Arrows for Parallel Computations

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Abstract

Arrows form a general interface for computation and pose therefore as an alternative to monads for API design. In the paper Hughes describes how arrows are a generalization of monads and how they are not as restrictive. We express parallelism using this concept. We define an Arrows-based interface for parallel computations and implement it using multiple parallel Haskells.

This new way of writing parallel programs has a benefit of being portable across flavours of parallel Haskells used. With our framework we define several parallel skeletons. We also introduce some syntactic sugar to provide parallelism-aware arrow combinators similar to the traditional ones. Benchmarks shows that our framework does not induce too much overhead performance-wise.

Contents

1 Introduction

blablabla arrows, parallel, haskell.

Contribution HIT HERE REALLY STRONG

Structure The remaining text is structures as follows. Section 2 briefly introduces known parallel Haskell flavours and gives an overview of Arrows to the reader (Sec. 3.1). Section 3 discusses related work. Section 4 defines Parallel Arrows and presents a basic interface. Section 5 defines futures for Parallel Arrows. Futures are a further concept that helps to define more advanced interfaces. Section 6 presents some basic algorithmic skeletons in our newly defined dialect. More advanced ones are showcased in Section 7. Section 8 shows the benchmark results. Section 9 concludes.

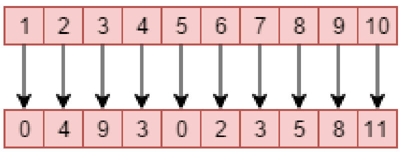
2 Background

2.1 Short introduction to parallel Haskells

There are already several ways to write parallel programs in Haskell. As we will base our parallel arrows on existing parallel Haskells, we will now give a short introduction to the ones we use as backends in this paper.

In its purest form, parallel computation (on functions) can be looked at as the execution of some functions `a -> b` in parallel:

```
parEvalN :: [a -> b] -> [a] -> [b]
```

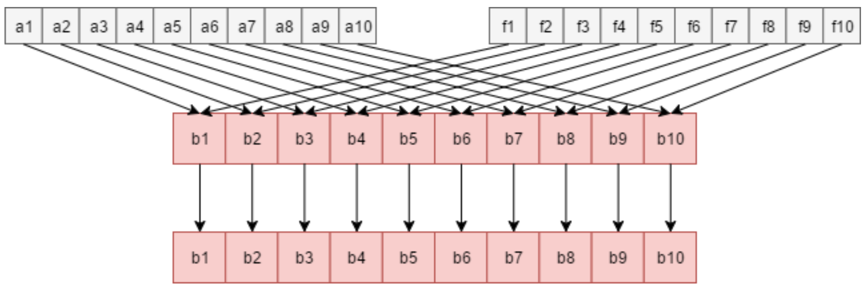


We will now implement parEvalN with the different parallel Haskells.

2.1.1 Multicore Haskell

Multicore Haskell (Marlow et al., 2009) is way to do parallel processing found in standard GHC.[1] It ships with parallel evaluation strategies for several types which can be applied with `using :: a -> Strategy a -> a`. For parEvalN this means that we can just apply the list of functions `[a -> b]` to the list of inputs `[a]` by zipping them with the application operator `$`. This lazy list `[b]` is then forcibly evaluated in parallel with the strategy `Strategy [b]` by the `using` operator. This strategy can be constructed with `parList :: Strategy a -> Strategy [a]` and `rdeepseq :: NFData a => Strategy a`.

```
parEvalN :: (NFData b) => [a -> b] -> [a] -> [b]
parEvalN fs as = zipWith ($) fs as `using` parList rdeepseq
```



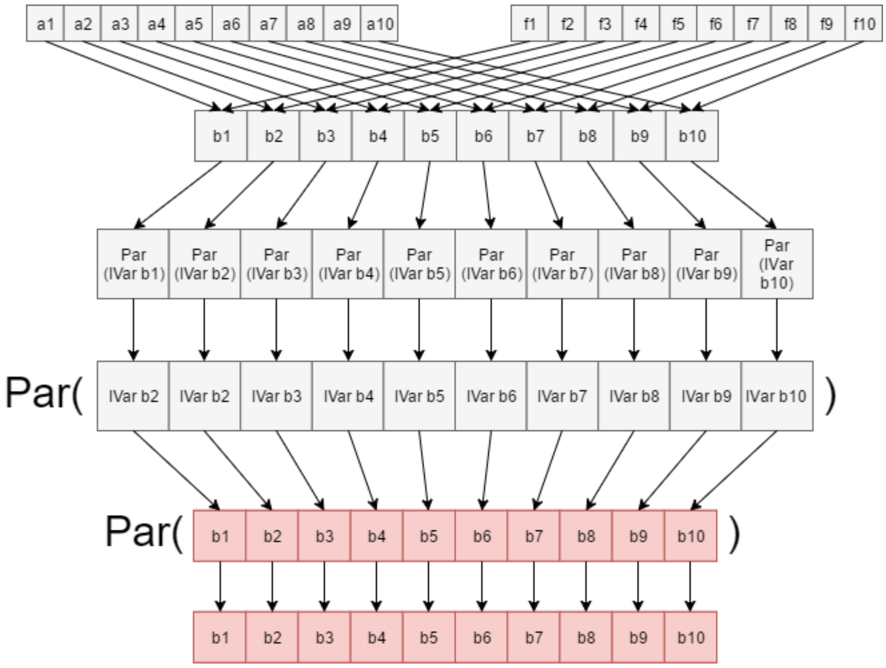
2.1.2 ParMonad

The Par monad[2] introduced by Marlow et al. (2011), is a monad designed for composition of parallel programs.

Our parallel evaluation function parEvalN can be defined by zipping the list of `[a -> b]` with the list of inputs `[a]` with the application operator `$` just like with Multicore Haskell. Then, we map over this not yet evaluated lazy list of results `[b]` with `spawnP :: NFData a => a -> Par (IVar a)` to transform them to a list of not yet evaluated forked away computations `[Par (IVar b)]`, which we convert to `Par [IVar b]` with `sequenceA`. We wait for the computations to finish by mapping over the `IVar b`'s inside the `Par` monad with `get`. This results in `Par [b]`. We finally execute this process with `runPar` to finally get `[b]` again.

```
parEvalN :: (NFData b) => [a -> b] -> [a] -> [b]
parEvalN fs as = runPar $
  (sequenceA $ map (spawnP) $ zipWith ($) fs as) >>= mapM get
```

[1] See `https://hackage.haskell.org/package/parallel-3.2.1.0`.
[2] It can be found in the `monad-par` package on hackage under `https://hackage.haskell.org/package/monad-par-0.3.4.8/`.



