*Under consideration for publication in J. Functional Programming* 

# *Arrows for Parallel Computations*

MARTIN BRAUN
University Bayreuth, 95440 Bayreuth, Germany
PHIL TRINDER
Glasgow University, Glasgow, G12 8QQ, Scotland
and OLEG LOBACHEV
University Bayreuth, 95440 Bayreuth, Germany

**Abstract**

Arrows form a general interface for computation and pose therefore as an alternative to monads for API design. We express parallelism using this concept. This is a new way to represent parallel computation. We define an Arrows-based interface for parallelism and implement it using multiple parallel Haskells. In this manner we are able to bridge across various parallel Haskells with a common interface.

This new way of writing parallel programs has a benefit of being portable across flavours of parallel Haskells. Each parallel computation is an arrow, they can be composed and transformed as such. We thus introduce some syntactic sugar to provide parallelism-aware arrow combinators.

We also define several parallel skeletons with our framework. Benchmarks shows that our framework does not induce too much overhead performance-wise.

**Contents**

**1 Introduction** 2
**2 Background** 2
2.1 Short introduction to parallel Haskells 2
2.2 Arrows 5
**3 Related Work** 7
3.1 Parallel Haskells 7
3.2 Algorithmic skeletons 8
3.3 Arrows 8
3.4 Other languages 8
**4 Parallel Arrows** 9
4.1 The ArrowParallel typeclass 9
4.2 Multicore Haskell 10
4.3 ParMonad 10
4.4 Eden 11
4.5 Impact of parallel Arrows 11
4.6 Extending the Interface 12
4.7 Lazy parEvalN 12
4.8 Heterogenous tasks 12

4.9 Syntactic Sugar 13
**5 Futures** 13
**6 Map-based Skeletons** 15
6.1 Parallel map 15
6.2 Lazy parallel map 16
6.3 Statically load-balancing parallel map 16
6.4 farmChunk 17
6.5 parMapReduce 17
**7 Topological Skeletons** 17
7.1 Parallel pipe 18
7.2 Ring skeleton 19
7.3 Torus skeleton 21
**8 Benchmarks** 23
**9 Conclusion** 23
**A Utility Functions** 28

**Contents**

## 1 Introduction

blablabla arrows, parallel, haskell.

**Contribution** HIT HERE REALLY STRONG

**Structure** The remaining text is structures as follows. Section 2 briefly introduces known parallel Haskell flavours. and gives an overview of Arrows to the reader (Sec. 2.2). Section 3 discusses related work. Section 4 defines Parallel Arrows and presents a basic interface. Section 5 defines futures for Parallel Arrows, this concept enables better communication. Section 6 presents some basic algorithmic skeletons (parallel **map** with and without load balancing, **map**−`reduce`) in our newly defined dialect. More advanced ones are showcased in Section 7 (`pipe`, `ring`, `torus`). Section 8 shows the benchmark results. Section 9 discusses future work and concludes.

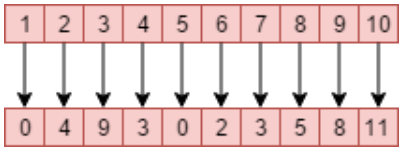
## 2 Background

### *2.1 Short introduction to parallel Haskells*

There are already several ways to write parallel programs in Haskell. As we will base our parallel arrows on existing parallel Haskells, we will now give a short introduction to the ones we use as backends in this paper.

In its purest form, parallel computation (on functions) can be looked at as the execution of some functions `a` $\rightarrow$ `b` in parallel:

```
parEvalN :: [a → b] → [a] → [b]
```

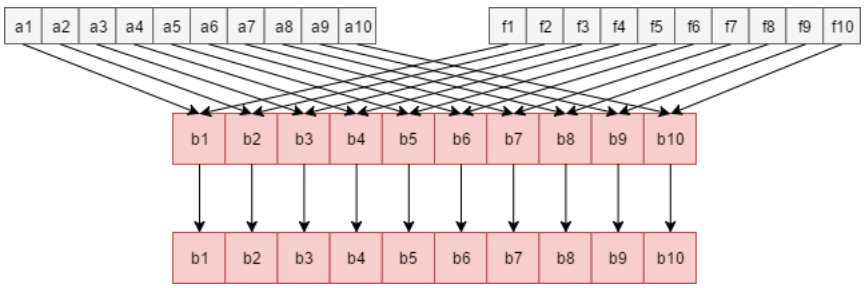



Before we go into detail on how we can use this idea of parallelism for parallel Arrows, as a short introduction to parallelism in Haskell we will now implement parEvalN with several different parallel Haskells.

### *2.1.1 Multicore Haskell*

Multicore Haskell (Marlow *et al.*, 2009; Trinder *et al.*, 1999) is way to do parallel processing found in standard GHC.[1] It ships with parallel evaluation strategies (Trinder *et al.*, 1998; Marlow *et al.*, 2010) for several types which can be applied with `using :: a → Strategy a → a`. For parEvalN this means that we can just apply the list of functions `[a → b]` to the list of inputs `[a]` by zipping them with the application operator `$`. We then evaluate this lazy list `[b]` according to a `Strategy [b]` with the `using :: a → Strategy a → a` operator. We construct this strategy with `parList :: Strategy a → Strategy [a]` and `rdeepseq :: NFData a ⇒Strategy a` where the latter is a strategy which evalutes to normal form. To ensure that programs that use parEvalN have the correct evaluation order, we annotate the computation with `pseq :: a → b → b` which forces the compiler to not reorder multiple parEvalN computations. This is particularly necessary in circular communication topologies like in the torus or ring skeleton that we will see in chapter 7 which resulted in deadlock scenarios when executed without pseq during testing for this paper.

```
parEvalN :: (NFData b) ⇒[a → b] → [a] → [b]
parEvalN fs as = let bs = zipWith ($) fs as in
  (bs `using` parList rdeepseq) `pseq` bs
```




[1] Multicore Haskell on Hackage is available under `https://hackage.haskell.org/package/parallel-3.2.1.0`, compiler support is integrated in the stock GHC.

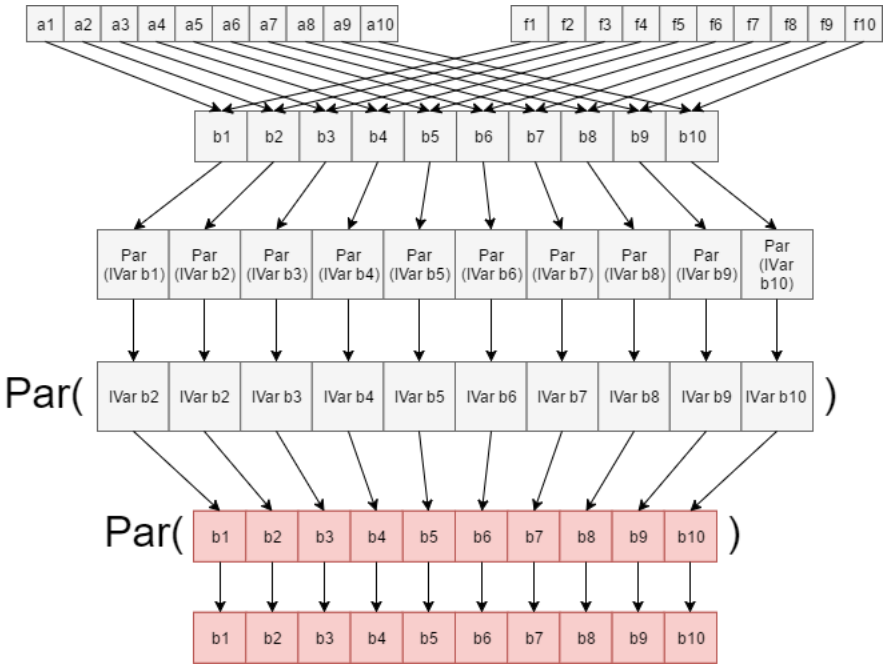
### 2.1.2 ParMonad

The Par monad[2] introduced by Marlow *et al.* (2011a), is a monad designed for composition of parallel programs.

Our parallel evaluation function parEvalN can be defined by zipping the list of [a → b] with the list of inputs [a] with the application operator $ just like with Multicore Haskell. Then, we map over this not yet evaluated lazy list of results [b] with spawnP :: NFData a ⇒a → Par (IVar a) to transform them to a list of not yet evaluated forked away computations [Par (IVar b)], which we convert to Par [IVar b] with sequenceA. We wait for the computations to finish by mapping over the IVar b's inside the Par monad with get. This results in Par [b]. We finally execute this process with runPar to finally get [b] again.

**explain problems with laziness here. Problems with torus**

```
1 parEvalN :: (NFData b) ⇒[a → b] → [a] → [b]
2 parEvalN fs as =runPar $
3   (sequenceA $ map (spawnP) $ zipWith ($) fs as) >>= mapM get
```




### 2.1.3 Eden

Eden (Loogen *et al.*, 2005; Loogen, 2012) is a parallel Haskell for distributed memory and comes with a MPI and a PVM backends.[3] This means that it works on clusters as well so it makes sense to have a Eden-based backend for our new parallel Haskell flavour.

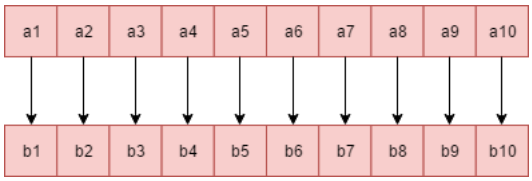
[2] It can be found in the `monad-par` package on hackage under `https://hackage.haskell.org/package/monad-par-0.3.4.8/`.

