

Signals and Systems

Project Report

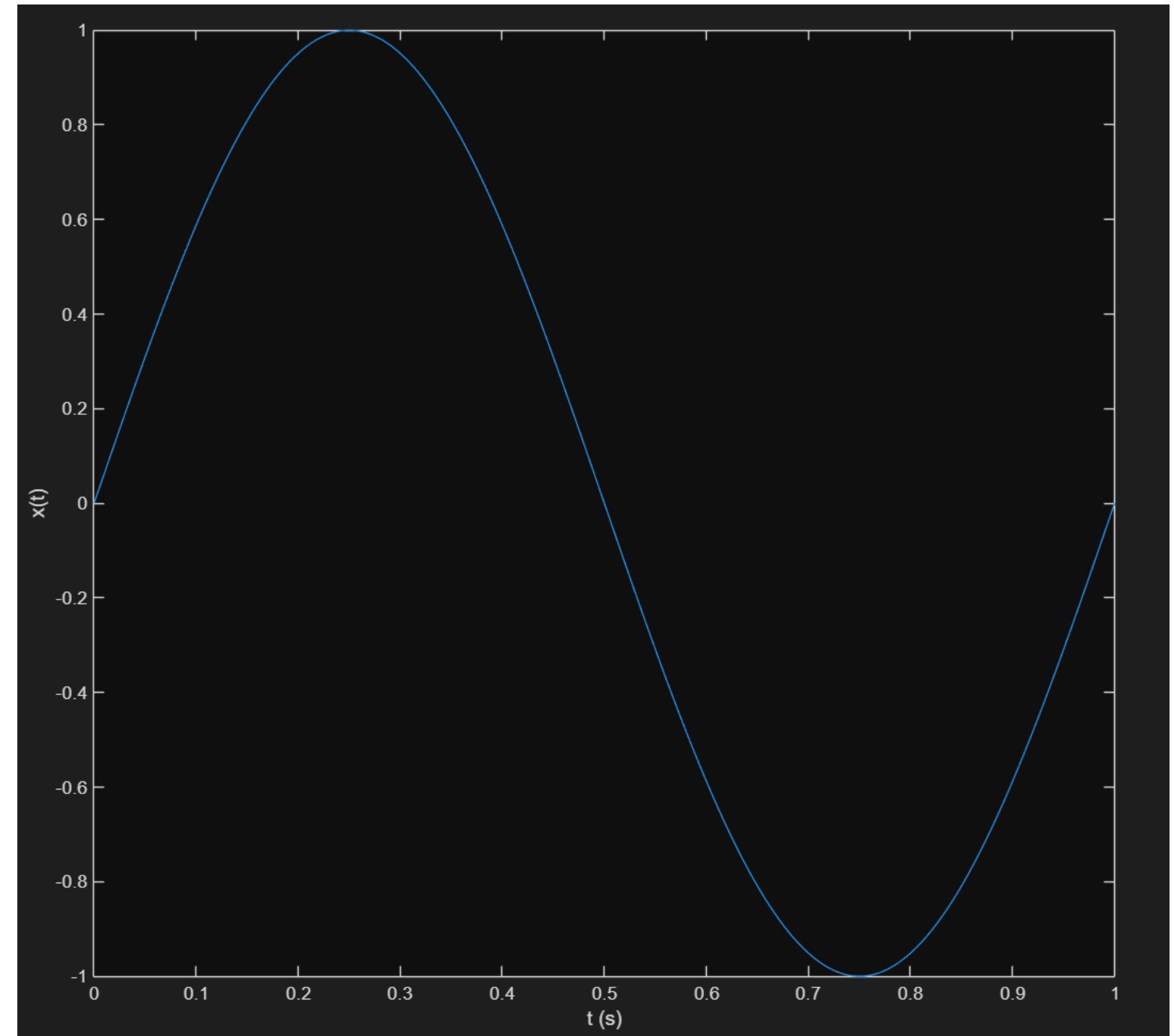
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CA1

