In the Name of God

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Contents:

An Introduction to Motor Imagery

Motor imagery (MI) is regarded as a pivotal cognitive process with broad implications across disciplines such as sports psychology, cognitive neuroscience, and rehabilitation. It involves the mental simulation of movements without physical execution, providing insights into motor performance enhancement and rehabilitation facilitation. Jeannerod's work (1994, 1995) highlights MI as a conscious access to movement intentions, akin to unconscious motor preparation. This functional equivalence between conscious imagery and unconscious preparation underlies observed motor performance improvements in athletes. Brain areas akin to actual movement are activated through MI, facilitating plastic changes in the motor system. Such activation, crucial in neurological rehabilitation, offers patients sensory experiences similar to physical exertion, aiding recovery and skill re-acquisition without movement. Therapeutically, MI's efficacy is substantiated by its capacity to enhance motor performance and recovery. Its synergy with action observation further underscores its utility in rehabilitation, aligning with empirical evidence and theoretical foundations. The integration of MI into therapeutic protocols holds promise for inducing neural adaptations and improving motor functions.

Figure 1 A diagrammatic representation of the presented BCI system

The importance of MI is further emphasized through its application in clinical settings, such as PTSD and social anxiety disorder. Its therapeutic potential, coupled with neural mechanisms akin to physical movements, underscores its significance. Moreover, the influence of physical training on MI's efficacy, highlighting the specificity of motor representations and the impact of prior physical experiences, accentuates the intricate relationship between imagery and action. Understanding how past experiences shape the brain's ability to simulate future events underscores MI's role in facilitating motor skill acquisition and rehabilitation.

The applications of motor imagery (MI) were examined, with a focus on brain-computer interfaces (BCIs). The impact of the level of abstraction in visual-guided systems on MI-BCI systems was investigated. The processes during motor imagery, changes in user brain activity, brain load, and overall BCI system performance induced by varying levels of abstraction in visual guidance were studied. The study aimed to understand user mechanisms and whether BCI performance is affected by brain activity and mental load. Indicators such as event-related desynchronization (ERD) and synchronization were used to assess the study's objectives. The results demonstrated that user brain activity and mental load during MI tasks were significantly affected by visual guidance with different levels of abstraction. Increased brain activity and reduced mental load for users were observed with lower levels of abstraction in visual guidance. Furthermore, MI-BCI performance was influenced by the level of abstraction in visual guidance, with better classification performance observed with low-abstraction visual guidance. The findings highlighted the correlation between MI-BCI performance, user brain activity, and mental load, suggesting the importance of suitable visual guidance in enhancing BCI usability and functionality.

Challenges of Motor Imagery

Identifying and explaining the primary challenges present in research and applications of motion visualization:

Identifying and explaining the primary challenges present in research and applications of motion visualization encompasses a multifaceted exploration into the intricacies of brain-computer interface (BCI) technology. One of the foremost challenges lies in the recognition of issues such as the non-stationarity inherent in electroencephalogram (EEG) signals, which significantly impacts the consistency of BCI classifiers over time. Additionally, BCI illiteracy poses a substantial hurdle, as users often grapple with achieving the desired classification accuracy. Moreover, the necessity for reducing or eliminating lengthy calibration phases emerges as another critical concern, as these phases can prove cumbersome and disheartening for users, hindering the seamless integration of BCIs into everyday life.

Delving deeper into the realm of motion visualization research and applications, the literature review unravels a tapestry of challenges that researchers encounter. From the complexities of processing non-linear, unstable, and artifact-sensitive EEG signals to the intricate task of integrating diverse datasets characterized by varying sampling rates and electrode configurations, the landscape is rife with obstacles. Moreover, the endeavor to select optimal classification techniques presents an additional layer of complexity, particularly when catering to individuals with brain injuries or illnesses. In such cases, the EEG motor imagery (MI) features may diverge significantly from those observed in healthy individuals, necessitating tailored approaches for accurate classification.

Reviewing potential solutions and current research to overcome these challenges:

In the quest to overcome these challenges, researchers are engaged in a continuous cycle of innovation and exploration. A myriad of potential solutions and ongoing research endeavors aim to pave the way for more robust and effective BCI systems. Adaptive BCI systems, for instance, stand out as a promising avenue, adapting dynamically to changes in EEG signals over time. Similarly, subject-specific optimization techniques hold considerable potential for enhancing classification accuracy by tailoring algorithms to individual users' unique physiological characteristics.

Transfer learning methods represent another frontier in BCI research, leveraging data from diverse subjects to bolster the robustness of classifiers. Concurrently, subject-independent approaches seek to circumvent the need for laborious individual calibration sessions, streamlining the user experience and enhancing accessibility. Furthermore, innovative modifications to training protocols, such as the incorporation of engaging activities and multiple sessions, aim to cultivate users' skills and performance levels.

However, the journey towards overcoming these challenges is not without its hurdles. Researchers must navigate the complexities of developing more efficient training approaches aimed at reducing the calibration time required for BCI utilization. While endeavors such as leveraging covariance matrices associated with common spatial patterns (CSP) features show promise in aiding the decoding of EEG signals, there remains a need to fully harness the geometric properties of these matrices to extract salient information effectively.

