Introduction to stellar astrophysics

Volume 1

Basic stellar observations and data

9

Masses and radii of stars

9.1 General discussion of binaries

For the stars for which angular diameters have been measured we can, of course, determine the radii by multiplying the angular radius by the distance to the star, provided we know the distance. For stars further away than about 20 pc we cannot measure trigonometric parallaxes accurately, we can therefore determine distances only in indirect ways. Star stream parallaxes (see for instance Becker, 1950) have been used to determine the distance to the Hyades cluster, but it turned out that the photometric parallaxes were more accurate. Fortunately, we can also determine radii for stars in special binary systems, namely in eclipsing binaries. For these binaries we can also determine the masses of the stars. We shall therefore devote this section to the discussion of binaries in general, and the next sections to the discussion of the special types of binaries for which stellar radii can be determined, namely eclipsing binaries, and for those binaries for which stellar masses can also be determined, again the eclipsing binaries and also the visual binaries.

We can determine stellar masses for binaries using Kepler's third law. Let us briefly discuss the mechanics of a binary system.

From the solar system we are accustomed to the case that one body, the sun, has a much larger mass than the other bodies, the planets. For binaries with two stars we have to take into account that both bodies have elliptical (or perhaps circular) orbits around their center of gravity S (see Fig. 9.1). From the definition of the center of gravity we know that

$$M_1 r_1 = M_2 r_2 \tag{9.1}$$

where M_1 and M_2 are the masses and r_1 and r_2 are the distances of the stars from the center of gravity. In a binary system there must be equilibrium between gravitational and centrifugal forces which means

$$\frac{GM_1 M_2}{(r_1 + r_2)^2} = M_1 \omega_1^2 r_1 = M_2 \omega_2^2 r_2 \tag{9.2}$$

and where ω_1 and ω_2 are the angular velocities, r_1 and r_2 are the orbital radii of the two stars, and G is the gravitational constant. For the two stars to remain in phase, which means for the center of gravity to remain at constant velocity, we must have $\omega_1 = \omega_2 = \omega$.

For (9.2) to be true, we must then also have $M_1 r_1 = M_2 r_2$ which means that both stars must orbit the center of gravity.

For *circular* orbits ω can be replaced by

$$\omega = 2\pi/P \tag{9.3}$$

where P is the orbital period. We then derive

$$\frac{GM_1 M_2}{(r_1 + r_2)^2} = M_1 r_1 \frac{4\pi^2}{P^2}, \tag{9.4}$$

division by $M_1 r_1 4\pi^2$ yields

$$\frac{G}{4\pi^2} \cdot \frac{M_2}{r_1} \frac{1}{(r_1 + r_2)^2} = \frac{1}{P^2}. \tag{9.5}$$

Using (9.1) we find $M_1 = (M_2 \cdot r_2)/r_1$ and

$$(M_1 + M_2) = M_2\left(\frac{r_2}{r_1} + 1\right) \quad \text{or} \quad M_1 + M_2 = \frac{M_2}{r_1}(r_2 + r_1). \tag{9.6}$$

This yields

$$\frac{M_2}{r_1} = \frac{M_1 + M_2}{r_2 + r_1}. \tag{9.7}$$

Inserting (9.7) into (9.5) yields the general form of Kepler's third law

$$M_1 + M_2 = \frac{(r_1 + r_2)^3}{P^2} \cdot \frac{4\pi^2}{G}. \tag{9.8}$$

If distances are measured in astronomical units and times in years and masses in units of solar masses, the factor $4\pi^2/G$ becomes unity.

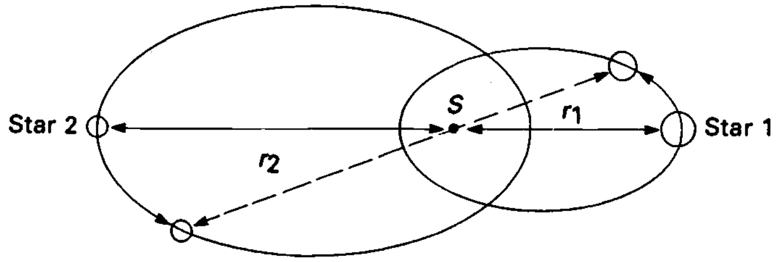

Fig. 9.1. In a binary system the two stars, Star 1 and Star 2 orbit their center of gravity, S.

While we have derived (9.8) only for circular orbits (otherwise r_1, r_2 and ω would be functions of time) it can be shown that this law also holds for elliptical orbits, if r_1 and r_2 are replaced by the semi-major axes a_1 and a_2 of the orbits around the center of gravity.

In order to determine the masses, we then have to determine a_1 and a_2, or at least the sum. If we look at visual binaries, i.e., binaries for which we can observe both stars, we will generally only be able to observe the orbit of one star relative to the other. For circular orbits of the two stars, the relative orbits of each star around the other will also be a circle with radius $r_1 + r_2$. Making a plot of the relative positions of one star around the other for elliptical orbits one can see that each star will also go around the other in an ellipse and the second star will be in the focal point of the true elliptical orbit.* The semi-major axis of the orbit of one star around the other is $a_1 + a_2$. This can be seen from Fig. 9.2 as follows.

The major axis of the relative orbit is

$$b_1 + b_2 + d_2 + d_1 = b_1 + d_1 + b_2 + d_2 = (a_1 + a_2)2 = 2a \qquad (9.9)$$

which is the sum of the major axes of the two orbits around the center of gravity. We have written a_1 and a_2 for the semi-major axes of the stellar orbits around the center of gravity, and a for the semi-major axis of the relative orbit of one star around the other.

