-thermally through misalignment mechanism [12] and we estimate[1]

$$T_{osc} \simeq 1.2 \text{ keV} \left(\frac{m_\theta}{10^{-20}\text{eV}}\right)^{1/2}\left(\frac{3.9}{g_*(T_{osc})}\right)^{1/2} , \tag{3.2}$$

where T_{osc} needs to be greater than temperature at matter-radiation equality ($T_{eq} = 0.5$ eV) [4, 12].

When a discrete symmetry is spontaneous breaking we have the formation of Domain Walls (DW). Then, when m_θ becomes comparable with Hubble and the pNGB field begins to oscillate at the minimum of potential, we have the SSB of Z_2 that emerges from SSB of $U(1)_G$ which leads to the formation of stable domain walls [18, 19], at this time each string becomes attached by domain walls, what is called by string-wall networks [20]. However, the energy density of stable domain walls evolves slower than radiation and matter, and it can dominate the energy density of the Universe. This is called a domain wall problem [21], and can be avoided as we will mention bellow.

One way to avoid the domain wall problem is to assume that the end of inflation occur after the SSB of $U(1)_G$ (scenario I), in this case the pNGB field is homogenized by inflation and there are no string-wall networks [22]. Thus, in the scenario I we will only have the contribution of misalignment mechanism for the final relic abundance of DM. Still, in the scenario II we will deal with the domain wall problem through another mechanism.

[1] In this work we assume the Standard Cosmology scenario, where the early universe is dominated by radiation, with $H \simeq 1.66\sqrt{g_*}T^2/M_{Pl}$, after the inflation.

3.1 Scenario I

Here we will deal with the case where the SSB of $U(1)_G$ occur before the end of inflation. As we mentioned above, in this scenario the θ will be produced only through misalignment mechanism, which we will explain. The temperature at inflation is given by the Gibbons-Hawking [11, 12, 23]

$$T_I = \frac{H_I}{2\pi}\,, \tag{3.3}$$

where H_I is the inflationary Hubble parameter and there is a upper limit to it that come from Planck and BICEP2 [12, 24]

$$H_I < 8.8 \times 10^{13}\ \text{GeV}\,, \tag{3.4}$$

in this work we will assume that $H_I = 1 \times 10^{13}$ GeV.

The SSB of $U(1)_G$ before the end of inflation lead to isocurvature perturbations [12]. The cosmic microwave background constraint the amplitude of isocurvature, imposing a limit at inflationary Hubble parameter H_I as function of vacuum expectation value

$$\frac{H_I}{\nu_\sigma} \lesssim 3 \times 10^{-5}\,, \tag{3.5}$$

then it will imply naturally that $\nu_\sigma \gtrsim 2.9 \times 10^{18}$ GeV.

In order to illustrate the misalignment mechanism, we will define the misalignment angle as

$$\Theta(x) \equiv \frac{\theta(x)}{\nu_\sigma/2}\,. \tag{3.6}$$

And the potential described by Eq. (3.1) near to the minimum is

$$V_{soft} \simeq \frac{\mu_s^2}{2}\left(\frac{\nu_\sigma}{2}\right)^2\frac{\Theta^2}{2}\,, \tag{3.7}$$

and the lagrangian can be written as

$$\mathscr{L} = \left(\frac{\nu_\sigma}{2}\right)^2\left[\frac{\dot{\Theta}^2}{2} - \frac{\mu_s^2}{2}\frac{\Theta^2}{2}\right]\,. \tag{3.8}$$

The equation of motion for Θ can be obtained varying the action, $S = \int d^4xR^3\mathscr{L}$, where R is the scale factor, and computing the D'Alembertian for the Friedmann–Robertson–Walker(FRW) metric we obtain

$$\ddot{\Theta} + 3H\dot{\Theta} + \frac{\mu_s^2}{2}\Theta = 0\,. \tag{3.9}$$

When the coherent Θ oscillations commence, we can replace $\dot{\Theta}$ by its average over an oscillation: $\langle\dot{\Theta}\rangle = \rho_\theta$, where ρ_θ is the energy density of θ

$$\rho_\theta = \left(\frac{\nu_\sigma}{2}\right)^2\left[\frac{\dot{\Theta}^2}{2} + \frac{\mu_s^2}{2}\frac{\Theta^2}{2}\right]\,. \tag{3.10}$$

Where, from Eq. (3.9) we obtain

$$\rho_\theta = \text{const}\frac{m_\theta}{R^3} \,, \tag{3.11}$$

which means that the coherent oscillation behave like non-relativist matter, and its number density is given by $n_\theta = \rho_\theta / m_\theta$.

The initial energy density in the misaligned scalar field is

$$\rho_\theta = \frac{m_\theta^2 \Theta_i^2}{2} \left(\frac{\nu_\sigma}{2}\right)^2 \,, \tag{3.12}$$

where Θ_i^2 is the initial misalignment angle, which is defined as the misalignment angle at the begin of oscillations T_{osc} (Eq. (3.2)).

