Arrows for Parallel Computations

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Abstract

Arrows form**OL: are?** a general interface for computation and pose therefore as an alternative to monads for API design. We express parallelism using this concept. This is a new way to represent parallel computation. We define an Arrows-based interface for parallelism and implement it using multiple parallel Haskells. **OL: Benefits:** In this manner we are able to bridge across various parallel Haskells with a common interface.

This new way of writing parallel programs has a benefit of being portable across flavours of parallel Haskells.**OL: Wdh?** Each parallel computation is an arrow, they can be composed and transformed as such. We thus introduce some syntactic sugar to provide parallelism-aware arrow combinators.

We also define several parallel skeletons with our framework. Benchmarks shows that our framework does not induce too much overhead performance-wise.

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1 Introduction

OL: todo, reuse 5.5, and more

blablabla arrows, parallel, haskell.

Contribution **OL: HIT HERE REALLY STRONG**

We wrap parallel Haskells inside of our *ArrowParallel* interface, but why do we aim to abstract parallelism this way and what does this approach do better than the other parallel Haskells?

- **Arrow API benefits**: With the *ArrowParallel* typeclass we do not lose any benefits of using arrows as *parEvalN* is just yet another arrow combinator. The resulting arrow can be used in the same way a potential serial version could be used. This is a big advantage of this approach, especially compared to the monad solutions as we do not introduce any new types. We can just 'plug' in parallel parts into our sequential programs without having to change anything.
- **Abstraction**: With the *ArrowParallel* typeclass, we abstracted all parallel implementation logic away from the business logic. This gives us the beautiful situation of being able to write our code against the interface the typeclass gives us without being bound to any parallel Haskell. So as an example, during development, we can run the code on the simple Multicore version and afterwards deploy it on a cluster by converting it into an Eden version, by just replacing the actual *ArrowParallel* instance.

Structure The remaining text is structures as follows. Section 2 briefly introduces known parallel Haskell flavours and gives an overview of Arrows to the reader (Sec. 2.2). Section 3 discusses related work. Section 4 defines Parallel Arrows and presents a basic interface. Section 5 defines Futures for Parallel Arrows, this concept enables better communication. Section 6 presents some basic algorithmic skeletons (parallel *map* with and without load

balancing, *map − reduce*) in our newly defined dialect. More advanced ones are showcased in Section 7 (*pipe*, *ring*, *torus*). Section 8 shows the benchmark results. Section 9 discusses future work and concludes.

2 Background

2.1 Short introduction to parallel Haskells

There are already several ways to write parallel programs in Haskell. As we will base our parallel arrows on existing parallel Haskells, we will now give a short introduction to the ones we use as backends in this paper.

In its purest form, parallel computation (on functions) can be looked at as the execution of some functions $a \rightarrow b$ in parallel, as also Figure **??** symbolically shows:
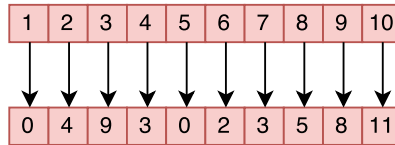
$parEvalN :: [a \rightarrow b] \rightarrow [a] \rightarrow [b]$

Figure 1: Schematic illustration of `parEvalN`.

Before we go into detail on how we can use this idea of parallelism for parallel Arrows, as a short introduction to parallelism in Haskell we will now implement *parEvalN* with several different parallel Haskells.

2.1.1 Multicore Haskell

Multicore Haskell (Marlow *et al.*, 2009; Trinder *et al.*, 1999) is way to do parallel processing found in standard GHC.[1] It ships with parallel evaluation strategies (Trinder *et al.*, 1998; Marlow *et al.*, 2010) for several types which can be applied with *using* :: $a \rightarrow Strategy\ a \rightarrow a$. For *parEvalN* this means that we can just apply the list of functions $[a \rightarrow b]$ to the list of inputs $[a]$ by zipping them with the application operator \$. We then evaluate this lazy list $[b]$ according to a *Strategy* $[b]$ with the *using* :: $a \rightarrow Strategy\ a \rightarrow a$ operator. We construct this strategy with *parList* :: $Strategy\ a \rightarrow Strategy\ [a]$ and *rdeepseq* :: $NFData\ a \Rightarrow Strategy\ a$ where the latter is a strategy which evalutes to normal form. To ensure that programs that use *parEvalN* have the correct evaluation order, we annotate the computation with *pseq* :: $a \rightarrow b \rightarrow b$ which forces the compiler to not reorder multiple *parEvalN* computations. This is particularly necessary in circular communication topologies like in the *torus* or *ring* skeleton that we will see in Section 7 which resulted in deadlock scenarios when executed without *pseq* during testing for this paper.

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[1] Multicore Haskell on Hackage is available under `https://hackage.haskell.org/package/parallel-3.2.1.0`, compiler support is integrated in the stock GHC.

$parEvalN :: (NFData\ b) \Rightarrow [a \rightarrow b] \rightarrow [a] \rightarrow [b]$
$parEvalN\ fs\ as = \mathbf{let}\ bs = zipWith\ (\$)\ fs\ as$
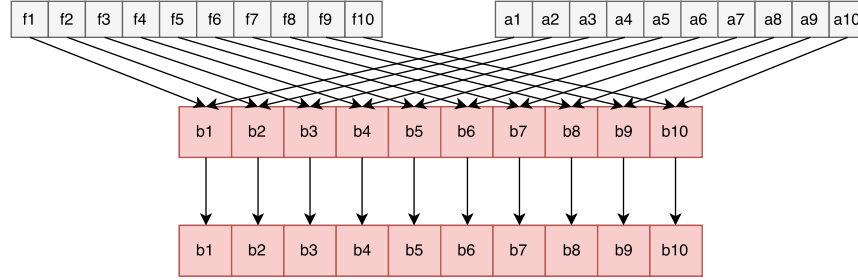$\quad \mathbf{in}\ (bs\ `using`\ parList\ rdeepseq)\ `pseq`\ bs$

Figure 2: Dataflow of the Multicore Haskell parEvalN version
OL: Evaluation step explicitly shown?

2.1.2 ParMonad

The *Par* monad[2] introduced by Marlow *et al.* (2011a), is a monad designed for composition of parallel programs.

Our parallel evaluation function *parEvalN* can be defined by zipping the list of $[a \rightarrow b]$ with the list of inputs $[a]$ with the application operator \$ just like with Multicore Haskell. Then, we map over this not yet evaluated lazy list of results $[b]$ with *spawnP* :: $NFData\ a \Rightarrow a \rightarrow Par\ (IVar\ a)$ to transform them to a list of not yet evaluated forked away computations $[Par\ (IVar\ b)]$, which we convert to $Par\ [IVar\ b]$ with *sequenceA*. We wait for the computations to finish by mapping over the *IVar b* values inside the *Par* monad with *get*. This results in *Par* $[b]$. We finally execute this process with *runPar* to finally get $[b]$ again.
MB: explain problems with laziness here. Problems with torus

$parEvalN :: (NFData\ b) \Rightarrow [a \rightarrow b] \rightarrow [a] \rightarrow [b]$
$parEvalN\ fs\ as = runPar\ \$$
$\quad (sequenceA\ \$\ map\ (spawnP)\ \$\ zipWith\ (\$)\ fs\ as) \ggg mapM\ get$

2.1.3 Eden

Eden (Loogen *et al.*, 2005; Loogen, 2012) is a parallel Haskell for distributed memory and comes with a MPI and a PVM backends.[3] This means that it works on clusters as well so it makes sense to have a Eden-based backend for our new parallel Haskell flavour.

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[2] It can be found in the `monad-par` package on hackage under `https://hackage.haskell.org/package/monad-par-0.3.4.8/`.

[3] See also `http://www.mathematik.uni-marburg.de/~eden/` and `https://hackage.haskell.org/package/edenmodules-1.2.0.0/`.
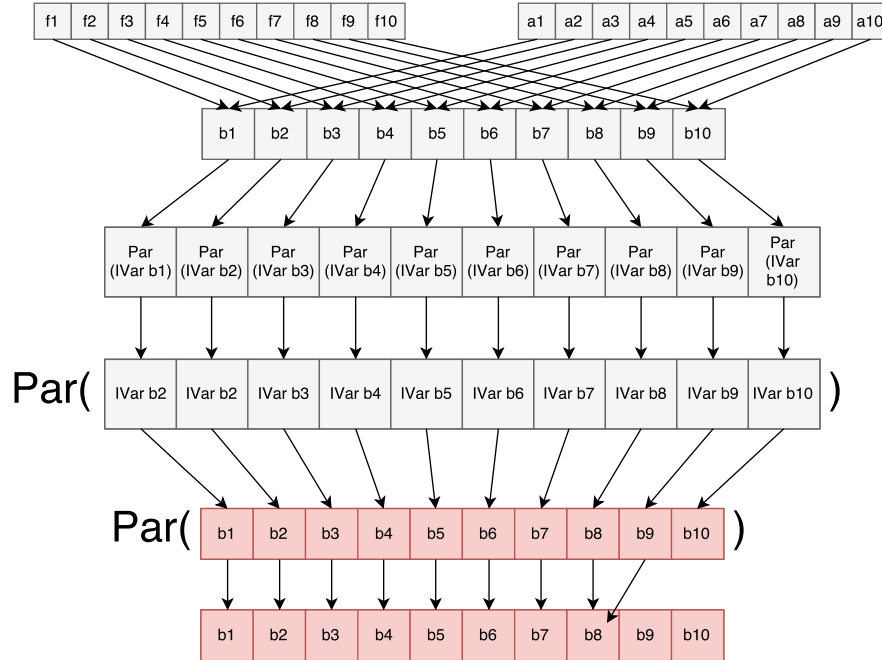
Figure 3: Dataflow of the Par Monad parEvalN version

Eden was designed to work on clusters, but with a further simple backend it operates on multicores. However, in contrast to many other parallel Haskells, in Eden each process has its own heap. This seems to be a waste of memory, but with distributed programming paradigm and individual GC per process, Eden yields good performance results also on multicores (Berthold *et al.*, 2009a; Aswad *et al.*, 2009).