What we actually observe for a visual binary system is unfortunately not

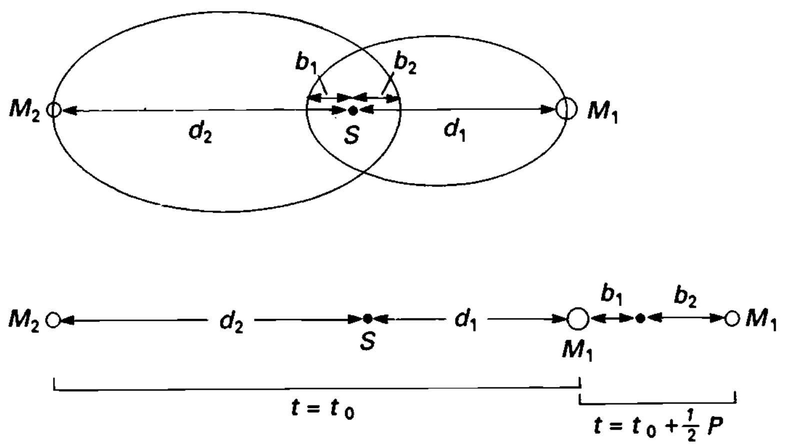

Fig. 9.2. The major axis of the relative orbit of M_1 around M_2 is the sum of the major axes of the orbit of each star around the center of gravity, S. t stands for time and P for the period, t_0 is the time of maximum distance apart of the two stars.

* What we see is, of course, generally not the true orbit but is the projection of the true orbit against the background sky. We will outline below how the true orbit can be determined from the projected orbit and the measured velocities.

the major axis itself but the angular major axis, namely $2a$/distance. In order to convert this into the actual major axis we have to determine the distance or find another way of conversion. As we know good parallaxes can only be determined for nearby stars. Fortunately the Doppler effect is independent of distance and provides a geometrical scale.

9.2 The Doppler effect

The Doppler effect is very important for many branches of astronomical research because it permits us to measure the component of the velocity along the line of sight, the so-called radial velocity (see Fig. 9.3). For the stars, we cannot of course measure their velocity by measuring the distance which they travel and then dividing by the time it takes them to do so, as we do for objects here on Earth. Remember that we can only observe the light from the star, from which we have to derive everything. Fortunately the frequency of the light that we see changes when the light source moves towards us or away from us. This shift in frequency or wavelength is called the Doppler effect. Fortunately the stellar spectra have very narrow wavelength bands in which they have very little light intensity, the so-called absorption lines, which mark distinct wavelengths whose Doppler shift can be measured. If the light source moves towards the observer the wavelength becomes shorter by $\Delta\lambda$ where

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v_r}{c}, \tag{9.10}$$

here λ_0 is the so-called rest wavelength of the line (i.e., the wavelength which the line has, if the light source and the observer have the same velocity). v_r is the component of the velocity in the direction of the observer, the line of sight or the radial velocity (see Fig. 9.3).

If the light source moves away from us, the observers, the wavelength becomes longer by the same relative amount. It is this wavelength shift which permits us to measure the radial velocities of the stars in cm/s.

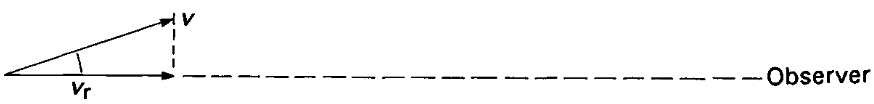

Fig. 9.3. The component of the velocity in the direction of the observer, the radial velocity, v_r, leads to a shift in wavelength of the light, given by (9.10).

9.3 Radial velocities and masses of binaries

In order to measure the radial velocities of the stars we have to observe the energy distributions of the stars in very narrow wavelength bands, i.e., we have to take so-called high ressolution spectra, such that we can see and measure the exact wavelengths of the spectral lines. If our line of sight happens to be in the orbital plane of the binary system, consisting of stars A and B orbiting in circular orbits around their center of gravity S (see Fig. 9.4), then at the time of largest angular distance of the two stars, star B will move towards us while star A moves away from us. If stars A and B have the same mass, as assumed in Fig. 9.4, then the velocities will be of the same magnitude but in opposite directions. At this time, the stars have the largest radial velocity component. While the speed of the two stars in their circular orbits remains the same, the component in the line of sight decreases when they move on and is zero when they reach points P_1 and P_2, where their motions are perpendicular to the line of sight. At this time, the spectral lines, also called Fraunhofer lines, appear at the wavelength λ_0 corresponding to the velocity of the center of gravity. The radial velocity components then change directions for both stars and the Doppler shift changes sign. The lines of both stars are now shifted in the opposite direction. Altogether the lines of each star show a periodic shift whose amount changes sinusoidally in time if the orbits are circular.

In Fig. 9.5 we show the spectra of a binary system, in which the stars are too close to be observed separately. We can only obtain spectra of both stars together, which show the spectral lines of both stars. We see a time series of spectra, proceeding through one orbital period from the top spectra to the

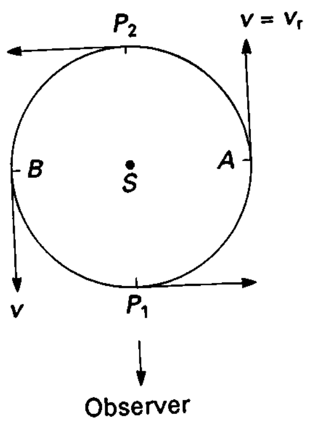

Fig. 9.4. At points A and B the spectra show Doppler shifts of the lines corresponding to the orbital velocities. At the positions P_1 and P_2 the radial velocities are zero.

