Linear Control Systems
Estimate black box and design controller
Fall 2024
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This file contains the report and results of the simulations conducted.

Abstract

In the analysis and design of controllers for systems, the first step is having an appropriate mathematical model of the system. This allows us to thoroughly examine the key characteristics of the real system through the model. For different systems, a specific model does not necessarily exist, and various models can be defined based on the understanding of the system and its application. Moreover, in some cases, due to insufficient knowledge in that domain or lack of access to system details, it is not possible to define a model based on the physics of the system. In such situations, the system must be identified using its input and output data.

In industry, a model for the system is not always available. Therefore, it is necessary to first identify the system using various methods based on the conditions and estimate a model, which can then be used for required analyses and designs. For a system, its model can be estimated by examining its impulse response, step response, response to sinusoidal inputs at different frequencies, and so on. Depending on the system, only some of these methods may be applicable.

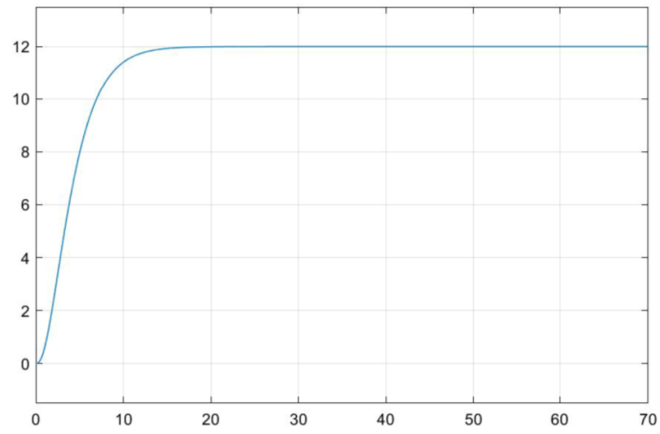
In cases where a model is estimated solely based on input and output data, the resulting model is referred to as a "black-box" model of the system. This method has diverse applications, including in machine learning techniques that operate solely based on data. It is also used in control systems for system identification.

In this project, we are dealing with a system for which internal details are not accessible. Initially, we aim to derive a model for this system using our knowledge of linear systems in the time and frequency domains. Then, we will analyze this model, design an appropriate controller based on it, and finally, connect the controller to the original system and evaluate its performance.

G(s) Estimation

In this section, we begin with system identification. We apply a constant value along with a unit step signal as the input and examine the output.

The output result is as follows:



First, by analyzing the output, we attempt to fit a first-order system to it.

$$G_1(s) = \frac{k}{Ts + 1}$$

Given that the steady-state value is 12, we consider the estimated transfer function of the system to be in standard form and set its gain to k=12.

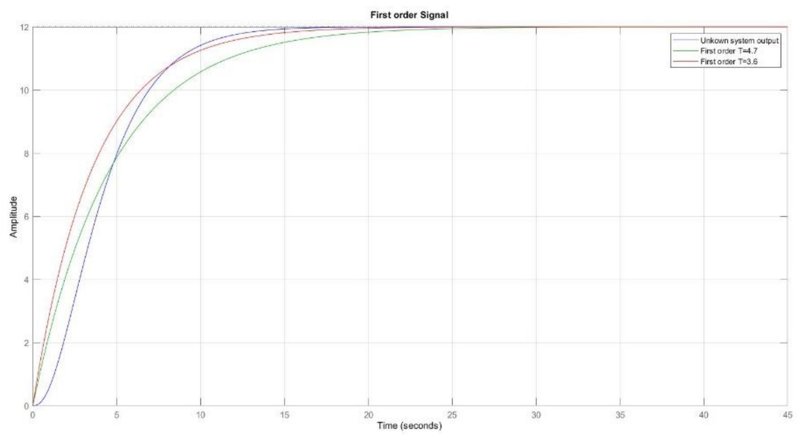
Then, noting that 63% of the steady-state value occurs at time T, we initially determine T=4.7s. By examining various values of T between the time constants for 63% and 98%, we estimated the best-fitting transfer function and finalized T as 3.6 seconds.

$$T = 4.7 \; for \; Amplitude = \frac{63}{100} * 12(steady - state \; value)$$

$$T = 3.04 \; for \; Amplitude = \frac{98}{100} * 12(steady - state \; value)$$

Note that we choose T from the plot. The result is:

$$G_1(s) = \frac{12}{3.6s + 1}$$



Next, we proceed to estimate the second-order transfer function of the system. Since there is no overshoot, we assume the damping ratio (ζ) to be greater than or equal to 1.

$$G(s) = k\,\frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

If ζ = 1, the following relationship holds:

$$c(t) = 1 - e^{-w_n t}(1 + w_n t)$$

Using the output plot of the black-box system and setting ζ=1, we select a specific time and its corresponding magnitude to determine the value of w_n.