The energy density of θ at some instant after T_{osc} can be described as [8, 12]

$$\rho_\theta(R) = \rho_\theta\left(R_{osc}\right)\left(\frac{R_{osc}}{R}\right)^3 \,, \tag{3.13}$$

which means the read-shift of energy density, then

$$\rho_\theta(R) = \frac{m_\theta^2 \Theta_i^2}{2} \left(\frac{\nu_\sigma}{2}\right)^2 \left(\frac{R_{osc}}{R}\right)^3 \,, \tag{3.14}$$

and the energy density of θ is controlled by initial value of θ.

Using the definition of entropy density and assuming that it is constant, we obtain

$$\left(\frac{a_{osc}}{a}\right)^3 = \frac{g_s(T_0)}{g_s(T_{osc})}\left(\frac{T_0}{T_{osc}}\right)^3 \,, \tag{3.15}$$

where $g_s(T_0) = 3.91$ where $T_0 = 2.3 \times 10^{-4}$ eV is the temperature today.

The final relic density is defined as

$$\Omega_\theta h^2 = \frac{\rho_\theta}{\rho_{crit}} \tag{3.16}$$

where $\rho_{crit} = 1.05 \times 10^{-5}\ h^2\ \text{GeV/cm}^3 \simeq 8.15 \times 10^{-47}\ h^2\ \ \text{GeV}^4$. We have disregard the anharmonicities in the potential, as we are assuming that $\theta_i \gg \nu_\sigma$, and obtain

$$\Omega_\theta h^2 = 0.11 \left(\frac{m_\theta}{10^{-14}\ \text{eV}}\right)^{1/2} \left(\frac{\nu_\sigma}{5\times 10^{17}\ \text{GeV}}\right)^2 \left(\frac{\Theta_i}{10^{-3}}\right)^2 \left(\frac{3.91}{g_s(T_{osc})}\right)\left(\frac{g_*(T_{osc})}{3.4}\right)^{3/4} \tag{3.17}$$

Where the results of Eq. (3.17) is shown in Fig. 1 for $m_\theta = 10^{-10}$ eV, 10^{-15} eV and 10^{-20} eV for blue, green and red curves, respectively.

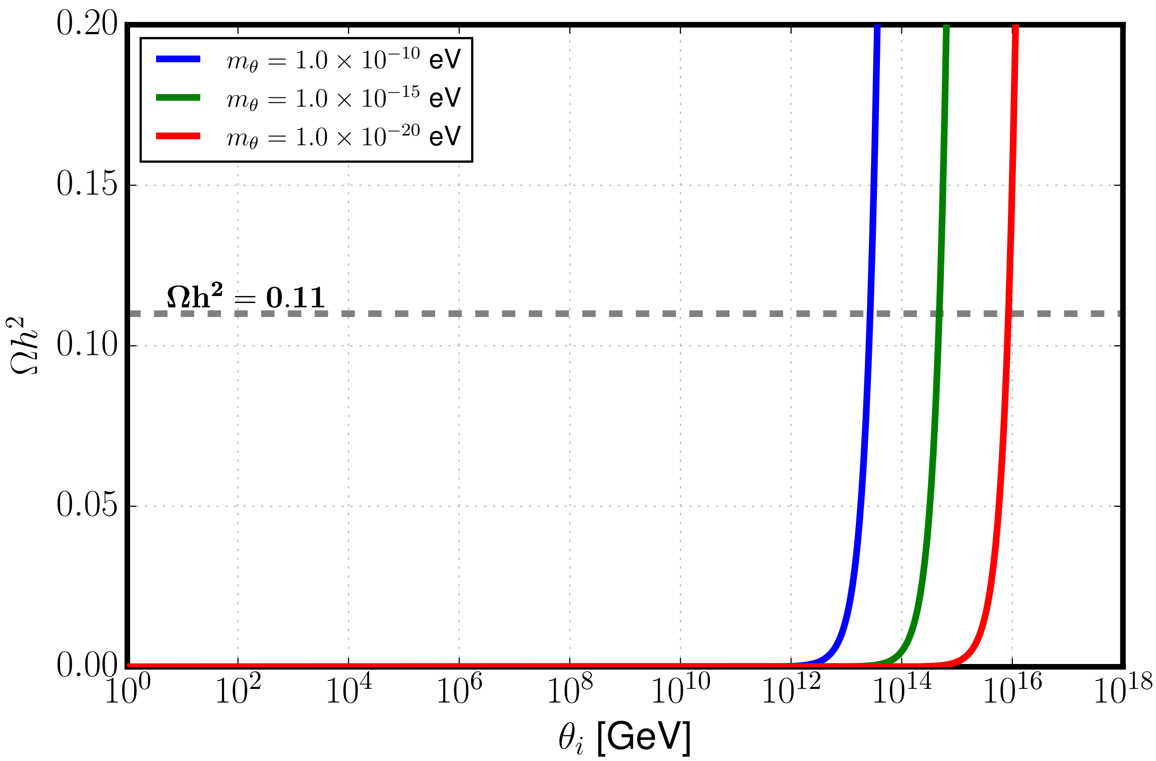

Figure 1: Abundance of ultralight dark matter for the case where the spontaneous symmetry breaking occur before the end of inflation.

4 Conclusions

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