Moreover, the quest for more dependable systems with stable performance necessitates a holistic approach to BCI system design. This entails not only refining signal processing techniques and classification approaches but also innovating in the realm of control systems and environmental adaptability. By embracing these challenges head-on and fostering a spirit of collaboration and innovation, researchers can chart a course towards a future where BCIs seamlessly integrate into everyday life, enriching the lives of users across diverse demographics and contexts.

Preprocessing EEG Signals

EEG signals are widely used due to their safety, portability, ease of use, high temporal resolution, and low cost. These signals effectively reflect brain activities, making them valuable for both brain-computer interfaces (BCIs) and clinical applications. One common application of EEG-BCI systems is motor imagery using brain signals, which can significantly benefit both disabled individuals and the general population by enhancing their quality of life. Consequently, processing and analyzing EEG signals is crucial to accommodate various BCI applications. This processing generally involves three main steps: preprocessing, feature extraction, and classification.

EEG signals are gathered using multiple scalp electrodes arranged in a montage, either bipolar or referential. A bipolar montage records differences between neighboring electrodes, while a referential montage compares all electrodes to a single reference. The collected EEG data is represented as 2D tensors with dimensions $C \times T$, where C is the number of channels and T is the number of time samples. Many datasets use referential montage, meaning each channel corresponds to one electrode. EEG signals, which reflect various brain activities, are categorized into frequency bands: delta (1–4 Hz), theta (4–8 Hz), alpha (8–12 Hz), beta (13–25 Hz), and gamma (≥25 Hz). Different EEG paradigms relate to specific tasks or stimuli, such as P300, Motor Imagery (MI), and Steady-State Visual Evoked Potential (SSVEP). For example, imagining limb movement (MI paradigm) causes specific EEG changes useful for tasks like device control.

To effectively process EEG signals, several inherent characteristics must be considered:

Low spatial resolution and low Signal-to-Noise Ratio (SNR): EEG signals are prone to interference and artifacts. Effective signal processing must separate noise from abnormal signals and extract meaningful features.

Dimensionality challenge: With multiple channels, EEG signal acquisition leads to exponentially increasing computational demands as dimensionality increases.

Non-stationariness: The statistical properties of EEG signals change rapidly over time.

Limited labeled training samples: High participant focus is required during data acquisition, making it difficult to gather large amounts of brain data. For instance, frequent visual stimulation during Visual Evoked Potential (VEP) acquisition can cause visual fatigue, resulting in smaller datasets.

Subject-specificity: EEG signals vary significantly between individuals, causing poor stability and generalization. Models trained on specific subjects may not perform well on new subjects.

Furthermore, unlike image processing and natural language processing, we lack detailed knowledge of the brain's physiological activities. This limits our ability to intuitively understand EEG signals or apply pre-existing knowledge to interpret them.

After collecting EEG signals, it's essential to preprocess the data to filter out unwanted noise and simplify computational tasks. Here are some preprocessing techniques we'll discuss next.

Fundamental preprocessing methods are grounded in core EEG signal characteristics. These include filtering, electrode placement, data cleaning, baseline correction, referencing, down sampling, artifact removal, and segment rejection. Filtering is a commonly used preprocessing method. Given the low Signal-to-Noise Ratio (SNR) and various rhythms in EEG signals, band-pass filtering is effective for removing noise of different frequencies and isolating useful rhythms from the source.

Data augmentation is a valuable technique for addressing the challenge of small datasets. It includes both conventional methods like sliding windows, noise injection, and segmentation and recombination, as well as advanced techniques such as Generative Adversarial Networks (GAN) and Variation Auto Encoder. Many models, particularly deep learning models, require abundant training data to achieve high classification accuracy and avoid overfitting. However, due to the inherent characteristics of EEG signals, collecting a large amount of data is challenging. Data augmentation allows for the generation of additional data from a limited dataset, thereby providing sufficient training data for effective model training.

Channel selection is a process aimed at optimizing EEG signal processing. During acquisition, each electrode records a channel, resulting in C channels in the raw EEG data, reflecting the multi-dimensional nature of EEG signals. Different channels correspond to various brain regions, and for specific tasks, some channels may contain irrelevant or redundant information. This increases data size and processing time, potentially reducing the performance of Brain-Computer Interfaces (BCIs). Channel selection involves choosing the most relevant channels from task-related brain regions to enhance performance and efficiency. However, multi-channel EEG data exhibit complex correlations beyond simple adjacencies. Therefore, selection criteria should be based on channel features such as correlation, electrode distance, and task characteristics. This ensures that selected channels retain maximum signal features for effective processing.

Dimensionality reduction techniques are employed to streamline EEG signal processing, as EEG signals are inherently multi-dimensional. Unlike traditional 1D signals, EEG data processing is computationally intensive. To enhance feature extraction and classification robustness, it is essential to apply constraint assumptions that align with the structure of EEG signals and reduce their dimensionality effectively.