[3] See also `http://www.mathematik.uni-marburg.de/~eden/` and `https://hackage.haskell.org/package/edenmodules-1.2.0.0/`.

Eden was designed to work on clusters, but with a further simple backend it operates on multicores. However, in contrast to many other parallel Haskells, in Eden each process has its own heap. This seems to be a waste of memory, but with distributed programming paradigm and individual GC per process, Eden yields good performance results also on multicores (Berthold *et al.*, 2009a; Aswad *et al.*, 2009).

While Eden also comes with a monad `PA` for parallel evaluation, it also ships with a completely functional interface that includes a `spawnF` function that allows us to define `parEvalN` directly:

```
parEvalN :: (Trans a, Trans b) ⇒ [a → b] → [a] → [b]
parEvalN =spawnF
```




**Eden TraceViewer.** To comprehend the efficiency and the lack thereof in a parallel program, an inspection of its execution is extremely helpful. While some large-scale solutions exist (Geimer *et al.*, 2010), the parallel Haskell community mainly utilises the tools Threadscope (Wheeler & Thain, 2009) and Eden TraceViewer[4] (Berthold & Loogen, 2007). In the next sections we will present some *traces*, the post-mortem process diagrams of Eden processes and their activity.

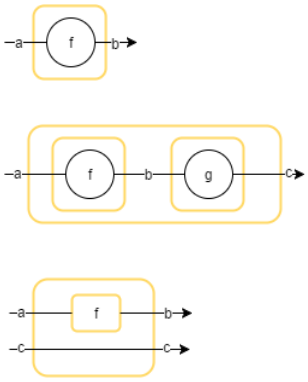
In a trace, the *x* axis shows the time, the *y* axis enumerates the machines and processes. A trace shows a running process in green, a blocked process is red. If the process is 'runnable', i.e. it may run, but does not, it is yellow. The typical reason for then is GC. An inactive machine where no processes are started yet, or all are already terminated, is shows as a blue bar. A comminication from one process to another is represented with a black arrow. A stream of communications, e.g. a transmitted list is shows as a dark shading between sender and receiver processes.

### *2.2 Arrows*


Arrows were introduced by Hughes (2000) as a general interface for computation. An arrow `arr a b` represents a computation that converts an input `a` to an output `b`. This is defined in the arrow typeclass:

[4] See ..... on hackage for the last available version of Eden TraceViewer. There was an effort to implement the TraceViewer using modern web technologies (**?**).

```
class Arrow arr where
arr :: (a → b) → arr a b



(>>> ) :: arr a b → arr b c → arr a c



first  :: arr a b → arr (a,c) (b,c)
```




arr is used to lift an ordinary function to an arrow type, similarly to the monadic return. The >>> operator is analogous to the monadic composition >>= and combines two arrows arr a b and arr b c by "wiring" the outputs of the first to the inputs to the second to get a new arrow arr a c. Lastly, the first operator takes the input arrow from b to c and converts it into an arrow on pairs with the second argument untouched. It allows us to to save input across arrows.

The most prominent instances of this interface are regular functions (→),

```
instance Arrow (→) where
  arr f = f
  f >>> g = g ∘ f
  first f = λ(a, c) → (f a, c)
```

and the Kleisli type

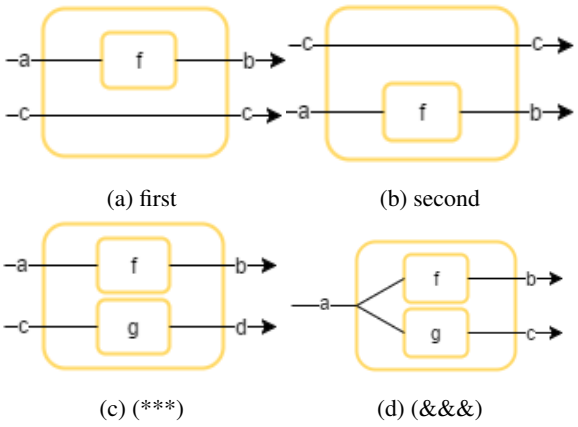
```
data Kleisli m a b = Kleisli { run :: a → m b }

instance Monad m ⇒Arrow (Kleisli m) where
  arr f = Kleisli $ return ∘ f
  f >>> g = Kleisli $ λa → f a >>= g
  first f = Kleisli $ λ(a,c) → f a >>= λb → return (b,c)
```

With this typeclass in place, Hughes also defined some syntactic sugar: The mirrored version of first , called second,

```
second :: Arrow arr ⇒ arr a b → arr (c, a) (c, b)
second f = arr swap >>> first f >>> arr swap
  where swap (x, y) = (y, x)
```

the *** combinator which combines first and second to handle two inputs in one arrow,

```
(***) :: Arrow arr ⇒ arr a b → arr c d → arr (a, c) (b, d)
f *** g = first f >>> second g
```




(a) first (b) second

(c) (***) (d) (&&&)

Figure 1: Syntactic sugar for arrows.

and the &&& combinator that constructs an arrow which outputs two different values like ***, but takes only one input.

```
(&&&) :: Arrow arr ⇒ arr a b → arr a c → a a (b, c)
f &&& g =arr (λa →(a, a)) >>> (f *** g)
```

A short example given by Hughes on how to use this is add over arrows:

```
add :: Arrow arr ⇒ arr a Int → arr a Int → arr a Int
add f g =(f &&& g) >>> arr (λ(u, v) →u +v)
```

The more restrictive interface of arrows (a monad can be *anything*, an arrow is a process of doing something, a *computation*) allows for more elaborate composition and transformation combinators. One of the major problems in parallel computing is composition of parallel processes.

## 3 Related Work

### *3.1 Parallel Haskells*

Of course, the three parallel Haskell flavours we have presented above: the GpH (Trinder *et al.*, 1998, 1999) parallel Haskell dialect and its multicore version (Marlow *et al.*, 2009), the Par monad (Marlow *et al.*, 2011b; Foltzer *et al.*, 2012), and Eden (Loogen *et al.*, 2005; Loogen, 2012) are related to this work. We use these languages as backends: our library can switch from one to other at user's command.

HdpH (Maier *et al.*, 2014; Stewart *et al.*, 2016) is an extension of Par monad to heterogeneous clusters. LVish (Kuper *et al.*, 2014) is a communication-centred extension of Par monad. Further parallel Haskell approaches include pH (Nikhil & Arvind, 2001), research work done on distributed variants of GpH (Trinder *et al.*, 1996; Aljabri *et al.*, 2014, 2015) and low-level Eden implementation (Berthold, 2008; Berthold *et al.*, 2016). Skeleton composition (Dieterle *et al.*, 2016), communication (Dieterle *et al.*, 2010a), and generation of process networks (Horstmeyer & Loogen, 2013) are recent in-focus research topics in

Eden. This also includes the definitions of new skeletons (Hammond *et al.*, 2003; Berthold & Loogen, 2006; Berthold *et al.*, 2009b,c; Brown & Hammond, 2010; Dieterle *et al.*, 2010b; de la Encina *et al.*, 2011; Dieterle *et al.*, 2013).

More different approaches include data parallelism (Chakravarty *et al.*, 2007; Keller *et al.*, 2010; , n.d.), GPU-based approaches (Mainland & Morrisett, 2010; Svensson, 2011), software transactional memory (Harris *et al.*, 2005; Perfumo *et al.*, 2008). The Haskell–GPU bridge Accelerate (Chakravarty *et al.*, 2011; Clifton-Everest *et al.*, 2014; McDonell *et al.*, 2015) deserves a special mention. Accelerate is completely orthogonal to our approach. Marlow authored a recent book 2013 on parallel Haskells.

### *3.2 Algorithmic skeletons*

Algorithmic skeletons were introduced by Cole (1989). Early efforts include (Darlington *et al.*, 1993; Botorog & Kuchen, 1996; Danelutto *et al.*, 1997; Gorlatch, 1998; Lengauer *et al.*, 1997). Rabhi & Gorlatch (2003) consolidated early reports on high-level programming approaches. The effort is ongoing, including topological skeletons (Berthold & Loogen, 2006), special-purpose skeletons for computer algebra (Berthold *et al.*, 2009c; Brown & Hammond, 2010; Lobachev, 2011, 2012), iteration skeletons (Dieterle *et al.*, 2013). The idea of Linton *et al.* (2010) is to use a parallel Haskell to orchestrate further software systems to run in parallel. Dieterle *et al.* (2016) compare the composition of skeleton to stable process networks.

### *3.3 Arrows*

Arrows were introduced by Hughes (2000), basically they are a generalised function arrow $\rightarrow$. Hughes (2005a) is a tutorial on arrows. Some theoretical details on arrows (Jacobs *et al.*, 2009; Lindley *et al.*, 2011; Atkey, 2011) are viable. Paterson (2001) introduced a new notation for arrows. Arrows have applications in information flow research (Li & Zdancewic, 2006, 2010; Russo *et al.*, 2008), invertible programming (Alimarine *et al.*, 2005), and quantum computer simulation (Vizzotto *et al.*, 2006). But perhaps most prominent application of arrows is functional reactive programming (Hudak *et al.*, 2003).

Liu *et al.* (2009) formally define a more special kind of arrows that capsule the computation more than regular arrows do and thus enable optimizations. Their approach would allow parallel composition, as their special arrows would not interfere with each other in concurrent execution. In a contrast, we capture a whole parallel computation as a single entity: our main instantiation function `parEvalN` makes a single (parallel) arrow out of list of arrows. Huang *et al.* (2007) utilise arrows for parallelism, but strikingly different from our approach. They basically use arrows to orchestrate several tasks in robotics. We propose a general interface for parallel programming, remaining completely in Haskell.

