Department of Computer Engineering

Artificial Intelligence

Assignment 5 part 2

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1.1

This is a trick which add a fix number to occurrence of each variable value. It will garauntee that all probabilities are non-zero. So the model is not overfitted.

1.2

First I write Naive Bayes based leaning result (table 1).
Now we can calculate the probability of each labels for given the query.

$$\begin{aligned}P(Y = Go|X_1 = 1, X_2 = 1, X_3 = 1, X_4 = 0)\\ = P(Y = Go) \times P(Y = Go|X_1 = 1) \times P(X_2 = 1|Y = Go)\\ \times P(X_3 = 1|Y = Go) \times P(X_4 = 0|Y = Go)\end{aligned}$$

$$\begin{aligned}P(Y = Stop|X_1 = 1, X_2 = 1, X_3 = 1, X_4 = 0)\\ = P(Y = Stop) \times P(Y = Stop|X_1 = 1) \times P(X_2 = 1|Y = Stop)\\ \times P(X_3 = 1|Y = Stop) \times P(X_4 = 0|Y = Stop)\end{aligned}$$

Both $P(Y = Go|X_1 = 1)$ and $P(X_3 = 1|Y = Stop)$ are zero. So the probability of all labels are zero, which is because of overfitting.
So we must use Laplace smoothing (a generalization technic) to avoid this problem. So I wrote Naive Bayes with Laplace smoothing result (table 2). Based on this table and above equations, we calcualte each label probability.

$$\begin{aligned}Query = X_1 = 1, X_2 = 1, X_3 = 1, X_4 = 0\\ P(Y = Go|Query) \propto \frac{5}{9} \times \frac{1}{7} \times \frac{4}{7} \times \frac{6}{7} \times \frac{4}{7} = \frac{480}{21609}\\ P(Y = Stop|Query) \propto \frac{4}{9} \times \frac{4}{6} \times \frac{4}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{64}{11664}\\ P(Y = Go|Query) > P(Y = Stop|Query)\end{aligned}$$

Finally robot must **Go** for given query.

Y	P($X_1 = 0$|Y)	P($X_1 = 1$|Y)
Go	$\frac{5}{5} = 1$	0
Stop	$\frac{1}{4}$	$\frac{3}{4}$

Y	P($X_2 = 0$|Y)	P($X_2 = 1$|Y)
Go	$\frac{2}{5}$	$\frac{3}{5}$
Stop	$\frac{1}{4}$	$\frac{3}{4}$

Y	P($X_3 = 0$|Y)	P($X_3 = 1$|Y)
Go	$\frac{0}{5} = 0$	1
Stop	$\frac{4}{4} = 1$	0

Y	P($X_4 = 0$|Y)	P($X_4 = 1$|Y)
Go	$\frac{3}{5}$	$\frac{2}{5}$
Stop	$\frac{0}{4} = 0$	1

Table 1: Naive Bayes

Y	P($X_1 = 0$|Y)	P($X_1 = 1$|Y)
Go	$\frac{6}{7}$	$\frac{1}{7}$
Stop	$\frac{2}{6}$	$\frac{4}{6}$

Y	P($X_2 = 0$|Y)	P($X_2 = 1$|Y)
Go	$\frac{3}{7}$	$\frac{4}{7}$
Stop	$\frac{2}{6}$	$\frac{4}{6}$

Y	P($X_3 = 0$|Y)	P($X_3 = 1$|Y)
Go	$\frac{1}{7}$	$\frac{6}{7}$
Stop	$\frac{5}{6}$	$\frac{1}{6}$

Y	P($X_4 = 0$|Y)	P($X_4 = 1$|Y)
Go	$\frac{4}{7}$	$\frac{3}{7}$
Stop	$\frac{1}{6}$	$\frac{5}{6}$

Table 2: Naive Bayes with Laplace smoothing