$2 \to 2 \quad (\chi + \bar{\chi} \to A + \bar{A})$

$$\int \hat{C}[f_\chi]\, d\pi_\chi = -\int d\pi_\chi\, d\pi_{\bar{\chi}}\, d\pi_A\, d\pi_{\bar{A}}\, (2\pi)^4 \delta^4(p_\chi + p_{\bar{\chi}} - p_A - p_{\bar{A}}) \times$$

$$\left[|\mathcal{M}|^2_{\bar{\chi}+\chi \to A+\bar{A}} f_\chi f_{\bar{\chi}} (1 \pm f_A)(1 \pm f_{\bar{A}}) - |\mathcal{M}|^2_{A\bar{A} \to \chi\bar{\chi}} f_A f_{\bar{A}} (1 \pm f_\chi)(1 \pm f_{\bar{\chi}}) \right]$$

- $d\pi \equiv g \dfrac{1}{(2\pi)^3} \dfrac{d^3 p}{2E}$
- $(1 \pm f) \approx 1$
- $|\mathcal{M}|^2_{\bar{\chi}+\chi \to A+\bar{A}} = |\mathcal{M}|^2_{A+\bar{A} \to \chi\bar{\chi}} = |\mathcal{M}|^2$

Assumindo A e $\bar{A}$ em equilíbrio.

$$\int \hat{C}[f_\chi]\, d\pi_\chi = -\int d\pi_\chi\, d\pi_{\bar{\chi}}\, d\pi_A\, d\pi_{\bar{A}}\, (2\pi)^4 \delta^4(p_\chi + p_{\bar{\chi}} - p_A - p_{\bar{A}}) \times |\mathcal{M}|^2 \left[f_\chi f_{\bar{\chi}} - f_\chi^{eq} f_{\bar{\chi}}^{eq} \right]$$

Como:

$$\sigma = \frac{1}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2}} \int |\mathcal{M}|^2 \, (2\pi)^4 \delta^4(p_{\bar{\chi}} + p_\chi - p_A - p_{\bar{A}}) \frac{d^3 p_3}{2E_3} \cdot \frac{d^3 p_4}{2E_4}$$

Então

- $\displaystyle\int d\pi_A\, d\pi_{\bar{A}}\, (2\pi)^4 \delta^4(p_\chi + p_{\bar{\chi}} - p_A - p_{\bar{A}})\, |\mathcal{M}|^2 = 4\, g_A g_{\bar{A}}\, \sigma \sqrt{(p_\chi \cdot p_{\bar{\chi}})^2 - (m_\chi m_{\bar{\chi}})^2}$

- $|\bar{\mathcal{M}}|^2 = \dfrac{1}{g_A g_{\bar{A}}} |\mathcal{M}|^2$

Logo

$$\int \hat{C}[f_\chi]\, d\pi_\chi = -4 \int \frac{d\pi_\chi}{g_\chi} \frac{d\pi_{\bar{\chi}}}{g_{\bar{\chi}}}\, \sigma \sqrt{(p_\chi \cdot p_{\bar{\chi}})^2 - (m_\chi m_{\bar{\chi}})^2} \left[f_\chi f_{\bar{\chi}} - f_\chi^{eq} f_{\bar{\chi}}^{eq} \right]$$

- Møller velocity: $v_{Møl_{ij}} = \dfrac{\sqrt{(p_i \cdot p_j)^2 - (m_i m_j)^2}}{E_i E_j}$

$$\int \hat{C}[f_\chi]\, d\pi_\chi = -\frac{1}{(2\pi)^6} \int d^3 p_\chi\, d^3 p_{\bar{\chi}}\, (\sigma v_{Møl})_{\chi\bar{\chi}} \left[f_\chi f_{\bar{\chi}} - f_\chi^{eq} f_{\bar{\chi}}^{eq} \right]$$

Como

$$\langle \mathcal{O}(\vec{p}_k) \rangle = \frac{1}{n_k} \cdot \frac{g}{(2\pi)^3} \int d\vec{p}_k\, \mathcal{O}(\vec{p}_k) f_k(\vec{p}_k)$$

Temos

$$\int \hat{C}[f_\chi]\, d\pi_\chi = -\langle \sigma v_{Møller} \rangle \left[n_\chi n_{\bar{\chi}} - n_\chi^{eq} n_{\bar{\chi}}^{eq} \right]$$