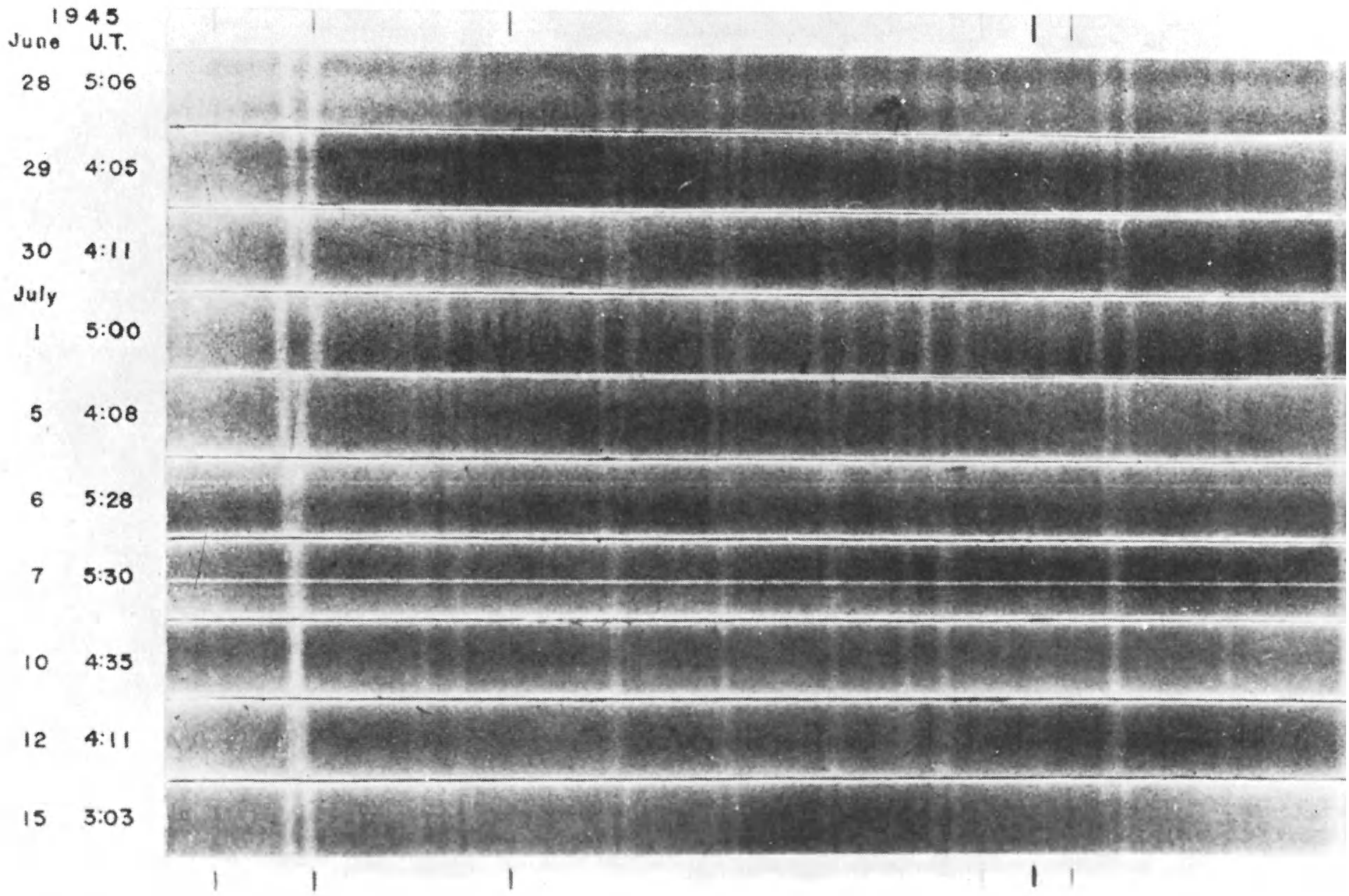

Fig. 9.5. From top to bottom a time sequence of the spectra of the binary star system Mizar is shown. Both stars are too close together to separate their spectra. The line systems of both stars are seen on each spectrum. The line positions of both stars shift sinusoidally due to the Doppler shift which results from the orbital motions. The wavelengths of the spectral lines of stars 1 and 2 vary in antiphase. (From Binnendijk 1960.)

bottom spectra. The approximately sinusoidal change in separation between the lines of the two stars can be seen.

Radial velocity curves as measured for the two stars in the binary system ζ Phoenicis are reproduced in Fig. 9.6B.

Stars which cannot be observed separately but whose binary nature can be seen from the periodic Doppler shifts of their spectral lines are called spectroscopic binaries.

For binaries, for which both spectra can be obtained, and for *circular orbits* with period P we must have

$$v_1 = \frac{2\pi r_1}{P} \quad \text{and} \quad v_2 = \frac{2\pi r_2}{P} \tag{9.11}$$