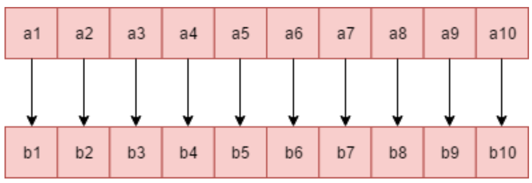
2.1.3 Eden

Eden (Loogen et al., 2005; Loogen, 2012a) is a parallel Haskell for distributed memory and comes with a MPI and a PVM backends.[3] This means that it works on clusters as well so it makes sense to have a Eden-based backend for our new parallel Haskell flavour.

Eden was designed to work on clusters, but with a further simple backend it operates on multicores. However, in contrast to many other parallel Haskells, in Eden each process has its own heap. This seems to be a waste of memory, but with distributed programming paradigm and individual GC per process, Eden yields good performance results also on multicores (Berthold et al., 2009b; **?**).

While Eden also comes with a monad PA for parallel evaluation, it also ships with a completely functional interface that includes
`spawnF :: (Trans a, Trans b) =>[a -> b] -> [a] -> [b]`.
This allows us to define parEvalN directly:

```
parEvalN :: (Trans a, Trans b) => [a -> b] -> [a] -> [b]
parEvalN = spawnF
```



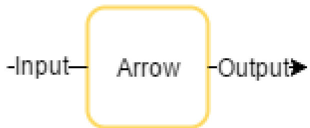
[3] See also http://www.mathematik.uni-marburg.de/~eden/ and https://hackage.haskell.org/package/edenmodules-1.2.0.0/.

Eden TraceViewer To comprehend the efficiency and the lack thereof in a parallel program, an inspection of its execution is extremely helpful. While some large-scale solutions exist (Geimer et al., 2010), in parallel Haskell community mainly utilises the tools Threadscope (Wheeler and Thain, 2009) and Eden TraceViewer[4] (Berthold and Loogen, 2007). In the next sections we will present some *traces*, the post-mortem process diagrams of Eden processes and their activity.

In a trace, the x axis shows the time, the y axis enumerates the machines and processes. A trace shows a running process in green, a blocked process is red. If the process is 'runnable', i.e. it may run, but does not, it is yellow. The typical reason for then is GC. An inactive machine where no processes are started yet, or all are already terminated, is shows as a blue bar. A comminication from one process to another is represented with a black arrow. A stream of communications, e.g. a transmitted list is shows as a dark shading between sender and receiver processes.

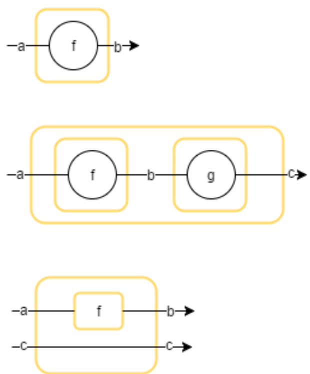
3 Related Work

3.1 Arrows



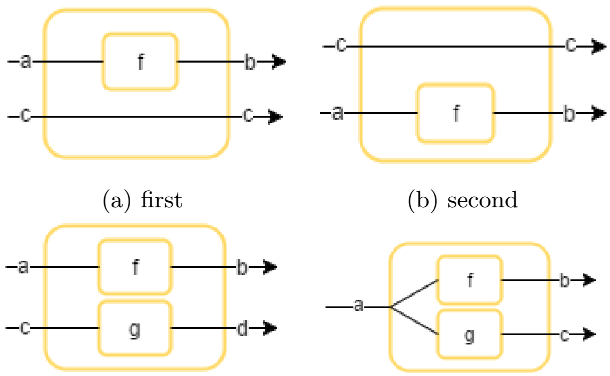
Arrows were introduced by Hughes (Hughes, 2000) as a general interface for computation. An arrow `arr a b` represents a computation that converts an input `a` to an output `b`. This is defined in the arrow typeclass:

```
class Arrow arr where
arr :: (a -> b) -> arr a b



(>>>) :: arr a b -> arr b c -> arr a c




first :: arr a b -> arr (a,c) (b,c)
```



`arr` is used to lift an ordinary function to an arrow type, similarly to the monadic **return**. The >>> operator is analogous to the monadic composition >>= and combines two arrows `arr a b` and `arr b c` by "wiring" the outputs of the first to

[4]See ..... on hackage for the last available version of Eden TraceViewer. There was an effort to implement the TraceViewer using modern web technologies (**?**).



(a) first (b) second

(c) (***) (d) (&&&)

Figure 1: Syntactic sugar for arrows.

the inputs to the second to get a new arrow `arr a c`. Lastly, the `first` operator takes the input arrow from `b` to `c` and converts it into an arrow on pairs with the second argument untouched. It allows us to to save input across arrows.

The most prominent instances of this interface are regular functions (−>),

```
instance Arrow (->) where
  arr f = f
  f >>> g = g . f
  first  f = \(a, c) -> (f a, c)
```

and the Kleisli type

```
data Kleisli m a b = Kleisli { run :: a -> m b }

instance Monad m => Arrow (Kleisli m) where
  arr f = Kleisli $ return . f
  f >>> g = Kleisli $ \a -> f a >>= g
  first  f = Kleisli $ \(a,c) -> f a >>= \b -> return (b,c)
```

With this typeclass in place, Hughes also defined some syntactic sugar: The mirrored version of `first`, called `second`,

```
second :: Arrow arr => arr a b -> arr (c, a) (c, b)
second f = arr swap >>> first f >>> arr swap
  where swap (x, y) = (y, x)
```

the *** combinator which combines first and second to handle two inputs in one arrow,

```
(***) :: Arrow arr => arr a b -> arr c d -> arr (a, c) (b, d)
f *** g = first f >>> second g
```

and the &&& combinator that constructs an arrow which outputs 2 different values like ***, but takes only one input.

```
(&&&) :: Arrow arr => arr a b -> arr a c -> a a (b, c)
f &&& g = arr (\a -> (a, a)) >>> (f *** g)
```

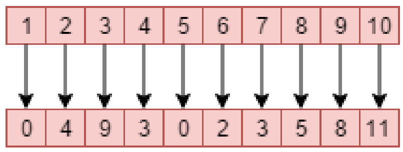
A short example given by Hughes on how to use this is add over arrows:

```
add :: Arrow arr => arr a Int -> arr a Int -> arr a Int
add f g = (f &&& g) >>> arr (\(u, v) -> u + v)
```

The more restrictive interface of arrows (a monad can be *anything*, an arrow is a process of doing something, a *computation*) allows for more elaborate composition and transformation combinators. One of the major problems in parallel computing is composition of parallel processes.