Section 2

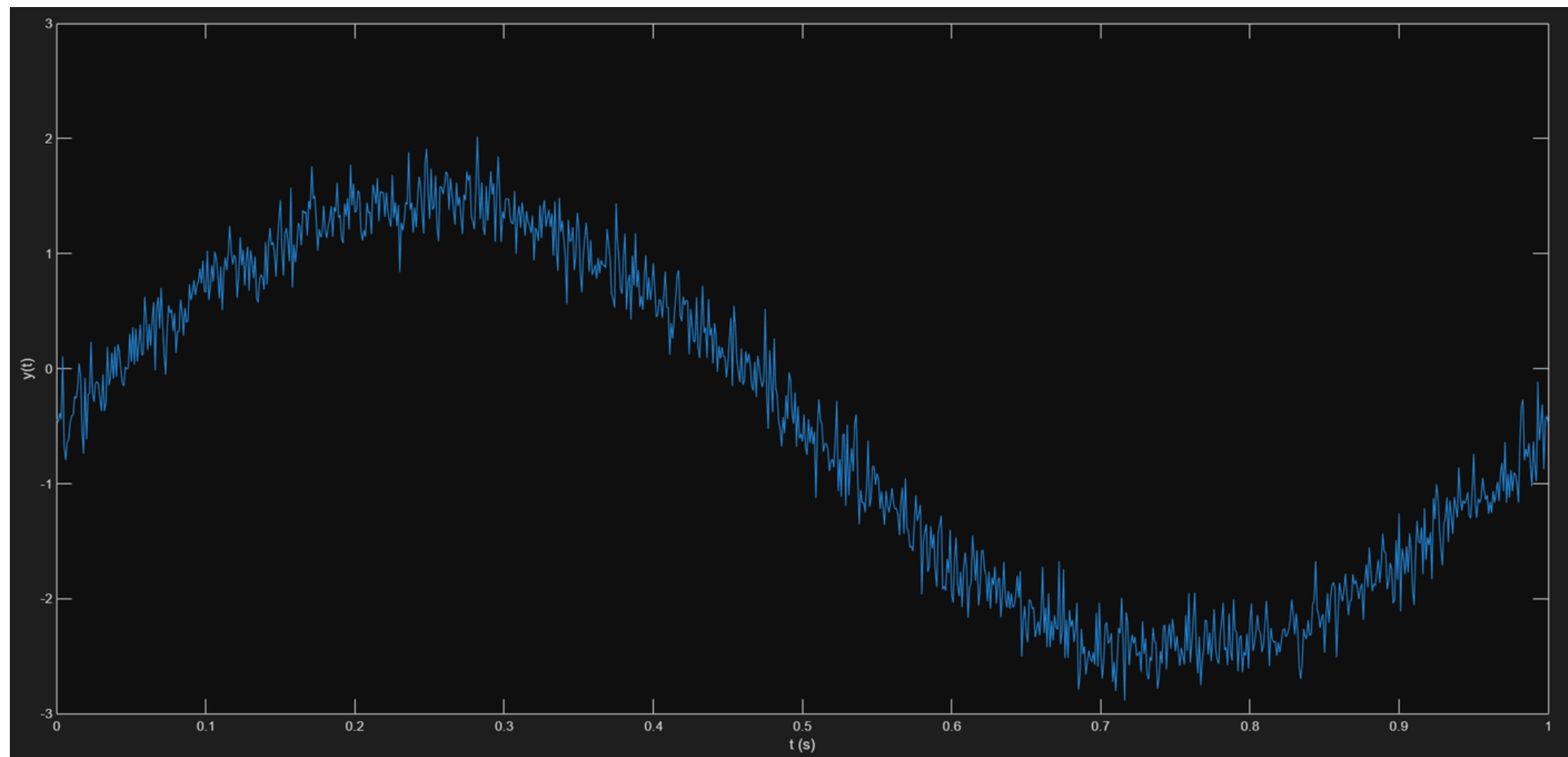
part 2.1:

```
1 clear; clc; close all;  
2  
3 load p2.mat  
4 plot(t, x);  
5 xlabel('t (s)'); ylabel('x(t)');
```