While Eden also comes with a monad *PA* for parallel evaluation, it also ships with a completely functional interface that includes a *spawnF* :: (*Trans a*, *Trans b*) $\Rightarrow [a \rightarrow b] \rightarrow [a] \rightarrow [b]$ function that allows us to define *parEvalN* directly:

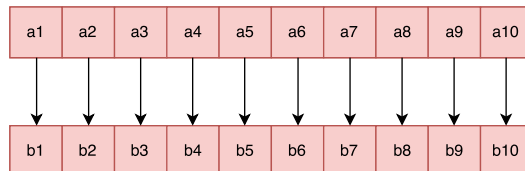$$parEvalN :: (Trans\ a, Trans\ b) \Rightarrow [a \rightarrow b] \rightarrow [a] \rightarrow [b]$$
$$parEvalN = spawnF$$

Figure 4: Dataflow of the Eden parEvalN version

Eden TraceViewer. To comprehend the efficiency and the lack thereof in a parallel program, an inspection of its execution is extremely helpful. While some large-scale solutions exist (Geimer *et al.*, 2010), the parallel Haskell community mainly utilises the tools Threadscope (Wheeler & Thain, 2009) and Eden TraceViewer[4] (Berthold & Loogen, 2007). In the next sections we will present some *traces*, the post-mortem process diagrams of Eden processes and their activity.

In a trace, the *x* axis shows the time, the *y* axis enumerates the machines and processes. A trace shows a running process in green, a blocked process is red. If the process is 'runnable', i.e. it may run, but does not, it is yellow. The typical reason for then is GC. An inactive machine where no processes are started yet, or all are already terminated, is shows as a blue bar. A comminication from one process to another is represented with a black arrow. A stream of communications, e.g. a transmitted list is shows as a dark shading between sender and receiver processes.

OL: show example trace or refer to a trace in later figures
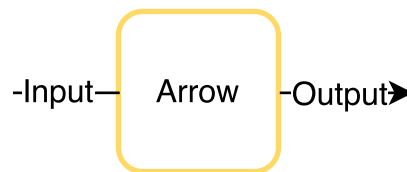
2.2 Arrows

Figure 5: Schematic depiction of an arrow

Arrows were introduced by Hughes (2000) as a general interface for computation. An arrow *arr a b* represents a computation that converts an input *a* to an output *b*. This is defined in the arrow typeclass:

arr is used to lift an ordinary function to an arrow type, similarly to the monadic *return*. The $>\!\!>\!\!>$ operator is analogous to the monadic composition $>\!\!>\!\!=$ and combines two arrows *arr a b* and *arr b c* by "wiring" the outputs of the first to the inputs to the second to get a new arrow *arr a c*. Lastly, the *first* operator takes the input arrow from *b* to *c* and converts it into an arrow on pairs with the second argument untouched. It allows us to to save input across arrows.

The most prominent instances of this interface are regular functions ($\rightarrow$):

instance *Arrow* ($\rightarrow$) **where**
    arr f = *f*
    f $>\!\!>\!\!>$ *g* = *g* $\circ$ *f*
    first f = $\lambda (a, c) \rightarrow (f\ a, c)$

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[4] See `http://hackage.haskell.org/package/edentv` on Hackage for the last available version of Eden TraceViewer.

class *Arrow arr* **where**
   arr :: $(a \rightarrow b) \rightarrow arr\ a\ b$
   $(\ggg) :: arr\ a\ b \rightarrow arr\ b\ c \rightarrow arr\ a\ c$
   first :: $arr\ a\ b \rightarrow arr\ (a,c)\ (b,c)$

Figure 6: Arrow class definition

Figure 7: The schematic depiction of *Arrow*
combinators

and the Kleisli type:

  data *Kleisli m a b* = *Kleisli* $\{\,run :: a \rightarrow m\ b\,\}$

  instance *Monad m* $\Rightarrow$ *Arrow* (*Kleisli m*) **where**
    arr f = *Kleisli* (*return* $\circ$ *f*)
    $f \ggg g = Kleisli\ (\lambda a \rightarrow f\ a \ggg g)$
    first f = *Kleisli* $(\lambda (a,c) \rightarrow f\ a \ggg \lambda b \rightarrow return\ (b,c))$

With this typeclass in place, Hughes also defined some syntactic sugar: the functions

Figure 8: Visual depiction of syntactic sugar for Arrows.

second, $***$ and &&&. The mirrored version of *first*, called *second* (Fig. **??**) is:

$$second :: Arrow\ arr \Rightarrow arr\ a\ b \rightarrow arr\ (c,a)\ (c,b)$$
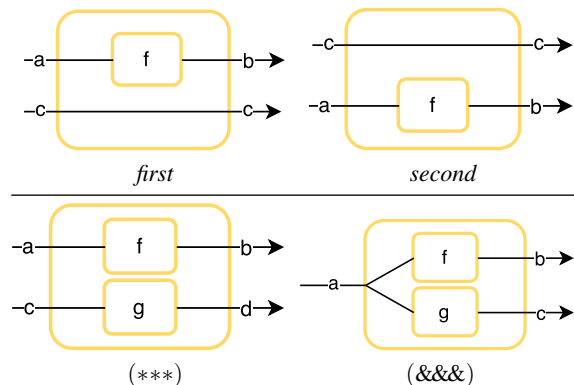$$second\ f = arr\ swap >>> first\ f >>> arr\ swap$$
$$\textbf{where}\ swap\ (x,y) = (y,x)$$

the $***$ combinator that combines *first* and *second* to handle two inputs in one arrow, (Fig.**??**) is defined as

$$(***) :: Arrow\ arr \Rightarrow arr\ a\ b \rightarrow arr\ c\ d \rightarrow arr\ (a,c)\ (b,d)$$
$$f *** g = first\ f >>> second\ g$$

and the **&&&** combinator that constructs an arrow which outputs two different values like $***$, but takes only one input (Fig. **??**) is:

$$(\&\&\&) :: Arrow\ arr \Rightarrow arr\ a\ b \rightarrow arr\ a\ c \rightarrow a\ a\ (b,c)$$
$$f \&\&\& g = arr\ (\lambda a \rightarrow (a,a)) >>> (f *** g)$$

A short example given by Hughes on how to use this is addition with arrows:

$$add :: Arrow\ arr \Rightarrow arr\ a\ Int \rightarrow arr\ a\ Int \rightarrow arr\ a\ Int$$
$$add\ f\ g = (f \&\&\& g) >>> arr\ (\lambda(u,v) \rightarrow u+v)$$

The more restrictive interface of arrows (a monad can be *anything*, an arrow is a process of doing something, a *computation*) allows for more elaborate composition and transformation combinators. One of the major problems in parallel computing is composition of parallel processes.

3 Related Work

OL: arrows or Arrows?

3.1 Parallel Haskells

Of course, the three parallel Haskell flavours we have presented above: the GpH (Trinder *et al.*, 1998, 1999) parallel Haskell dialect and its multicore version (Marlow *et al.*, 2009), the *Par* monad (Marlow *et al.*, 2011b; Foltzer *et al.*, 2012), and Eden (Loogen *et al.*, 2005; Loogen, 2012) are related to this work. We use these languages as backends: our library can switch from one to other at user's command.

HdpH (Maier *et al.*, 2014; Stewart *et al.*, 2016) is an extension of *Par* monad to heterogeneous clusters. LVish (Kuper *et al.*, 2014) is a communication-centred extension of *Par* monad. Further parallel Haskell approaches include pH (Nikhil & Arvind, 2001), research work done on distributed variants of GpH (Trinder *et al.*, 1996; Aljabri *et al.*, 2014, 2015) and low-level Eden implementation (Berthold, 2008; Berthold *et al.*, 2016). Skeleton composition (Dieterle *et al.*, 2016), communication (Dieterle *et al.*, 2010a), and generation of process networks (Horstmeyer & Loogen, 2013) are recent in-focus research topics in Eden. This also includes the definitions of new skeletons (Hammond *et al.*, 2003; Berthold & Loogen, 2006; Berthold *et al.*, 2009b,c; Dieterle *et al.*, 2010b; de la Encina *et al.*, 2011; Dieterle *et al.*, 2013; Janjic *et al.*, 2013).

More different approaches include data parallelism (Chakravarty *et al.*, 2007; Keller *et al.*, 2010; , n.d.), GPU-based approaches (Mainland & Morrisett, 2010; Svensson, 2011),

software transactional memory (Harris *et al.*, 2005; Perfumo *et al.*, 2008). The Haskell–GPU bridge Accelerate (Chakravarty *et al.*, 2011; Clifton-Everest *et al.*, 2014; McDonell *et al.*, 2015) deserves a special mention. Accelerate is completely orthogonal to our approach. Marlow authored a recent book 2013 on parallel Haskells.