4 Parallel Arrows

We have seen what Arrows are and how they can be used as a general interface to computation. In the following section we will discuss how Arrows constitute a general interface not only to computation, but to **parallel computation** as well. We start by introducing the interface and explaining the reasonings behind it. Then, we discuss some implementations using exisiting parallel Haskells. Finally, we explain why using Arrows for expressing parallelism is beneficial.



4.1 The ArrowParallel typeclass

As we have seen earlier, in its purest form, parallel computation (on functions) can be seen as the execution of some functions a -> b in parallel:

```
parEvalN :: [a -> b] -> [a] -> [b]
```

Translating this into arrow terms gives us a new operator parEvalN that lifts a list of arrows [arr a b] to a parallel arrow arr [a] [b] (This combinator is similar to our utility function listApp, but does parallel instead of serial evaluation).

```
parEvalN :: (Arrow arr) => [arr a b] -> arr [a] [b]
```

With this definition of parEvalN, parallel execution is yet another arrow combinator. But as the implementation may differ depending on the actual type of the arrow arr and we want this to be an interface for different backends, we have to introduce the new typeclass ArrowParallel to host this combinator.

```
class Arrow arr => ArrowParallel arr a b where
  parEvalN :: [arr a b] -> arr [a] [b]
```

Sometimes parallel Haskells require additional configuration parameters, e.g. an information about the execution environment. For this reason we also introduce an additional conf parameter to the function. We also do not want conf to be a fixed type, as the configuration parameters can differ for different instances of ArrowParallel. So we add it to the type signature of the typeclass as well.

```
class Arrow arr => ArrowParallel arr a b conf where
  parEvalN :: conf -> [arr a b] -> arr [a] [b]
```

Note that we don't require the conf parameter in every implementation. If it is not needed, we usually want to allow the conf type parameter to be of any type and don't even evaluate it by blanking it in the type signature of the implemented parEvalN, as we will see in the implementation of the Multicore and the ParMonad backend.

4.2 Multicore Haskell

The Multicore Haskell implementation of this class is straightforward using listApp from chapter A combined with the using operator from Multicore Haskell.

```
instance (NFData b, ArrowApply arr, ArrowChoice arr) =>
  ArrowParallel arr a b conf where
    parEvalN _ fs = listApp fs >>> arr (flip using $ parList rdeepseq)
```

We hardcode the parList rdeepseq strategy here, as in this context it is the only one making sense, since we usually want the output list to be fully evaluated to its normal form.

4.3 ParMonad

The ParMonad implementation makes use of Haskells laziness and ParMonad's spawnP :: NFData a =>a -> Par (IVar a) function. The latter forks away the computation of a value and returns an IVar containing the result in the Par monad.

We therefore apply each function to its corresponding input value with app and then fork the computation away with arr spawnP inside a zipWithArr call. This yields a list [Par (IVar b)], which we then convert into Par [IVar b] with arr sequenceA. In order to wait for the computation to finish, we map over the IVars inside the ParMonad with arr (>>= mapM get). The result of this operation is a Par [b] from which we can finally remove the monad again by running arr runPar to get our output of [b].

```
instance (NFData b, ArrowApply arr, ArrowChoice arr) =>
  ArrowParallel arr a b conf where
    parEvalN _ fs =
      (arr $ \as -> (fs, as)) >>>
      zipWithArr (app >>> arr spawnP) >>>
      arr sequenceA >>>
      arr (>>= mapM get) >>>
      arr runPar
```

4.4 Eden

For the Multicore and ParMonad implementation we could use general instances of ArrowParallel that just require the ArrowApply and ArrowChoice typeclasses. With Eden this is not the case as we can only spawn a list of functions and we cannot extract simple functions out of arrows. While we could still manage to have only one class in the module by introducing a typeclass like

```
class (Arrow arr) => ArrowUnwrap arr where
  arr a b -> (a -> b)
```

we don't do it here, for aesthetic resons. For now, we just implement ArrowParallel for normal functions

```
instance (Trans a, Trans b) => ArrowParallel (->) a b conf where
parEvalN _ fs as = spawnF fs as
```

and the Kleisli type.

```
instance (Monad m, Trans a, Trans b, Trans (m b)) =>
  ArrowParallel (Kleisli m) a b conf where
parEvalN conf fs =
  (arr $ parEvalN conf (map (\(Kleisli f) -> f) fs)) >>>
  (Kleisli $ sequence)
```

4.5 Impact of parallel Arrows

We have seen that we can wrap parallel Haskells inside of the ArrowParallel interface, but why do we abstract parallelism this way and what does this approach do better than the other parallel Haskells?

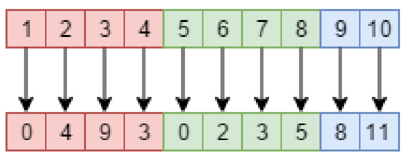
- **Arrow API benefits**: With the ArrowParallel typeclass we do not lose any benefits of using arrows as parEvalN is just yet another arrow combinator. The resulting arrow can be used in the same way a potential serial version could be used. This is a big advantage of this approach, especially compared to the monad solutions as we do not introduce any new types. We can just 'plug' in parallel parts into our sequential programs without having to change anything.

- **Abstraction**: With the ArrowParallel typeclass, we abstracted all parallel implementation logic away from the business logic. This gives us the beautiful situation of being able to write our code against the interface the typeclass gives us without being bound to any parallel Haskell. So as an example, during development, we can run the code on the simple Multicore version and afterwards deploy it on a cluster by converting it into an Eden version, by just replacing the actual ArrowParallel instance.

4.6 Extending the Interface

With the ArrowParallel typeclass in place and implemented, we can now implement some further basic parallel interface functions. These are algorithmic skeletons that, however, mostly serve as a foundation to further, more specific algorithmic skeletons.