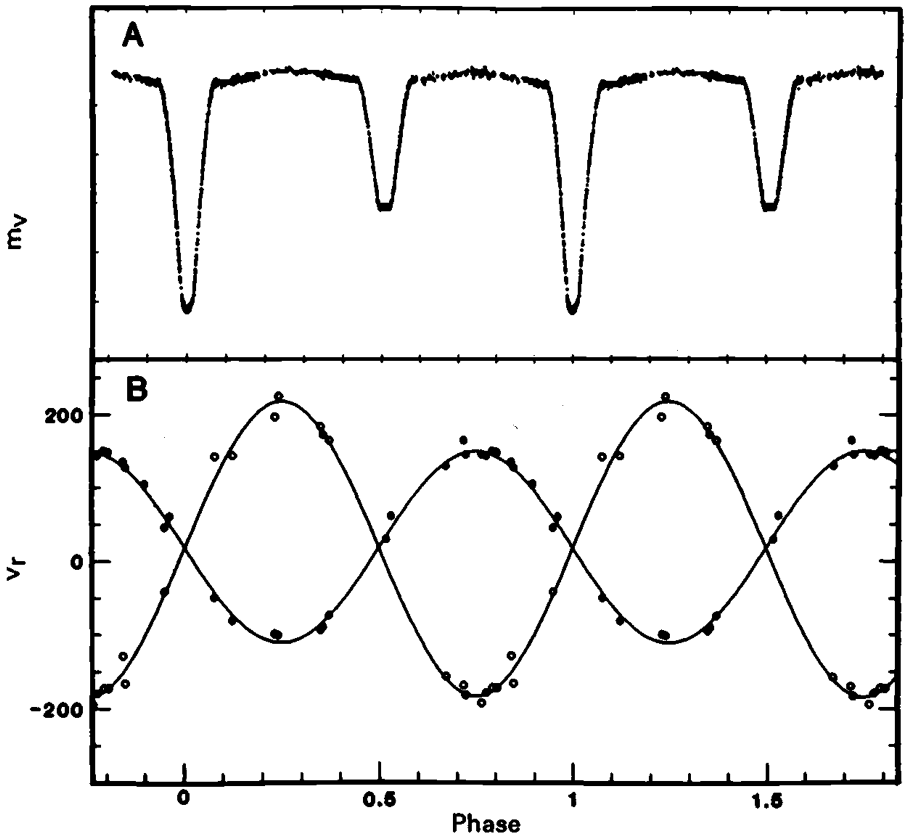

Fig. 9.6. Light and velocity curves for the eclipsing binary ζ Phoenicis are shown as a function of orbital phase. During the times when one star is behind the other we see a reduced light intensity. This happens twice during one period, once if star 1 is in front of star 2, and once if star 2 is in front of star 1. If star 2 has the larger surface brightness, i.e., if star 2 emits more light per cm^2, then less light will be received when star 2 is eclipsed during the first eclipse than will be received when star 1 is eclipsed, i.e., during the second eclipse. The first light minimum is then deeper than the second one. (From Paczinski 1985.)

and

$$\frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{M_2}{M_1}. \tag{9.12}$$

The periods must of course be the same for both stars. v_1 and v_2 can be measured at maximum angular distance of the two stars. With the measured period, r_1 and r_2 and the sum $r_1 + r_2$ can be determined in cm, which then also gives the sum of the masses according to (9.8). From (9.12) the mass ratio can be obtained. With the mass sum and the mass ratio known it is a simple mathematical problem to determine both masses.

Unfortunately, most stars do not have circular orbits and our line of sight usually is not in the orbital plane.

If the stars are not in circular orbits, their radial velocity curves are in general not sinusoidal, the shape depends on the degree of eccentricity and on the orientation of the major axis of the orbit with respect to the line of sight. It also depends on the tilt of the orbital plane with respect to the line of sight, which is described by the angle i between the normal **n** on the orbital plane and the line of sight.

If the inclination angle i between the normal **n** on the orbital plane and the line of sight is not 90°, which means the line of sight is not in the orbital plane, then all measured radial velocities only measure the component of the velocity projected onto the line of sight. This means all velocities are reduced by the factor $\sin i$. Even for circular orbits the radial velocity measured at the largest distance is then only $V \sin i$. (see Fig. 9.7). Then we cannot determine the masses without knowing $\sin i$. For $i = 90°$ the stars appear to move in a straight line. Only for $\sin i = 0$ are we able to see the true shape of the orbit, but then we cannot measure the velocities. For angles i between 0 and 90° orbits appear as ellipses even if they should be circular. In order to be able to derive semi-major axes and masses we therefore have to determine the

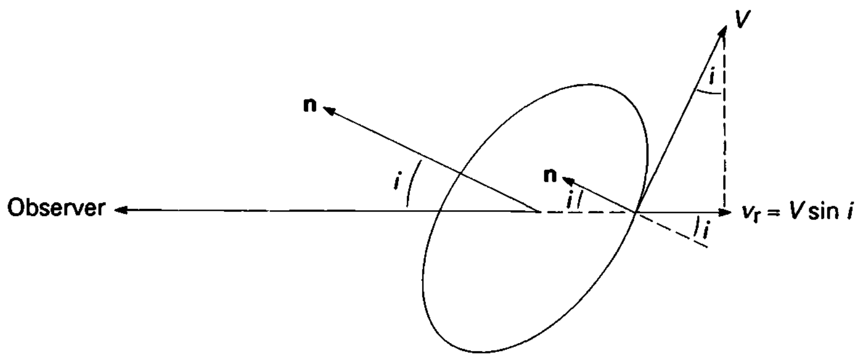

Fig. 9.7. If the normal **n** to the orbital plane is inclined with respect to the line of sight by the angle i, then the radial velocity v_r is $V \sin i$ if V is the true orbital velocity.

orbital parameters like ellipticities, orientation of the ellipse and the inclination angle i of the orbital plane.

9.4 Determination of orbital parameters for binaries

9.4.1 General discussion

Before we discuss in detail the procedure for determining the orbital parameters for binaries let us see how the radial velocity curves change for different orbits. The orbit is determined if we know the ellipticity, the orientation of the ellipse and the inclination angle i between the line of sight and the normal $\mathbf{n}$ on the orbital plane.

The ellipticity e is defined by

$$e^2 = 1 - \frac{b^2}{a^2} \tag{9.13}$$

where a and b are the semi-major and semi-minor axes respectively. The orientation of the ellipse is described by the angle ω, which is the angle between the minor axis of the ellipse and the projection of the line of sight onto the orbital plane.

In Fig. 9.8 we show radial velocity curves to be observed for a circular orbit, $e = 0.0(a)$, and for $e = 0.5$ for different orientations of the ellipse. For different angles i all the velocities would be multiplied by the factor $\sin i$ but the shape would remain the same. We cannot therefore determine the angle i from the shape of the velocity curve. The ellipticity can however be determined from the shape of the radial velocity curve. If $e \neq 0$ the curve is not sinusoidal because the stars have larger velocities when they are closer together. In Fig. 9.8(*b*) we see the high peaks of the radial velocity when the star moves through point b, the perihelion of its orbit. When it moves towards us at point d it is far away from the other star and therefore has a much lower velocity. We measure a broad, flat valley for the negative velocities. A very asymmetric velocity curve is obtained if $\omega = 45°$ as seen in Fig. 9.8(*c*), but we still see larger positive velocities than negative ones. If $\omega = 90°$ (Fig. 9.8(*d*)) the position of the ellipse is symmetric with respect to the line of sight. The largest positive and negative velocities have therefore equal absolute values, but it takes the star a much longer time to get from the largest negative velocity to the largest positive velocity, because the distance is larger and the speed is smaller when going from d to b than when going from b to d. From Fig. 9.8 we can easily see that we can determine e and ω from the deviations of the radial velocity curve from a sinusoidal curve.