3.2 Algorithmic skeletons

Algorithmic skeletons were introduced by Cole (1989). Early efforts include (Darlington *et al.*, 1993; Botorog & Kuchen, 1996; Danelutto *et al.*, 1997; Gorlatch, 1998; Lengauer *et al.*, 1997). Rabhi & Gorlatch (2003) consolidated early reports on high-level programming approaches. The effort is ongoing, including topological skeletons (Berthold & Loogen, 2006), special-purpose skeletons for computer algebra (Berthold *et al.*, 2009c; Lobachev, 2011, 2012; Janjic *et al.*, 2013), iteration skeletons (Dieterle *et al.*, 2013). The idea of Linton *et al.* (2010) is to use a parallel Haskell to orchestrate further software systems to run in parallel. Dieterle *et al.* (2016) compare the composition of skeleton to stable process networks.

3.3 Arrows

Arrows were introduced by Hughes (2000), basically they are a generalised function arrow $\rightarrow$. Hughes (2005a) is a tutorial on arrows. Some theoretical details on arrows (Jacobs *et al.*, 2009; Lindley *et al.*, 2011; Atkey, 2011) are viable. Paterson (2001) introduced a new notation for arrows. Arrows have applications in information flow research (Li & Zdancewic, 2006, 2010; Russo *et al.*, 2008), invertible programming (Alimarine *et al.*, 2005), and quantum computer simulation (Vizzotto *et al.*, 2006). But perhaps most prominent application of arrows is functional reactive programming (Hudak *et al.*, 2003).**OL: cite more!**

Liu *et al.* (2009) formally define a more special kind of arrows that capsule the computation more than regular arrows do and thus enable optimizations. Their approach would allow parallel composition, as their special arrows would not interfere with each other in concurrent execution. In a contrast, we capture a whole parallel computation as a single entity: our main instantiation function *parEvalN* makes a single (parallel) arrow out of list of arrows.**OL: ugh, take care!** Huang *et al.* (2007) utilise arrows for parallelism, but strikingly different from our approach. They basically use arrows to orchestrate several tasks in robotics. We propose a general interface for parallel programming, remaining completely in Haskell.

3.4 Other languages

Although this work is centred on Haskell implementation of arrows, it is applicable to any functional programming language where parallel evaluation and arrows can be defined. Our experiments with our approach in Frege language (which is basically Haskell on the JVM) were quite successful, we were able to use typical Java libraries for parallelism. However, it is beyond the scope of this work.

Achten *et al.* (2004, 2007) use an arrow implementation in Clean for better handling of typical GUI tasks. Dagand *et al.* (2009) used arrows in OCaml in the implementation of a distributed system.

4 Parallel Arrows

We have seen what Arrows are and how they can be used as a general interface to computation. In the following section we will discuss how Arrows constitute a general interface not only to computation, but to **parallel computation** as well. We start by introducing the interface and explaining the reasonings behind it. Then, we discuss some implementations using exisiting parallel Haskells. Finally, we explain why using Arrows for expressing parallelism is beneficial.

4.1 The ArrowParallel typeclass

As we have seen earlier, in its purest form, parallel computation (on functions) can be seen as the execution of some functions $a \rightarrow b$ in parallel, *parEvalN* (Fig. **??**, 1). Translating this into arrow terms gives us a new operator *parEvalN* that lifts a list of arrows $[arr\ a\ b]$ to a parallel arrow $arr\ [a]\ [b]$. This combinator is similar to our utility function *listApp* from Appendix A, but does parallel instead of serial evaluation.

$$parEvalN :: (Arrow\ arr) \Rightarrow [arr\ a\ b] \rightarrow arr\ [a]\ [b]$$

With this definition of *parEvalN*, parallel execution is yet another arrow combinator. But as the implementation may differ depending on the actual type of the arrow *arr* and we want this to be an interface for different backends, we introduce a new typeclass *ArrowParallel arr a b* to host this combinator:

class *Arrow arr* ⇒ *ArrowParallel arr a b* **where**
    parEvalN :: $[arr\ a\ b] \rightarrow arr\ [a]\ [b]$

Sometimes parallel Haskells require or allow for additional configuration parameters, e.g. an information about the execution environment or the level of evaluation (weak-head normalform vs. normalform). For this reason we also introduce an additional *conf* parameter to the function. We also do not want *conf* to be a fixed type, as the configuration parameters can differ for different instances of *ArrowParallel*. So we add it to the type signature of the typeclass as well and get *ArrowParallel arr a b conf* : **OL:** *ArrowParallel arr a b conf* **or** *ArrowParallel conf arr a b***?**

class *Arrow arr* ⇒ *ArrowParallel arr a b conf* **where**
    parEvalN :: $conf \rightarrow [arr\ a\ b] \rightarrow arr\ [a]\ [b]$

Note that we don't require the *conf* parameter in every implementation. If it is not needed, we usually just default the *conf* type parameter to $()$ and even blank it out in the parameter list of the implemented *parEvalN*, as we will see in the implementation of the Multicore and the *Par* Monad backend.

4.2 ArrowParallel instances

4.2.1 Multicore Haskell

The Multicore Haskell implementation of this class is implemented in a straightforward manner by using *listApp* from Appendix A combined with the *withStrategy* :: *Strategy a* → *a* → *a* and *pseq* :: *a* → *b* → *b* combinators from Multicore Haskell, where *withStrategy* is the same as *using* :: *a* → *Strategy a* → *a* but with flipped parameters. For most cases a

> **instance** (*NFData b*, *ArrowApply arr*, *ArrowChoice arr*) ⇒ *ArrowParallel arr a b* () **where**
>   *parEvalN* _ *fs* =
>   *listApp fs* >>>
>   *arr* (*withStrategy* (*parList rdeepseq*)) &&& *arr id* >>>
>   *arr* (*uncurry pseq*)

Figure 9: Fully evaluating ArrowParallel instance for the Multicore Haskell backend

fully evaluating version like in Fig. 9 would probably suffice, but as the Multicore Haskell interface allows the user to specify the level of evaluation to be done via the *Strategy* interface, we want to the user not to lose this ability because of using our API. We therefore introduce the *Conf a* data-type that simply wraps a *Strategy a* (Fig. **??**). We can't directly use the *Strategy a* type here as GHC (at least in the versions used for development in this paper) does not allow type synonyms in type class instances:

> **data** *Conf a* = *Conf* (*Strategy a*)

To get our configurable *ArrowParallel* instance, we simply unwrap the strategy and pass it to *parList* like in the fully evaluating version (Fig. 10).

> **instance** (*NFData b*, *ArrowApply arr*, *ArrowChoice arr*) ⇒
>   *ArrowParallel arr a b* (*Conf b*) **where**
>   *parEvalN* (*Conf strat*) *fs* =
>   *listApp fs* >>>
>   *arr* (*withStrategy* (*parList strat*)) &&& *arr id* >>>
>   *arr* (*uncurry pseq*)

Figure 10: Configurable ArrowParallel instance for the Multicore Haskell backend

4.2.2 Par Monad

OL: introduce a newcommand for par-monad, "arrows", "parrows" and replace all mentions to them to ensure uniform typesetting The *Par* monad implementation (Fig. 11) makes use of Haskells laziness and *Par* monad's *spawnP* :: *NFData a* ⇒ *a* → *Par* (*IVar a*) function. The latter forks away the computation of a value and returns an *IVar* containing the result in the *Par* monad.

We therefore apply each function to its corresponding input value with *listApp* (Fig. A 5) and then fork the computation away with *arr spawnP* inside a *zipWithArr* call. This yields

a list $[Par\,(IVar\,b)]$, which we then convert into $Par\,[IVar\,b]$ with *arr sequenceA*. In order
to wait for the computation to finish, we map over the *IVar*s inside the *Par* monad with
$arr\,(\gg=mapM\,get)$. The result of this operation is a $Par\,[b]$ from which we can finally
remove the monad again by running *arr runPar* to get our output of $[b]$.