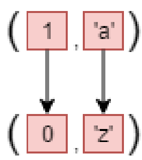
4.7 Lazy parEvalN



The function parEvalN is 100% strict, which means that it fully evaluates all passed arrows. Sometimes this might not be feasible, as it will not work on infinite lists of functions like e.g. **map** (arr . (+)) [1..] or just because we need the arrows evaluated in chunks. parEvalNLazy fixes this. It works by first chunking the input from [a] to [[a]] with the given ChunkSize in arr (chunksOf chunkSize). These chunks are then fed into a list [arr [a] [b]] of parallel arrows created by feeding chunks of the passed ChunkSize into the regular parEvalN by using listApp. The resulting [[b]] is lastly converted into [b] with arr **concat**.

```
parEvalNLazy :: (ArrowParallel arr a b conf, ArrowChoice arr, ArrowApply arr) =>
  conf -> ChunkSize -> [arr a b] -> (arr [a] [b])
parEvalNLazy conf chunkSize fs =
  arr (chunksOf chunkSize) >>>
  listApp fchunks >>>
  arr concat
  where fchunks = map (parEvalN conf) $ chunksOf chunkSize fs
```

4.8 Heterogenous tasks



We have only talked about the paralellization arrows of the same type until now. But sometimes we want to paralellize heterogenous types as well. However, we can implement such a parEval2 combinator which combines two arrows arr a b and arr c d into a new parallel arrow arr (a, c) (b, d) quite easily with the help of the ArrowChoice typeclass. The idea is to use the +++ combinator which combines two arrows arr a b and arr c d and transforms them into arr (**Either** a c) (**Either** b d) to get a common arrow type that we can then feed into parEvalN.

We start by transforming the (a, c) input into a 2-element list [**Either** a c] by first tagging the two inputs with **Left** and **Right** and wrapping the right element in a singleton list with **return** so that we can combine them with arr (**uncurry** (:)). Next, we feed this list into a parallel arrow running on 2 instances of f +++ g as described above. After the calculation is finished we convert the resulting [**Either** b d] into ([b], [d]) with arr partitionEithers. The two lists in this tuple contain only 1 element each by construction, so we can finally just convert the tuple to (b, d) in the last step.

```
parEval2 :: (ArrowChoice arr,
   ArrowParallel arr (Either a c) (Either b d) conf) =>
  conf -> arr a b -> arr c d -> arr (a, c) (b, d)
parEval2 conf f g =
  arr Left *** (arr Right >>> arr return) >>>
  arr (uncurry (:)) >>>
  parEvalN conf (replicate 2 (f +++ g)) >>>
  arr partitionEithers >>>
```

```
arr head *** arr head
```

4.9 Syntactic Sugar

For basic arrows, we have the *** combinator which allows us to combine two arrows `arr a b` and `arr c d` into an arrow `arr (a, c) (b, d)` which does both computations at once. This can easily be translated into a parallel version with parEval2, but for this we require a backend which has an implementation that does not require any configuration (hence the () as the conf parameter in the following code snippet).

```
(|***|) :: (ArrowChoice arr, ArrowParallel arr (Either a c) (Either b d) ())) =>
  arr a b -> arr c d -> arr (a, c) (b, d)
(|***|) = parEval2 ()
```

We define the parallel |&&&| in a similar manner to its sequential prototype.

```
(|&&&|) :: (ArrowChoice arr, ArrowParallel arr (Either a a) (Either b c) ()) =>
  arr a b -> arr a c -> arr a (b, c)
(|&&&|) f g = (arr $ \a -> (a, a)) >>> f |***| g
```

5 Futures

Consider the following parallel arrow combinator:

```
someCombinator :: (Arrow arr) => [arr a b] -> [arr b c] -> arr [a] [c]
someCombinator fs1 fs2 = parEvalN () fs1 >>> rightRotate >>> parEvalN () fs2
```

In a distributed environment, the resulting arrow of this combinator first evaluates all `[arr a b]` in parallel, sends the results back to the master node, rotates the input once and then evaluates the `[arr b c]` in parallel to then gather the input once again on the master node. Such situations arise, e. g. in scientific computations when the data distributed across the nodes needs to be transposed. A concrete example is 2D FFT computation (Gorlatch and Bischof, 1998; Berthold et al., 2009c).

While this could be rewritten into only one parEvalN call by directly wiring the arrows properly together, this example illustrates an important problem: When using a ArrowParallel backend that resides on multiple computers, all communication between the nodes is done via the master node, as shown in the Eden trace in Figure 2. This can become a serious bottleneck for larger amount of data and number of processes (showcases Berthold et al., 2009a, as, e. g.). This motivates for an approach that allows the nodes to communicate directly with each other. Thankfully, Eden, the distributed parallel Haskell we have used in this paper so far, already ships with the concept of RD (remote data) that enables this behaviour (Alt and Gorlatch, 2003; Dieterle et al., 2010). But as we want code written against our API to be implementation agnostic, we have to wrap this context. We do this with the Future typeclass:

```
class Future fut a | a -> fut where
    put :: (Arrow arr) => arr a (fut a)
    get :: (Arrow arr) => arr (fut a) a
```

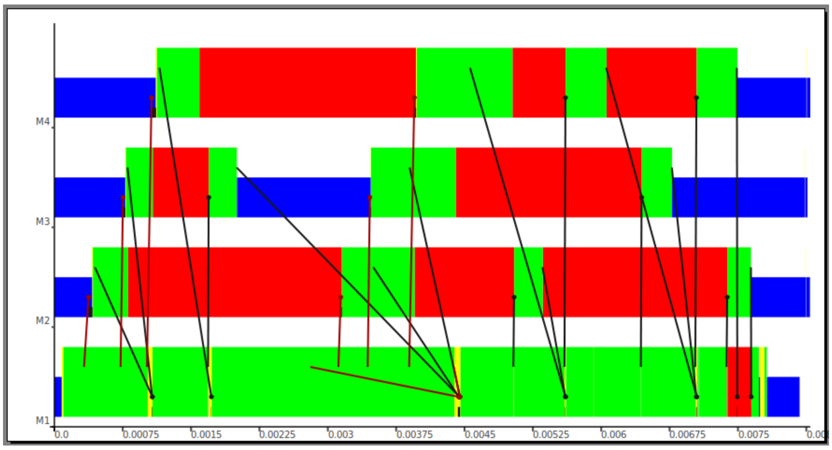


Figure 2: Communication between 4 threads without Futures

As RD is only type synonym for communication type that Eden uses internally, we have to use some wrapper classes to fit that definition, though:

```
data RemoteData a = RD { rd :: RD a }

instance (Trans a) => Future RemoteData a where
    put = arr (\a -> RD { rd = release a })
    get = arr rd >>> arr fetch
```

For ParMonad and Multicore we can use a basic dummy wrapper because we have shared memory in a single node:

```
data BasicFuture a = BF { val :: a }

instance (NFData a) => Future BasicFuture a where
    put = arr (\a -> BF { val = a })
    get = arr val
```

To fit the ArrowParallel instances we gave earlier, we also have to give the necessary NFData and Trans instances - the latter only being needed in Eden. We need this implementation for our RemoteData wrapper

```
instance NFData (RemoteData a) where
    rnf = rnf . rd
instance Trans (RemoteData a)
```

and the following for the BasicFuture dummy type:

```
instance (NFData a) => NFData (BasicFuture a) where
  rnf = rnf . val
```

Going back to our communication example we can use this Future concept in order to enable direct communications between the nodes in the following way:

```
someCombinator :: (Arrow arr) => [arr a b] -> [arr b c] -> arr [a] [c]
someCombinator fs1 fs2 =
  parEvalN () (map (>>> put) fs1) >>>
```

```
  rightRotate >>>
  parEvalN () (map (get >>>) fs2)
```

In a distributed environment, this gives us a communication scheme with messages going through the master node only if it is needed - similar to what is shown in the trace in Fig. 3.