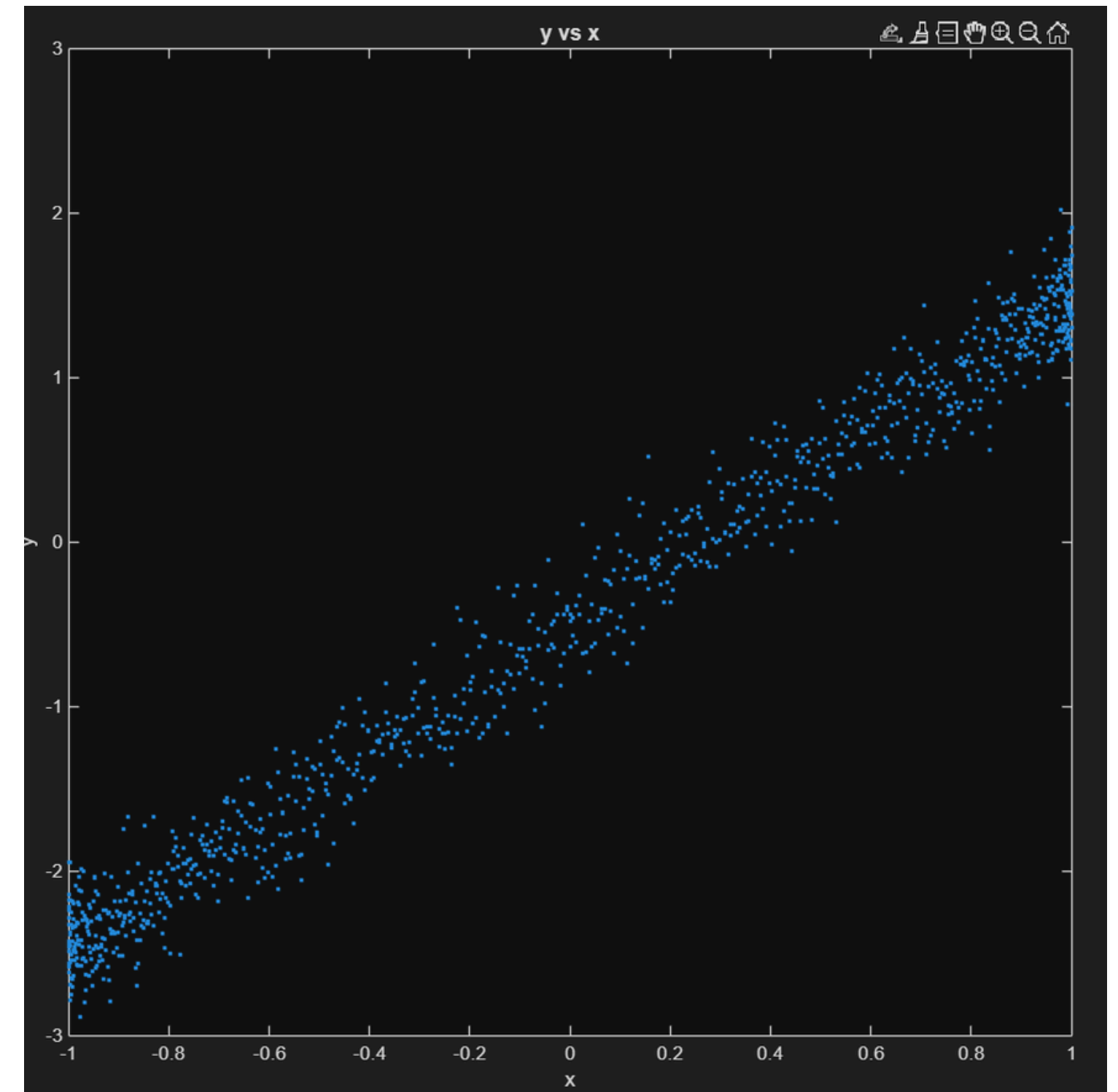
part 2.2:

```
1 clear; clc; close all;  
2  
3 load p2.mat  
4 plot(t, y);  
5 xlabel('t (s)'); ylabel('y(t)');
```



part 2.3:

```
1 clear; clc; close all;  
2  
3 load p2.mat  
4 plot(x, y, '.');  
5 xlabel('x'); ylabel('y');  
6 title('y vs x');
```



The scatter plot of y versus x shows an approximately linear trend, consistent with the idle model $y = \alpha x + \beta$. The slope of the best-fit line estimates α (system gain) and the intercept at $x=0$ estimates β . Spread around the line is due to noise.

part 2.4:

To estimate the parameters α and β in the linear model $y=\alpha x+\beta$, the least squares method was applied by minimizing ----->

$$f(\alpha, \beta) = \sum (y(t) - \alpha x(t) - \beta)^2$$

We minimize the total squared error By taking the partial derivatives of the cost function $f(\alpha, \beta)$ with respect to α and β and setting them to zero, we get two linear equations ----->

$$\alpha = \frac{N \sum xy - (\sum x)(\sum y)}{N \sum x^2 - (\sum x)^2}, \quad \beta = \frac{\sum y - \alpha \sum x}{N}$$

```
1 clear; clc; close all;
2
3 function [alpha, beta] = func(x, y)
4     N = length(x);
5     sumx = sum(x);
6     sumy = sum(y);
7     sumxy = sum(x.*y);
8     sumx2 = sum(x.^2);
9
10    alpha = (N*sumxy - sumx*sumy) / (N*sumx2 - sumx^2);
11    beta = (sumy - alpha*sumx) / N;
12 end
13
14 load p2.mat
15 [a,b] = func(x,y)
```