$1 \longrightarrow 2$

$$\hat{C}_{1\leftrightarrow 2,3}[f_1] = -\int d\Pi_2\, d\Pi_3\, (2\pi)^4 \delta^4(p_1 - p_2 - p_3)\left[\, |\bar{\mathcal{M}}|^2_{1\to 23} f_1 (1-f_2)(1-f_3) \; + \right.$$

$$d\Pi_i \equiv g_i \frac{d^3 p_i}{2E_i} \qquad \left. - |\bar{\mathcal{M}}|^2_{23\to 1} f_2 f_3 (1-f_1) \right]$$

- $|\mathcal{M}|^2_{1\to 2,3} = |\mathcal{M}|^2_{2,3\to 1}$ e $(1-f_i) \approx 1$

$$\hat{C}_{1\leftrightarrow 23}[f_1] = \int d\Pi_2\, d\Pi_3\, (2\pi)^4 \delta^4(p_1 - p_2 - p_3)\, |\bar{\mathcal{M}}|^2 \left[f_1 - f_2 f_3 \right]$$

- Se 2 e 3 estão em eq. $\Rightarrow$ $f_2 \cdot f_3 = f_2^{eq} f_3^{eq} = f_1^{eq}$

$$\hat{C}_{1\leftrightarrow 23}[f_1] = \int d\Pi_2\, d\Pi_3\, (2\pi)^4 \delta^4(p_1 - p_2 - p_3)\, |\bar{\mathcal{M}}|^2 \left[f_1 - f_1^{eq} \right]$$

- $|\bar{\mathcal{M}}|$ já inclui os fatores de simetria S_{23}

$$\hat{C}_{1\leftrightarrow 23}[f_1] = \left[f_1 - f_1^{eq} \right] \cdot \overbrace{\left[\frac{S_{23}}{2} \int d\Pi_2\, d\Pi_3\, (2\pi)^4 \delta^4(p_1 - p_2 - p_3)\, |\mathcal{M}|^2 \right]}^{E_1 \Gamma_{S_2}}$$

$$\hat{C}_{1\leftrightarrow 23}[f_1] = E_1 \Gamma \left[f_1 - f_1^{eq} \right]$$

$$\Gamma = \frac{\sqrt{3}\, M_{Pl}}{\sqrt{m Y_S}}$$

$$\Gamma^2 = \frac{3 M_P^2}{m Y_S}$$

$$\frac{2\pi^2}{45} g_S(T) T^3 = \frac{3 M_P}{m Y \Gamma^2}$$

$$T = \left(\frac{45 \cdot 3}{2\pi^2} \right)^{1/3}$$

$$\int \hat{C}_{1\leftrightarrow 23}[f_1]\, d\Pi_1 = \int d\Pi_1\, E_1 \Gamma \left[f_1 - f_1^{eq} \right]$$

$$= \frac{g}{(2\pi)^3} \int d\vec{p}_1\, \Gamma \left[f_1 - f_1^{eq} \right]$$

$$= \frac{g}{(2\pi)^3} \left[\int d\vec{p}_1\, \Gamma f_1 - \int d\vec{p}_1\, \Gamma f_1^{eq} \right]$$

- $\langle \mathcal{O} \rangle \equiv \frac{1}{n} \frac{g}{(2\pi)^3} \int d\vec{p}\, \mathcal{O} f$

$$\langle \Gamma \rangle n = \frac{g}{(2\pi)^3} \int d\vec{p}\, \Gamma f$$

$$= \left[\langle \Gamma \rangle n_1 - \langle \Gamma \rangle n_1^{eq} \right]$$

$$= \langle \Gamma \rangle \left[n_1 - n_1^{eq} \right]$$

$\bullet \langle \Gamma \rangle$

$$\langle \Gamma \rangle \equiv \frac{\int d^3p\, \Gamma(E)\, e^{-E/T}}{\int d^3p\, e^{-E/T}}$$

$$\int_{-\infty}^{+\infty} d^3p\, e^{-E/T} = \int_{-\infty}^{+\infty} d^3p \exp\left(-\sqrt{\frac{p^2}{T^2}+\frac{m^2}{T^2}}\right) = 2\int_0^{\infty} d^3p \exp\left(-\sqrt{\frac{p^2}{T^2}+\frac{m^2}{T^2}}\right) = 8\pi \int_0^{\infty} dp\cdot e^{-\sqrt{\frac{p^2}{T^2}+\frac{m^2}{T^2}}}\, p^2$$

$$= 8\pi \int_0^{\infty} dp\, e^{-\frac{m}{T}\sqrt{\frac{p^2}{m^2}+1}} \cdot p^2$$

$\operatorname{Senh} x(p) = \frac{p}{m}$

$\frac{d}{dp}(\operatorname{Senh} x) = \cosh x \cdot \frac{dx}{dp} = \frac{1}{m}$