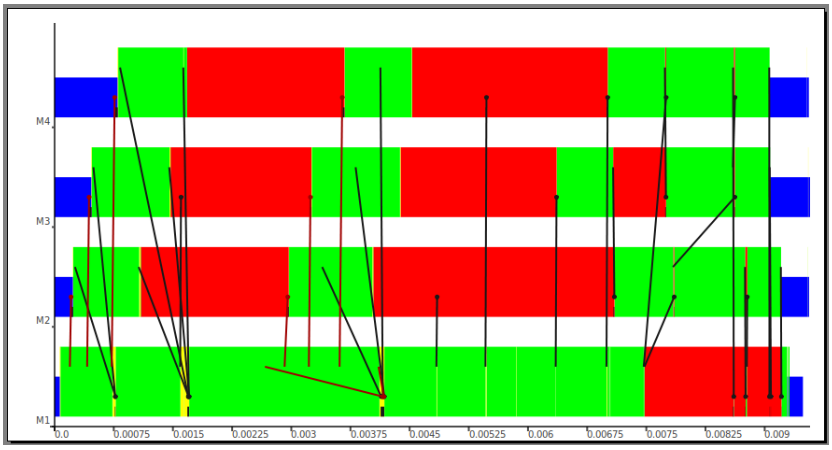
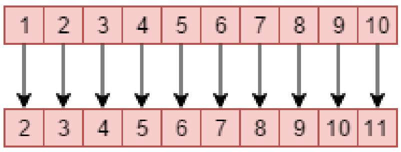


Figure 3: Communication between 4 threads with Futures

6 Map-based Skeletons

Now we have developed Parallel Arrows further enough to define some algorithmic skeletons useful to an application programmer.

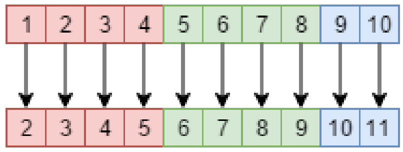
6.1 Parallel map



parMap is probably the most common skeleton for parallel programs. We can implement it with ArrowParallel by repeating an arrow arr a b and then passing it into parEvalN to get an arrow arr [a] [b]. Just like parEvalN, parMap is 100 % strict.

```
parMap :: (ArrowParallel arr a b conf) =>
  conf -> (arr a b) -> (arr [a] [b])
parMap conf f = parEvalN conf (repeat f)
```

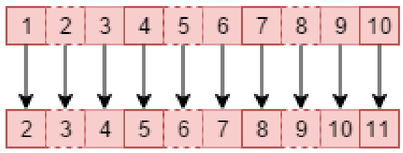
6.2 Lazy parallel map



As parMap is 100% strict it has the same restrictions as parEvalN compared to parEvalNLazy. So it makes sense to also have a parMapStream which behaves like parMap, but uses parEvalNLazy instead of parEvalN.

```
parMapStream :: (ArrowParallel arr a b conf, ArrowChoice arr, ArrowApply arr) =>
  conf -> ChunkSize -> arr a b -> arr [a] [b]
parMapStream conf chunkSize f = parEvalNLazy conf chunkSize (repeat f)
```

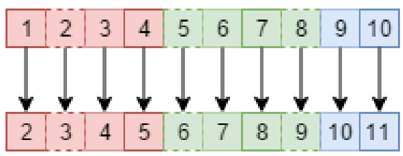
6.3 Statically load-balancing parallel map



parMap spawns every single computation in a new thread (at least for the instances of ArrowParallel we gave in this paper). This can be quite wasteful and a farm that equally distributes the workload over numCores workers (if numCores is greater than actualProcessorCount, the fastest processor(s) to finish will get more tasks) seems useful.

```
farm :: (ArrowParallel arr a b conf,
   ArrowParallel arr [a] [b] conf, ArrowChoice arr) =>
  conf -> NumCores -> arr a b -> arr [a] [b]
farm conf numCores f =
   unshuffle numCores >>>
  parEvalN conf (repeat (mapArr f)) >>>
   shuffle
```

6.4 farmChunk



As farm is basically just parMap with a different work distribution, it is, again, 100% strict. So we define farmChunk which uses parEvalNLazy instead of parEvalN like this:

```
farmChunk :: (ArrowParallel arr a b conf, ArrowParallel arr [a] [b] conf,
  ArrowChoice arr, ArrowApply arr) =>
  conf -> ChunkSize -> NumCores -> arr a b -> arr [a] [b]
```

```
farmChunk conf chunkSize numCores f =
    unshuffle numCores >>>
    parEvalNLazy conf chunkSize (repeat (mapArr f)) >>>
    shuffle
```

6.5 parMapReduce

– this does not completely adhere to Google's definition of Map Reduce as it – the mapping function does not allow for "reordering" of the output – The original Google version can be found at https://de.wikipedia.org/wiki/MapReduce

```
parMapReduceDirect :: (ArrowParallel arr [a] b conf,
    ArrowApply arr, ArrowChoice arr) =>
    conf -> ChunkSize -> arr a b -> arr (b, b) b -> b -> arr [a] b
parMapReduceDirect conf chunkSize mapfn foldfn neutral =
    arr (chunksOf chunkSize) >>>
    parMap conf (mapArr mapfn >>> foldlArr foldfn neutral) >>>
    foldlArr foldfn neutral
```

7 Topological Skeletons

Even though many algorithms can be expressed by parallel maps, some problems require more sophisticated skeletons. The Eden library leverages this problem and already comes with more predefined skeletons, among them a `pipe`, a `ring` and a `torus` implementation (Loogen, 2012b; ede, b). These seem like reasonable candidates to be ported to our arrow based parallel Haskell to showcase that we can express such skeletons with Parallel Arrows as well.