There are several algorithms available for dimensionality reduction. For example, Principal Component Analysis (PCA) can decompose EEG signals into linearly uncorrelated components with maximum variance. PCA can effectively separate redundant components, such as interference from eyes and muscles, before reconstructing the EEG signal. Independent Component Analysis (ICA) is another method that separates artifacts from EEG signals as independent components based on data features. However, ICA may inadvertently remove valuable signals as noise if the algorithm is not trained to distinguish noise characteristics.

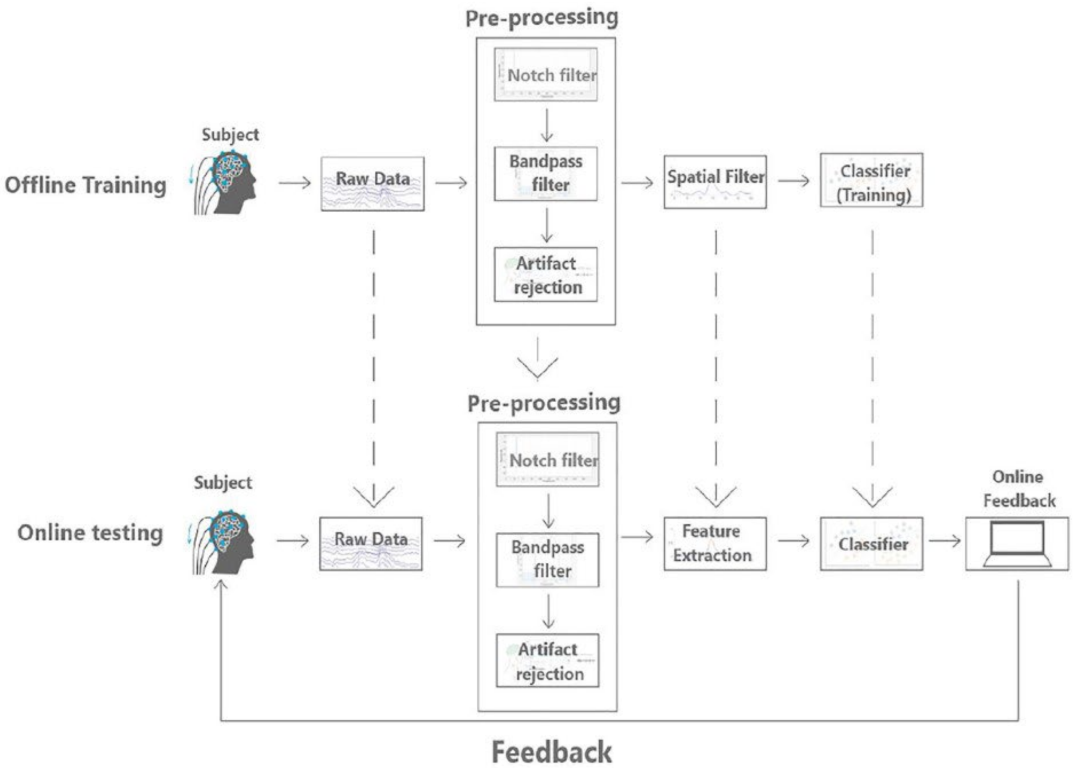

Figure 2 General working of EEG-based BCI

Continuing with the explanation of preprocessing steps using bandpass filters and spatial filters, we will review some of the existing methods.

Band-Pass Filter (8-30 Hz):

The band-pass filter with a range of 8-30 Hz plays a crucial role in isolating specific frequency bands, specifically the mu and beta frequency bands, which are important in the context of motor imagery. Here's an explanation of its roles:

1. ***Separation of Frequency Bands:***
   Mu Rhythm (8-12 Hz): The mu rhythm is a prominent EEG rhythm observed over the sensorimotor cortex, particularly during rest and in the absence of movement. It

attenuates or desynchronizes during motor tasks or motor imagery. Filtering the EEG signal to isolate the mu band (8-12 Hz) allows researchers to focus on these frequency components specifically associated with motor imagery tasks.

Beta Rhythm (13-30 Hz): The beta rhythm is another EEG frequency band that is observed over the sensorimotor cortex and is associated with motor execution and motor imagery tasks. It tends to increase in amplitude during motor planning and execution. Filtering the EEG signal to isolate the beta band (13-30 Hz) allows researchers to focus on these frequency components associated with motor-related cortical activity.

2. ***Enhanced SNR and Signal Specificity:***

   Improving Signal-to-Noise Ratio (SNR): By filtering out unwanted frequencies outside the 8-30 Hz range, the band-pass filter enhances the SNR of the mu and beta bands. This makes it easier to detect and analyze these specific frequency components related to motor imagery.

   Reducing Interference: EEG signals can contain various sources of noise, including power line interference (e.g., 50 or 60 Hz) and muscle artifacts. The band-pass filter attenuates frequencies outside the 8-30 Hz range, effectively reducing these types of interference and improving the clarity and specificity of the mu and beta bands.