Then, based on the standard form of the second-order transfer function and the fact that our poles have real values, the transfer function can be decomposed accordingly.

$$G(s) = \frac{2.4}{s^2 + 0.9s + 0.2}$$

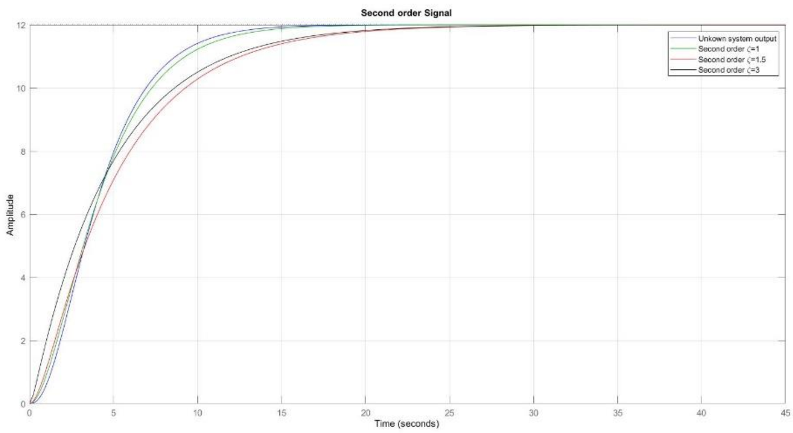
If ζ > 1, the relationship is given by:

$$c(t) = 1 - e^{-\left(\zeta - \sqrt{\zeta^2 - 1}\right) w_n t}$$

For two different values of ζ, we select a specific point to determine the corresponding w_n. For example we have:

$$G(s) = \frac{3.63}{s^2 + 1.65s + 0.3025}$$

The results of all three estimated transfer functions compared with the black-box output are as follows:



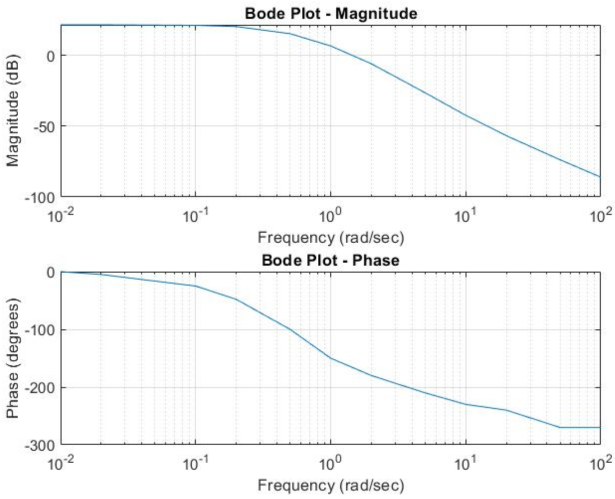
After analyzing the plot, the best-estimated transfer function is as follows:

$$G_2(s) = \frac{2.4}{s^2 + 0.9s + 0.2}$$

Then we apply a sinusoidal input to the system to estimate its transfer function based on the information, similar to constructing a Bode plot. A sinusoidal input with different frequencies is fed into the system, and the output magnitude and output phase are measured. The results are as follows:

ω	Amplitude	Phase
0.01	12	0
0.02	12	-5
0.1	11.52	-25
0.2	10.39	-48
0.5	5.831	-100
1	2.155	-150
2	0.5061	-180
5	4.78e-2	-210
10	7.639e-3	-230
20	1.450e-3	-240
50	2.055e-4	-270
100	5e-5	-270

Based on the obtained results of magnitude, phase, and their corresponding frequencies, we plot the Bode diagram as shown below:



Based on the diagram, it can be understood that at low frequencies, the slope is zero, indicating a first-order system. Moreover, from both magnitude and phase, we determine that the difference between the zero and the pole is 2. Additionally, by further analyzing the magnitude and phase, we observe that another pole is introduced at mid frequencies.