For the details of the mathematical procedure see for instance Binnendijk 1960.

9.4.2 *Determination of i for visual binaries*

From the study of the radial velocities we have no means of determining the inclination angle i between the line of sight and the normal **n** to the orbital plane. Since the shape of the radial velocity curve is not changed by a change in i we have no way to determine i if we have only radial velocity curves. If we are however dealing with a visual binary for which we can measure the shape of the orbits we can determine i. We know that for binaries the second star must be in the focal point of the orbit for the first star. Suppose the stars have circular orbits which appear as ellipses because i is not 0. For a circular orbit the secondary star will therefore appear in the

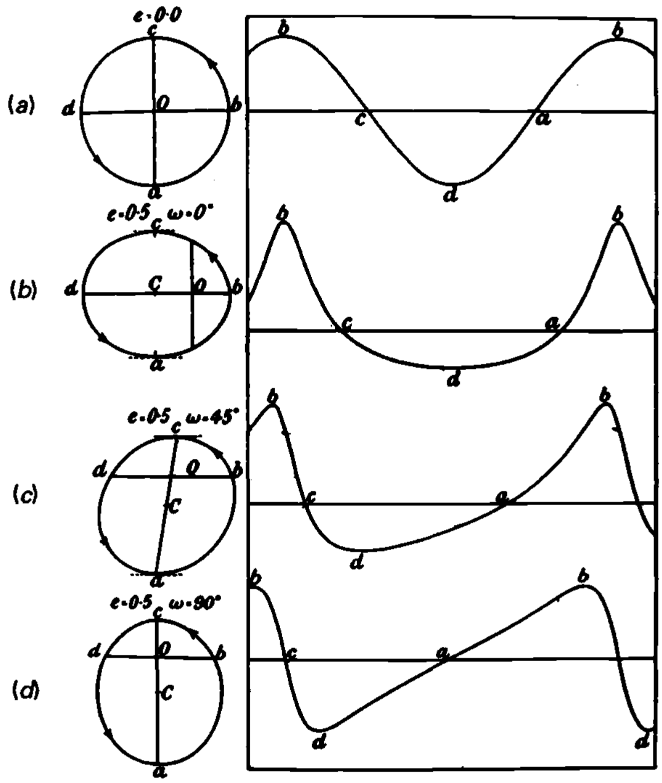

Fig. 9.8. Plotted are radial velocity curves to be seen for stellar orbits with zero ellipticities (*a*) and with an ellipticity $e = 0.5$ for $\omega = 0$ (*b*), and for the same ellipticity but $\omega = 45°$ (*c*), and also for $\omega = 90°$ (*d*). $e \neq 0$ generally introduces a difference between maximum positive and negative velocities (see (*c*) and (*d*)) except in the case when $\omega = 90°$. In this case the time difference between points b and d on the one side and between points d and b on the other side (d) gives a measure of the ellipticity. (From Becker 1950.)

middle of the major axis and not in the focal point (see Fig. 9.9). We know immediately that the true orbit must be a circle. From the apparent ratio of the minor to the major axis we can in this special case, determine how large i is. If the second star appears off center but not in the focal point we also know that i is not 90°, but that the true orbit is an ellipse. We can figure out how much we have to tilt the observed projected ellipse in order to fit the second star onto the focal point of the orbital ellipse.

For the procedure and details of the orbit determinations see for instance Binnendijk 1960.

9.4.3 Distance to visual binaries

In Section 9.3 we saw that we can determine the orbital radii in cm if we can determine the true orbital velocity by measuring the radial velocities, determining the orbital parameters, and determining $\sin i$. If we know the true orbital velocities then the orbital radii and the velocities are related as given by (9.11) for the case of a circular orbit. From the two Equations (9.11) we can determine r_1 and r_2 and thereby $r_1 + r_2$. Observing the orbit of one star around the other permits us to measure the angular semi-major axis $(r_1 + r_2)/d = \alpha$, where d is the distance to the binary system. Once we have determined r_1 and r_2 in cm we can also find d in cm (or in pc) from

$$d = \frac{r_1 + r_2}{\alpha}. \tag{9.14}$$

For visual binaries we can therefore determine the distance even if we cannot measure trigonometric parallaxes.*

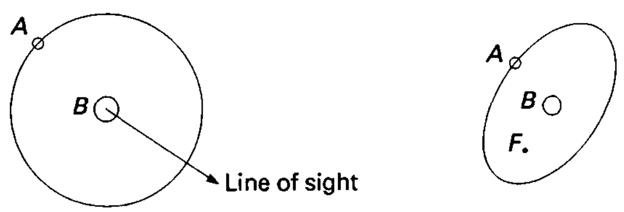

Fig. 9.9. For a circular orbit of star A around star B whose normal **n** is inclined at an angle i with respect to the line of sight, the apparent orbit projected against the background sky is an ellipse with the second star B in the middle of the major axis, instead of in the focal point F.

* In practice there are only a few systems which are close enough for the two stars to be seen separately while the periods are not too long to be determined in a (wo)man's lifetime and for which the radial velocities are large enough to be accurately measurable.

9.4.4 *Eclipsing binaries*

a. *Mass determination.* Another group of binaries for which i can be determined are the eclipsing binaries. For these stars the line of sight is almost in the orbital plane such that at some time the secondary star is in front of the primary (see Fig. 9.10). Half a period later the primary star will be in front of the secondary. If along the line of sight one star gets in front of the other, we can be fairly certain that $\sin i$ has to be close to 1 otherwise the 'front' star would remain above or below the 'back' star, except if the stars are very close (see Fig. 9.11). By far the most binaries are separated far enough that eclipses will only occur if $i > 75°$ which means $\sin i \geqslant 0.96$ and $\sin^3 i \geqslant 0.89$. This means that if any sign of an eclipse is seen $\sin^3 i$ cannot differ by more than 10% from 1. Assuming $\sin^3 i = 1$ will at most cause a 10%

