Department of Computer Engineering

Artificial Intelligence

Mini Project 3 Theory Questions

Dr. Rohban

Parsa Mohammadian — 98102284

December 20, 2021

Contents

1

1.1

We decide if a move violates game rules or not by using d-separation algroithm to find out if A and B are still independent. A valid move for each of the graphs is illustrated in figure 1.

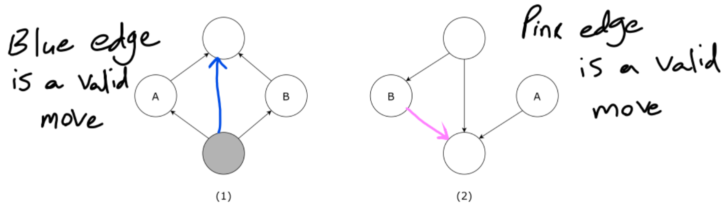

Figure 1: Valid Moves

1.2

In a tree structure, I propose a move for first player and consider every possible move for second player and evaluate outcome from the leaves. If I can find a tree with all win outcomes, then I find a win strategy for first player.
First graph tree is shown in figure 2. So this graph has a win strategy for first player.
Second graph tree is shown in figure 3. So this graph has a win strategy for first player too.

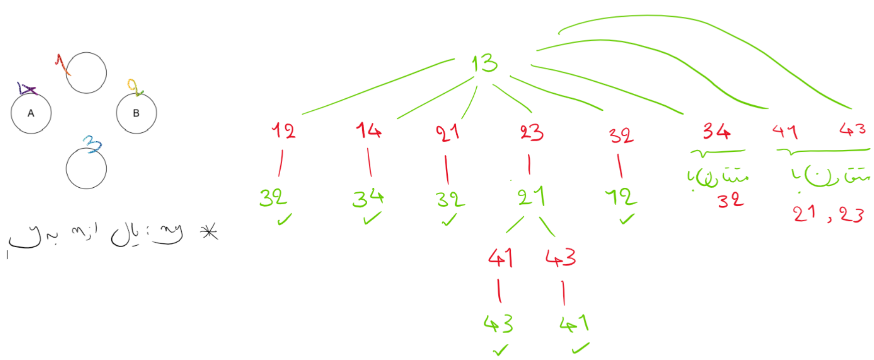

Figure 2: First Graph Tree

2

Base factor headers are:

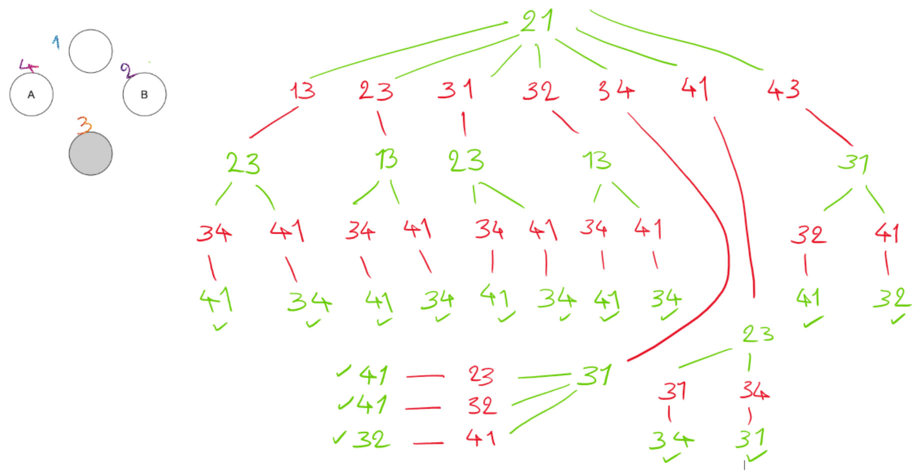

Figure 3: Second Graph Tree

$$
\begin{aligned}
A &: \begin{bmatrix} A & C & D & P(A|C,D) \end{bmatrix}\\
B &: \begin{bmatrix} B & D & E & G & P(B|D,E,G) \end{bmatrix}\\
C &: \begin{bmatrix} C & F & I & P(C|F,I) \end{bmatrix}\\
D &: \begin{bmatrix} D & G & H & P(D|G,H) \end{bmatrix}\\
E &: \begin{bmatrix} E & P(E) \end{bmatrix}\\
F &: \begin{bmatrix} F & H & P(F|H) \end{bmatrix}\\
G &: \begin{bmatrix} G & H & P(G|H) \end{bmatrix}\\
H &: \begin{bmatrix} H & I & P(H|I) \end{bmatrix}\\
I &: \begin{bmatrix} I & P(I) \end{bmatrix}
\end{aligned}
$$