3. ***Motor Imagery Analysis:***

   Task-Specific Analysis: During motor imagery tasks, the mu and beta bands exhibit distinct patterns. The mu band typically shows desynchronization (reduction in amplitude) over the sensorimotor cortex, while the beta band may show synchronization (increase in amplitude) depending on the specific motor task or imagery being performed.

   Feature Extraction: Filtering the EEG signal into mu (8-12 Hz) and beta (13-30 Hz) bands facilitates the extraction of features relevant to motor imagery. These features can include spectral power, event-related desynchronization/synchronization (ERD/ERS), and coherence between brain regions, which are used in machine learning algorithms for classification and prediction tasks related to motor imagery.

4. ***Clinical and Research Applications:***

   Brain-Computer Interface (BCI):

   BCIs that rely on motor imagery tasks often use mu and beta band analysis interpret the intentions of the user. Filtering the EEG signal to isolate these bands is crucial for accurate decoding of motor intentions in real-time applications.

   Neurofeedback and Rehabilitation:

   In clinical settings, band-pass filtering helps in designing neurofeedback protocols and rehabilitation programs that aim to modulate mu and beta rhythms for therapeutic purposes, such ad motor recovery after stroke.

Spatial Filter:

The EEG has numerous regular rhythms. The most famous are the occipital α rhythm, the central mu and beta rhythm. People can desynchronize the rhythm and more increase activity by motor imagery. This desynchronization reflects a decrease of oscillatory activity related to an internally or externally-paced event and is known as Event-Related Desynchronization (ERD). The opposite, namely the increase of rhythmic activity, was termed Event-Related Synchronization (ERS). ERD and ERS are characterized by fairly localized topographic and frequency specificity. The ERD/ERS patterns can be volitionally produced by motor imagery, which is the imagination of movement of limbs without actual movement. In general, the EEGs are recorded over primary sensorimotor cortical areas often displays 8–10 Hz (μ rhythm) and 18–26 Hz (β rhythm) activity. Some published paper had shown that people can learn to control the amplitude of μ/β rhythm in the absence of actual movement or sensation. Because μ/β rhythms are included in the ERD and ERS of the brain signals. It can be as part of spatial features for motor imagery. Before feature extraction step, we need a spatial filtering acted on EEG signals. The purpose of the spatial filter is to reduce the effect of spatial blurring from the raw signal. The spatial blurring occurs as effect of the distance between the sensor and the signal sources in the brain, because of the inhomogeneity of the tissues between the brain areas.

1. ***Common Average Reference (CAR):***

   CAR a spatial filter could be considered as the subtraction of the common activity of EEG, which left only the idle activity of each individual EEG in specific electrode (Fig. 1). The potential of each electrode after the filter could be computed as:

$$x_i^{CAR}(t) = x_i(t) - \frac{1}{C}\sum_{j=1}^{C} x_j(t)$$

   Where $x_i^{CAR}(t)$ is the filtered output of electrode ith, $x_j(t)$ is the potential between jth electrode and the reference, C is total number of all electrode on the scalp.

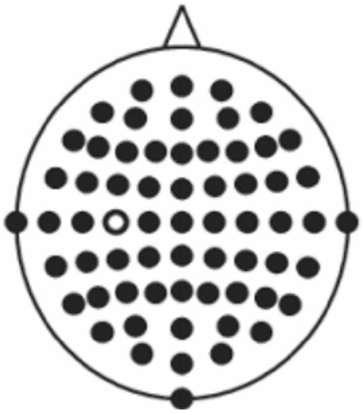

Figure 3 Common average reference (CAR) filter sketch map

2. ***Laplacian Filter:***

   The Laplacian is a 2D isotropic measure that calculates the second spatial derivative of an image. It's typically applied to an image that has been smoothed using a Gaussian filter to reduce noise sensitivity, which also positively impacts signal processing. This method assumes a Gaussian distribution on the scalp surface and tries to reverse the blurring effect that occurs when brain activities are detected on the scalp. The process is further simplified:

$$x_i^{LAP}(t) = x_i(t) - \sum_{j \in S_i} \omega_{ij} x_j(t)$$

Where $x_i^{LAP}(t)$ represents the filtered signal of electrode i, and $x_i(t)$ is the potential of electrode i compared to the reference electrode. The constant weight ω_{ij} is calculated:

$$\omega_{ij} = \frac{\frac{1}{d_{ij}}}{\sum_{j \in S_j} \frac{1}{d_{ij}}}$$

with d_{ij} being the Euclidean distance between electrode i and electrode j. S_i is the set of neighboring electrodes around the central electrode i. The layout of these neighboring electrodes can be small or large, as shown in Figures 2 and 3. Typically, the neighborhood size includes 4 electrodes chosen vertically and horizontally, with x_i being the central electrode.