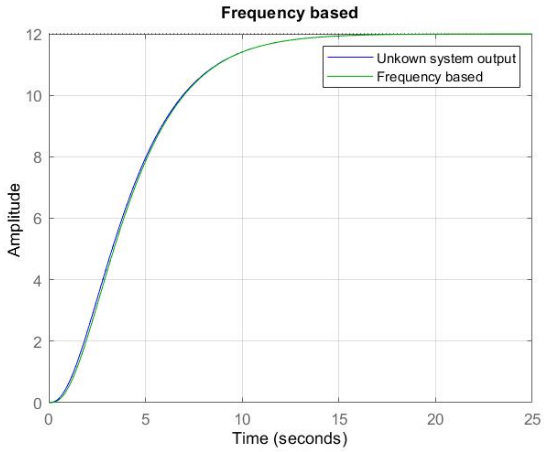
Thus, our system has three poles. Using cftool, we analyze the break frequencies using the magnitude formula.

The breakpoints are:

$$\omega_1 = 1.971,\ \omega_2 = 1.975,\ \omega_3 = 0.5455$$

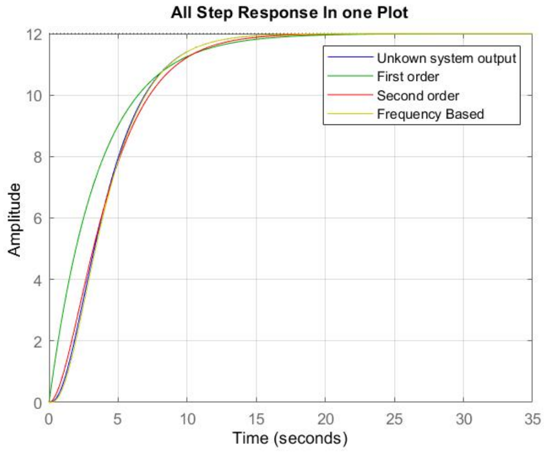
The system's step response and black box are as follows:

$$G_3(s) = \frac{12}{2.123\,s^3 + 6.045\,s^2 + 4.492\,s + 1}$$



Time Characteristics of The Estimated Systems

All estimated signals are shown in the plot below.



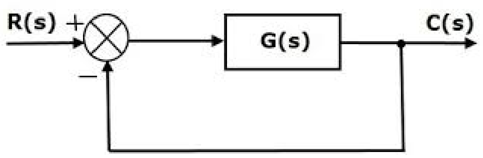
The time characteristics of the estimated systems:

System	t_s	os	t_r
Black Box	17.83	0	7.33
First order	14.08	0	7.9
Second order	13.20	0	7.57
Bode based	12.12	0	6.78

Root Locus

In this section, we plot the root locus of the negative feedback systems to determine their stability for different gain values.

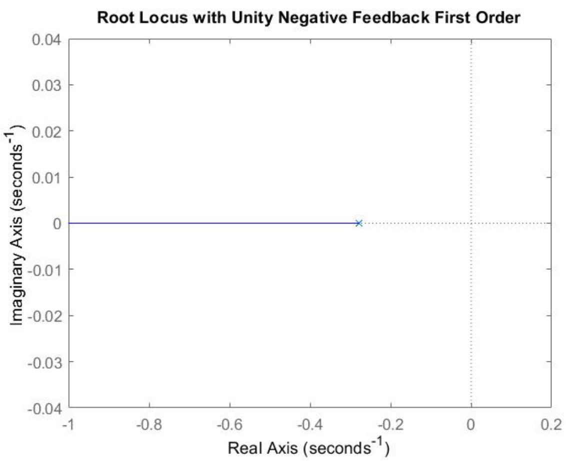
We know that the structure of negative feedback systems is as shown below.



