Task

Roulette Simulation and Profit Analysis

Roulette is a popular casino game played with a wheel that has numbered slots colored red, black, or green. In American roulette, the wheel has 38 slots: 18 red slots, 18 black slots, and 2 green slots labeled "0" and "00".

Players can place various types of bets, including betting on whether the outcome will be a red or black slot. In this exercise, we focus on a simple bet: betting on black.

If you place a bet on black and the outcome is indeed black, you win and double your money. However, if the outcome is red or green, you lose the amount you bet. For example, if you bet 1 dollar on black and win, you gain 1 dollar. If you lose, you forfeit your 1-dollar bet.

Because there are three colors, the chance of landing on the black isn't exactly $\frac{1}{2}$, it is less, specifically $\frac{18}{38} = \frac{9}{19}$.

Consider the following tasks to simulate this game and analyze the expected outcomes of betting on black:

1. Write a function that simulates this game for N rounds, where each round consists of betting 1 dollar on black. The function should return your total earnings S_N after N rounds.
2. Use Monte Carlo simulation to study the distribution of total earnings S_N for N = 10, 25, 100, 1000. For each N, simulate 100,000 rounds and plot the distribution of total earnings. Analyze whether the distributions appear similar to a normal distribution and observe how the expected values and standard errors change with N.
3. Repeat the previous simulation but for the average winnings $\frac{S_N}{N}$ instead of S_N. For each N, plot the distribution of average winnings and examine the changes in expected values and standard errors with different values of N.
   ($N = 10, 25, 100, 1000$)
4. Calculate the theoretical expected values and standard errors of S_N for each N, and compare these theoretical values with your Monte Carlo simulation results. Report any differences between the theoretical and simulated values for each N.
5. Use the Central Limit Theorem (CLT) to approximate the probability that the casino loses money when you play $N = 25$ rounds, and verify this approximation using a Monte Carlo simulation.
6. Plot the probability that the casino loses money as a function of N for values N ranging from 25 to 1000. Discuss why casinos might encourage players to continue betting in light of these results.