> **instance** $(NFData\,b,ArrowApply\,arr,ArrowChoice\,arr) \Rightarrow$
>   *ArrowParallel arr a b conf* **where**
> $parEvalN\,\_fs =$
> $(arr\,\$\,\lambda\,as \rightarrow (fs,as)) \ggg$
> $zipWithArr\,(app \ggg arr\,spawnP) \ggg$
> $arr\,sequenceA \ggg$
> $arr\,(\gg=mapM\,get) \ggg$
> $arr\,runPar$

Figure 11: ArrowParallel instance for the Par Monad backend

4.2.3 Eden

For both the Multicore Haskell and *Par* Monad implementations we could use general
instances of *ArrowParallel* that just require the *ArrowApply* and *ArrowChoice* typeclasses.
With Eden this is not the case as we can only spawn a list of functions and we cannot extract
simple functions out of arrows. While we could still manage to have only one class in the
module by introducing a typeclass:

> **class** $(Arrow\,arr) \Rightarrow ArrowUnwrap\,arr$ **where**
> $arr\,a\,b \rightarrow (a \rightarrow b)$

We don't do it here for aesthetic resons, though. For now, we just implement *ArrowParallel*
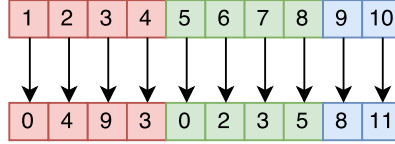for normal functions:

> **instance** $(Trans\,a,Trans\,b) \Rightarrow ArrowParallel\,(\rightarrow)\,a\,b\,conf$ **where**
> $parEvalN\,\_fs\,as = spawnF\,fs\,as$

and the Kleisli type:

> **instance** $(Monad\,m,Trans\,a,Trans\,b,Trans\,(m\,b)) \Rightarrow$
>   *ArrowParallel (Kleisli m) a b conf* **where**
> $parEvalN\,conf\,fs =$
> $(arr\,\$\,parEvalN\,conf\,(map\,(\lambda\,(Kleisli\,f) \rightarrow f)\,fs)) \ggg$
> $(Kleisli\,\$\,sequence)$

4.3 Extending the Interface

With the *ArrowParallel* typeclass in place and implemented, we can now implement some
further basic parallel interface functions. These are algorithmic skeletons that, however,
mostly serve as a foundation to further, more specific algorithmic skeletons.
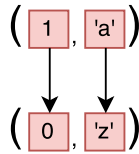
4.3.1 Lazy parEvalN

Figure 12: Schematic depiction of parEvalNLazy

The function *parEvalN* is 100% strict, which means that it fully evaluates all passed arrows. Sometimes this might not be feasible, as it will not work on infinite lists of functions like e.g. *map* (*arr* ∘ (+)) [1..] or just because we need the arrows evaluated in chunks. *parEvalNLazy* (Fig. 12, 13) fixes this. It works by first chunking the input from [*a*] to [[*a*]] with the given *ChunkSize* in *arr* (*chunksOf chunkSize*). These chunks are then fed into a list [*arr* [*a*] [*b*]] of parallel arrows created by feeding chunks of the passed *ChunkSize* into the regular parEvalN by using *listApp* (Fig. A 5). The resulting [[*b*]] is lastly converted into [*b*] with *arr concat*.

$$parEvalNLazy :: (ArrowParallel\ arr\ a\ b\ conf, ArrowChoice\ arr, ArrowApply\ arr) \Rightarrow$$
$$conf \rightarrow ChunkSize \rightarrow [arr\ a\ b] \rightarrow (arr\ [a]\ [b])$$
$$parEvalNLazy\ conf\ chunkSize\ fs =$$
$$arr\ (chunksOf\ chunkSize) \ggg$$
$$listApp\ fchunks \ggg$$
$$arr\ concat$$
$$\textbf{where}\ fchunks = map\ (parEvalN\ conf)\ \$\ chunksOf\ chunkSize\ fs$$

Figure 13: Definition of *parEvalNLazy*.

4.3.2 Heterogenous tasks

Figure 14: Schematic depiction of *parEval2*.

We have only talked about the paralellization arrows of the same type until now. But sometimes we want to paralellize heterogenous types as well. However, we can implement such a *parEval2* combinator (Fig. 14, 15) which combines two arrows *arr a b* and *arr c d* into a new parallel arrow *arr* (*a,c*) (*b,d*) quite easily with the help of the *ArrowChoice* typeclass. The idea is to use the +++ combinator which combines two arrows *arr a b* and

arr c d and transforms them into *arr* (*Either a c*) (*Either b d*) to get a common arrow type
that we can then feed into parEvalN.

We start by transforming the (a,c) input into a 2-element list [*Either a c*] by first tagging
the two inputs with *Left* and *Right* and wrapping the right element in a singleton list with
return so that we can combine them with *arr* (*uncurry* (:)). Next, we feed this list into a
parallel arrow running on 2 instances of $f +\!+\!+ g$ as described above. After the calculation
is finished, we convert the resulting [*Either b d*] into ([*b*], [*d*]) with *arr partitionEithers*.
The two lists in this tuple contain only 1 element each by construction, so we can finally
just convert the tuple to (b,d) in the last step.

$$
\begin{aligned}
&parEval2 :: (ArrowChoice\ arr, \\
&\quad ArrowParallel\ arr\ (Either\ a\ c)\ (Either\ b\ d)\ conf) \Rightarrow \\
&\quad conf \rightarrow arr\ a\ b \rightarrow arr\ c\ d \rightarrow arr\ (a,c)\ (b,d) \\
&parEval2\ conf\ f\ g = \\
&\quad arr\ Left *\!*\!* (arr\ Right \ggg arr\ return) \ggg \\
&\quad arr\ (uncurry\ (:)) \ggg \\
&\quad parEvalN\ conf\ (replicate\ 2\ (f +\!+\!+ g)) \ggg \\
&\quad arr\ partitionEithers \ggg \\
&\quad arr\ head *\!*\!* arr\ head
\end{aligned}
$$

Figure 15: Definition of parEval2

4.3.3 Syntactic Sugar

For basic arrows, we have the $*\!*\!*$ combinator (Fig. **??**, **??**) which allows us to combine
two arrows *arr a b* and *arr c d* into an arrow *arr* (a,c) (b,d) which does both computations
at once. This can easily be translated into a parallel version |$*\!*\!*$| (Fig. **??**) with the use
of *parEval2*, but for this we require a backend which has an implementation that does not
require any configuration (hence the () as the *conf* parameter):

$$
\begin{aligned}
&(|*\!*\!*|) :: (ArrowChoice\ arr, ArrowParallel\ arr\ (Either\ a\ c)\ (Either\ b\ d)\ ())) \Rightarrow \\
&\quad arr\ a\ b \rightarrow arr\ c\ d \rightarrow arr\ (a,c)\ (b,d) \\
&(|*\!*\!*|) = parEval2\ ()
\end{aligned}
$$

We define the parallel |&&&| in a similar manner to its sequential pendant &&& (Fig. **??**):

$$
\begin{aligned}
&(|\&\&\&|) :: (ArrowChoice\ arr, ArrowParallel\ arr\ (Either\ a\ a)\ (Either\ b\ c)\ ()) \Rightarrow \\
&\quad arr\ a\ b \rightarrow arr\ a\ c \rightarrow arr\ a\ (b,c) \\
&(|\&\&\&|)\ f\ g = (arr\ \$\ \lambda a \rightarrow (a,a)) \ggg f\ |*\!*\!*|\ g
\end{aligned}
$$

5 Futures

Consider the parallel arrow combinator in Fig. 16 In a distributed environment, the resulting
arrow of this combinator first evaluates all [*arr a b*] in parallel, sends the results back to
the master node, rotates the input once and then evaluates the [*arr b c*] in parallel to then

$$someCombinator :: (Arrow\ arr) \Rightarrow [arr\ a\ b] \rightarrow [arr\ b\ c] \rightarrow arr\ [a]\ [c]$$
$$someCombinator\ fs1\ fs2 = parEvalN\ ()\ fs1 \ggg rightRotate \ggg parEvalN\ ()\ fs2$$

Figure 16: An example parallel Arrow combinator without Futures

gather the input once again on the master node. Such situations arise, e.g. in scientific computations when the data distributed across the nodes needs to be transposed. A concrete example is 2D FFT computation (Gorlatch & Bischof, 1998; Berthold *et al.*, 2009c).

While the example in Fig. 16 could be rewritten into only one *parEvalN* call by directly wiring the arrows properly together, this example illustrates an important problem: When using a *ArrowParallel* backend that resides on multiple computers, all communication between the nodes is done via the master node, as shown in the Eden trace in Figure 17. This can become a serious bottleneck for larger amount of data and number of processes (showcases Berthold *et al.*, 2009c, as, e.g.).
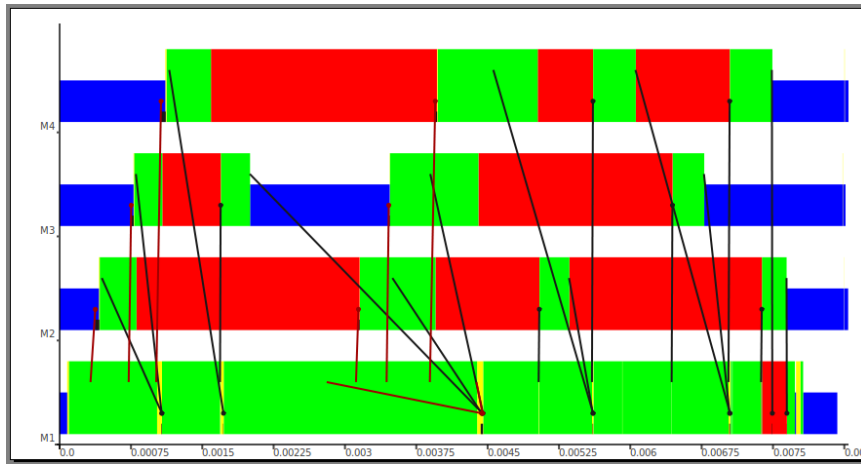

Figure 17: Communication between 4 threads without Futures. All communication goes through the master node. Each bar represents one process. Black lines between processes represent communication. Colors: blue $\hat{=}$ idle, green $\hat{=}$ running, red $\hat{=}$ blocked, yellow $\hat{=}$ suspended.

OL: more practical and heavy-weight example! fft (I have the code)?