### *3.4 Other languages*

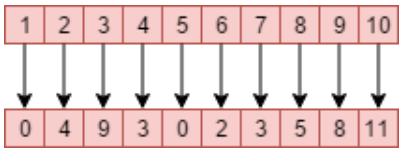
Although this work is centred on Haskell implementation of arrows, it is applicable to any functional programming language where parallel evaluation and arrows can be defined. Our experiments with our approach in Frege language (which is basically Haskell on the JVM)

were quite successful, we were able to use typical Java libraries for parallelism. However, it is beyond the scope of this work.

Achten *et al.* (2004, 2007) use an arrow implementation in Clean for better handling of typical GUI tasks. Dagand *et al.* (2009) used arrows in OCaml in the implementation of a distributed system.

## 4 Parallel Arrows

We have seen what Arrows are and how they can be used as a general interface to computation. In the following section we will discuss how Arrows constitute a general interface not only to computation, but to **parallel computation** as well. We start by introducing the interface and explaining the reasonings behind it. Then, we discuss some implementations using exisiting parallel Haskells. Finally, we explain why using Arrows for expressing parallelism is beneficial.




### *4.1 The ArrowParallel typeclass*

As we have seen earlier, in its purest form, parallel computation (on functions) can be seen as the execution of some functions `a → b` in parallel:

```
parEvalN :: [a → b] → [a] → [b]
```

Translating this into arrow terms gives us a new operator `parEvalN` that lifts a list of arrows `[arr a b]` to a parallel arrow `arr [a] [b]` (This combinator is similar to our utility function `listApp`, but does parallel instead of serial evaluation).

```
parEvalN :: (Arrow arr) ⇒ [arr a b] → arr [a] [b]
```

With this definition of `parEvalN`, parallel execution is yet another arrow combinator. But as the implementation may differ depending on the actual type of the arrow `arr` and we want this to be an interface for different backends, we have to introduce the new typeclass `ArrowParallel` to host this combinator.

```
class Arrow arr ⇒ ArrowParallel arr a b where
  parEvalN :: [arr a b] → arr [a] [b]
```

Sometimes parallel Haskells require or allow for additional configuration parameters, e. g. an information about the execution environment or the level of evaluation (weak-head normalform vs. normalform). For this reason we also introduce an additional `conf` parameter to the function. We also do not want `conf` to be a fixed type, as the configuration parameters can differ for different instances of `ArrowParallel`. So we add it to the type signature of the typeclass as well.

```
class Arrow arr ⇒ ArrowParallel arr a b conf where
  parEvalN :: conf → [arr a b] → arr [a] [b]
```

Note that we don't require the conf parameter in every implementation. If it is not needed, we usually just default the conf type parameter to () and even blank it out in the parameter list of the implemented parEvalN, as we will see in the implementation of the Multicore and the ParMonad backend.

### 4.2 Multicore Haskell

The Multicore Haskell implementation of this class is implemented in a straightforward manner by using listApp from chapter A combined with the withStrategy :: Strategy a → a → a and pseq :: a → b → b combinators from Multicore Haskell, where withStrategy is the same as using :: a → Strategy a → a but with flipped parameters.

```
instance (NFData b, ArrowApply arr, ArrowChoice arr) ⇒
  ArrowParallel arr a b () where
    parEvalN _ fs =
        listApp fs >>>
        arr (withStrategy (parList rdeepseq)) &&& arr id >>>
        arr (uncurry pseq)
```

For most cases this fully evaluating version would probably suffice, but as the Multicore Haskell interface allows the user to specify the level of evaluation to be done via the Strategy interface, we want to the user not to lose this ability because of using our API. We therefore introduce the Conf a data-type that simply wraps a Strategy a. We can't directly use the Strategy a type here as GHC (at least in the versions used for development in this paper) does not allow type synonyms in type class instances.

```
data Conf a = Conf (Strategy a)
```

We simply unwrap the strategy out of this wrapper and get the following (configurable) ArrowParallel instance:

```
instance (NFData b, ArrowApply arr, ArrowChoice arr) ⇒
  ArrowParallel arr a b (Conf b) where
    parEvalN (Conf strat) fs =
        listApp fs >>>
        arr (withStrategy (parList strat)) &&& arr id >>>
        arr (uncurry pseq)
```

### 4.3 ParMonad

The ParMonad implementation makes use of Haskells laziness and ParMonad's spawnP :: NFData a ⇒ a → Par (IVar function. The latter forks away the computation of a value and returns an IVar containing the result in the Par monad.

We therefore apply each function to its corresponding input value with app and then fork the computation away with arr spawnP inside a zipWithArr call. This yields a list

[Par (IVar b)], which we then convert into Par [IVar b] with arr sequenceA. In order to wait for the computation to finish, we map over the IVars inside the ParMonad with arr (>>= **mapM** get). The result of this operation is a Par [b] from which we can finally remove the monad again by running arr runPar to get our output of [b].

```
instance (NFData b, ArrowApply arr, ArrowChoice arr) ⇒
  ArrowParallel arr a b conf where
    parEvalN _ fs =
      (arr $ λas → (fs, as)) >>>
      zipWithArr (app >>> arr spawnP) >>>
      arr sequenceA >>>
      arr (>>= mapM get) >>>
      arr runPar
```

### *4.4 Eden*

For the Multicore and ParMonad implementation we could use general instances of ArrowParallel that just require the ArrowApply and ArrowChoice typeclasses. With Eden this is not the case as we can only spawn a list of functions and we cannot extract simple functions out of arrows. While we could still manage to have only one class in the module by introducing a typeclass like

```
class (Arrow arr) ⇒ ArrowUnwrap arr where
  arr a b → (a → b)
```

we don't do it here, for aesthetic resons. For now, we just implement ArrowParallel for normal functions

```
instance (Trans a, Trans b) ⇒ ArrowParallel (→) a b conf where
parEvalN _ fs as =spawnF fs as
```

and the Kleisli type.

```
instance (Monad m, Trans a, Trans b, Trans (m b)) ⇒
  ArrowParallel (Kleisli m) a b conf where
parEvalN conf fs =
  (arr $ parEvalN conf (map (λ(Kleisli f) → f) fs)) >>>
  (Kleisli $ sequence)
```

### *4.5 Impact of parallel Arrows*

We have seen that we can wrap parallel Haskells inside of the ArrowParallel interface, but why do we abstract parallelism this way and what does this approach do better than the other parallel Haskells?

- **Arrow API benefits**: With the ArrowParallel typeclass we do not lose any benefits of using arrows as parEvalN is just yet another arrow combinator. The resulting arrow can be used in the same way a potential serial version could be used. This is a big advantage of this approach, especially compared to the monad solutions as we do

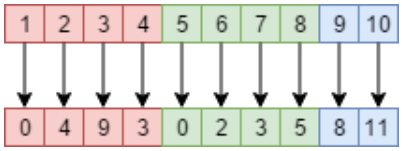
not introduce any new types. We can just 'plug' in parallel parts into our sequential programs without having to change anything.

- **Abstraction**: With the ArrowParallel typeclass, we abstracted all parallel implementation logic away from the business logic. This gives us the beautiful situation of being able to write our code against the interface the typeclass gives us without being bound to any parallel Haskell. So as an example, during development, we can run the code on the simple Multicore version and afterwards deploy it on a cluster by converting it into an Eden version, by just replacing the actual ArrowParallel instance.

### 4.6 Extending the Interface

With the ArrowParallel typeclass in place and implemented, we can now implement some further basic parallel interface functions. These are algorithmic skeletons that, however, mostly serve as a foundation to further, more specific algorithmic skeletons.

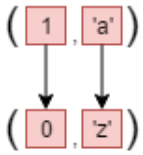
### 4.7 Lazy parEvalN




The function parEvalN is 100% strict, which means that it fully evaluates all passed arrows. Sometimes this might not be feasible, as it will not work on infinite lists of functions like e.g. **map** (arr ∘ (+)) [1..] or just because we need the arrows evaluated in chunks. parEvalNLazy fixes this. It works by first chunking the input from [a] to [[a]] with the given ChunkSize in arr (chunksOf chunkSize). These chunks are then fed into a list [arr [a] [b]] of parallel arrows created by feeding chunks of the passed ChunkSize into the regular parEvalN by using listApp. The resulting [[b]] is lastly converted into [b] with arr **concat**.

```
parEvalNLazy :: (ArrowParallel arr a b conf, ArrowChoice arr, ArrowApply arr) ⇒
  conf → ChunkSize → [arr a b] → (arr [a] [b])
parEvalNLazy conf chunkSize fs =
  arr (chunksOf chunkSize) >>>
  listApp fchunks >>>
  arr concat
  where fchunks = map (parEvalN conf) $ chunksOf chunkSize fs
```

### 4.8 Heterogenous tasks



We have only talked about the paralellization arrows of the same type until now. But sometimes we want to paralellize heterogenous types as well. However, we can implement such a parEval2 combinator which combines two arrows `arr a b` and `arr c d` into a new parallel arrow `arr (a, c) (b, d)` quite easily with the help of the ArrowChoice typeclass. The idea is to use the `+++` combinator which combines two arrows `arr a b` and `arr c d` and transforms them into `arr (Either a c) (Either b d)` to get a common arrow type that we can then feed into parEvalN.