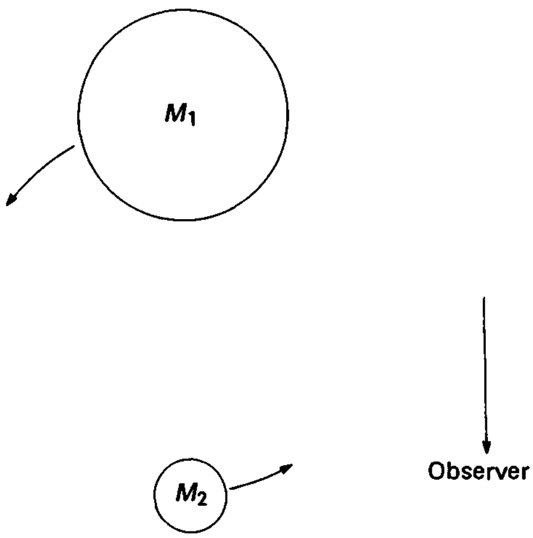

Fig. 9.10. If along the line of sight one star is in front of the other star system the total amount of light of the system is reduced. We see the eclipse of the star.

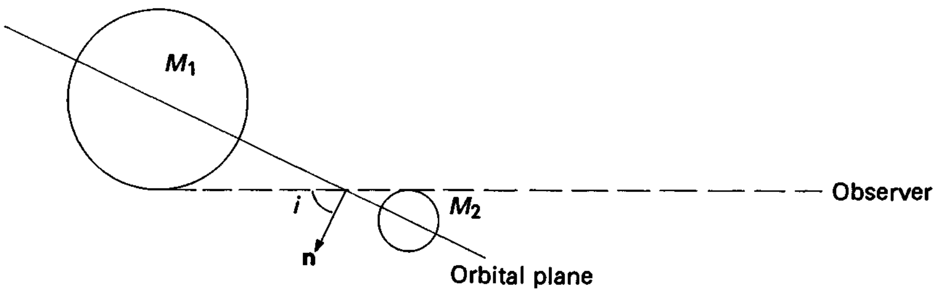

Fig. 9.11. If the line of sight is inclined too much to the orbital plane then star M_2 is not in front of star M_1 along the line of sight. The angle i has to be very close to 90° for an eclipse.

error in the masses (see equation 9.26). With refined technique we can even determine $\sin i$ for partial eclipses. We can usually assume $\sin i = 1$ for eclipsing binaries.

Such eclipses can be recognized by the decreasing amount of light received during the phases of occultation. In Fig. 9.6 we showed the light curve for the eclipsing binary ζ Phoenicis. In Fig. 9.12 we reproduce schematic light curves for binaries with different ratios of the size of their radii and with different surface brightnesses. In Fig. 9.13 we relate the geometrical situation to the observed light curve as a function of time. From Fig. 9.14 we can see, that for a circular orbit of star A around star B the ratio of the total length of the eclipse time t_e to the length of the period P is given by

$$t_e/P = \frac{2R_A + 2R_B}{2\pi r}, \tag{9.15}$$

where R_A and R_B are the radii of stars A and B respectively, and r is the orbital radius of star A around star B. Equation (9.15) assumes that star B is much more massive than star A such that the velocity of star B can be neglected. It also assumes $r \gg R_A$ and R_B.

If $t_e/P \ll 1$ the stellar radii must be much smaller than the orbital radius and the stars cannot be very close. Eclipses must then mean that $i \approx 90°$ and

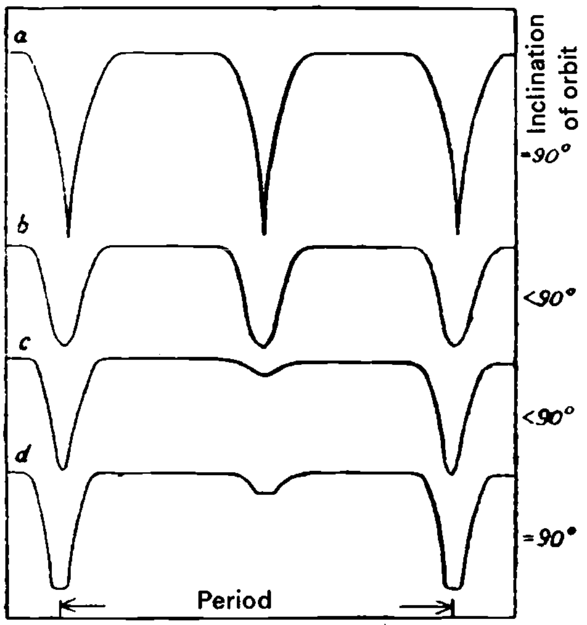

Fig. 9.12. Schematic light curves are shown for different ratios of stellar radii and different surface brightnesses of the eclipsing stars. Curves (*a*) and (*b*) show the light curves for eclipsing binaries for which both stars have the same radius and the same surface brightnesses. Curves (*c*) and (*d*) apply to binaries with different radii and surface brightnesses. From Becker (1950).

$\sin i \approx 1$. We can get even more accurate information about i if we study the shape of the light curve in detail. The decrease of the light intensity will be less steep if the secondary star does not pass over the equator of the other star (see Fig. 9.13(*b*)).

For such eclipsing binaries we then also know the inclination angle i. We can therefore determine the masses of both components as described in Sections 9.4.1 and 9.4.4.

b. *Radius determination.* Eclipsing binaries are also very useful for radius determinations, especially if we see a total eclipse.