$\Rightarrow dp = m \cosh x\, dx.$

$$= 8\pi \cdot m^3 \int_0^{\infty} dx \cosh x\, e^{-\frac{m}{T}\cdot \cosh x} \cdot \operatorname{Senh}^2 x$$

$$E\left(\partial_t - H\vec{p}\,\nabla_{\vec{p}}\right) f(t,\vec{p}) = \hat{C}[f]$$

- $$\langle \vec{p} \rangle = \frac{g}{(2\pi)^3} \int \frac{d\vec{p}}{E} \cdot \vec{p} f = 3.15\, T$$

$$\partial_t T_x + H T_x = \frac{1}{3} \int d\pi_x \hat{C}[p_x f]$$

$$\frac{d}{dt} = aH \frac{d}{da} = AH \frac{d}{A} \qquad A \equiv a.T_r$$

$$AH \frac{dT_x}{dA} + H T_x = \frac{1}{3} \int d\pi_x \hat{C}[p_x f]$$

- P/ $x + \Delta \rightarrow x + \Delta$:

$$\int d\pi\, \hat{C}[p_x f] = -\int d\pi \dots d\pi \quad \tau)^4 |M|^2 p_x \left[f^i_x f^i_\Delta - f^f_x f^f_\Delta \right]$$

$$AH \frac{dT_x}{dA} + H T_x = \frac{1}{3}\left[n^i_\Delta \int d\pi_x p_x f \sigma_x - n^f_\Delta \int d\pi_x p_x f^f_x \sigma \right]$$

$$= \frac{1}{3}\left[n^i_\Delta \int d\pi_x p_x f \sigma v - n^f_\Delta \int d\pi_x p_x f^f_x \sigma v \right]$$

$$= \frac{1}{3} \Big[$$

• $\dot{\rho}$

$$\left[E\frac{\partial}{\partial t} - H|\vec{p}|^2\frac{\partial}{\partial E}\right] = \hat{C}$$

$$dE = \frac{p\,dp}{E}$$

$$\frac{d}{dE} = \frac{E}{p}$$

$$\Rightarrow \quad E\frac{\partial f}{\partial t} - H|\vec{p}|E\frac{\partial f}{\partial p} = \hat{C}$$

$$d\pi \equiv \frac{g}{(2\pi)^3}d\vec{p}$$

$$\frac{\partial}{\partial p}(pEf) = Ef + pf\frac{\partial E}{\partial p} + pE\frac{\partial f}{\partial p}$$

$$\int d\pi\, E\frac{\partial f}{\partial t} - H\int d\pi\,|\vec{p}|E\frac{\partial f}{\partial p} = \int d\pi\,\hat{C}$$

$$\frac{\partial}{\partial p}(p^3Ef) = 3p^2Ef + p^3f\frac{\partial E}{\partial p} + p^3E\frac{\partial f}{\partial p}$$

• $$\int d\pi\,|\vec{p}|E\frac{\partial f}{\partial|\vec{p}|} = 4\pi\int dp\,p^3E\frac{\partial f}{\partial p}$$

Teoreme de Gauss $\to \int ds(pEf)$ $p, E \to 0$
Conforme $f \to 0$ mais rapido que $s \to \infty$

$$= \underbrace{4\pi\int dp\,\frac{\partial}{\partial p}(p^3Ef)}_{\to 0} - 3\cdot4\pi\int dp\,p^2Ef - 4\pi\int dp\,p^3f\underbrace{\frac{\partial E}{\partial p}}_{=\frac{p}{E}}$$

$$= -3\int d\pi\,Ef - \int d\pi\cdot\frac{p^2}{E}f$$

$$\int d\pi\, E\frac{\partial f}{\partial t} - H\int d\pi\,|\vec{p}|E\frac{\partial f}{\partial p} = \int d\pi\,\hat{C}$$

$$\frac{\partial\rho}{\partial t} + 3H(\rho + P) = \int d\pi\,\hat{C}[f]$$

$$\frac{\partial}{\partial p}(p^4f) = 4p^3f + p^4\frac{\partial f}{\partial p}$$

$$\int dp\,p^3\frac{\partial f}{\partial p} =$$

$$\hat{L}[f] \equiv \left[\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}_{\vec{x}} + \frac{\vec{F}}{m} \cdot \vec{\nabla}_{\vec{v}}\right] f$$

$$= \left[\frac{\partial}{\partial t} + \frac{d\vec{x}}{dt} \cdot \frac{\partial}{\partial \vec{x}} + \frac{d\vec{p}}{dt} \cdot \frac{\partial}{\partial \vec{p}}\right] f$$