Department of Computer Engineering

Artificial Intelligence

Mini Project 3 Theory Questions

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1.1

We decide if a move violates game rules or not by using d-separation algroithm to find out if A and B are still independent. A valid move for each of the graphs is illustrated in figure 1.

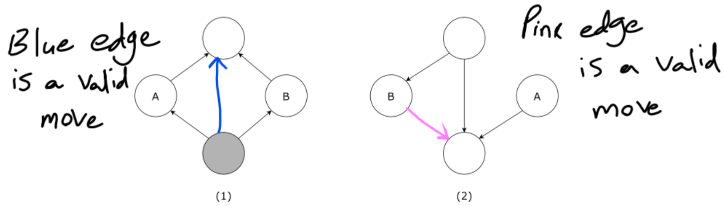

Figure 1: Valid Moves

1.2

In a tree structure, I propose a move for first player and consider every possible move for second player and evaluate outcome from the leaves. If I can find a tree with all win outcomes, then I find a win strategy for first player.
First graph tree is shown in figure 2. So this graph has a win strategy for first player.
Second graph tree is shown in figure 3. So this graph has a win strategy for first player too.

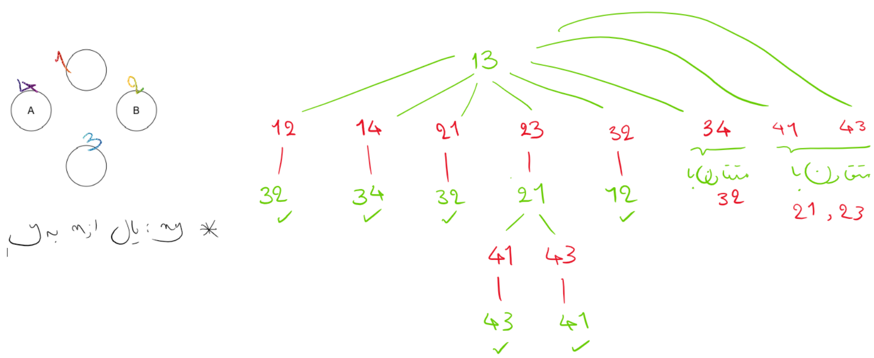

Figure 2: First Graph Tree

2

Base factor headers are:

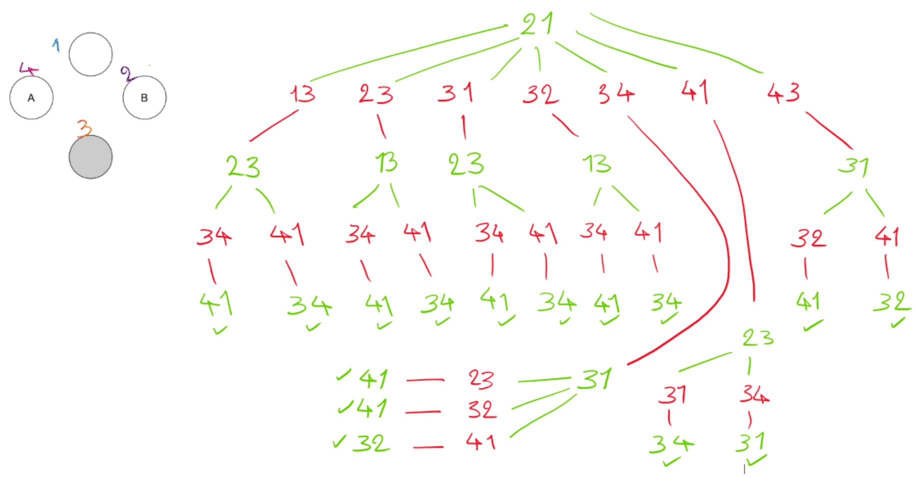

Figure 3: Second Graph Tree

$$\begin{aligned}
A &: \begin{bmatrix} A & C & D & P(A|C,D) \end{bmatrix} \\
B &: \begin{bmatrix} B & D & E & G & P(B|D,E,G) \end{bmatrix} \\
C &: \begin{bmatrix} C & F & I & P(C|F,I) \end{bmatrix} \\
D &: \begin{bmatrix} D & G & H & P(D|G,H) \end{bmatrix} \\
E &: \begin{bmatrix} E & P(E) \end{bmatrix} \\
F &: \begin{bmatrix} F & H & P(F|H) \end{bmatrix} \\
G &: \begin{bmatrix} G & H & P(G|H) \end{bmatrix} \\
H &: \begin{bmatrix} H & I & P(H|I) \end{bmatrix} \\
I &: \begin{bmatrix} I & P(I) \end{bmatrix}
\end{aligned}$$