code implementation

```
a =
    1.9736
b =
   -0.4983
```

output

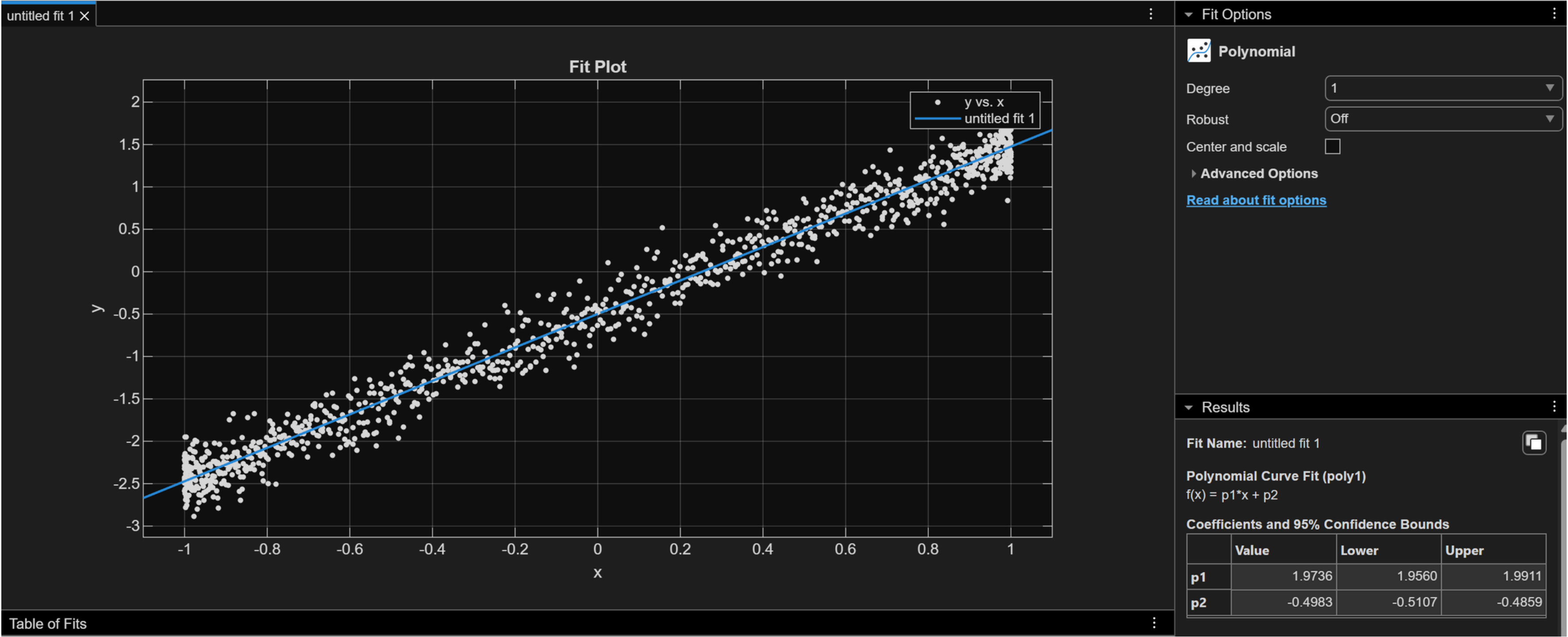
```
19 % Checking if the function is correct
20
21 x2 = linspace(0,10,100);
22 alpha_true = 3; beta_true = 1;
23 y2 = alpha_true*x2 + beta_true + 0.1*randn(size(x2));
24 [alpha_test,beta_test] = func(x2,y2)
25
```