This motivates for an approach that allows the nodes to communicate directly with each other. Thankfully, Eden, the distributed parallel Haskell we have used in this paper so far, already ships with the concept of *RD* (remote data) that enables this behaviour (Alt & Gorlatch, 2003; Dieterle *et al.*, 2010a).

But as we want code written against our API to be implementation agnostic, we have to wrap this context. We do this with the *Future* typeclass (Fig. 18). Since *RD* is only a type synonym for communication type that Eden uses internally, we have to use some wrapper classes to fit that definition, though, as seen in Appendix in Fig. A 6. This is due to the same reason we had to introduce a wrapper for *Strategy a* in the Multicore Haskell implementation of *ArrowParallel* in Section 4.2.1.

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$$\textbf{class } \textit{Future fut a @} | @ \textit{ a} \rightarrow \textit{fut } \textbf{where}$$
$$\textit{put} :: (\textit{Arrow arr}) \Rightarrow \textit{arr a (fut a)}$$
$$\textit{get} :: (\textit{Arrow arr}) \Rightarrow \textit{arr (fut a) a}$$

Figure 18: Definition of the Future typeclass

For our Par Monad and Multicore Haskell backends, we can simply use *MVar*s (Jones *et al.*, 1996) (Fig. 19), because we have shared memory in a single node and don't require Eden's sophisticated communication channels. **MB: explain MVars**

```
{-# NOINLINE putUnsafe #-}
putUnsafe :: a → MVar a
putUnsafe a = unsafePerformIO $ do
  mVar ← newEmptyMVar
  putMVar mVar a
  return mVar
instance (NFData a) ⇒ Future MVar a where
  put = arr putUnsafe
  get = arr takeMVar >>> arr unsafePerformIO
```

Figure 19: MVar instance of the Future typeclass for the Par Monad and Multicore Haskell backends

Furthermore, in order for these *Future* types to fit with the *ArrowParallel* instances we gave earlier, we have to give the necessary *NFData* and *Trans* instances, the latter are only needed in Eden. Because *MVar* already ships with a *NFData* instance, we only have to supply two simple instances for our *RemoteData* type:

```
instance NFData (RemoteData a) where
  rnf = rnf ∘ rd
instance Trans (RemoteData a)
```

The *Trans* instance does not have any functions declared as the default implementation suffices here.

Going back to our communication example we can use this *Future* concept in order to enable direct communications between the nodes in the following way: In a distributed

```
someCombinator :: (Arrow arr) ⇒ [arr a b] → [arr b c] → arr [a] [c]
someCombinator fs1 fs2 =
  parEvalN () (map (>>>put) fs1) >>>
  rightRotate >>>
  parEvalN () (map (get>>>) fs2)
```

Figure 20: The combinator from Fig. 16 in parallel

environment, this gives us a communication scheme with messages going through the master

node only if it is needed - similar to what is shown in the trace in Fig. 21.**OL: Fig. 3 is not really clear. Do Figs 2-3 with a lot of load?**
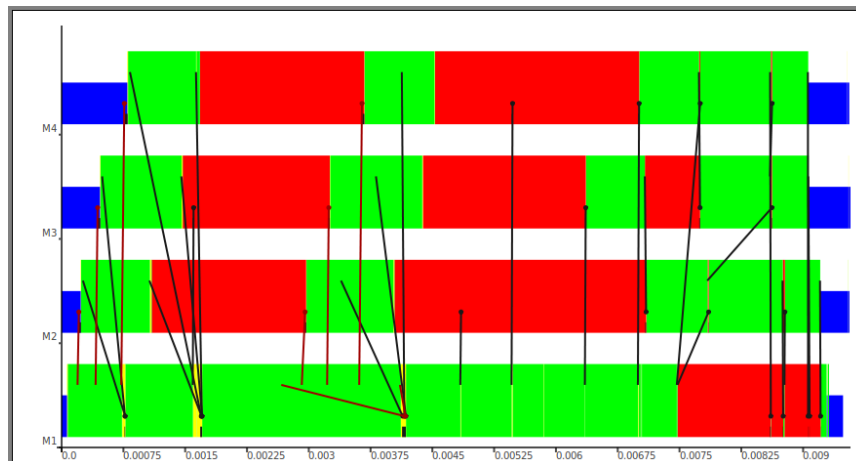

Figure 21: Communication between 4 threads with Futures. Other than in Fig. 17, threads communicate directly (black lines between the bars) instead of always going through the master node (bottom bar).

6 Map-based Skeletons

Now we have developed Parallel Arrows far enough to define some algorithmic skeletons useful to an application programmer.
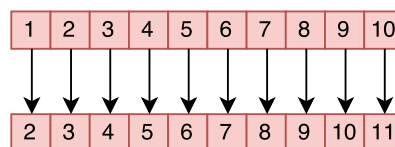
6.1 Parallel map

Figure 22: Schematic depiction of *parMap*.

The *parMap* skeleton (Fig. 22, 23) is probably the most common skeleton for parallel programs. We can implement it with *ArrowParallel* by repeating an arrow *arr a b* and then passing it into *parEvalN* to get an arrow *arr* [*a*] [*b*]. Just like *parEvalN*, *parMap* is 100% strict.

$$parMap :: (ArrowParallel\ arr\ a\ b\ conf) \Rightarrow conf \to (arr\ a\ b) \to (arr\ [a]\ [b])$$
$$parMap\ conf\ f = parEvalN\ conf\ (repeat\ f)$$
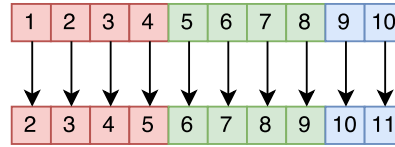
Figure 23: Definition of parMap

Figure 24: Schematic depiction of parMapStream

6.2 *Lazy parallel map*

As *parMap* (Fig. 22, 23) is 100% strict it has the same restrictions as *parEvalN* compared to *parEvalNLazy*. So it makes sense to also have a *parMapStream* (Fig. 24, 25) which behaves like *parMap*, but uses *parEvalNLazy* instead of *parEvalN*.

$$parMapStream :: (ArrowParallel\ arr\ a\ b\ conf, ArrowChoice\ arr, ArrowApply\ arr) \Rightarrow$$
$$conf \to ChunkSize \to arr\ a\ b \to arr\ [a]\ [b]$$
$$parMapStream\ conf\ chunkSize\ f = parEvalNLazy\ conf\ chunkSize\ (repeat\ f)$$

Figure 25: Definition of *parMapStream*.

6.3 *Statically load-balancing parallel map*

A *parMap* (Fig. 22, 23) spawns every single computation in a new thread (at least for the instances of *ArrowParallel* we gave in this paper). This can be quite wasteful and a *farm* (Fig. 26, 27) that equally distributes the workload over *numCores* workers (if numCores is greater than the actual processor count, the fastest processor(s) to finish will get more tasks) seems useful. The definitions of helper functions *unshuffle*, *takeEach*, *shuffle* (shown in Appendix) originate from an Eden skeleton[5].

6.4 *The* farmChunk *Skeleton*

Since a *farm* (Fig. 26, 27) is basically just *parMap* with a different work distribution, it is, again, 100% strict. So we can define *farmChunk* (Fig. 28, A 7) which uses *parEvalNLazy* instead of *parEvalN*. It is basically the same definition as for *farm*, with *parEvalN* replaced with *parEvalNLazy*, as Appendix shows.

---

[5] Available on Hackage under `https://hackage.haskell.org/package/edenskel-2.1.0.0/docs/src/Control-Parallel-Eden-Map.html`.

Figure 26: Schematic depiction of a *farm*, a statically load-balanced *parMap*.

$$
\begin{array}{l}
\textit{farm} :: (\textit{ArrowParallel arr a b conf}, \\
\quad \textit{ArrowParallel arr} [a] [b] \textit{conf}, \textit{ArrowChoice arr}) \Rightarrow \\
\quad \textit{conf} \rightarrow \textit{NumCores} \rightarrow \textit{arr a b} \rightarrow \textit{arr} [a] [b] \\
\textit{farm conf numCores f} = \\
\quad \textit{unshuffle numCores} \ggg \\
\quad \textit{parEvalN conf} (\textit{repeat} (\textit{mapArr f})) \ggg \\
\quad \textit{shuffle} \\[4pt]
\textit{unshuffle} :: (\textit{Arrow arr}) \Rightarrow \textit{Int} \rightarrow \textit{arr} [a] [[a]] \\
\textit{unshuffle n} = \textit{arr} (\lambda xs \rightarrow [\textit{takeEach n} (\textit{drop i xs}) \, @ | @ \, i \leftarrow [0 .. n-1]]) \\[4pt]
\textit{takeEach} :: \textit{Int} \rightarrow [a] \rightarrow [a] \\
\textit{takeEach n} [\,] = [\,] \\
\textit{takeEach n} (x : xs) = x : \textit{takeEach n} (\textit{drop} (n-1) xs) \\[4pt]
\textit{shuffle} :: (\textit{Arrow arr}) \Rightarrow \textit{arr} [[a]] [a] \\
\textit{shuffle} = \textit{arr} (\textit{concat} \circ \textit{transpose})
\end{array}
$$

Figure 27: The definition of *farm*.

6.5 *Map and reduce*

A simple *map–reduce* can be written like in Figure 29. Notice that the performance of the $\ggg$ combinator is essential for the performance of the skeleton. A definitive version would use Futures.