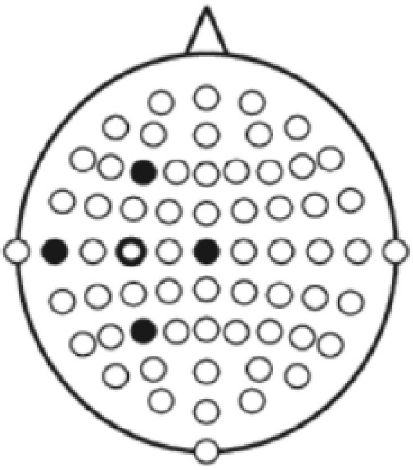

Figure 4 Large Laplacian

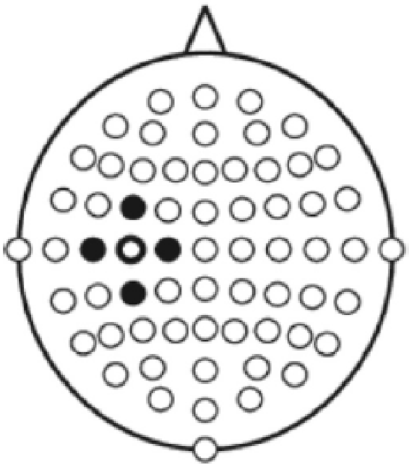

Figure 5 Small Laplacian

3. ***Independent Component Analysis (ICA):***
   Independent Component Analysis (ICA) is a computational technique used for separating a multivariate signal into additive, independent components. This method is widely applied in various fields, including biomedical signal processing, particularly for analyzing electroencephalography (EEG) data. In the context of EEG, ICA is employed to isolate and remove artifacts (such as those caused by eye movements, muscle activity, and line noise) and to identify underlying brain sources that contribute to the recorded signals. By decomposing the EEG data into statistically independent components, ICA enhances the accuracy and interpretability of neural signals, facilitating better analysis and understanding of brain activity. It is especially effective in identifying and removing artifacts from EEG data and in isolating brain-related components. ICA assumes that the observed EEG signals are linear mixtures of unknown, statistically independent source signals. By leveraging this assumption, ICA separates these mixed signals into their independent components. This process enhances the quality of EEG data, making it

easier to study and interpret the underlying neural dynamics. we can define the measured signals $x_i(t)$ as a linear combination:

$$x_i^{ICA}(t) = \sum_j a_{ij} S_j$$

where S_j are independent components or sources and a_{ij} are some weights. Similarly, we can express sources S_i as a linear combination of signals x_i:

$$S_i = \sum_j \omega_{ij} x_j$$

Where ω_{ij} are weights. Using matrix notation, source signals S would be equal to $S = WX$ where W is a weight matrix, and X are measured signals. To estimate one of the source signals, we'll consider a linear combination of x_i signals. Let's denote that estimation with y:

$$y = \omega^T X$$

where ω is a weight vector. Next, if we define $z = A^T \omega$ we have that:

$$y = \omega^T X = \omega^T AS = z^T S$$

From the central limit theorem, $z^T S$is more Gaussian than any of the S_i and it's least Gaussian if it's equal to one of theS_i. It means that maximizing the non-Gaussianity of $\omega^T X$ will give us one of the independent components.

4. ***Minimum Norm Estimation (MNE):***

Minimum Norm Estimation (MNE) is a powerful technique used in the field of neuroimaging to estimate the distribution of neural activity within the brain from electroencephalography (EEG) data. Unlike methods that focus solely on the scalp-recorded signals, MNE seeks to reconstruct the underlying sources of these signals in the brain, providing a more detailed and accurate picture of neural activity. By solving an inverse problem, MNE helps to map the EEG signals back to their origin in the brain, making it possible to localize and visualize brain activity with higher spatial resolution. This approach is particularly useful for understanding the brain's functional organization and for clinical applications such as epilepsy diagnosis and brain-computer interfaces. Express the EEG signal X as a linear combination pf neural source activities S and the lead field matrix:

$$X = LS + N$$

Where N is the noise. To fine the most likely distribution of neural activity, minimize the norm of the source activity S. This is done by solving the following optimization problem:

$$\hat{S} = arg\min_S \left\{ \left| |X - LS| \right|^2 + \lambda \left| |S| \right|^2 \right\}$$

Where λ is a regularization parameter that controls the trade-off between fitting the data and minimizing the source activity norm. Compute the inverse operator W that transforms the EEG data into source estimates:

$$W = (L^T L + \lambda I)^{-1} L^T$$

Apply the inverse operator to the preprocessed EEG data to estimate the source activity:

$$\hat{S} = WX$$

By following these steps, MNE provides a detailed estimate of the spatial distribution of neural activity in the brain from EEG recordings, enhancing our ability to study brain function and diagnose neurological conditions.