Fig. 9.12 shows light curves which are obtained for stars with different radii and different surface brightnesses. Since these light curves look very different we can use the light curves to determine the properties of the stars. For simplicity we again assume circular orbits. Generally orbits are elliptic

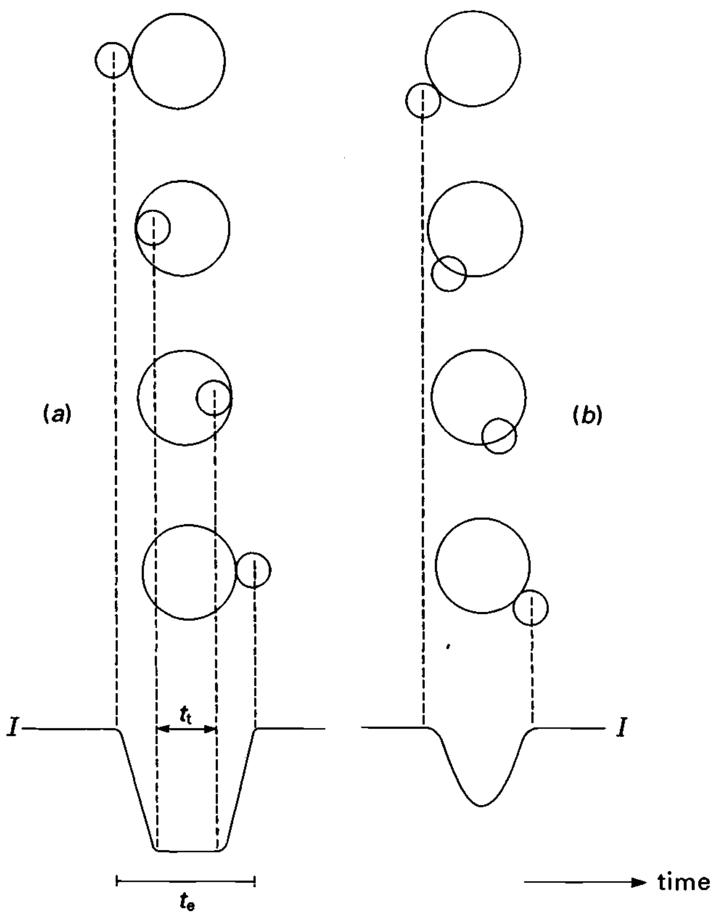

Fig. 9.13. For a total eclipse (*a*) the light curve shows a flat bottom (except if the radii are equal), while for a partial eclipse (*b*) the light curve does not have a flat bottom.

but the method does not change, only the mathematics gets more complicated.

If radial velocities for the stars are measured we can determine the radii of both stars by means of (9.15). The maximum radial velocity of star A is

$$V_{r(\max)} = 2\pi r_A / P. \tag{9.16}$$

We also know that for a total eclipse the duration for the flat part of the light curve t_t, the time during which star A is totally in front of star B, is related to the length of the period P by

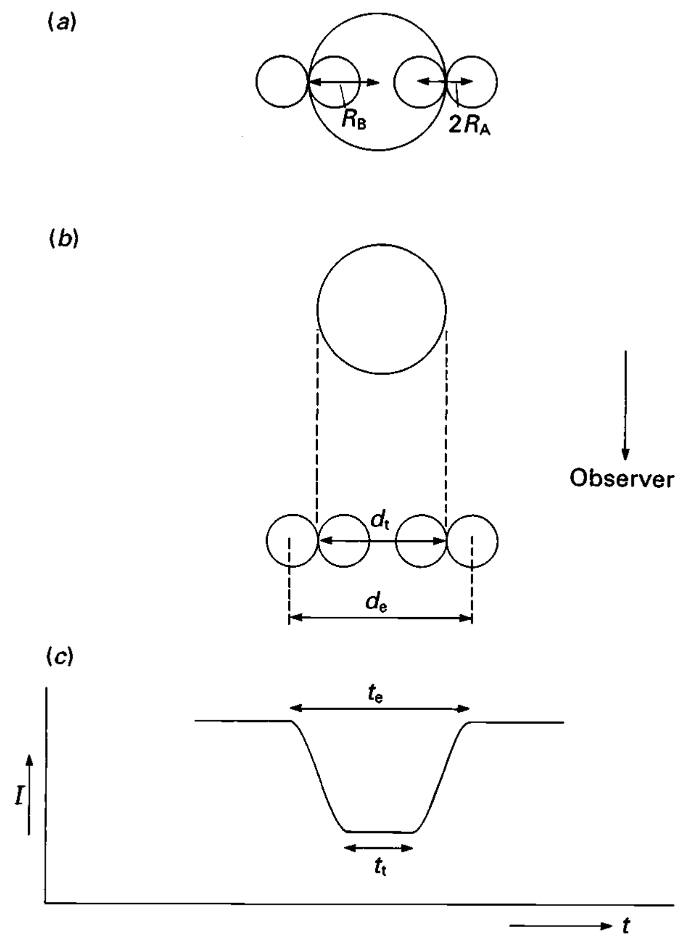

Fig. 9.14. Figure (*a*) shows the positions of the eclipsing star at first, second, third, and fourth contact as seen from an observer above the plane of the paper. Figure (*b*) shows the geometry of the eclipse for an observer whose line of sight is in the orbital plane (which is the plane of the paper), not drawn to scale. For a large distance between stars A and B the path of star B can be approximated by a straight line. Figure (*c*) shows the light curve as a function of time for the duration of the eclipse. During the time t_e star A travels a distance $d_e = 2R_A + 2R_B$. During the time t_t star A travels the distance $d_t = 2R_B - 2R_A$.

$$t_t/P = (2R_B - 2R_A)/2\pi r_A. \tag{9.17}$$

if again the mass of star B is much larger than the mass of star A. Equation (9.16) determines the orbital radius r_A. From (9.15) and (9.17) we obtain

$$(t_e - t_t)/P = \frac{4R_A}{2\pi r_A} \tag{9.18}$$

and

$$(t_e + t_t)/P = \frac{4R_B}{2\pi r_A}. \tag{9.19}$$

These equations determine the radii of both stars.

If stars A and B have comparable masses we have to consider the motions of both stars. The eclipse times will then be shorter, because the relative velocities of the two stars will be larger, namely $V_A + V_B$, with $V_A = 2\pi r_A/P$ and $V_B = 2\pi r_B/P$.

If the stars have elliptical orbits we have to know in which part of the ellipse the occultation occurs, such that we can relate the maximum measured radial velocity to the relative velocity at the time of the eclipse. The mathematics becomes more involved but the principle of determining the radii of both stars remains the same.

c. *Surface brightness of eclipsing binaries.* From the light curves of eclipsing binaries we can also determine the ratio of the surface brightnesses of the stars.