> **OL: it appears STRANGE. are the data really left alone and noded after map and taken from there by reduce? It makes sense only is no communication through master takes place. ELSE: CUT!**
> **MB: this requires some work. Either change this to use futures or cut, yes.**
> **OL: now rewritten as motivation for futures. maybe still cut?**

7 Topological Skeletons

Even though many algorithms can be expressed by parallel maps, some problems require more sophisticated skeletons. The Eden library leverages this problem and already comes[6] with more predefined skeletons, among them a *pipe*, a *ring*, and a *torus* implementations (Loogen, 2012). These seem like reasonable candidates to be ported to our Arrow-based parallel Haskell. We aim to showcase that we can express more sophisticated skeletons with Parallel Arrows as well.

---

[6] Available on Hackage: `https://hackage.haskell.org/package/edenskel-2.0.0.2/docs/Control-Parallel-Eden-Topology.html`.
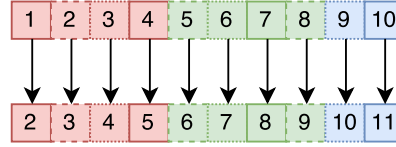
Figure 28: Schematic depiction of farmChunk

parMapReduceDirect :: (*ArrowParallel arr* [*a*] *b conf*, *ArrowApply arr*, *ArrowChoice arr*) ⇒
   conf → *ChunkSize* → *arr a b* → *arr* (*b*,*b*) *b* → *b* → *arr* [*a*] *b*
parMapReduceDirect conf chunkSize mapfn foldfn neutral =
   arr (*chunksOf chunkSize*) >>>
   parMap conf (*mapArr mapfn* >>> *foldlArr foldfn neutral*) >>>
   foldlArr foldfn neutral

Figure 29: Definition of parMapReduceDirect

7.1 Parallel pipe

The parallel *pipe* skeleton is semantically equivalent to folding over a list [*arr a a*] of arrows with >>>, but does this in parallel, meaning that the arrows do not have to reside on the same thread/machine. We implement this skeleton using the *ArrowLoop* typeclass which gives us the *loop* :: *arr* (*a*,*b*) (*c*,*b*) → *arr a c* combinator which allows us to express recursive fix-point computations in which output values are fed back as input. For example **MB: das kann man hier so lassen, oder?OL: sicherlich!**

   loop (*arr* (*λ*(*a*,*b*) → (*b*,*a*:*b*)))

which is the same as

   loop (*arr snd* &&& *arr* (*uncurry* (:)))

defines an arrow that takes its input *a* and converts it into an infinite stream [*a*] of it. Using this to our advantage gives us a first draft of a pipe implementation (Fig. 30) by plugging in the parallel evaluation call *parEvalN conf fs* inside the second argument of &&& and then only picking the first element of the resulting list with *arr last*.

However, using this definition directly will make the master node a potential bottleneck in distributed environments as described in Section 5. Therefore, we introduce a more sophisticated version that internally uses Futures and get the final definition of *pipe*:

   pipe :: (*ArrowLoop arr*, *ArrowParallel arr* (*fut a*) (*fut a*) *conf*, *Future fut a*) ⇒
     conf → [*arr a a*] → *arr a a*
   pipe conf fs = *unliftFut* (*pipeSimple conf* (*map liftFut fs*))

Sometimes, this *pipe* definition can be a bit inconvenient, especially if we want to pipe arrows of mixed types together, i.e. *arr a b* and *arr b c*. By wrapping these two arrows inside a common type we obtain *pipe2* (Fig. 31).

Note that extensive use of this combinator over *pipe* with a hand-written combination data type will probably result in worse performance because of more communication overhead from the many calls to parEvalN. Nonetheless, we can define a parallel piping operator

$$pipeSimple :: (ArrowLoop\ arr, ArrowParallel\ arr\ a\ a\ conf) \Rightarrow$$
$$conf \rightarrow [arr\ a\ a] \rightarrow arr\ a\ a$$
$$pipeSimple\ conf\ fs =$$
$$loop\ (arr\ snd \&\&\&$$
$$(arr\ (uncurry\ (:) \ggg lazy) \ggg parEvalN\ conf\ fs)) \ggg$$
$$arr\ last$$
$$lazy :: (Arrow\ arr) \Rightarrow arr\ [a]\ [a]$$
$$lazy = arr\ (\lambda \sim (x:xs) \rightarrow x:lazy\ xs)$$

Figure 30: A first implementation of the *pipe* skeleton expressed with Parallel Arrows. Note that the use of *lazy* is essential as without it programs using this definition would never halt. We need to enforce that the evaluation of the input [*a*] terminates before passing it into *parEvalN*.

OL: I swapped the type classes here:

$$pipe2 :: (ArrowLoop\ arr, ArrowChoice\ arr, Future\ fut\ (([a],[b]),[c]),$$
$$ArrowParallel\ arr\ (fut\ (([a],[b]),[c]))\ (fut\ (([a],[b]),[c]))\ conf) \Rightarrow$$
$$conf \rightarrow arr\ a\ b \rightarrow arr\ b\ c \rightarrow arr\ a\ c$$
$$pipe2\ conf\ f\ g =$$
$$(arr\ return \&\&\& arr\ (const\ [\,])) \&\&\& arr\ (const\ [\,]) \ggg$$
$$pipe\ conf\ (replicate\ 2\ (unify\ f\ g)) \ggg$$
$$arr\ snd \ggg arr\ head\ \mathbf{where}$$
$$unify :: (ArrowChoice\ arr) \Rightarrow arr\ a\ b \rightarrow arr\ b\ c \rightarrow arr\ (([a],[b]),[c])\ (([a],[b]),[c])$$
$$unify\ f\ g =$$
$$(mapArr\ f *** mapArr\ g) *** arr\ (\backslash\_ \rightarrow [\,]) \ggg$$
$$arr\ (\lambda ((a,b),c) \rightarrow ((c,a),b))$$

Figure 31: Definition of *pipe2*.

|$\ggg$| (Fig. 32, which is semantically equivalent to $\ggg$ similarly to other parallel syntactic sugar from Section 4.3.3.

OL: swapped type classes

$$(|\ggg|) :: (ArrowLoop\ arr, ArrowChoice\ arr, Future\ fut\ (([a],[b]),[c]),$$
$$ArrowParallel\ arr\ (fut\ (([a],[b]),[c]))\ (fut\ (([a],[b]),[c]))\ ()) \Rightarrow$$
$$arr\ a\ b \rightarrow arr\ b\ c \rightarrow arr\ a\ c$$
$$(|\ggg|) = pipe2\ ()$$
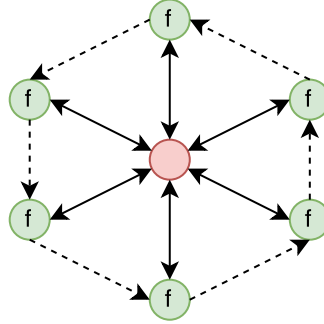
Figure 32: Definition of |$\ggg$|.

Figure 33: Schematic depiction of the ring skeleton

7.2 Ring skeleton

Eden comes with a ring skeleton[7] (Fig. 33) implementation that allows the computation of a function $[i] \rightarrow [o]$ with a ring of nodes that communicate in a ring topology with each other. Its input is a node function $i \rightarrow r \rightarrow (o, r)$ in which r serves as the intermediary output that gets send to the neighbour of each node. This data is sent over direct communication channels, so called 'remote data'. We depict it in Appendix, Fig. A 8.

We can rewrite this functionality easily with the use of *loop* as the definition of the node function, *arr* (i, r) (o, r), after being transformed into an arrow, already fits quite neatly into the *loop*'s *arr* (a, b) $(c, b) \rightarrow arr\ a\ c$. In each iteration we start by rotating the intermediary input from the nodes $[fut\ r]$ with *second* (*rightRotate* $>>>$ *lazy*). Similarly to the *pipe* (Fig. 30, **??**), we have to feed the intermediary input into our *lazy* arrow here, or the evaluation would hang.**OL: meh, wording** The reasoning is explained by Loogen (2012):

> *Note that the list of ring inputs ringIns is the same as the list of ring outputs ringOuts rotated by one element to the right using the auxiliary function rightRotate. Thus, the program would get stuck without the lazy pattern, because the ring input will only be produced after process creation and process creation will not occur without the first input.*

Next, we zip the resulting $([i], [fut\ r])$ to $[(i, fut\ r)]$ with *arr* (*uncurry zip*) so we can feed that into a our input arrow *arr* (i, r) (o, r), which we transform into *arr* $(i, fut\ r)$ $(o, fut\ r)$ before lifting it to *arr* $[(i, fut\ r)]$ $[(o, fut\ r)]$ to get a list $[(o, fut\ r)]$. Finally we unzip this list into $([o], [fut\ r])$. Plugging this arrow *arr* $([i], [fut\ r])$ $([o], fut\ r)$ into the definition of *loop* from earlier gives us *arr* $[i]$ $[o]$, our ring arrow (Fig. 34). This combinator can, for example, be used to calculate the shortest paths in a graph using Warshall's algorithm. **OL: let's do it?**
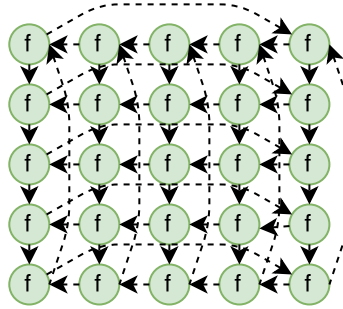
7.3 Torus skeleton

If we take the concept of a ring from 7.2 one dimension further, we get a torus (Fig. 35, 36). Every node sends ands receives data from horizontal and vertical neighbours in each

---

[7] Available on Hackage: `https://hackage.haskell.org/package/edenskel-2.0.0.2/docs/Control-Parallel-Eden-Topology.html`

ring :: (*ArrowLoop arr*, *Future fut r*, *ArrowParallel arr* (*i*,*fut r*) (*o*,*fut r*) *conf*) ⇒
    conf → *arr* (*i*,*r*) (*o*,*r*) → *arr* [*i*] [*o*]
ring conf f =
    loop (*second* (*rightRotate* >>> *lazy*) >>> *arr* (*uncurry zip*) >>>
    parMap conf (*second get* >>> *f* >>> *second put*) >>> *arr unzip*)
rightRotate :: (*Arrow arr*) ⇒ *arr* [*a*] [*a*]
rightRotate = *arr* $ λ *list* → **case** *list* **of**
    [] → []
    xs → *last xs* : *init xs*
lazy :: (*Arrow arr*) ⇒ *arr* [*a*] [*a*]
lazy = *arr* (λ ∼(*x* : *xs*) → *x* : *lazy xs*)

Figure 34: Final definition of the *ring* skeleton.