function check

```
alpha_test =
    3.0013
beta_test =
    0.9860
```

output

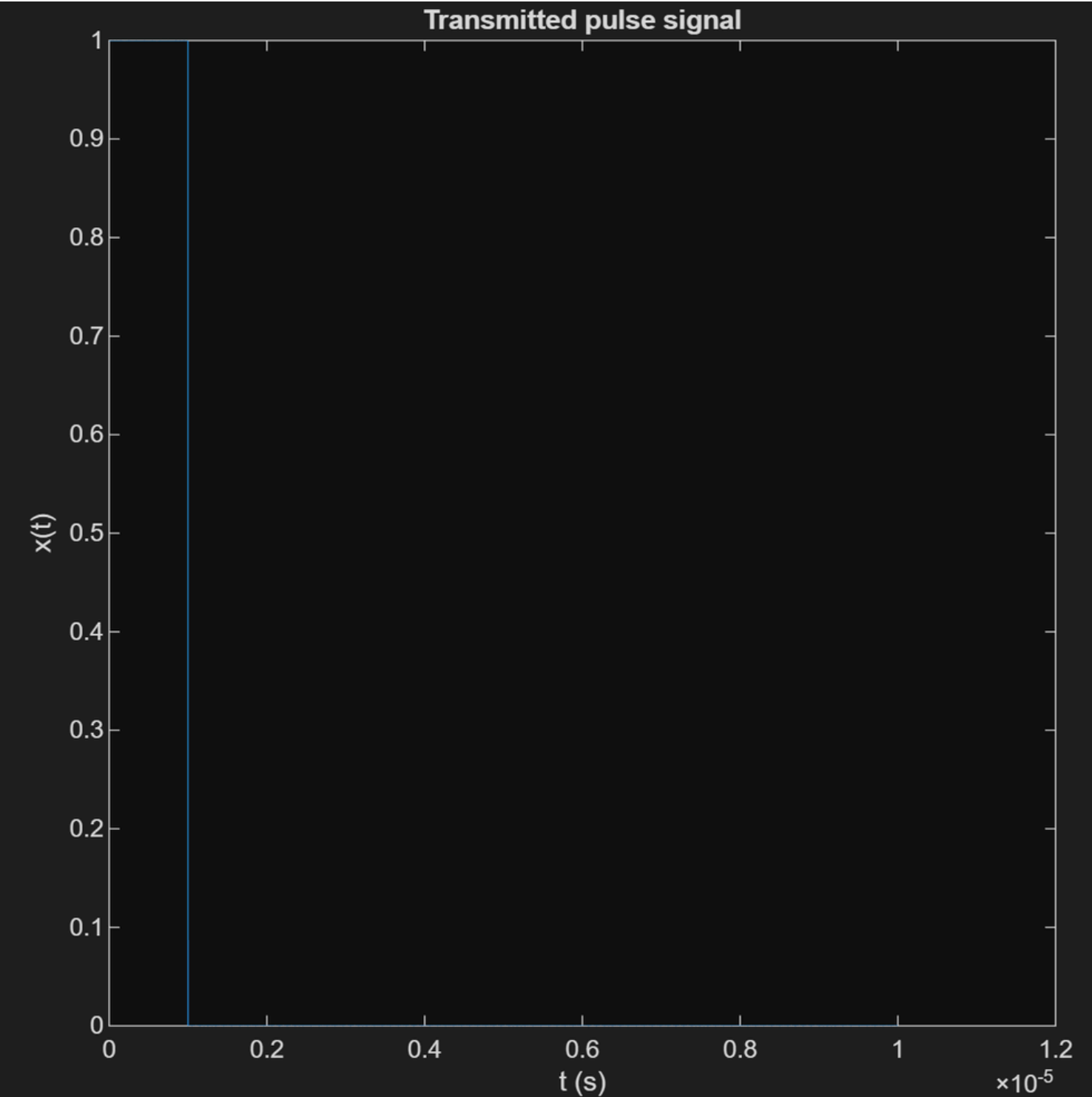
part 2.5:



Section 3

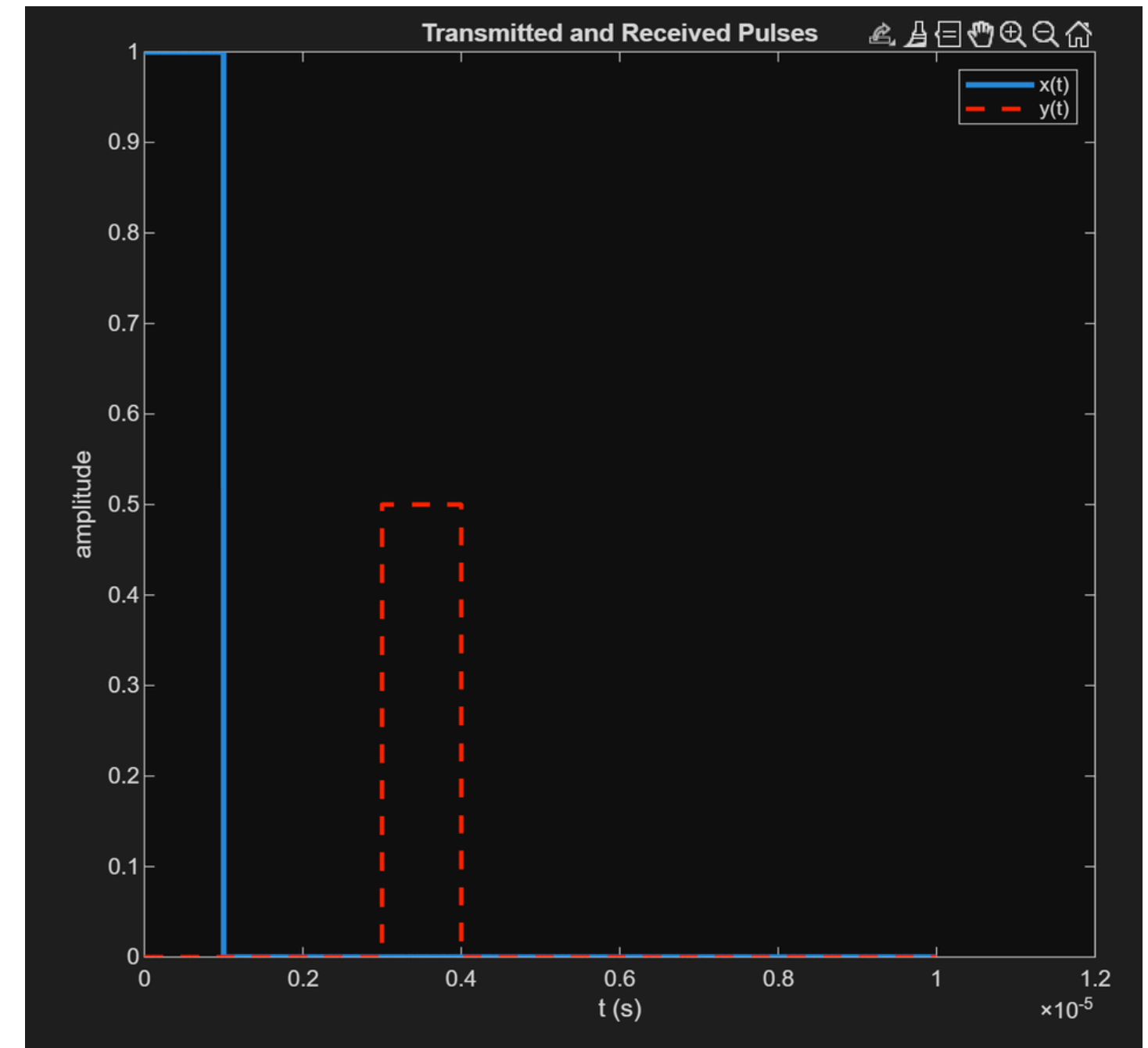
part 3.1:

```
1 clear; clc; close all;
2
3
4 ts = 1e-9;
5 T = 1e-5;
6 tau = 1e-6;
7
8 t = 0:ts:T;
9
10 x = double(t <= tau);
11
12 plot(t, x); xlabel('t (s)'); ylabel('x(t)');
13 title('Transmitted pulse signal');
```