2.1 B, E, D, C, H, I

$$
\begin{aligned}
f_1 = \text{Join}(B) \to \text{Eliminate}(B) &: \begin{bmatrix} D & E & G & P(D,E,G) \end{bmatrix}\\
f_2 = \text{Join}(f_1, E) \to \text{Eliminate}(E) &: \begin{bmatrix} D & G & P(D,G) \end{bmatrix}\\
f_3 = \text{Join}(f_2, A, D) \to \text{Eliminate}(D) &: \begin{bmatrix} A & C & G & H & P(A,C,G,H) \end{bmatrix}\\
f_4 = \text{Join}(f_3, C) \to \text{Eliminate}(C) &: \begin{bmatrix} A & G & H & F & I & P(A,G,H,F,I) \end{bmatrix}\\
f_5 = \text{Join}(f_4, F, G, H) \to \text{Eliminate}(H) &: \begin{bmatrix} A & F & G & I & P(A,F,G,I) \end{bmatrix}\\
f_6 = \text{Join}(f_4, I) \to \text{Eliminate}(I) &: \begin{bmatrix} A & G & F & P(A,G,F) \end{bmatrix}
\end{aligned}
$$

2.2 I, H, C, D, E, B

$$
\begin{aligned}
f_1 = \text{Join}(C, H, I) \to \text{Eliminate}(I) &: \begin{bmatrix} C & F & H & P(C, F, H) \end{bmatrix} \\
f_2 = \text{Join}(f_1, D, F, G) \to \text{Eliminate}(H) &: \begin{bmatrix} C & F & D & G & P(C, F, D, G) \end{bmatrix} \\
f_3 = \text{Join}(f_2, A) \to \text{Eliminate}(C) &: \begin{bmatrix} A & D & F & G & P(A, D, F, G) \end{bmatrix} \\
f_4 = \text{Join}(f_3, B) \to \text{Eliminate}(D) &: \begin{bmatrix} A & F & G & B & E & P(A, F, G, B, E) \end{bmatrix} \\
f_5 = \text{Join}(f_4, E) \to \text{Eliminate}(E) &: \begin{bmatrix} A & F & G & B & P(A, F, G, B) \end{bmatrix} \\
f_6 = \text{Join}(f_4) \to \text{Eliminate}(B) &: \begin{bmatrix} A & F & G & P(A, F, G) \end{bmatrix}
\end{aligned}
$$

2.3 Comparison

The maximum size of the factor is 2^3 for both orderings (Consider we know F, G). So we should compare each factor size. If we ignore evidence variables which we already know their values, clearly the second order is better one and need less time to be computed.

3

3.1

$$
\begin{gathered}
T : \text{Today} \\
Y : \text{Yesterday} \\
P(T = D) = P(T = D|Y = D) \times P(Y = D) \\
+P(T = D|Y = N) \times P(Y = N) \\
Y \text{ and } T \text{ have the same distribution} \\
\Rightarrow P(T = D) = 0.3 \times P(T = D) + 0.1 \times P(T = N) \\
P(T = N) = 1 - P(T = D) \\
\Rightarrow P(T = D) = 0.3 \times P(T = D) + 0.1 \times (1 - P(T = D)) \\
= 0.3 \times P(T = D) + 0.1 - 0.1 \times P(T = D) \\
= 0.2 \times P(T = D) + 0.1 \\
\Rightarrow 0.8 \times P(T = D) = 0.1 \Rightarrow P(T = D) = \frac{0.1}{0.8} = 0.125
\end{gathered}
$$

3.2

$$
\begin{gathered}
P(X = x|Dizinnes, Shortnessof Breath) \\
\propto P(X = x, Dizinnes, Shortnessof Breath) \\
= \sum_{y} P(X = x) \times P(Shortnessof Breath|X = x) \\
\times P(Dizinnes|Y) \times P(Y|X = x) \\
\Rightarrow P(X = x|Dizinnes, Shortnessof Breath) \\
= \frac{P(X = x, Dizinnes, Shortnessof Breath)}{\sum_x P(X = x, Dizinnes, Shortnessof Breath)} \\
\Rightarrow P(X = Sepsis, Dizinnes, Shortnessof Breath) \\
= 0.003 \times 0.85 \times 0.8 \times 0.7 + 0.003 \times 0.85 \times 0.7 \times 0.5 = 0.0023205 \\
\Rightarrow P(X = HeartMuscleWeakness, Dizinnes, Shortnessof Breath) \\
= 0.02 \times 0.5 \times 0.8 \times 0.7 + 0.02 \times 0.5 \times 0.7 \times 0.1 = 0.0063 \\
\Rightarrow P(X = Sepsis|Dizinnes, Shortnessof Breath) \\
= \frac{0.0023205}{0.0023205 + 0.0063} = 0.26918392204628505 \\
\Rightarrow P(X = HeartMuscleWeakness|Dizinnes, Shortnessof Breath) \\
= \frac{0.0063}{0.0023205 + 0.0063} = 0.7308160779537151
\end{gathered}
$$

So the patient is more likely to have Heart Muscle Weakness.