Figure 35: Schematic depiction of the *torus* skeleton.

communication round. With our Parallel Arrows we re-implement the *torus* combinator[8]
from Eden—yet again with the help of the *ArrowLoop* typeclass.

Similar to the *ring*, we once again start by rotating the input, but this time not only in one
direction, but in two. This means that the intermediary input from the neighbour nodes has
to be stored in a tuple ([[*fut a*]], [[*fut b*]]) in the second argument (loop only allows for two
arguments) of our looped arrow *arr* ([[*c*]], ([[*fut a*]], [[*fut b*]])) ([[*d*]], ([[*fut a*]], [[*fut b*]]))
and our rotation arrow becomes *second* ((*mapArr rightRotate* >>> *lazy*) ∗∗∗ (*arr rightRotate* >>>
lazy)) instead of the singular rotation in the ring as we rotate [[*fut a*]] horizontally and
[[*fut b*]] vertically. Then, we once again zip the inputs for the input arrow with *arr* (*uncurry3 zipWith3 lazyzip3*)
from ([[*c*]], ([[*fut a*]], [[*fut b*]])) to [[(*c*,*fut a*,*fut b*)]], which we then feed into our parallel
execution.

This, however, is more complicated than in the ring case as we have one more dimension
of inputs to be transformed. We first have to *shuffle* all the inputs to then pass it into
parMap conf (*ptorus f*) which yields us [(*d*,*fut a*,*fut b*)]. We can then unpack this shuffled
list back to its original ordering by feeding it into the specific unshuffle arrow we created
one step earlier with *arr length* >>> *arr unshuffle* with the use of *app* :: *arr* (*arr a b*,*a*) *c*

---

[8] Available on Hackage: `https://hackage.haskell.org/package/edenskel-2.0.0.2/`
`docs/Control-Parallel-Eden-Topology.html`.

from the *ArrowApply* typeclass. Finally, we unpack this matrix $[[[(d, \textit{fut a}, \textit{fut b})]]]$ with *arr* (*map unzip3*) $\ggg$ *arr unzip3* $\ggg$ *threetotwo* to get $([[d]], ([[\textit{fut a}]], [[\textit{fut b}]]))$.

OL: swapped type classes

torus :: (*ArrowLoop arr*, *ArrowChoice arr*, *ArrowApply arr*, *Future fut a*, *Future fut b*,
    ArrowParallel arr (*c*, *fut a*, *fut b*) (*d*, *fut a*, *fut b*) *conf*) $\Rightarrow$
    conf $\rightarrow$ *arr* (*c*, *a*, *b*) (*d*, *a*, *b*) $\rightarrow$ *arr* $[[c]]$ $[[d]]$
torus conf f =
    loop (*second* ((*mapArr rightRotate* $\ggg$ *lazy*) $***$ (*arr rightRotate* $\ggg$ *lazy*)) $\ggg$
    arr (*uncurry3* (*zipWith3 lazyzip3*)) $\ggg$
    (*arr length* $\ggg$ *arr unshuffle*) &&& (*shuffle* $\ggg$ *parMap conf* (*ptorus f*)) $\ggg$ *app* $\ggg$
    arr (*map unzip3*) $\ggg$ *arr unzip3* $\ggg$ *threetotwo*)
ptorus :: (*Arrow arr*, *Future fut a*, *Future fut b*) $\Rightarrow$
    arr (*c*, *a*, *b*) (*d*, *a*, *b*) $\rightarrow$ *arr* (*c*, *fut a*, *fut b*) (*d*, *fut a*, *fut b*)
ptorus f = *arr* ($\lambda{\sim}(c, a, b) \rightarrow (c, \textit{get a}, \textit{get b})$) $\ggg$ *f* $\ggg$ *arr* ($\lambda{\sim}(d, a, b) \rightarrow (d, \textit{put a}, \textit{put b})$)
uncurry3 :: ($a \rightarrow b \rightarrow c \rightarrow d$) $\rightarrow$ (a, (b, c)) $\rightarrow d$
uncurry3 f (a, (b, c)) = *f a b c*
lazyzip3 :: $[a] \rightarrow [b] \rightarrow [c] \rightarrow [(a, b, c)]$
lazyzip3 as bs cs = *zip3 as* (*lazy bs*) (*lazy cs*)
threetotwo :: (*Arrow arr*) $\Rightarrow$ *arr* (a, b, c) (a, (b, c))
threetotwo = *arr* \$ $\lambda{\sim}(a, b, c) \rightarrow (a, (b, c))$

Figure 36: Definition of the *torus* skeleton.

As an example of using this skeleton (Loogen, 2012) showed the matrix multiplication using the Gentleman algorithm (Gentleman, 1978). Their instantiation of the skeleton *nodefunction* can be adapted as shown in Fig. 37. If we compare the trace from a call using

nodefunction :: *Int* $\rightarrow$ ((*Matrix*, *Matrix*), [*Matrix*], [*Matrix*]) $\rightarrow$ ([*Matrix*], [*Matrix*], [*Matrix*])
nodefunction n ((*bA*, *bB*), *rows*, *cols*) = ([*bSum*], *bA* : *nextAs*, *bB* : *nextBs*)
   where *bSum* = *foldl'* *matAdd* (*matMult bA bB*) (*zipWith matMult nextAs nextBs*)
     nextAs = *take* ($n - 1$) *rows*
     nextBs = *take* ($n - 1$) *cols*

Figure 37: Adapted *nodefunction* for matrix multiplication with the *torus* from Fig. 36

our arrow definition of the torus (Fig. 38) with the Eden version (Fig. 39) we can see that the behaviour of the arrow version is comparable. **OL: much more details on this!**
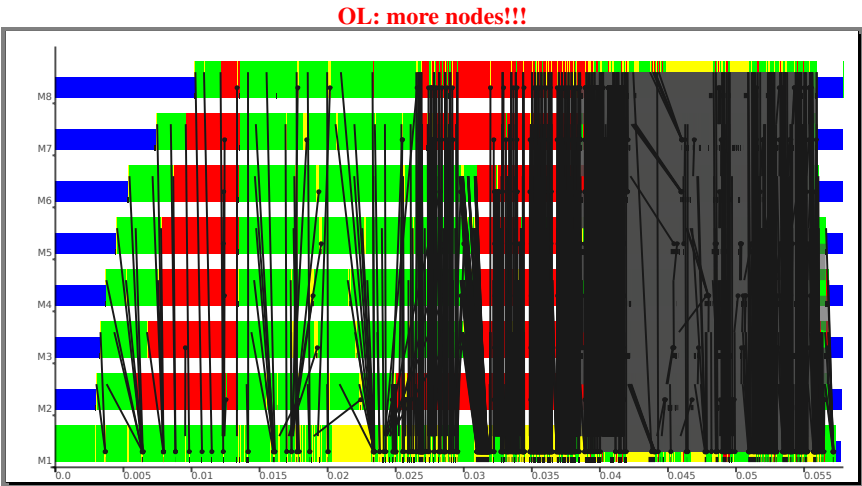
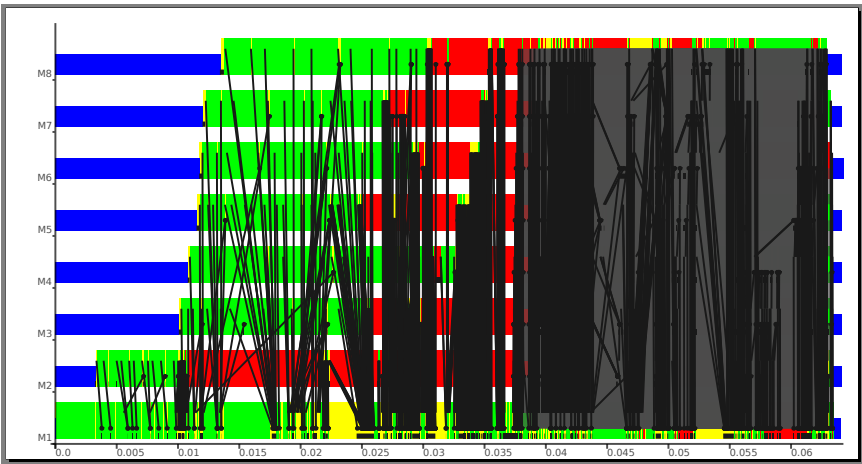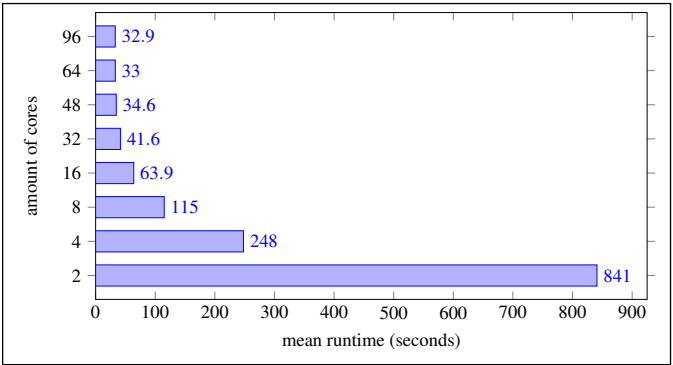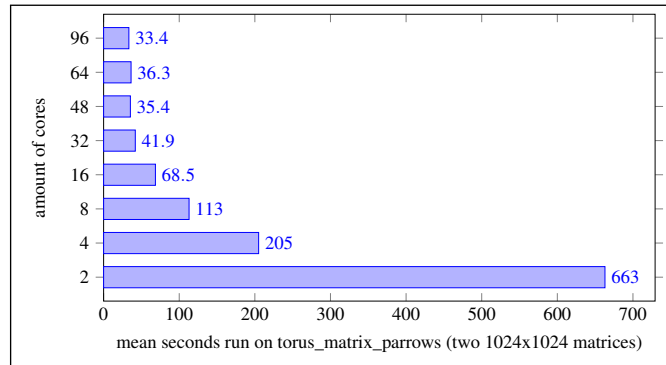Figure 38: Matrix Multiplication with a torus (Parrows)