7.1 Parallel pipe

The parallel pipe skeleton is semantically equivalent to folding over a list `[arr a a]` of arrows with >>>, but does this in parallel, meaning that the arrows do not have to reside on the same thread/machine. We implement this skeleton using the `ArrowLoop` typeclass which gives us the `loop :: arr (a, b) (c, b) -> arr a c` combinator which allows us to express loop like computations. For example this

```
loop (arr (\(a, b) -> (b, a:b)))
```

,which is the same as

```
loop (arr snd &&& arr (uncurry (:)))
```

defines an arrow that takes its input `a` and converts it into an infinite stream `[a]` of it. Using this to our advantage gives us a first draft of a pipe implementation by plugging in the parallel evaluation call `parEvalN conf fs` inside the second argument of &&& and then only picking the first element of the resulting list with `arr last`:

```
pipeSimple :: (ArrowLoop arr, ArrowParallel arr a a conf) =>
    conf -> [arr a a] -> arr a a
pipeSimple conf fs =
```

```
  loop (arr snd &&&
    (arr (uncurry (:) >>> lazy) >>> parEvalN conf fs)) >>>
  arr last
```

where lazy is defined as:

```
lazy :: (Arrow arr) => arr [a] [a]
lazy = arr (\ ~(x:xs) -> x : lazy xs)
```

Note that here the use of lazy is essential as without it programs using this definition would never halt. We need to enforce that the evaluation of the input [a] terminates before passing it into parEvalN.

However, using this definition directly, will make the master node a potential bottleneck in distributed environments as described in Section 5. Therefore, we introduce a more sophisticated version that internally uses Futures.

```
pipe :: (ArrowLoop arr, ArrowParallel arr (fut a) (fut a) conf,
  Future fut a) =>
  conf -> [arr a a] -> arr a a
pipe conf fs = unliftFut (pipeSimple conf (map liftFut fs))
```

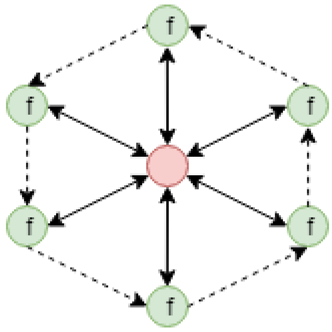
Sometimes, this pipe definition can be a bit inconvenient, especially if we want to pipe arrows of mixed types together, i.e. arr a b and arr b c. By wrapping these two arrows inside a common type we obtain

```
pipe2 :: (ArrowLoop arr, ArrowChoice arr,
  ArrowParallel arr (fut (([a], [b]), [c])) (fut (([a], [b]), [c])) conf,
  Future fut (([a], [b]), [c])) =>
  conf -> arr a b -> arr b c -> arr a c
pipe2 conf f g =
  (arr return &&& arr (const [])) &&& arr (const []) >>>
  pipe conf (replicate 2 (unify f g)) >>>
  arr snd >>>
  arr head
    where
      unify :: (ArrowChoice arr) =>
        arr a b -> arr b c -> arr (([a], [b]), [c]) (([a], [b]), [c])
      unify f g =
        (mapArr f *** mapArr g) *** arr (\_ -> []) >>>
        arr (\((a, b), c) -> ((c, a), b))
```

Note that extensive use of this combinator over pipe with a hand-written combination data-type will probably result in worse performance because of more communication overhead from the many calls to parEvalN. Nonetheless, we can define a parallel piping operator |>>>| which is semantically equivalent to >>> in a similar manner to the other parallel syntactic sugar from Section 4.9:

```
(|>>>|) :: (ArrowLoop arr, ArrowChoice arr,
  ArrowParallel arr (fut (([a], [b]), [c])) (fut (([a], [b]), [c])) (),
  Future fut (([a], [b]), [c])) =>
  arr a b -> arr b c -> arr a c
(|>>>|) = pipe2 ()
```

7.2 Ring skeleton



Eden comes with a ring skeleton implementation that allows the computation of a function `[i] -> [o]` with a ring of nodes that communicate in a ring topology with each other. Its input is a node function `i -> r -> (o, r)` in which `r` serves as the intermediary output that gets send to the neighbour of each node. This data is sent over direct communication channels (remote data). They define it as (ede, b)

```haskell
ringSimple :: (Trans i, Trans o, Trans r) =>
   (i -> r -> (o,r))
   -> [i] -> [o]
ringSimple f is = os
  where (os,ringOuts) = unzip (parMap (toRD $ uncurry f)
                                 (zip is $ lazy ringIns))
        ringIns = rightRotate ringOuts

toRD :: (Trans i, Trans o, Trans r) =>
        ((i,r) -> (o,r))
        -> ((i, RD r) -> (o, RD r))
toRD f (i, ringIn) = (o, release ringOut)
  where (o, ringOut) = f (i, fetch ringIn)

rightRotate    :: [a] -> [a]
rightRotate [] = []
rightRotate xs = last xs : init xs
```

We can rewrite its functionality easily with the use of `loop` as the definition of the node function, `arr (i, r) (o, r)`, after being transformed into an arrow, already fits quite neatly into the `loop`'s `arr (a, b) (c, b) -> arr a c`. In each iteration we start by rotating the intermediary input from the nodes `[fut r]` with `second (rightRotate >>> lazy)`. Similarly to the pipe, we have to feed the intermediary input into our `lazy` arrow here, or the evaluation would hang. The reasoning is explained by (Loogen, 2012b):

> Note that the list of ring inputs ringIns is the same as the list of ring outputs ringOuts rotated by one element to the right using the auxiliary function rightRotate. Thus, the program would get stuck without the lazy pattern, because the ring input will only be produced after process creation and process creation will not occur without the first input.

Next, we zip the resulting `([i], [fut r])` to `[(i, fut r)]` with `arr (uncurry zip)` so we can feed that into a our input arrow `arr (i, r) (o, r)`, which we transform into `arr (i, fut r) (o, fut r)` before lifting it to `arr [(i, fut r)] [(o, fut r)]` to get a list `[(o, fut r)]`. Finally we unzip this list into `([o], [fut r])`. Plugging this arrow `arr ([i], [fut r]) ([o], fut r)` into the definition of `loop` from earlier gives us `arr [i] [o]`, our ring arrow.