5. ***Principal Component Analysis (PCA):***

Due to the instability of signal collection methods, we collected signal on the scalp have mixed with many interfering signal and have high dimensions. Principal component analysis (PCA) is a statistical method that transforms a set of potentially correlated variables into a set of linearly uncorrelated variables known as principal components, using an orthogonal transformation. Consider a random vector x with components $x = (x_1, x_2, \ldots, x_n)^T$ and a population mean denoted by $\bar{x} = E(x)$. The covariance matrix of this data set is defined as $\Sigma = E[(x - \bar{x})(x - \bar{x})^T]$. The elements of Σ, denoted by Σ_{ii}, represent the covariance between the random variables x_i and x_j. The diagonal elements Σ_{ii} are the variance of the corresponding variables x_i. The covariance matrix is symmetric, and the variance of a component indicated the spread of its values around the mean. If two components x_i and x_j are uncorrelated, their covariance Σ_{ij} is zero. From a sample of vectors $x_1, x_2, \ldots, x_N$, the sample mean and sample covariance matrix can be calculated as estimates of the population mean and covariance matrix. For symmetric matrix like the covariance matrix, we can find an orthogonal basis by computing its eigenvalues and eigenvectors. The eigen vectors e_i $(for\ i = 1, \ldots, k \leq n)$ and their corresponding eigenvalues λ_i are the solutions to the characteristic equation. This orthogonal basis provides the principal components, transforming the original data into a new set of uncorrelated variables.

$$\Sigma_i e_i = \lambda_i e_i$$

If the data vector has n components, the characteristic equation is of order n, which is straightforward to solve only when n is small. Finding the eigenvalues and corresponding eigenvectors is a complex task, and there are various methods to achieve this. This transformation is structured so that the first principal component captures the maximum possible variance in the data, representing the greatest amount of variability. Each subsequent principal component then captures the highest variance possible, with the constraint that it must be orthogonal to all previous components. Suppose we have a data set for which the sample means and the covariance matrix have been calculated. Let A be a transformation matrix composed of the eigenvectors of the covariance matrix as its row vectors. By applying this transformation to an EEG data set, converting it into a vector, we obtain the output y, which represents the data after dimensionality reduction.

$$y = A(x - \bar{x})$$

The component of y can be seen as the coordinates in the orthogonal base. We can reconstruct the original data vector x from y using the property of an orthogonal matrix $A^{-1} = A^T$.

$$x = A^T y + \bar{x}$$

The original vector x was projected on the coordinate axes defined by the orthogonal basis. The original vector was then reconstructed by a linear combination of the orthogonal basis vectors. Instead of using all the eigenvectors of the covariance matrix, we may represent the data in terms of only a few basis vectors of the orthogonal basis. If we denote the matrix having K first eigenvectors as rows by A_K^T, we can create a similar transformation as:

$$y = A_K(x - \bar{x})$$
$$x = A_K^T y + \bar{x}$$

This means that we project the original data vector on the coordinate axes having the dimension K and transforming the vector by a linear combination of the basis vectors. This minimizes the mean-square error (MSE) between the data and this representation with the given number of eigenvectors.

Feature Extraction Techniques

In this part, we aim to identify the best features of EEG signals for classifying them into different classes with the highest accuracy. To understand these features and their corresponding algorithms, we must first learn some terms related to EEG signals that is useful in BCI systems.

The event-related desynchronization (ERD) & event-related synchronization (ERS) are prompted by performing mental tasks, such as motor imagery, mental arithmetic, or mental rotation. In the case of the motor imagery paradigm, the mu (8–13Hz) and beta (14–30Hz) rhythms of the sensorimotor cortex are used. During physical and motor imagery of right and left hand movements, beta band brain activation β-ERD happens predominantly over the contralateral left and right motor areas and a β-ERS ipsilaterally. The post movement ERS related to ceasing to move can also be found over the contralateral motor areas.

Steady-state visual evoked potentials (SSVEP) is a response to a visual stimulus modulated at a frequency greater than 6 Hz, which can be read from the occipital area.

P300 is a localized response to a joined visual, auditory, or tactile stimulus, and it is mainly read from the parietal lobe during 300ms after starting of the stimulus.

Some of the features we use include variance and energy, entropy and logarithmic band power (LBP).

Now, we will examine algorithms and their features based on papers from recent years.

Common Space Pattern Algorithm (CSP):

CSP algorithm is a spatial filtering method, which is to find the optimal projection direction in space, so that the variance of the projection signal of one kind in this direction reaches the maximum, and the variance of the projection signal of another kind reaches the minimum. The CSP algorithm entails conducting eigenvalue decomposition on the matrix to derive the whitening matrix. This matrix is subsequently employed to perform a whitening operation on the two standardized covariance matrices. The algorithm then identifies the projection space that maximizes the variance difference between the two whitened matrices.

Now we want to talk about the mathematics logic. Calculating the projection matrix involves creating a set of common spatial pattern filters. The algorithm begins by determining the normalized spatial covariance for both classes, using the following equations.

$$C_{c1} = \frac{E_{c1}E_{c1}^T}{tr(E_{c1}E_{c1}^T)} \qquad C_{c2} = \frac{E_{c2}E_{c2}^T}{tr(E_{c2}E_{c2}^T)}$$

where ECI and ECII denote two single trials, under two conditions classes, of size $n \times T$, where n is the number of channels and T is the number of samples per channel.