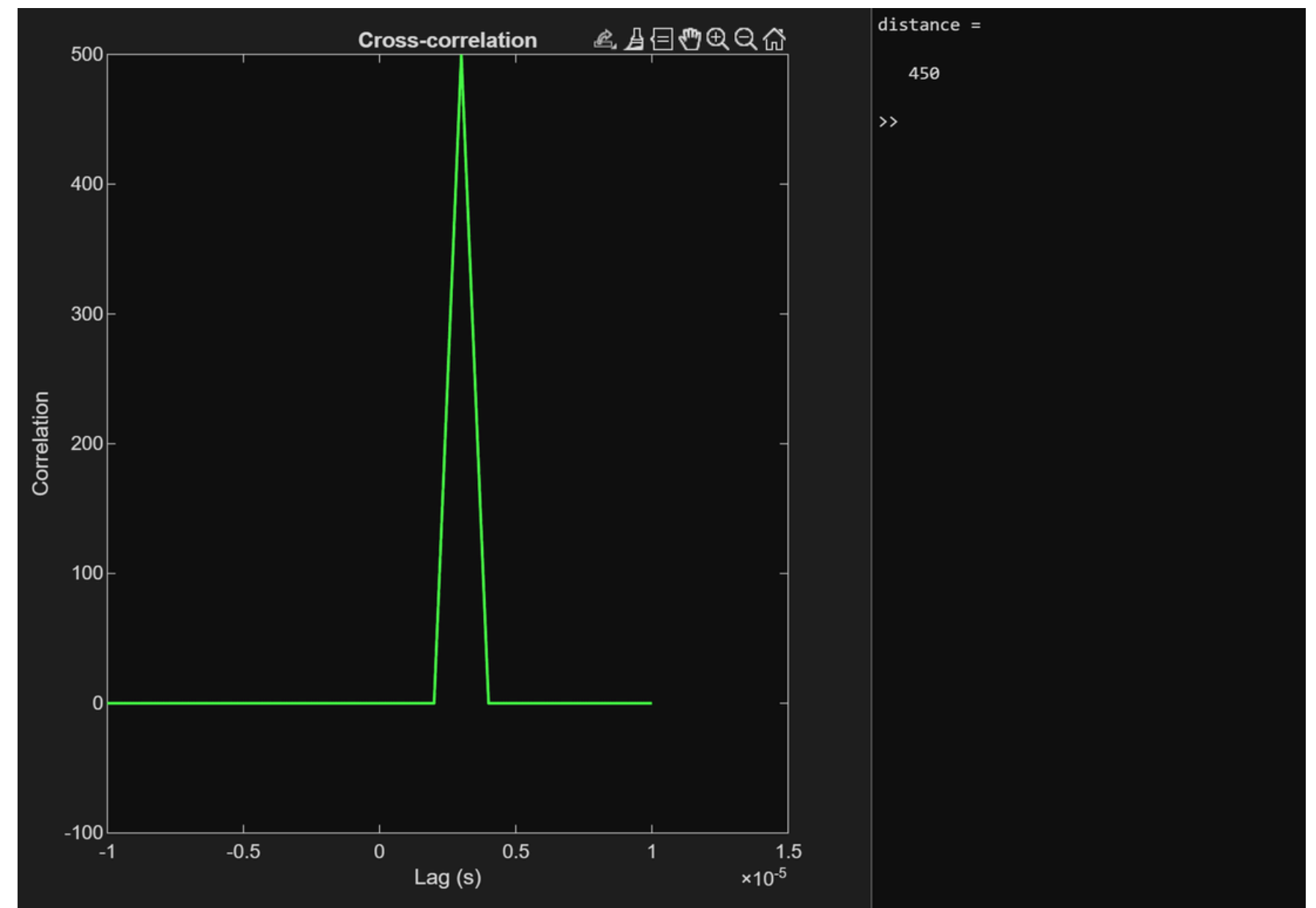
part 3.2:

```
1 clear; clc; close all;
2
3 ts = 1e-9;
4 T = 1e-5;
5 tau = 1e-6;
6
7 t = 0:ts:T;
8
9 x = double(t <= tau);
10 |
11 plot(t, x, 'LineWidth', 2.5); hold on;
12
13 c = 3e8;
14 R = 450;
15 alpha = 0.5;
16
17 delay = 2*R/c;
18 y = alpha * double((t - delay) <= tau & (t - delay) >= 0);
19
20 plot(t, y, 'r--', 'LineWidth', 2);
21 xlabel('t (s)'); ylabel('amplitude');
22 legend('x(t)', 'y(t)'); title('Transmitted and Received Pulses');
```



part 3.3:

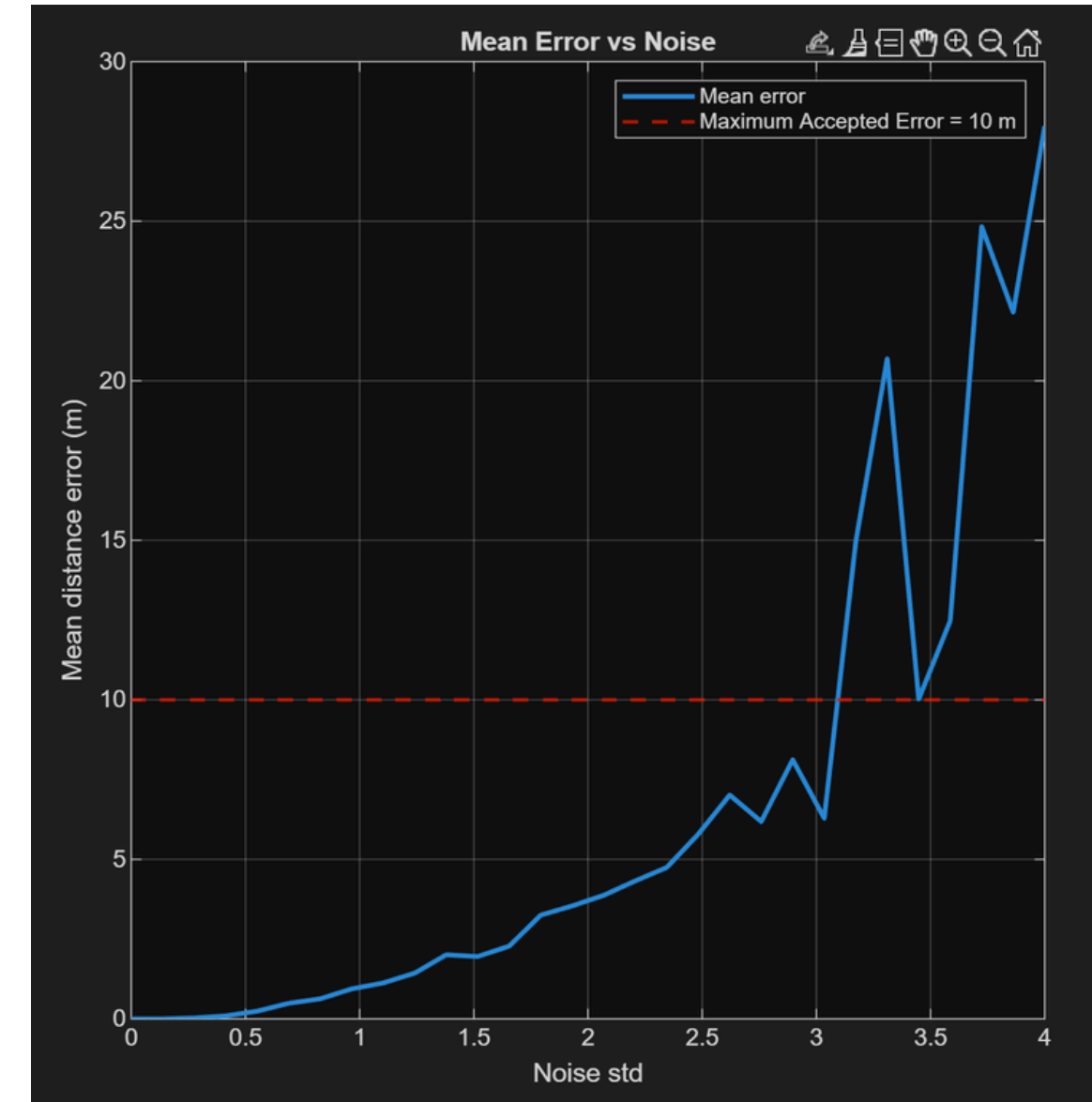
```
1 clear; clc; close all;
2
3 ts = 1e-9;
4 T = 1e-5;
5 tau = 1e-6;
6 c = 3e8;
7 R = 450;
8 alpha = 0.5;
9
10 t = 0:ts:T;
11 delay = 2*R/c;
12
13 x = double(t <= tau);
14 y = alpha * double((t - delay) <= tau & (t - delay) >= 0);
15
16 [r,lags] = xcorr(y, x);
17 [dc,k] = max(r); % peak index
18 delay_hat = lags(k)*ts; % convert to seconds
19 distance = c*delay_hat/2
20
21 plot(lags*ts, r, 'k', 'LineWidth', 1.5, 'Color', 'green');
22 xlabel('Lag (s)'); ylabel('Correlation');
23 title('Cross-correlation');
```



We calculate the cross-correlation between the transmitted and received pulses. The correlation peak shows where the two signals best align — this gives the time delay of the echo. From that delay, the target distance is estimated as **$R = c \times \text{delay} / 2$**

part 3.4:

```
1 clear; clc; close all;
2
3 ts = 1e-9;
4 T = 1e-5;
5 tau = 1e-6;
6 c = 3e8;
7 R = 450;
8 alpha = 0.5;
9
10 t = 0:ts:T;
11 delay = 2*R/c;
12
13 x = double(t <= tau);
14 y = alpha * double((t - delay) >= 0 & (t - delay) <= tau);
15
16 noiseStd = linspace(0, 4, 30);
17 trials = 100;
18 err = zeros(size(noiseStd));
19
20 for i = 1:numel(noiseStd)
21     e = zeros(trials,1);
22     for j = 1:trials
23         y_noisy = y + noiseStd(i)*randn(size(y));
24         [r,lags] = xcorr(y_noisy, x);
25         [~,k] = max(r);
26         delay_hat = lags(k)*ts;
27         R_hat = c*delay_hat/2;
28         e(j) = abs(R_hat - R);
29     end
30     err(i) = mean(e);
31 end
32
33 figure;
34 plot(noiseStd, err, 'LineWidth', 2); hold on
35 yline(10, 'r--', 'LineWidth', 1.5); % accepted error = 10 m
36 grid on
37 xlabel('Noise std'); ylabel('Mean distance error (m)');
38 title('Mean Error vs Noise');
39 legend('Mean error','Maximum Accepted Error = 10 m');
40
```



We gradually increase the noise level and, for each level, repeat the distance estimation 100 times with new random noise. The average error of these trials is taken as the error for that noise level. Finally, the mean error is plotted versus noise strength — as noise increases, estimation accuracy decreases.

From roughly noise std ≈ 3 onward, the error crosses the 10 m limit, meaning accurate distance detection is no longer reliable beyond that noise level.