From Fig. 9.14 we see that maximum light is given by

$$I_{\max} = (\pi R_B^2 \cdot F_B + \pi R_A^2 \cdot F_A)\text{const.} \tag{9.20}$$

where F_A and F_B are the amounts of radiation emitted per cm^2 of the stellar surface of star A and star B respectively.

The constant is determined by the distance and the sensitivity of the receiving instrument as well as the transmission of the Earth's atmosphere. During the first eclipse the minimum intensity I_1 is given by (see Fig. 9.15)

$$I_1 = [(\pi R_B^2 - \pi R_A^2) \cdot F_B + \pi R_A^2 \cdot F_A]\text{const.}. \tag{9.21}$$

During the second eclipse the minimum intensity I_2 is given by

$$I_2 = (\pi R_B^2 \cdot F_B)\text{const.}. \tag{9.22}$$

From this we derive

$$I_{\max} - I_2 = (\pi R_A^2 \cdot F_A)\text{const.} \tag{9.23}$$

and

$$I_{\max} - I_1 = (\pi R_A^2 \cdot F_B)\text{const.}. \tag{9.24}$$

Knowing R_B and R_A from our previous discussion we can then determine F_A and F_B from (9.23) and (9.24), except that we have to know the constant, which means we have to know the distance, the transmission of the Earth's atmosphere, the sensitivity of the instrument, etc. So in general we can only determine ratios namely

$$\frac{I_{\max} - I_2}{I_{\max} - I_1} = \frac{F_A}{F_B}. \tag{9.25}$$

If we know the flux for one star, we can determine the flux of the other star. Since the radiative fluxes F_A and F_B determine the effective temperatures of the stars we can also determine the ratio of the effective temperatures of the two components of an eclipsing binary system.

9.4.5 *Spectroscopic binaries*

Most binaries are spectroscopic binaries, which means we can see their binary nature from the periodic line shifts due to the Doppler effect. Very often we see only lines from one star because the companion is too faint to be recognized in the combined spectrum. If we see both spectra we can determine the mass ratio of the two stars according to (9.2). It does not cause any problems that we can only measure $v_1 \sin i$ and $v_2 \sin i$, without knowing what $\sin i$ is. In the ratio the $\sin i$ factor cancels out. But if we want to determine the mass sum we are in trouble. We know that $r_1 = v_1 P/(2\pi)$ and similarly $r_2 = v_2 P/(2\pi)$. Since we only know $v_1 \sin i$ and $v_2 \sin i$ we also can

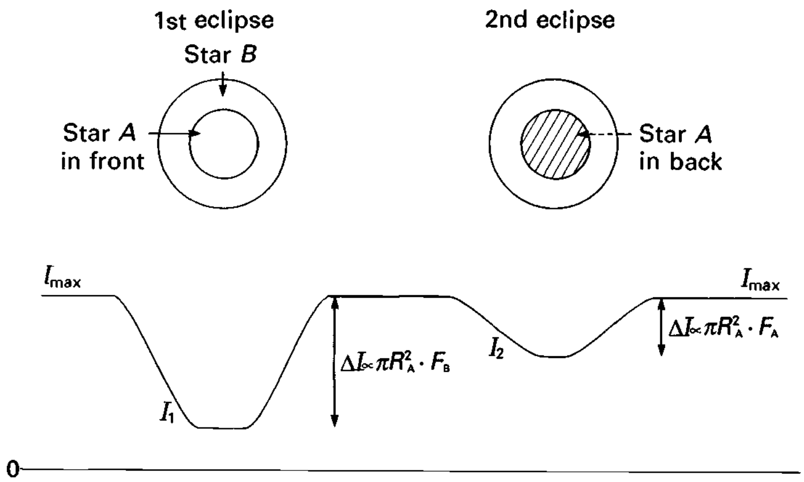

Fig. 9.15. During the first eclipse a fraction of the surface of star B is occulted. During the second eclipse star A is completely covered up. The depths of the light minima depend on the surface fluxes of star A and star B.

Table 9.1 *Masses and radii for some visual binaries according to D. Popper (1980)*

Star	S_p	$M/M_\odot$	$R/R_\odot$	π''
α CMaA	A1 V	2.20	1.68	0.377
α CMi	F5IV-V	1.77	2.06	0.287
ξ Her A	G0IV	1.25	2.24	0.104
ξ Her B	K0V	0.70	0.79	0.104
α Cen	G2V	1.14	1.27	0.743
γ Vir	F0V	1.08	1.35	0.094
η Cas A	G0V	0.91	0.98	0.172
η Cas B	M0V	0.56	0.59	0.172
ξ Boo	G8V	0.90	0.77	0.148

only determine $r_1 \sin i$ and $r_2 \sin i$. According to (9.8) we therefore can determine only

$$(M_1 + M_2)\sin^3 i = \frac{(r_1 + r_2)^3 \sin^3 i}{P^2} \frac{4\pi^2}{G}. \tag{9.26}$$

The mass sum remains uncertain by the factor $\sin^3 i$, and therefore both masses remain uncertain by the same factor.

9.5 Data for stellar masses, radii, and effective temperatures

In Table 9.1 we list stellar masses as determined from the study of visual binaries according to D. Popper (1980). In the same table we also list stellar radii determined for the same binary systems.

In Table 9.2 we have given average values of masses, radii and effective temperatures for main sequence stars of different B − V colors, corrected for interstellar extinction (see Chapter 19).

In Fig. 9.16 we have plotted a color magnitude diagram with the position of the average main sequence drawn in. On this average main sequence we have also given values for the mass, radius, and T_{eff} for stars at that particular position on the main sequence. Also given are spectral types, which will be discussed in the next chapter.

At the top of the main sequence we find the most massive stars, which are also the largest ones and the hottest ones. It becomes obvious that the main sequence is indeed a one parameter sequence: If we know the B − V color for a star we also know its absolute magnitude, its mass and radius and its