The overall composite spatial covariance is given by two formula that one of them factorized into eigenvalues and eigenvectors.

$$C_c = \overline{C_{c1}} + \overline{C_{c2}}$$
$$C_c = U_c \lambda_c U_c^T$$

U_c is the matrix of eigenvectors and λ_c is the diagonal matrix of eigenvalues. Note that Eigenvalues are arranged in descending order. Following this, the whitening transformation p is calculated.

$$p = \sqrt{\lambda_c^{-1}} U_c^T$$

There are three principals that we can test our calculation.

$$S_{c1} = B\lambda_{c1}B^T$$
$$S_{c2} = B\lambda_{c2}B^T$$
$$\lambda_{c1} + \lambda_{c2} = I$$

S_c is calculate from:

$$S_{c1} = P\overline{C_{c1}}P^T \qquad S_{c2} = P\overline{C_{c2}}P^T$$

B is any orthonormal matrix:

$$B^T(S_{c1} + S_{c2})B = I$$

This indicates that eigenvalues for one class will be maximized at a point whereas the other class will have eigenvalues minimum at that same point. Thus, the covariance between the two classes is successfully maximized.

The first CSP filter w_1, provides the maximum variance for class I, while the last CSP filter w_{ch}, provides the maximum variance for class II. So, we have

$$W_{CSP} = P^T B = [w_1 \quad w_2 \quad \dots \quad w_{ch}]$$

Note that $W_{CSP} \in R^{ch \times ch}$.

The filter signal is s(t). In this processing we need first and last m filters so filter dimension change to $W_{CSP} \in R^{2m \times ch}$. We have:

$$s(t) = W_{csp}e(t) = [s_1(t) \quad \dots \quad s_d(t)]^T$$

where $e(t)$ is the signal to be reduced, d is called reduction number and equal to $2m$. The reduction number is the number of channels that desired to reduce. For each class's EEG sample matrix, we will select only a small number of signals m, that are most important for distinguishing between the two classes. The feature vector is:

$$f_i = \log\left(\frac{var(s_i(t))}{\sum_{i=1}^{2m} var(s_i(t))}\right)$$

$$f_i = (f_1, f_2, \dots, f_{2m})^T$$

Now we can extract another feature that we want like:

Energy $Energy = \sum_{n=1}^{N} |S(n)|^2$

Entropy $Enteropy = \sum_{n=1}^{N} |S(n)|^2 \log|S(n)|$

Logarithmic band power $LBP = \log\left(\frac{1}{N}\left(\sum_{n=1}^{N} |S(n)|^2\right)\right)$

For all feature above, d features were obtained for each trial.

The primary steps of feature extraction involve the following: acquiring the mean-normalized spatial covariance matrix for the initial 80 sets of EEG data related to left and right hand motion imagination; obtaining the combined spatial covariance matrix for both types of motion imagination data. Subsequently, the projection matrix is derived. Decomposition of 160 EEG data sets was conducted to obtain the eigenvector matrix for each dataset. The variance of each row in the eigenvector matrix for every dataset was computed as the EEG feature.

Figure 6 Before and after using CSP

Another transformation called "Wavelet Transformation" exists, which complements the CSP algorithm.

Wavelet Transformation

Wavelet transforms (WT) to analyze various transient events in biomedical field. They have described that WT is suitable for nonstationary signals and has advantage over spectral analysis. It is useful because it has accuracy in frequency information at low frequence and accurate in time domain at high frequence.

To enhance EEG recognition accuracy, the Wavelet Packet Transform dissects the signal's low and high-frequency components in finer detail. This decomposition method, unlike the Wavelet Transform, offers superior time-frequency localization analysis without redundancy or omission, particularly beneficial for vibration signals encompassing medium to high-frequency data. Its fundamental concept revolves around spatial decomposition.

Frequency content in the EEG signal provides useful information as compared to time domain. The mother function (n) is convolved with the signal $x(n)$. Its function is given by formula, where a is called as scale coefficient and b is called shift coefficient. Formation of mother wavelet is important because when it is fixed then it is easy to understand signal at possible coefficients a and b.

$$w_{\varphi}X(a,b) = \sum_{n}^{N-1} x(n)\varphi * (n - \frac{b}{a})$$

We use WT to make better filter. It`s a highly effective method for decomposing a signal into multiple frequency sub-bands is available. First, we analyze the high-frequency components. In the second step, we increase the scale by a factor of two (decreasing the frequency by a factor of two), and in this process, we analyze the behavior around half the maximum frequency.