2.1 B, E, D, C, H, I

$$\begin{aligned}
f_1 = \text{Join}(B) \rightarrow \text{Eliminate}(B) &: \begin{bmatrix} D & E & G & P(D,E,G) \end{bmatrix} \\
f_2 = \text{Join}(f_1, E) \rightarrow \text{Eliminate}(E) &: \begin{bmatrix} D & G & P(D,G) \end{bmatrix} \\
f_3 = \text{Join}(f_2, A, D) \rightarrow \text{Eliminate}(D) &: \begin{bmatrix} A & C & G & H & P(A,C,G,H) \end{bmatrix} \\
f_4 = \text{Join}(f_3, C) \rightarrow \text{Eliminate}(C) &: \begin{bmatrix} A & G & H & F & I & P(A,G,H,F,I) \end{bmatrix} \\
f_5 = \text{Join}(f_4, F, G, H) \rightarrow \text{Eliminate}(H) &: \begin{bmatrix} A & F & G & I & P(A,F,G,I) \end{bmatrix} \\
f_6 = \text{Join}(f_4, I) \rightarrow \text{Eliminate}(I) &: \begin{bmatrix} A & G & F & P(A,G,F) \end{bmatrix}
\end{aligned}$$

2.2 I, H, C, D, E, B

$$
\begin{aligned}
f_1 = \text{Join}(C, H, I) \rightarrow \text{Eliminate}(I) &: \begin{bmatrix} C & F & H & P(C, F, H) \end{bmatrix} \\
f_2 = \text{Join}(f_1, D, F, G) \rightarrow \text{Eliminate}(H) &: \begin{bmatrix} C & F & D & G & P(C, F, D, G) \end{bmatrix} \\
f_3 = \text{Join}(f_2, A) \rightarrow \text{Eliminate}(C) &: \begin{bmatrix} A & D & F & G & P(A, D, F, G) \end{bmatrix} \\
f_4 = \text{Join}(f_3, B) \rightarrow \text{Eliminate}(D) &: \begin{bmatrix} A & F & G & B & E & P(A, F, G, B, E) \end{bmatrix} \\
f_5 = \text{Join}(f_4, E) \rightarrow \text{Eliminate}(E) &: \begin{bmatrix} A & F & G & B & P(A, F, G, B) \end{bmatrix} \\
f_6 = \text{Join}(f_4) \rightarrow \text{Eliminate}(B) &: \begin{bmatrix} A & F & G & P(A, F, G) \end{bmatrix}
\end{aligned}
$$

2.3 Comparison

The maximum size of the factor is 2^5 for both orderings. So we should compare each factor size. Clearly the first order is better one and need less time to be computed.

3

3.1

$$
\begin{gathered}
T : \text{Today} \\
Y : \text{Yesterday} \\
P(T = D) = P(T = D|Y = D) \times P(Y = D) \\
+P(T = D|Y = N) \times P(Y = N) \\
Y \text{ and } T \text{ have the same distribution} \\
\Rightarrow P(T = D) = 0.3 \times P(T = D) + 0.1 \times P(T = N) \\
P(T = N) = 1 - P(T = D) \\
\Rightarrow P(T = D) = 0.3 \times P(T = D) + 0.1 \times (1 - P(T = D)) \\
= 0.3 \times P(T = D) + 0.1 - 0.1 \times P(T = D) \\
= 0.2 \times P(T = D) + 0.1 \\
\Rightarrow 0.8 \times P(T = D) = 0.1 \Rightarrow P(T = D) = \frac{0.1}{0.8} = 0.125
\end{gathered}
$$

3.2

$$
\begin{gathered}
P(X = x|Dizinnes, Shortness of Breath) \\
\propto P(X = x, Dizinnes, Shortness of Breath) \\
= \sum_y P(X = x) \times P(Shortness of Breath|X = x) \\
\times P(Dizinnes|Y) \times P(Y|X = x) \\
\Rightarrow P(X = x|Dizinnes, Shortness of Breath) \\
= \frac{P(X = x, Dizinnes, Shortness of Breath)}{\sum_x P(X = x, Dizinnes, Shortness of Breath)} \\
\Rightarrow P(X = Sepsis, Dizinnes, Shortness of Breath) \\
= 0.003 \times 0.85 \times 0.8 \times 0.7 + 0.003 \times 0.85 \times 0.7 \times 0.5 = 0.0023205 \\
\Rightarrow P(X = HeartMuscleWeakness, Dizinnes, Shortness of Breath) \\
= 0.02 \times 0.5 \times 0.8 \times 0.7 + 0.02 \times 0.5 \times 0.7 \times 0.1 = 0.0063 \\
\Rightarrow P(X = Sepsis|Dizinnes, Shortness of Breath) \\
= \frac{0.0023205}{0.0023205 + 0.0063} = 0.26918392204628505 \\
\Rightarrow P(X = HeartMuscleWeakness|Dizinnes, Shortness of Breath) \\
= \frac{0.0063}{0.0023205 + 0.0063} = 0.7308160779537151
\end{gathered}
$$

So the patient is more likely to have Heart Muscle Weakness.