We start by transforming the `(a, c)` input into a 2-element list `[Either a c]` by first tagging the two inputs with **Left** and **Right** and wrapping the right element in a singleton list with `return` so that we can combine them with `arr (uncurry (:))`. Next, we feed this list into a parallel arrow running on 2 instances of `f +++g` as described above. After the calculation is finished we convert the resulting `[Either b d]` into `([b], [d])` with `arr partitionEithers`. The two lists in this tuple contain only 1 element each by construction, so we can finally just convert the tuple to `(b, d)` in the last step.

```
parEval2 :: (ArrowChoice arr,
  ArrowParallel arr (Either a c) (Either b d) conf) ⇒
  conf → arr a b → arr c d → arr (a, c) (b, d)
parEval2 conf f g =
  arr Left *** (arr Right >>> arr return) >>>
  arr (uncurry (:)) >>>
  parEvalN conf (replicate 2 (f +++g)) >>>
  arr partitionEithers >>>
  arr head *** arr head
```

### *4.9 Syntactic Sugar*

For basic arrows, we have the `***` combinator which allows us to combine two arrows `arr a b` and `arr c d` into an arrow `arr (a, c) (b, d)` which does both computations at once. This can easily be translated into a parallel version with parEval2, but for this we require a backend which has an implementation that does not require any configuration (hence the `()` as the conf parameter in the following code snippet).

```
(|***|) :: (ArrowChoice arr, ArrowParallel arr (Either a c) (Either b d) ()) ⇒
  arr a b → arr c d → arr (a, c) (b, d)
(|***|) = parEval2 ()
```

We define the parallel `|&&&|` in a similar manner to its sequential prototype.

```
(|&&&|) :: (ArrowChoice arr, ArrowParallel arr (Either a a) (Either b c) ()) ⇒
  arr a b → arr a c → arr a (b, c)
(|&&&|) f g =(arr $ λa → (a, a)) >>> f |***| g
```

## 5 Futures

Consider the following parallel arrow combinator:

```
someCombinator :: (Arrow arr) ⇒ [arr a b] → [arr b c] → arr [a] [c]
someCombinator fs1 fs2 = parEvalN () fs1 >>> rightRotate >>> parEvalN () fs2
```

In a distributed environment, the resulting arrow of this combinator first evaluates all [arr a b] in parallel, sends the results back to the master node, rotates the input once and then evaluates the [arr b c] in parallel to then gather the input once again on the master node. Such situations arise, e. g. in scientific computations when the data distributed across the nodes needs to be transposed. A concrete example is 2D FFT computation (Gorlatch & Bischof, 1998; Berthold *et al.*, 2009c).

While this could be rewritten into only one parEvalN call by directly wiring the arrows properly together, this example illustrates an important problem: When using a ArrowParallel backend that resides on multiple computers, all communication between the nodes is done via the master node, as shown in the Eden trace in Figure 2. This can become a serious bottleneck for larger amount of data and number of processes (showcases Berthold *et al.*, 2009c, as, e. g. ). This motivates for an approach that allows the nodes to communicate

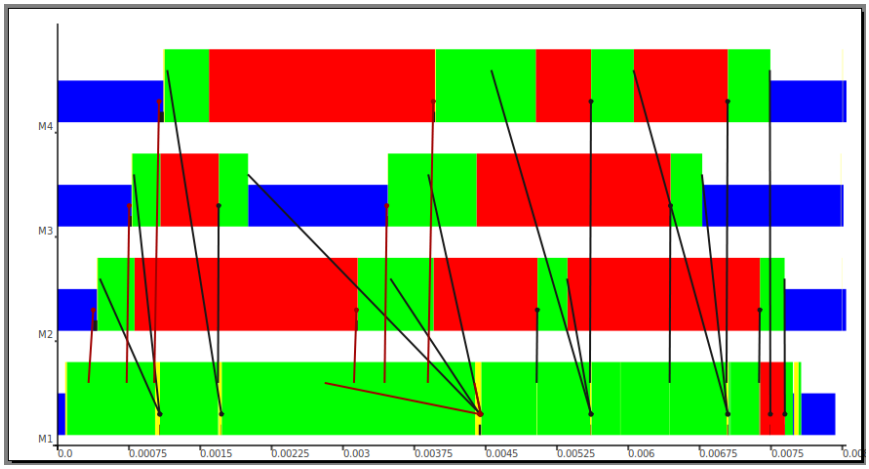



Figure 2: Communication between 4 threads without Futures

directly with each other. Thankfully, Eden, the distributed parallel Haskell we have used in this paper so far, already ships with the concept of RD (remote data) that enables this behaviour (Alt & Gorlatch, 2003; Dieterle *et al.*, 2010a). But as we want code written against our API to be implementation agnostic, we have to wrap this context. We do this with the Future typeclass:

```
class Future fut a | a → fut where
    put :: (Arrow arr) ⇒ arr a (fut a)
    get :: (Arrow arr) ⇒ arr (fut a) a
```

As RD is only type synonym for communication type that Eden uses internally, we have to use some wrapper classes to fit that definition, though:

```
data RemoteData a =RD { rd :: RD a }

```

```
instance (Trans a) ⇒ Future RemoteData a where
    put = arr (λa → RD { rd = release a })
    get = arr rd >>> arr fetch
```

For ParMonad and Multicore we can simply use MVars because we have shared memory in a single node:

```
{−# NOINLINE putUnsafe #−}
putUnsafe :: a → MVar a
putUnsafe a = unsafePerformIO $ do
    mVar ∈ newEmptyMVar
    putMVar mVar a
    return mVar

instance (NFData a) ⇒Future MVar a where
    put = arr putUnsafe
    get = arr takeMVar >>> arr unsafePerformIO
```

To fit the ArrowParallel instances we gave earlier, we also have to give the necessary NFData and Trans instances - the latter only being needed in Eden. We need this implementation for our RemoteData wrapper,

```
instance NFData (RemoteData a) where
    rnf = rnf ∘ rd
instance Trans (RemoteData a)
```

while MVar already has a suitable NFData implementation.

Going back to our communication example we can use this Future concept in order to enable direct communications between the nodes in the following way:

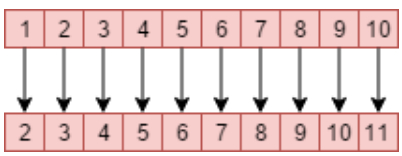
```
someCombinator :: (Arrow arr) ⇒ [arr a b] → [arr b c] → arr [a] [c]
someCombinator fs1 fs2 =
  parEvalN () (map (>>> put) fs1) >>>
  rightRotate >>>
  parEvalN () (map (get >>> ) fs2)
```

In a distributed environment, this gives us a communication scheme with messages going through the master node only if it is needed - similar to what is shown in the trace in Fig. 3.

## 6 Map-based Skeletons

Now we have developed Parallel Arrows far enough to define some algorithmic skeletons useful to an application programmer.

### *6.1 Parallel map*



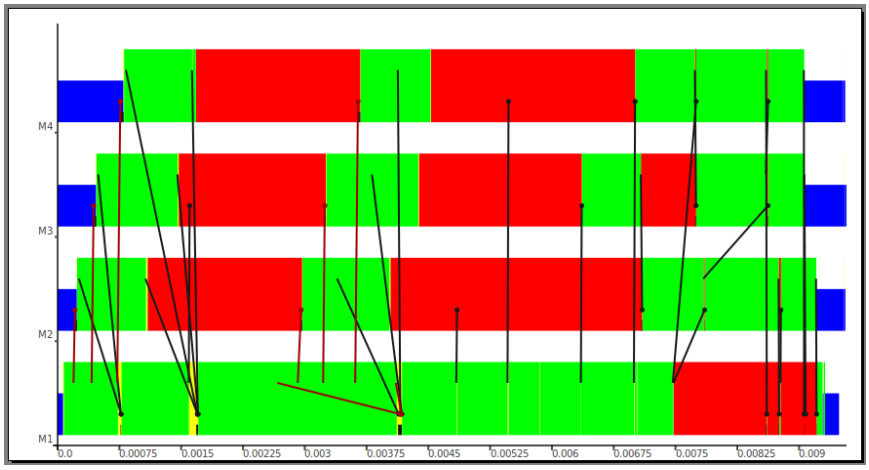
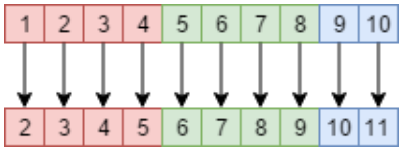



Figure 3: Communication between 4 threads with Futures

parMap is probably the most common skeleton for parallel programs. We can implement it with ArrowParallel by repeating an arrow arr a b and then passing it into parEvalN to get an arrow arr [a] [b]. Just like parEvalN, parMap is 100 % strict.

```
parMap :: (ArrowParallel arr a b conf) ⇒
  conf → (arr a b) → (arr [a] [b])
parMap conf f = parEvalN conf (repeat f)
```

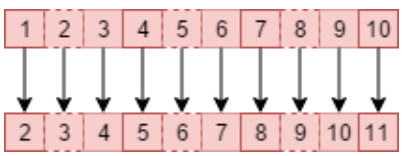
### 6.2 Lazy parallel map




As parMap is 100% strict it has the same restrictions as parEvalN compared to parEvalNLazy. So it makes sense to also have a parMapStream which behaves like parMap, but uses parEvalNLazy instead of parEvalN.

```
parMapStream :: (ArrowParallel arr a b conf, ArrowChoice arr, ArrowApply arr) ⇒
  conf → ChunkSize → arr a b → arr [a] [b]
parMapStream conf chunkSize f = parEvalNLazy conf chunkSize (repeat f)
```