This gives us the following complete definition for the `ring` combinator:

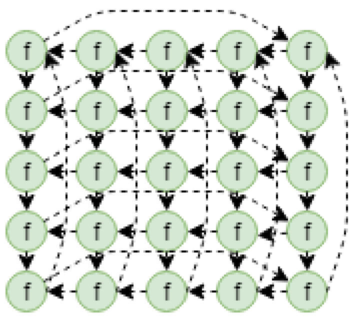
```
ring :: (ArrowLoop arr, Future fut r,
  ArrowParallel arr (i, fut r) (o, fut r) conf) =>
    conf ->
    arr (i, r) (o, r) ->
    arr [i] [o]
ring conf f =
  loop (second (rightRotate >>> lazy) >>>
    arr (uncurry zip) >>>
    parMap conf (second get >>> f >>> second put) >>>
    arr unzip)
```

with `rightRotate`:

```
rightRotate :: (Arrow arr) => arr [a] [a]
rightRotate = arr $ \list -> case list of
  [] -> []
  xs -> last xs : init xs
```

This combinator can, for example, be used to calculate the shortest paths in a graph using Warshall's algorithm.

7.3 Torus skeleton



If we take the concept of a ring one dimension further, we get a torus. Every node sends ands receives data from horizontal and vertical neighbours in each communication round.

With our parallel Arrows we implement a torus combinator yet again with the help of the `ArrowLoop` typeclass.

Similar to the ring, we once again start by rotating the input, but this time not only in one direction, but in two. This means that the intermediary input from the neighbour nodes has to be stored in a tuple `([[fut a]], [[fut b]])` in the second argument (loop only allows for 2 arguments) of our looped arrow arr ([[c]], ([[fut a]], [[fut b]])) ([[d]], ([[fut a]], [[fut b]])) and our rotation arrow becomes second ((mapArr rightRotate >>> lazy) *** (arr rightRotate >>> lazy)) instead of the singular rotation in the ring as we rotate [[fut a]] horizontally and [[fut b]] vertically. Then, we once again zip the inputs for the input arrow with arr (uncurry3 **zipWith3** lazyzip3) from ([[c]], ([[fut a]], [[fut b]])) to [[(c, fut a, fut b)]], which we then feed into our parallel execution.

This, however, is more complicated than in the ring case as we have one more dimension of inputs to be transformed. We first have to shuffle all the inputs to then pass it into parMap conf (ptorus f) which yields us [(d, fut a, fut b)]. We can then unpack this shuffled list back to its original ordering by feeding it into the specific unshuffle arrow we created one step earlier with arr **length** >>> arr unshuffle with the use of app from the ArrowApply typeclass. Finally, we unpack this matrix [[[(d, fut a, fut b)]] with arr (**map unzip3**) >>> arr **unzip3** >>> threetotwo to get ([[d]], ([[fut a]], [[fut b]])).

The complete definition of the torus combinator is:

```
torus :: (ArrowLoop arr, ArrowChoice arr, ArrowApply arr,
  ArrowParallel arr (c, fut a, fut b) (d, fut a, fut b) conf,
  Future fut a, Future fut b) =>
  conf -> arr (c, a, b) (d, a, b) -> arr [[c]] [[d]]
torus conf f =
  loop (second ((mapArr rightRotate >>> lazy) ***
    (arr rightRotate >>> lazy)) >>>
  arr (uncurry3 (zipWith3 lazyzip3)) >>>
  (arr length >>> arr unshuffle) &&&
    (shuffle >>> parMap conf (ptorus f)) >>>
  app >>>
  arr (map unzip3) >>> arr unzip3 >>> threetotwo)
```

with uncurry3, lazyzip3, and threetotwo from the Appendix, and

```
ptorus :: (Arrow arr, Future fut a, Future fut b) =>
  arr (c, a, b) (d, a, b) ->
  arr (c, fut a, fut b) (d, fut a, fut b)
ptorus f =
  arr (\ ~(c, a, b) -> (c, get a, get b)) >>> f >>>
  arr (\ ~(d, a, b) -> (d, put a, put b))
```

As an example of using this skeleton Loogen (2012b) showed the matrix multiplication using the Gentleman algorithm (Gentleman, 1978). Adapting this nodefunction to our Arrow API gives us:

```
nodefunction :: Int ->
  ((Matrix, Matrix), [Matrix], [Matrix]) ->
  ([Matrix], [Matrix], [Matrix])
nodefunction n ((bA, bB), rows, cols) =
  ([bSum], bA:nextAs , bB:nextBs)
    where bSum =
      foldl' matAdd (matMult bA bB) (zipWith matMult nextAs nextBs)
      nextAs = take (n-1) rows
      nextBs = take (n-1) cols
```

If we compare the trace from a call using our arrow definition of the torus (Fig. 4) with the Eden version (Fig. 5) we can see that the behaviour of the arrow version is comparable.

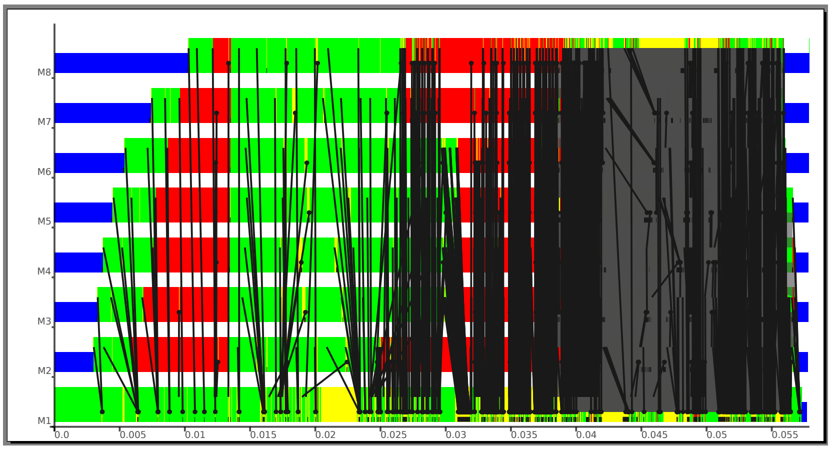


Figure 4: Matrix Multiplication with a torus (Parrows)