Figure 39: Matrix Multiplication with a torus (Eden)

8 Benchmarks

9 Conclusion

Arrows are a generic concept that allows for powerful composition combinators. To our knowledge we are the first ones to represent parallel computation with arrows.**OL: that strange arrows-based robot interaction paper from 1993 or so! clearly discuss in related work**

Arrows turn out to be a useful tool for composing in parallel programs. We do not have to introduce new monadic types that wrap the computation. Instead use arrows just like regular sequential pure functions. This work features multiple parallel backends: the already available parallel Haskell flavours. Parallel Arrows feature an implementation of the *ArrowParallel* interface for Multicore Haskell, *Par* Monad, and Eden. With our approach parallel programs can be ported across these flavours with no effort. Performancewise, Parallel Arrows are on par with existing parallel Haskells, as they do not introduce any notable overhead.**OL: PROOFS. Many proofs in benchmarks!**

MB: ArrowApply (or equivalent) are needed because we basically want to be able to produce intermediary results, this is by definition of the parallel evaluation combinators

OL: Remove websites from citations, put them into footnotes!

OL: Parrows + accelerate = love? Metion port to Frege. Mention the Par monad troubles.

9.1 Future Work

Our PArrows interface can be expanded to futher parallel Haskells. More specifically we target HdpH (Maier *et al.*, 2014) a modern distributed Haskell that would benefit from our Arrows notation. Future-aware special versions of Arrow combinated can be extended and further improved. We would look into more transparency of the API, it should basically infuse as little overhead as possible.

Of course, definitions of further skeletons are viable and needed. We are looking into more experiences with seamless porting of parallel PArrow-based programs across the backends.

Accelerate (Chakravarty *et al.*, 2011) is not related to our approach. It would be interesting to see a hybrid of both APIs.

OL: replace API with DSL globally? Bibliography

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A Utility Functions

To be able to go into detail on parallel arrows, we introduce some utility combinators first, that will help us later: *map*, *foldl* and *zipWith* on arrows.

The *mapArr* combinator (Fig. A 1) lifts any arrow *arr a b* to an arrow *arr* [*a*] [*b*] (Hughes, 2005b): Similarly, we can also define *foldlArr* (Fig. A 2) that lifts any arrow *arr* (*b*, *a*) *b* with

```
mapArr :: ArrowChoice arr ⇒ arr a b → arr [a] [b]
mapArr f =
  arr listcase >>>
  arr (const [ ]) ||| (f *** mapArr f >>> arr (uncurry (:)))
  where listcase [ ] = Left ()
    listcase (x : xs) = Right (x, xs)
```

Figure A 1: *map* over arrows

a neutral element *b* to *arr* [*a*] *b*: **MB: pipepipepipe does not work with lhs2TeX** Finally,

```
foldlArr :: (ArrowChoice arr, ArrowApply arr) ⇒ arr (b, a) b → b → arr [a] b
foldlArr f b =
  arr listcase >>>
  arr (const b) |||
    (first (arr (λa → (b, a)) >>> f >>> arr (foldlArr f)) >>> app)
  where listcase [ ] = Left [ ]
    listcase (x : xs) = Right (x, xs)
```

Figure A 2: *foldl* over arrows

with the help of *mapArr* (Fig. A 1), we can define *zipWithArr* (Fig. A 3) that lifts any arrow *arr* (a, b) c to an arrow *arr* $([a], [b])$ $[c]$. These combinators make use of the *ArrowChoice*

$$zipWithArr :: ArrowChoice\ arr \Rightarrow arr\ (a, b)\ c \rightarrow arr\ ([a], [b])\ [c]$$
$$zipWithArr\ f = (arr\ \$\ \lambda\ (as, bs) \rightarrow zipWith\ (,)\ as\ bs) >\!\!>\!\!> mapArr\ f$$

Figure A 3: *zipWith* over arrows

type class which provides the ⫴ **OL: CHECK!** combinator. It takes two arrows *arr a c* and *arr b c* and combines them into a new arrow *arr* (*Either a b*) *c* which pipes all *Left a*'s to the first arrow and all *Right b*'s to the second arrow. **MB: pipepipepipe does not work with**

$$(\text{⫴}) :: ArrowChoice\ arr\ a\ c \rightarrow arr\ b\ c \rightarrow arr\ (Either\ a\ b)\ c$$

Figure A 4: Type signature of ⫴

lhs2TeXOL: trying direct one

With the zipWithArr combinator we can also write a combinator *listApp* (Fig. A 5), that lifts a list of arrows [*arr a b*] to an arrow *arr* [*a*] [*b*]. . Note that this additionally makes use

$$listApp :: (ArrowChoice\ arr, ArrowApply\ arr) \Rightarrow [arr\ a\ b] \rightarrow arr\ [a]\ [b]$$
$$listApp\ fs = (arr\ \$\ \lambda\ as \rightarrow (fs, as)) >\!\!>\!\!> zipWithArr\ app$$

Figure A 5: Definition of *listApp*

of the *ArrowApply* typeclass that allows us to evaluate arrows with *app* :: *arr* (*arr a b*, *a*) *c*.

A.1 Omitted Funtion Definitions

We have omitted some function definitions in the main text for brevity, and redeem this here. We warp Eden's build-in Futures in PArrows as in Figure A 6.

The full definition of *farmChunk* is in Figure A 7.

Eden definition of *ring* skeleton is in Figure A 8.

data *RemoteData a = RD* { *rd* :: *RD a* }
instance (*Trans a*) ⇒ *Future RemoteData a* **where**
  put = arr (λ*a* → *RD* { *rd = release a* })
  get = arr rd >>> *arr fetch*

Figure A 6: *RD*-based *RemoteData* version of *Future* for the Eden backend.

farmChunk :: (*ArrowParallel arr a b conf*, *ArrowParallel arr* [*a*] [*b*] *conf*, *ArrowChoice arr*, *ArrowApply arr*) ⇒
  conf → *ChunkSize* → *NumCores* → *arr a b* → *arr* [*a*] [*b*]
farmChunk conf chunkSize numCores f =
  unshuffle numCores >>>
  parEvalNLazy conf chunkSize (*repeat* (*mapArr f*)) >>>
  shuffle

Figure A 7: Definition of *farmChunk*.

ringSimple :: (*Trans i*, *Trans o*, *Trans r*) ⇒
    (*i* → *r* → (*o*, *r*)) → [*i*] → [*o*]
ringSimple f is = os
  where (*os*, *ringOuts*) = *unzip* (*parMap* (*toRD* $ *uncurry f*) (*zip is* $ *lazy ringIns*))
      ringIns = rightRotate ringOuts
toRD :: (*Trans i*, *Trans o*, *Trans r*) ⇒
      ((*i*, *r*) → (*o*, *r*)) → ((*i*, *RD r*) → (*o*, *RD r*))
toRD f (*i*, *ringIn*) = (*o*, *release ringOut*)
    where (*o*, *ringOut*) = *f* (*i*, *fetch ringIn*)
rightRotate :: [*a*] → [*a*]
rightRotate [ ] = [ ]
rightRotate xs = last xs : *init xs*
lazy :: [*a*] → [*a*]
lazy∼(*x* : *xs*) = *x* : *lazy xs*

Figure A 8: Eden's definition of the *ring* skeleton.