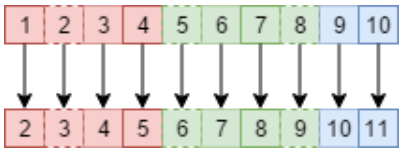
### 6.3 Statically load-balancing parallel map



parMap spawns every single computation in a new thread (at least for the instances of ArrowParallel we gave in this paper). This can be quite wasteful and a farm that equally distributes the workload over numCores workers (if numCores is greater than actualProcessorCount, the fastest processor(s) to finish will get more tasks) seems useful.

```
farm :: (ArrowParallel arr a b conf,
  ArrowParallel arr [a] [b] conf, ArrowChoice arr) ⇒
  conf → NumCores →arr a b → arr [a] [b]
farm conf numCores f =
  unshuffle numCores >>>
  parEvalN conf (repeat (mapArr f)) >>>
  shuffle
```

### *6.4 farmChunk*




As farm is basically just parMap with a different work distribution, it is, again, 100% strict. So we define farmChunk which uses parEvalNLazy instead of parEvalN like this:

```
farmChunk :: (ArrowParallel arr a b conf, ArrowParallel arr [a] [b] conf,
  ArrowChoice arr, ArrowApply arr) ⇒
  conf → ChunkSize → NumCores →arr a b → arr [a] [b]
farmChunk conf chunkSize numCores f =
  unshuffle numCores >>>
  parEvalNLazy conf chunkSize (repeat (mapArr f)) >>>
  shuffle
```

### *6.5 parMapReduce*

– this does not completely adhere to Google's definition of Map Reduce as it – the mapping function does not allow for "reordering" of the output – The original Google version can be found at https://de.wikipedia.org/wiki/MapReduce

```
parMapReduceDirect :: (ArrowParallel arr [a] b conf,
  ArrowApply arr, ArrowChoice arr) ⇒
  conf → ChunkSize → arr a b → arr (b, b) b → b → arr [a] b
parMapReduceDirect conf chunkSize mapfn foldfn neutral =
  arr (chunksOf chunkSize) >>>
  parMap conf (mapArr mapfn >>> foldlArr foldfn neutral) >>>
  foldlArr foldfn neutral
```

## 7 Topological Skeletons

Even though many algorithms can be expressed by parallel maps, some problems require more sophisticated skeletons. The Eden library leverages this problem and already comes

with more predefined skeletons, among them a pipe, a ring and a torus implementation (Loogen, 2012; ede, n.d.b). These seem like reasonable candidates to be ported to our arrow based parallel Haskell to showcase that we can express such skeletons with Parallel Arrows as well.

### 7.1 Parallel pipe

The parallel pipe skeleton is semantically equivalent to folding over a list [arr a a] of arrows with >>> , but does this in parallel, meaning that the arrows do not have to reside on the same thread/machine. We implement this skeleton using the ArrowLoop typeclass which gives us the loop :: arr (a, b) (c, b) → arr a c combinator which allows us to express loop like computations. For example this

```
loop (arr (λ(a, b) → (b, a:b)))
```

,which is the same as

```
loop (arr snd &&& arr (uncurry (:)))
```

defines an arrow that takes its input a and converts it into an infinite stream [a] of it. Using this to our advantage gives us a first draft of a pipe implementation by plugging in the parallel evaluation call parEvalN conf fs inside the second argument of &&& and then only picking the first element of the resulting list with arr **last**:

```
pipeSimple :: (ArrowLoop arr, ArrowParallel arr a a conf) ⇒
  conf → [arr a a] → arr a a
pipeSimple conf fs =
  loop (arr snd &&&
    (arr (uncurry (:) >>> lazy) >>> parEvalN conf fs)) >>>
  arr last
```

where lazy is defined as:

```
lazy :: (Arrow arr) ⇒ arr [a] [a]
lazy = arr (λ ~(x:xs) → x : lazy xs)
```

Note that here the use of lazy is essential as without it programs using this definition would never halt. We need to enforce that the evaluation of the input [a] terminates before passing it into parEvalN.

However, using this definition directly, will make the master node a potential bottleneck in distributed environments as described in Section 5. Therefore, we introduce a more sophisticated version that internally uses Futures.

```
pipe :: (ArrowLoop arr, ArrowParallel arr (fut a) (fut a) conf,
  Future fut a) ⇒
  conf → [arr a a] → arr a a
pipe conf fs = unliftFut (pipeSimple conf (map liftFut fs))
```

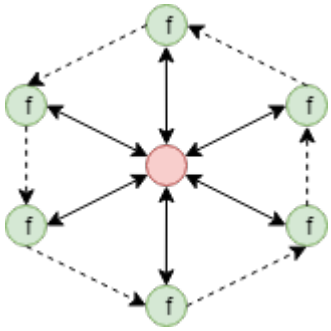
Sometimes, this pipe definition can be a bit inconvenient, especially if we want to pipe arrows of mixed types together, i.e. arr a b and arr b c. By wrapping these two arrows inside a common type we obtain

```
pipe2 :: (ArrowLoop arr, ArrowChoice arr,
  ArrowParallel arr (fut (([a], [b]), [c])) (fut (([a], [b]), [c])) conf,
  Future fut (([a], [b]), [c])) ⇒
  conf → arr a b → arr b c → arr a c
pipe2 conf f g =
  (arr return &&& arr (const [])) &&& arr (const []) >>>
  pipe conf (replicate 2 (unify f g)) >>>
  arr snd >>>
  arr head
    where
      unify :: (ArrowChoice arr) ⇒
        arr a b → arr b c → arr (([a], [b]), [c]) (([a], [b]), [c])
      unify f g =
        (mapArr f *** mapArr g) *** arr (λ_ → []) >>>
        arr (λ((a, b), c) → ((c, a), b))
```

Note that extensive use of this combinator over pipe with a hand-written combination data-type will probably result in worse performance because of more communication overhead from the many calls to parEvalN. Nonetheless, we can define a parallel piping operator |>>>| which is semantically equivalent to >>> in a similar manner to the other parallel syntactic sugar from Section 4.9:

```
(|>>>|) :: (ArrowLoop arr, ArrowChoice arr,
  ArrowParallel arr (fut (([a], [b]), [c])) (fut (([a], [b]), [c])) (),
  Future fut (([a], [b]), [c])) ⇒
  arr a b → arr b c → arr a c
(|>>>|) =pipe2 ()
```

### *7.2 Ring skeleton*




Eden comes with a ring skeleton implementation that allows the computation of a function [i] → [o] with a ring of nodes that communicate in a ring topology with each other. Its input is a node function i → r → (o, r) in which r serves as the intermediary output that gets send to the neighbour of each node. This data is sent over direct communication channels (remote data). They define it as (ede, n.d.b)

```
ringSimple :: (Trans i, Trans o, Trans r) ⇒
  (i → r → (o,r))
```

```
    → [i] → [o]
ringSimple f is = os
  where (os,ringOuts) = unzip (parMap (toRD $ uncurry f)
                                      (zip is $ lazy ringIns))
        ringIns = rightRotate ringOuts

toRD :: (Trans i, Trans o, Trans r) ⇒
        ((i,r) → (o,r))
        → ((i, RD r) → (o, RD r))
toRD f (i, ringIn) = (o, release ringOut)
  where (o, ringOut) = f (i, fetch ringIn)

rightRotate     :: [a] → [a]
rightRotate []  = []
rightRotate xs  = last xs : init xs
```

We can rewrite its functionality easily with the use of loop as the definition of the node function, arr (i, r) (o, r), after being transformed into an arrow, already fits quite neatly into the loop's arr (a, b) (c, b) → arr a c. In each iteration we start by rotating the intermediary input from the nodes [fut r] with second (rightRotate >>> lazy). Similarly to the pipe, we have to feed the intermediary input into our lazy arrow here, or the evaluation would hang. The reasoning is explained by Loogen (2012): 'Note that the list of ring inputs ringIns is the same as the list of ring outputs ringOuts rotated by one element to the right using the auxiliary function rightRotate. Thus, the program would get stuck without the lazy pattern, because the ring input will only be produced after process creation and process creation will not occur without the first input.' Next, we zip the resulting ([i], [fut r]) to [(i, fut r)] with arr (**uncurry zip**) so we can feed that into a our input arrow arr (i, r) (o, r), which we transform into arr (i, fut r) (o, fut r) before lifting it to arr [(i, fut r)] [(o, fut r)] to get a list [(o, fut r)]. Finally we unzip this list into ([o], [fut r]). Plugging this arrow arr ([i], [fut r]) ([o], fut r) into the definition of loop from earlier gives us arr [i] [o], our ring arrow.

This gives us the following complete definition for the ring combinator:

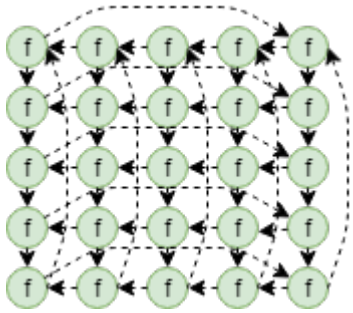
```
ring :: (ArrowLoop arr, Future fut r,
  ArrowParallel arr (i, fut r) (o, fut r) conf) ⇒
    conf →
    arr (i, r) (o, r) →
    arr [i] [o]
ring conf f =
  loop (second (rightRotate >>> lazy) >>>
    arr (uncurry zip) >>>
    parMap conf (second get >>> f >>> second put) >>>
    arr unzip)
```

with rightRotate:

```
rightRotate :: (Arrow arr) ⇒ arr [a] [a]
rightRotate = arr $ λ list → case list of
  [] → []
  xs → last xs : init xs
```

This combinator can, for example, be used to calculate the shortest paths in a graph using Warshall's algorithm.

### *7.3 Torus skeleton*




If we take the concept of a ring one dimension further, we get a torus. Every node sends ands receives data from horizontal and vertical neighbours in each communication round.