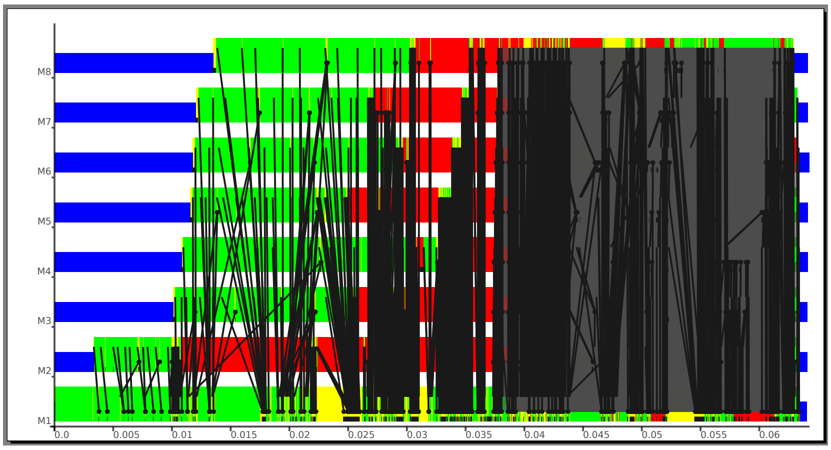
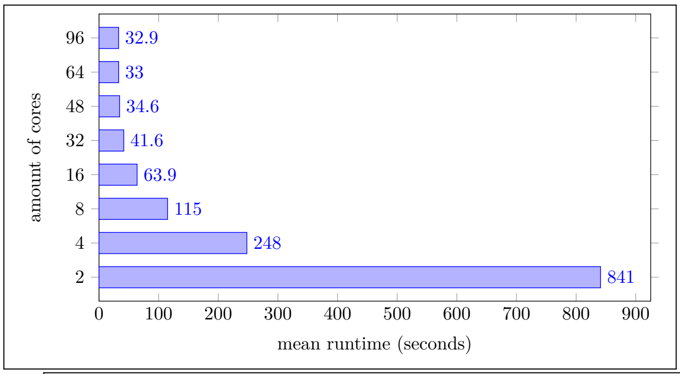
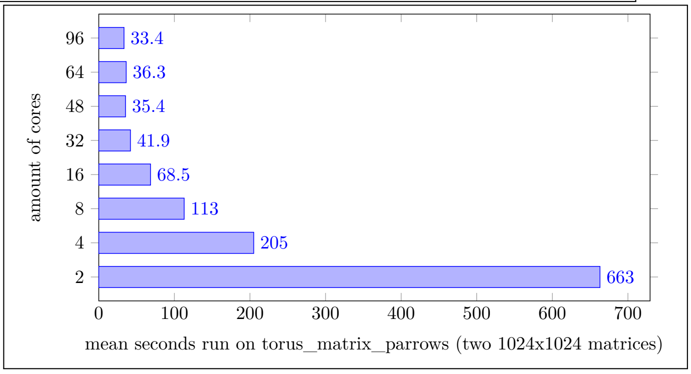


Figure 5: Matrix Multiplication with a torus (Eden)

8 Benchmarks





9 Conclusion

Arrows are a generic concept that allows for powerful composition combinators. To our knowledge we are the first ones to represent parallel computation with arrows.

Arrows turn out to be a useful tool for composing in parallel programs. We do not have to introduce new monadic types that wrap the computation. Instead use arrows just like regular sequential pure functions. This work features multiple parallel backends: the already available parallel Haskell flavours. Parallel Arrows feature an implementation of the ArrowParallel interface for Multicore Haskell, Par monad, and Eden. With our approach parallel programs can be ported across these flavours with no effort. Performancewise, Parallel Arrows are on par with existing parallel Haskells, as they do not introduce any notable overhead.

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A Utility Functions

To be able to go into detail on parallel arrows, we introduce some utility combinators first, that will help us later: **map**, **foldl** and **zipWith** on Arrows.

The mapArr combinator lifts any arrow arr a b to an arrow arr [a] [b] (Hughes, 2005):

```
mapArr :: ArrowChoice arr => arr a b -> arr [a] [b]
mapArr f =
  arr listcase >>>
  arr (const []) ||| (f *** mapArr f >>> arr (uncurry (:)))
  where listcase [] = Left ()
        listcase (x:xs) = Right (x,xs)
```

Similarly, we can also define foldlArr that lifts any arrow arr (b, a) b with a neutral element b to arr [a] b:

```
foldlArr :: (ArrowChoice arr, ArrowApply arr) => arr (b, a) b -> b -> arr [a] b
foldlArr f b =
  arr listcase >>>
```

```
  arr (const b) |||
    ( first (arr (\a -> (b, a)) >>> f >>> arr (foldlArr f)) >>> app)
  where listcase [] = Left []
        listcase (x:xs) = Right (x,xs)
```

Finally, with the help of mapArr, we can define zipWithArr that lifts any arrow `arr (a, b) c` to an arrow `arr ([a], [b]) [c]`.

```
zipWithArr :: ArrowChoice arr => arr (a, b) c -> arr ([a], [b]) [c]
zipWithArr f = (arr $ \(as, bs) -> zipWith (,) as bs) >>> mapArr f
```

These combinators make use of the ArrowChoice type class, it provides the ||| combinator. The latter takes two arrows `arr a c` and `arr b c` and combines them into a new arrow `arr (Either a b) c` which pipes all **Left** a's to the first arrow and all **Right** b's to the second arrow.

```
(|||) :: ArrowChoice arr a c -> arr b c -> arr (Either a b) c
```

With the zipWithArr combinator we can also write a combinator listApp, which lifts a list of arrows `[arr a b]` to an arrow `arr [a] [b]`.

```
listApp :: (ArrowChoice arr, ArrowApply arr) => [arr a b] -> arr [a] [b]
listApp fs = (arr $ \as -> (fs, as)) >>> zipWithArr app
```

This combinator also makes use of the ArrowApply typeclass that allows us to evaluate arrows with `app :: arr (arr a b, a) c`.

The definition of unshuffle is

```
unshuffle :: (Arrow arr) => Int -> arr [a] [[a]]
unshuffle n = arr (\xs -> [takeEach n (drop i xs) | i <- [0..n-1]])

takeEach :: Int -> [a] -> [a]
takeEach n [] = []
takeEach n (x:xs) = x : takeEach n (drop (n-1) xs)
```

while shuffle is defined as:

```
shuffle :: (Arrow arr) => arr [[a]] [a]
shuffle = arr (concat . transpose)
```

These functions were taken from Eden skeleton source code (ede, a) and lifted to Arrows.

The helper functions for torus are

```
uncurry3 :: (a -> b -> c -> d) -> (a, (b, c)) -> d
uncurry3 f (a, (b, c)) = f a b c

lazyzip3 :: [a] -> [b] -> [c] -> [(a, b, c)]
lazyzip3 as bs cs = zip3 as (lazy bs) (lazy cs)

threetotwo :: (Arrow arr) => arr (a, b, c) (a, (b, c))
threetotwo = arr $ \ ~(a, b, c) -> (a, (b, c))
```