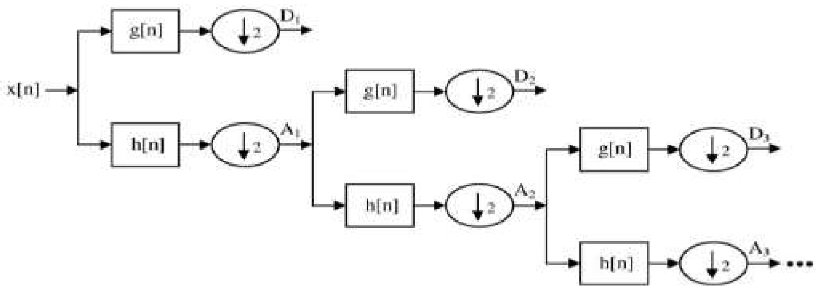

Figure 7 Wavelet decomposition process

Wavelet provides features such as maximum, minimum, mean and standard deviation coefficient of each sub-band. WT is used in the detection of mental tasks such as resting, multiplication, figure rotation and letter composition. So, it can help us in BCI analyzing. the frequency range of EEG data is (0~50) Hz. In the EEG signal analysis of left and right hands motor imagination, a wavelet packet decomposition relation was selected according to the ERD/ERS phenomenon and each component was decomposed at the same level with the same sampling rate and data length, so as to facilitate the extraction of μ and β EEG. We continue decomposing till we gain Maximum Decomposition Level.
Now, we examine whether these two mentioned methods are related to each other or not.
on the basis of analyzing channels and frequency bands closely related to event desynchronization, wavelet packet decomposition was carried out for EEG signals to extract the activity imagination EEG co-rhythms and beta rhythms. Spatial filtering was carried out to extract features through the CSP algorithm, and then the related nodes were selected to calculate the wavelet packet energy. When we talk about EEG signals, we only think about each channel`s energy. But one of the important features is correlation of energy between channels. By using two mentioned tools, the correlation information between different channels can be fully utilized.

Support Vector Machine (SVM):

SVM is useful because of their good prediction accuracy and ability to process large amounts of data. SVM are established and developed on the basis of computational learning theory. The principle of SVM is to map the indivisible data in the low-dimensional space to the high-dimensional space, and obtain an optimal surface through calculation, so as to linearly separate the samples. SVM offers strong generalization and nonlinear mapping abilities, though the selection of kernel functions and parameters can be challenging. Given its widespread use, there is limited scope for improving the single SVM algorithm in the field of EEG signal analysis for motion imagination.
You can observe the classification system in the figure below.

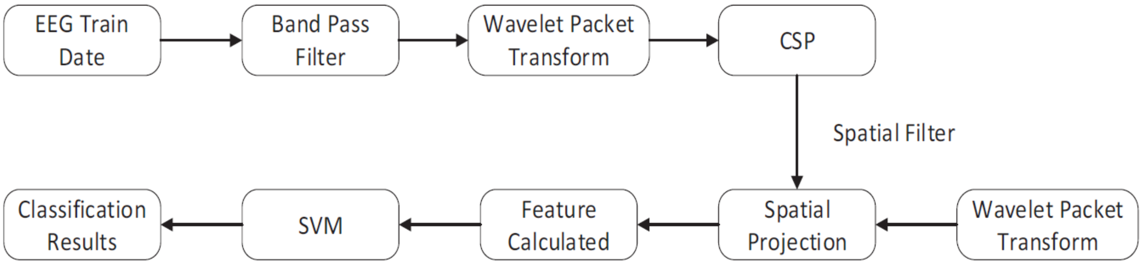

Figure 8 Classification's system

The CSP method and the MT method have accuracies of 75 percent and 83.5 percent, respectively. However, combining them gives us the best accuracy (93.75%).
So, it seems the best feature is to use CSP and WT. We can use linear discriminant analysis (LDA) and artificial neural network (ANN).

Now we want to talk about other features that use in recent project.

ICA vs. CSP:

As it was mentioned in the previous section, ICA tries to find a way to separate the original independent sources using a linear transformation (mixing matrix). The goal is to make the transformed data as independent from each other as possible, meaning the components should have minimal correlation. ICA works on the principle that the sources are non-Gaussian (not following a normal distribution), which helps in taking advantage of their statistical independence.
It`s feature can relate to Spatial Patterns or Statistical Measures or even ERB that mention in CSP method. The most important feature is Spatial patterns in MI. It represents the distribution of neural activity across electrodes. They directly relate to the brain regions involved in MI. Features derived from these patterns can be powerful discriminators.
ICA`s accuracy is in range of 70 to 90%. We observe the accuracy of CSP can be same as ICA. However, for MI, CSP is more effective because it has lower computational complexity and is optimized for detecting spatial patterns in the signals, making it simpler and faster.

There is some method that only work special condition. Example:

1. **Time frequency distribution**: we have Transient signal but it`s work for stationary signals. This method has accuracy for clean signal but we know that EEG is not clear signal.
2. **Autoregressive:** It has a good frequency resolution but has 2 problems.
   I. AR method will give poor spectral estimation once the estimated once the estimated model is not appropriate, and model's orders are incorrectly selected.
   II. It is readily susceptible to heavy biases and even large variability.
3. **Fast Fourier transform:** Like TFD it is useful for stationary signals and it suffers from large noise sensitivity, and it does not have shorter duration data record.

As we see there are many methods for extracting features that have own advantages and disadvantages. But it seems CSP + MT is the best method to get high accuracy and better feature in compare to another one. In the next phase, we will implement the code to identify the best methods mentioned.

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