With our parallel Arrows we implement a torus combinator yet again with the help of the ArrowLoop typeclass.

Similar to the ring, we once again start by rotating the input, but this time not only in one direction, but in two. This means that the intermediary input from the neighbour nodes has to be stored in a tuple `([[fut a]], [[fut b]])` in the second argument (loop only allows for 2 arguments) of our looped arrow `arr ([[c]], ([[fut a]], [[fut b]])) ([[d]], ([[fut a]], [[fut b]]))` and our rotation arrow becomes `second ((mapArr rightRotate >>> lazy) *** (arr rightRotate >>> lazy))` instead of the singular rotation in the ring as we rotate `[[fut a]]` horizontally and `[[fut b]]` vertically. Then, we once again zip the inputs for the input arrow with `arr (uncurry3 zipWith3 lazyzip3)` from `([[c]], ([[fut a]], [[fut b]]))` to `[[(c, fut a, fut b)]]`, which we then feed into our parallel execution.

This, however, is more complicated than in the ring case as we have one more dimension of inputs to be transformed. We first have to `shuffle` all the inputs to then pass it into `parMap conf (ptorus f)` which yields us `[(d, fut a, fut b)]`. We can then unpack this shuffled list back to its original ordering by feeding it into the specific unshuffle arrow we created one step earlier with `arr length >>> arr unshuffle` with the use of `app` from the ArrowApply typeclass. Finally, we unpack this matrix `[[[(d, fut a, fut b)]]` with `arr (map unzip3) >>> arr unzip3 >>> threetotwo` to get `([[d]], ([[fut a]], [[fut b]]))`.

The complete definition of the `torus` combinator is:

```
1 torus :: (ArrowLoop arr, ArrowChoice arr, ArrowApply arr,
2   ArrowParallel arr (c, fut a, fut b) (d, fut a, fut b) conf,
3   Future fut a, Future fut b) ⇒
4   conf → arr (c, a, b) (d, a, b) → arr [[c]] [[d]]
5 torus conf f =
6   loop (second ((mapArr rightRotate >>> lazy) ***
```

```
    (arr rightRotate >>> lazy)) >>>
  arr (uncurry3 (zipWith3 lazyzip3)) >>>
  (arr length >>> arr unshuffle) &&&
    (shuffle >>> parMap conf (ptorus f)) >>>
  app >>>
  arr (map unzip3) >>> arr unzip3 >>> threetotwo)
```

with uncurry3, lazyzip3, and threetotwo from the Appendix, and

```
ptorus :: (Arrow arr, Future fut a, Future fut b) ⇒
  arr (c, a, b) (d, a, b) →
  arr (c, fut a, fut b) (d, fut a, fut b)
ptorus f =
  arr (λ ~(c, a, b) → (c, get a, get b)) >>> f >>>
  arr (λ ~(d, a, b) → (d, put a, put b))
```

As an example of using this skeleton (Loogen, 2012) showed the matrix multiplication using the Gentleman algorithm (Gentleman, 1978). Adapting this nodefunction to our Arrow API gives us:

```
nodefunction :: Int →
  ((Matrix, Matrix), [Matrix], [Matrix]) →
  ([Matrix], [Matrix], [Matrix])
nodefunction n ((bA, bB), rows, cols) =
  ([bSum], bA:nextAs , bB:nextBs)
    where bSum =
      foldl' matAdd (matMult bA bB) (zipWith matMult nextAs nextBs)
      nextAs = take (n−1) rows
      nextBs = take (n−1) cols
```

If we compare the trace from a call using our arrow definition of the torus (Fig. 4) with the Eden version (Fig. 5) we can see that the behaviour of the arrow version is comparable.

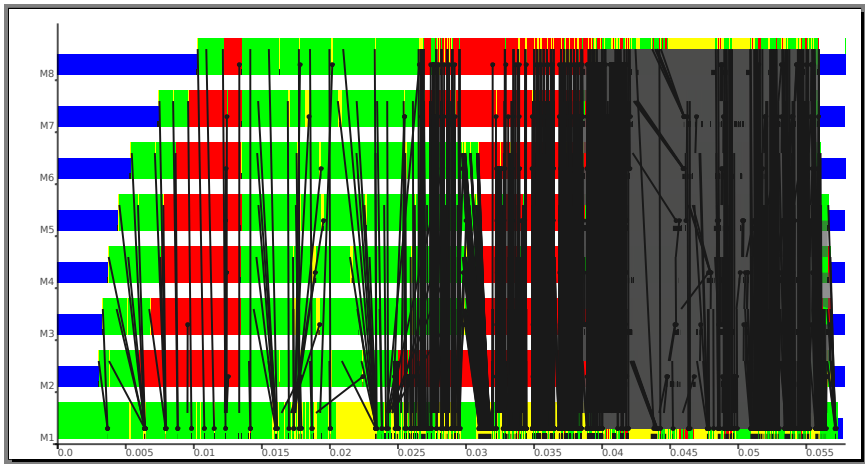



Figure 4: Matrix Multiplication with a torus (Parrows)

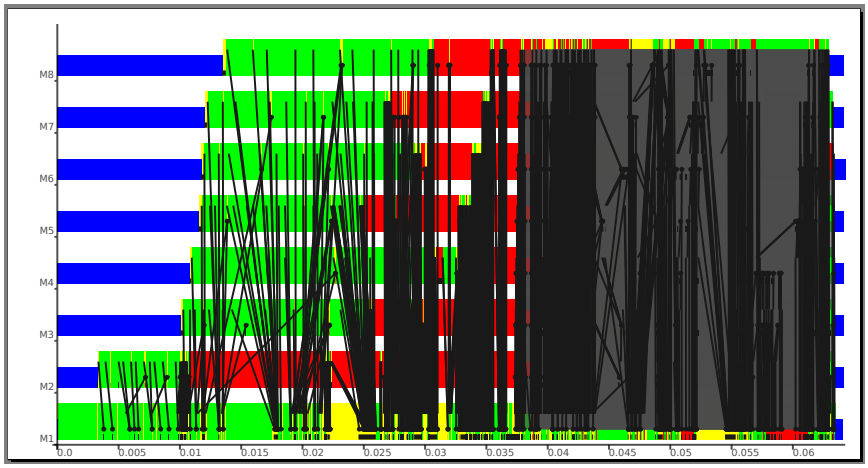
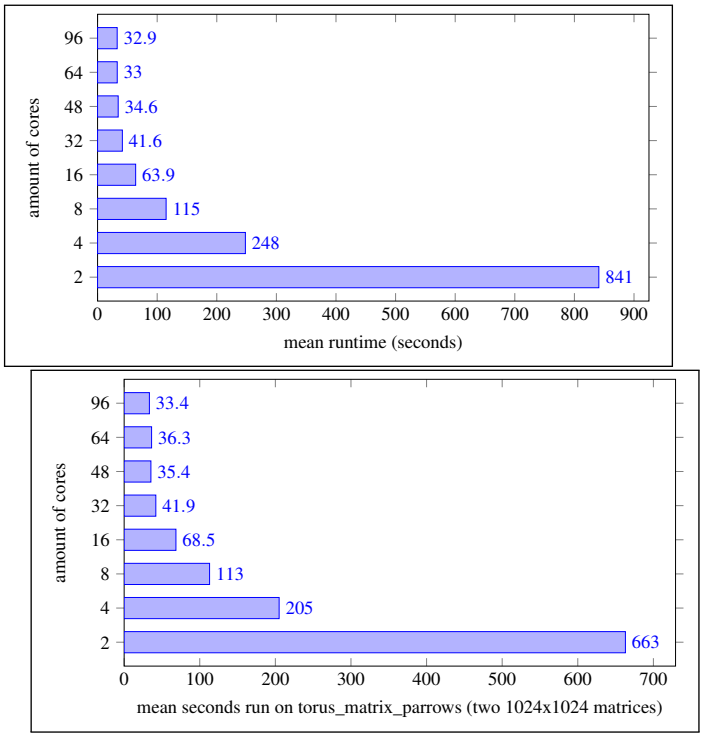



Figure 5: Matrix Multiplication with a torus (Eden)

## 8 Benchmarks




## 9 Conclusion

Arrows are a generic concept that allows for powerful composition combinators. To our knowledge we are the first ones to represent parallel computation with arrows.

Arrows turn out to be a useful tool for composing in parallel programs. We do not have to introduce new monadic types that wrap the computation. Instead use arrows just like regular sequential pure functions. This work features multiple parallel backends: the already available parallel Haskell flavours. Parallel Arrows feature an implementation of the ArrowParallel interface for Multicore Haskell, Par monad, and Eden. With our approach parallel programs can be ported across these flavours with no effort. Performancewise, Parallel Arrows are on par with existing parallel Haskells, as they do not introduce any notable overhead.

Bibliography

*Eden skeletons' control.parallel.eden.map package source code*. [Accessed on 02/12/2017].

*Eden skeletons' control.parallel.eden.topology package source code*. [Accessed on 03/17/2017].

Achten, Peter, Van Eekelen, Marko, De Mol, Maarten, & Plasmeijer, Rinus. (2007). An arrow based semantics for interactive applications. *Draft Proceedings of the Symposium on Trends in Functional Programming*. TFP '07.

Achten, PM, van Eekelen, Marko CJD, Plasmeijer, MJ, & Weelden, A van. (2004). *Arrows for generic graphical editor components*.

Alimarine, Artem, Smetsers, Sjaak, van Weelden, Arjen, van Eekelen, Marko, & Plasmeijer, Rinus. (2005). There and back again: Arrows for invertible programming. *Pages 86–97 of: Proceedings of the 2005 ACM SIGPLAN Workshop on Haskell*. Haskell '05. ACM.

Aljabri, Malak, Loidl, Hans-Wolfgang, & Trinder, Phil W. (2014). The design and implementation of gumsmp: A multilevel parallel haskell implementation. *Pages 37:37–37:48 of: Proceedings of the 25$^{th}$ Symposium on Implementation and Application of Functional Languages*. IFL '13. ACM.

Aljabri, Malak, Loidl, Hans-Wolfgang, & Trinder, Phil. (2015). *Balancing shared and distributed heaps on NUMA architectures*. Springer. Pages 1–17.

Alt, Martin, & Gorlatch, Sergei. (2003). Future-Based RMI: Optimizing compositions of remote method calls on the Grid. *Pages 682–693 of:* Kosch, Harald, Böszörményi, László, & Hellwagner, Hermann (eds), *Euro-Par 2003*. LNCS 2790. Springer-Verlag.

Aswad, Mustafa, Trinder, Phil, Al Zain, Abdallah, Michaelson, Greg, & Berthold, Jost. (2009). Low pain vs no pain multi-core Haskells. *Pages 49–64 of: Trends in Functional Programming*.

Atkey, Robert. (2011). What is a categorical model of arrows? *Electronic notes in theoretical computer science*, **229**(5), 19–37.

Berthold, Jost. (2008). *Explicit and implicit parallel functional programming — concepts and implementation*. Ph.D. thesis, Philipps-Universit"at Marburg.

Berthold, Jost, & Loogen, Rita. (2006). Skeletons for recursively unfolding process topologies. Joubert, Gerhard R., Nagel, Wolfgang E., Peters, Frans J., Plata, Oscar G., Tirado, P., & Zapata, Emilio L. (eds), *Parallel Computing: Current & Future Issues of High-End Computing, ParCo 2005, Malaga, Spain*. NIC Series 33. Central Institute for Applied Mathematics, Jülich, Germany.

Berthold, Jost, & Loogen, Rita. (2007). Visualizing Parallel Functional Program Executions: Case Studies with the Eden Trace Viewer. *ParCo '07. Parallel Computing: Architectures, Algorithms and Applications*. IOS Press.

Berthold, Jost, Dieterle, Mischa, Lobachev, Oleg, & Loogen, Rita. (2009a). Distributed Memory Programming on Many-Cores – A Case Study Using Eden Divide-&-Conquer Skeletons. *Pages 47–55 of:* Großpitsch, K.-E., Henkersdorf, A., Uhrig, S., Ungerer, T., & Hähner, J. (eds), *Workshop on Many-Cores at ARCS '09 – 22nd International Conference on Architecture of Computing Systems 2009*. VDE-Verlag.

Berthold, Jost, Dieterle, Mischa, & Loogen, Rita. (2009b). Implementing parallel Google map-reduce in Eden. *Pages 990–1002 of:* Sips, Henk, Epema, Dick, & Lin, Hai-Xiang (eds), *Euro-Par 2009 Parallel Processing*. LNCS 5704. Springer Berlin Heidelberg.

Berthold, Jost, Dieterle, Mischa, Lobachev, Oleg, & Loogen, Rita. (2009c). *Parallel FFT with Eden skeletons*. PaCT '09. Springer. Pages 73–83.

Berthold, Jost, Loidl, Hans-Wolfgang, & Hammond, Kevin. (2016). PAEAN: Portable and scalable runtime support for parallel Haskell dialects. *Journal of functional programming*, **26**.

Botorog, G. H., & Kuchen, H. (1996). *Euro-Par'96 Parallel Processing*. LNCS 1123. Springer-Verlag. Chap. Efficient parallel programming with algorithmic skeletons, pages 718–731.

Brown, C., & Hammond, K. (2010). Ever-decreasing circles: a skeleton for parallel orbit calculations in Eden. *Draft Proceedings of the Symposium on Trends in Functional Programming*. TFP '10.

Chakravarty, Manuel M. T., Leshchinskiy, Roman, Peyton Jones, Simon L., Keller, Gabriele, & Marlow, Simon. (2007). Data Parallel Haskell: a status report. *Pages 10–18 of: DAMP '07*. ACM Press.

Chakravarty, Manuel M.T., Keller, Gabriele, Lee, Sean, McDonell, Trevor L., & Grover, Vinod. (2011). Accelerating Haskell array codes with multicore GPUs. *Pages 3–14 of: Proceedings of the 6th Workshop on Declarative Aspects of Multicore Programming*. DAMP '11. ACM.

Clifton-Everest, Robert, McDonell, Trevor L, Chakravarty, Manuel M T, & Keller, Gabriele. (2014). Embedding Foreign Code. *PADL '14: The 16th International Symposium on Practical Aspects of Declarative Languages*. LNCS. Springer-Verlag.

Cole, M. I. (1989). Algorithmic skeletons: Structured management of parallel computation. *Research Monographs in Parallel and Distributed Computing*. Pitman.

Dagand, Pierre-Évariste, Kostić, Dejan, & Kuncak, Viktor. (2009). Opis: Reliable distributed systems in OCaml. *Pages 65–78 of: Proceedings of the 4th International Workshop on Types in Language Design and Implementation*. TLDI '09. ACM.

Danelutto, M., Pasqualetti, F., & Pelagatti, S. (1997). Skeletons for Data Parallelism in $P^3L$. *Pages 619–628 of:* Lengauer, C., Griebl, M., & Gorlatch, S. (eds), *Euro-Par'97*. LNCS 1300. Springer-Verlag.

Darlington, J., Field, AJ, Harrison, PG, Kelly, PHJ, Sharp, DWN, Wu, Q., & While, RL. (1993). Parallel programming using skeleton functions. *Pages 146–160 of: Parallel architectures and languages Europe*. Springer-Verlag.

de la Encina, Alberto, Hidalgo-Herrero, Mercedes, Rabanal, Pablo, & Rubio, Fernando. (2011). *A parallel skeleton for genetic algorithms*. IWANN '11. Springer. Pages 388–395.

Dieterle, M., Horstmeyer, T., & Loogen, R. (2010a). Skeleton composition using remote data. *Pages 73–87 of:* Carro, M., & Peña, R. (eds), *12th International Symposium on Practical Aspects of Declarative Languages*. PADL '10, vol. 5937. Springer-Verlag.

Dieterle, M., Horstmeyer, T., Loogen, R., & Berthold, J. (2016). Skeleton composition versus stable process systems in Eden. *Journal of functional programming*, **26**.

Dieterle, Mischa, Berthold, Jost, & Loogen, Rita. (2010b). *A skeleton for distributed work pools in Eden*. FLOPS '10. Springer. Pages 337–353.

Dieterle, Mischa, Horstmeyer, Thomas, Berthold, Jost, & Loogen, Rita. (2013). *Iterating skeletons*. IFL '12. Springer. Pages 18–36.

Foltzer, Adam, Kulkarni, Abhishek, Swords, Rebecca, Sasidharan, Sajith, Jiang, Eric, & Newton, Ryan. (2012). A meta-scheduler for the Par-monad: Composable scheduling for the heterogeneous cloud. *Sigplan not.*, **47**(9), 235–246.

Geimer, M., Wolf, F., Wylie, B. J. N., Ábrahám, E., Becker, D., & Mohr, B. (2010). The Scalasca performance toolset architecture. *Concurrency and computation: Practice and experience*, **22**(6).

Gentleman, W. Morven. (1978). Some complexity results for matrix computations on parallel processors. *Journal of the acm*, **25**(1), 112–115.

Gorlatch, Sergei. (1998). Programming with divide-and-conquer skeletons: A case study of FFT. *Journal of supercomputing*, **12**(1-2), 85–97.

Gorlatch, Sergei, & Bischof, Holger. (1998). A generic MPI implementation for a data-parallel skeleton: Formal derivation and application to FFT. *Parallel processing letters*, **8**(4).

Hammond, Kevin, Berthold, Jost, & Loogen, Rita. (2003). Automatic skeletons in Template Haskell. *Parallel processing letters*, **13**(03), 413–424.

Harris, Tim, Marlow, Simon, Peyton Jones, Simon, & Herlihy, Maurice. (2005). Composable memory transactions. *Pages 48–60 of: Proceedings of the 10th ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming*. PPoPP '05. ACM.

Horstmeyer, Thomas, & Loogen, Rita. (2013). Graph-based communication in Eden. *Higher-order and symbolic computation*, **26**(1), 3–28.

Huang, Liwen, Hudak, Paul, & Peterson, John. (2007). *HPorter: Using arrows to compose parallel processes*. Berlin, Heidelberg: Springer Berlin Heidelberg. Pages 275–289.

Hudak, Paul, Courtney, Antony, Nilsson, Henrik, & Peterson, John. (2003). *Arrows, robots, and functional reactive programming*. Springer. Pages 159–187.

Hughes, John. (2000). Generalising monads to arrows. *Science of computer programming*, **37**(1–3), 67–111.

Hughes, John. (2005a). *Programming with arrows*. AFP '04. Springer. Pages 73–129.

Hughes, John. (2005b). *Programming with arrows*. Berlin, Heidelberg: Springer Berlin Heidelberg. Pages 73–129.

Jacobs, Bart, Heunen, Chris, & Hasuo, Ichiro. (2009). Categorical semantics for arrows. *Journal of functional programming*, **19**(3-4), 403–438.

Keller, Gabriele, Chakravarty, Manuel M.T., Leshchinskiy, Roman, Peyton Jones, Simon, & Lippmeier, Ben. (2010). Regular, shape-polymorphic, parallel arrays in haskell. *Sigplan not.*, **45**(9), 261–272.

Kuper, Lindsey, Todd, Aaron, Tobin-Hochstadt, Sam, & Newton, Ryan R. (2014). Taming the parallel effect zoo: Extensible deterministic parallelism with LVish. *Sigplan not.*, **49**(6), 2–14.

Lengauer, Christian, Gorlatch, Sergei, & Herrmann, Christoph. (1997). The static parallelization of loops and recursions. *The journal of supercomputing*, **11**(4), 333–353.

Li, Peng, & Zdancewic, S. (2006). Encoding information flow in Haskell. *Pages 12–16 of: 19th IEEE Computer Security Foundations Workshop*. CSFW '06.

Li, Peng, & Zdancewic, Steve. (2010). Arrows for secure information flow. *Theoretical computer science*, **411**(19), 1974–1994.

Lindley, Sam, Wadler, Philip, & Yallop, Jeremy. (2011). Idioms are oblivious, arrows are meticulous, monads are promiscuous. *Electronic notes in theoretical computer science*, **229**(5), 97–117.

Linton, S., Hammond, K., Konovalov, A., Al Zain, A. D., Trinder, P., Horn, P., & Roozemond, D. (2010). Easy composition of symbolic computation software: a new lingua franca for symbolic computation. *Pages 339–346 of: Proceedings of the 2010 International Symposium on Symbolic and Algebraic Computation*. ISSAC '10. ACM Press.

Liu, Hai, Cheng, Eric, & Hudak, Paul. (2009). Causal commutative arrows and their optimization. *Sigplan not.*, **44**(9), 35–46.

Lobachev, Oleg. (2011). *Implementation and evaluation of algorithmic skeletons: Parallelisation of computer algebra algorithms*. Ph.D. thesis, Philipps-Universität Marburg.

Lobachev, Oleg. (2012). *Parallel computation skeletons with premature termination property*. FLOPS 2012. Springer. Pages 197–212.

Loogen, Rita. (2012). *Eden – parallel functional programming with Haskell*. Springer. Pages 142–206.

Loogen, Rita, Ortega-Mallén, Yolanda, & Peña-Marí, Ricardo. (2005). Parallel Functional Programming in Eden. *Journal of Functional Programming*, **15**(3), 431–475. Special Issue on Functional Approaches to High-Performance Parallel Programming.

Maier, Patrick, Stewart, Robert, & Trinder, Phil. (2014). The HdpH DSLs for scalable reliable computation. *Sigplan not.*, **49**(12), 65–76.

Mainland, Geoffrey, & Morrisett, Greg. (2010). Nikola: Embedding compiled GPU functions in Haskell. *Sigplan not.*, **45**(11), 67–78.

Marlow, S., Peyton Jones, S., & Singh, S. (2009). Runtime support for multicore Haskell. *Acm sigplan notices*, **44**(9), 65–78.

Marlow, S., Maier, P., Loidl, H. W., Aswad, M. K., & Trinder, P. (2010). Seq no more: better strategies for parallel Haskell. *Pages 91–102 of: Proceedings of the third ACM Haskell symposium on Haskell*. ACM Press.

Marlow, Simon. (2013). *Parallel and concurrent programming in Haskell: Techniques for multicore and multithreaded programming*. "O'Reilly Media, Inc.".

Marlow, Simon, Newton, Ryan, & Peyton Jones, Simon. (2011a). A monad for deterministic parallelism. *Sigplan not.*, **46**(12), 71–82.

Marlow, Simon, Newton, Ryan, & Peyton Jones, Simon. (2011b). A monad for deterministic parallelism. *Sigplan not.*, **46**(12), 71–82.

McDonell, Trevor L., Chakravarty, Manuel M. T., Grover, Vinod, & Newton, Ryan R. (2015). Type-safe runtime code generation: Accelerate to LLVM. *Sigplan not.*, **50**(12), 201–212.

Nikhil, R., & Arvind, L. A. (2001). *Implicit parallel programming in pH*. Morgan Kaufmann.

Paterson, Ross. (2001). A new notation for arrows. *Sigplan not.*, **36**(10), 229–240.

Perfumo, Cristian, Sönmez, Nehir, Stipic, Srdjan, Unsal, Osman, Cristal, Adrián, Harris, Tim, & Valero, Mateo. (2008). The limits of software transactional memory (STM): Dissecting Haskell STM applications on a many-core environment. *Pages 67–78 of: Proceedings of the 5th Conference on Computing Frontiers*. CF '08. ACM.

Rabhi, F. A., & Gorlatch, S. (eds). (2003). *Patterns and Skeletons for Parallel and Distributed Computing*. Springer-Verlag.

Russo, Alejandro, Claessen, Koen, & Hughes, John. (2008). A library for light-weight information-flow security in Haskell. *Pages 13–24 of: Proceedings of the 1st ACM SIGPLAN Symposium on Haskell*. Haskell '08. ACM.

Stewart, Robert, Maier, Patrick, & Trinder, Phil. (2016). Transparent fault tolerance for scalable functional computation. *Journal of functional programming*, **26**.

Svensson, Joel. (2011). *Obsidian: Gpu kernel programming in haskell*. Ph.D. thesis, Chalmers University of Technology.

Trinder, Phil W., Hammond, Kevin, Mattson Jr., James S., Partridge, Andrew S., & Peyton Jones, Simon L. (1996). GUM: a Portable Parallel Implementation of Haskell. *PLDI'96*. ACM Press.

Trinder, Philip W, Barry Jr, M, Hammond, Kevin, Junaidu, Sahalu B, Klusic, Ulrike, Loidl, Hans-Wolfgang, & Peyton Jones, Simon L. (1999). GPH: an architecture-independent functional language. *Ieee trans. software engineering*.

Trinder, P.W., Hammond, K., Loidl, H-W., & Peyton Jones, S. (1998). Algorithm + Strategy = Parallelism. *J. of functional programming*, **8**(1), 23–60.

Vizzotto, Juliana, Altenkirch, Thorsten, & Sabry, Amr. (2006). Structuring quantum effects: superoperators as arrows. *Mathematical structures in computer science*, **16**(3), 453–468.

Wheeler, K. B., & Thain, D. (2009). Visualizing massively multithreaded applications with ThreadScope. *Concurrency and computation: Practice and experience*, **22**(1), 45–67.

## A Utility Functions

To be able to go into detail on parallel arrows, we introduce some utility combinators first, that will help us later: **map**, **foldl** and **zipWith** on Arrows.

The mapArr combinator lifts any arrow `arr a b` to an arrow `arr [a] [b]` (Hughes, 2005b):

```
mapArr :: ArrowChoice arr ⇒ arr a b → arr [a] [b]
mapArr f =
  arr listcase >>>
  arr (const []) ||| (f *** mapArr f >>> arr (uncurry (:)))
  where listcase [] = Left ()
        listcase (x:xs) = Right (x,xs)
```

Similarly, we can also define `foldlArr` that lifts any arrow `arr (b, a) b` with a neutral element `b` to `arr [a] b`:

```
foldlArr :: (ArrowChoice arr, ArrowApply arr) ⇒ arr (b, a) b → b → arr [a] b
foldlArr f b =
  arr listcase >>>
  arr (const b) |||
    (first (arr (λa → (b, a)) >>> f >>> arr (foldlArr f)) >>> app)
```

```
  where listcase [] = Left []
        listcase (x:xs) = Right (x,xs)
```

Finally, with the help of mapArr, we can define zipWithArr that lifts any arrow arr (a, b) c to an arrow arr ([a], [b]) [c].

```
zipWithArr :: ArrowChoice arr ⇒ arr (a, b) c → arr ([a], [b]) [c]
zipWithArr f = (arr $ λ(as, bs) → zipWith (,) as bs) >>> mapArr f
```

These combinators make use of the ArrowChoice type class, it provides the ||| combinator. The latter takes two arrows arr a c and arr b c and combines them into a new arrow arr (Either a b) c which pipes all Left a's to the first arrow and all Right b's to the second arrow.

```
(|||) :: ArrowChoice arr a c → arr b c → arr (Either a b) c
```

With the zipWithArr combinator we can also write a combinator listApp, which lifts a list of arrows [arr a b] to an arrow arr [a] [b].

```
listApp :: (ArrowChoice arr, ArrowApply arr) ⇒ [arr a b] → arr [a] [b]
listApp fs = (arr $ λas → (fs, as)) >>> zipWithArr app
```

This combinator also makes use of the ArrowApply typeclass that allows us to evaluate arrows with app :: arr (arr a b, a) c.

The definition of unshuffle is

```
unshuffle :: (Arrow arr) ⇒ Int → arr [a] [[a]]
unshuffle n = arr (λxs → [takeEach n (drop i xs) | i ∈ [0..n−1]])

takeEach :: Int → [a] → [a]
takeEach n [] = []
takeEach n (x:xs) = x : takeEach n (drop (n−1) xs)
```

while shuffle is defined as:

```
shuffle :: (Arrow arr) ⇒ arr [[a]] [a]
shuffle = arr (concat ∘ transpose)
```

These functions were taken from Eden skeleton source code (ede, n.d.a) and lifted to Arrows.

The helper functions for torus are

```
uncurry3 :: (a → b → c → d) → (a, (b, c)) → d
uncurry3 f (a, (b, c)) = f a b c

lazyzip3 :: [a] → [b] → [c] → [(a, b, c)]
lazyzip3 as bs cs = zip3 as (lazy bs) (lazy cs)

threetotwo :: (Arrow arr) ⇒ arr (a, b, c) (a, (b, c))
threetotwo = arr $ λ ~(a, b, c) → (a, (b, c))
```