Moreover, the root locus is determined based on the following relationship.

$$1 + kG(s) = 0$$

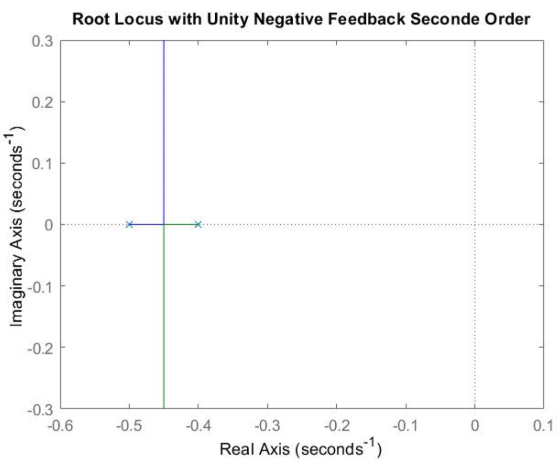
For the first-order system, we have:



We can assume that for all positive K, we have stable system.

$$K > 0 \;\; for \; G_1(s)$$

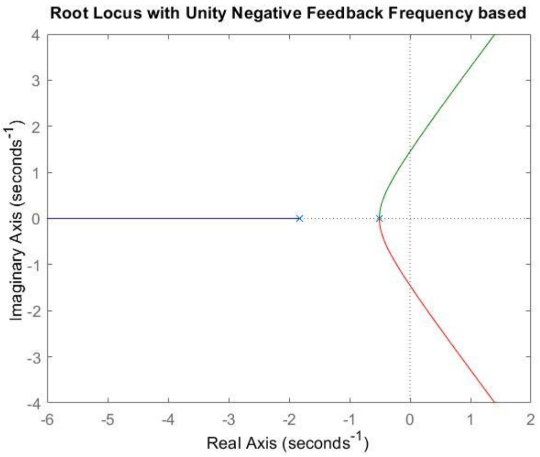
For the second-order system, we have:



Again, we can assume that for all positive K, we have stable system.

$$K > 0 \;\; for \; G_2(s)$$

For the Bode base system, we have:



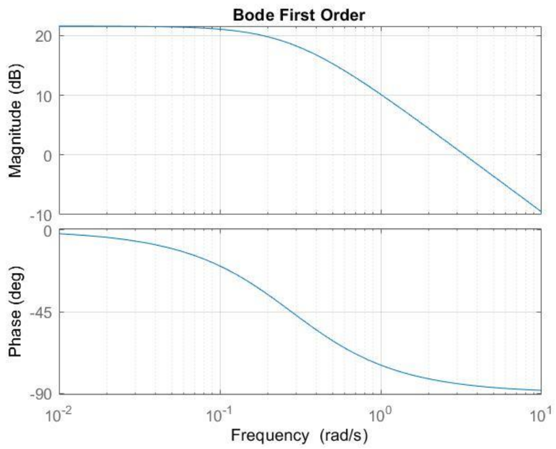
From the diagram, we observe that the system has an upper limit for positive K since the poles move to the right half of the imaginary axis. The range of K should be as follows:

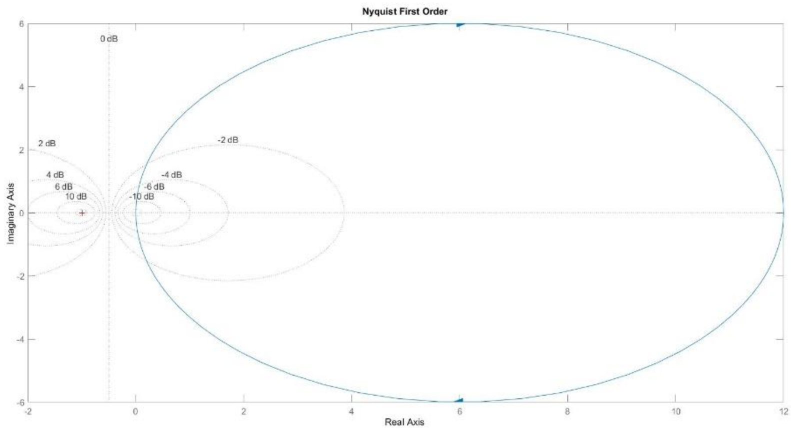
$$0.9 > K > 0 \quad for\ G_3(s)$$

Bode & Nyquist

In this section, we analyze the Bode and Nyquist plots, as well as calculate the phase margin, gain margin, and the range of different K values for system stability.

For the first-order system, we have:



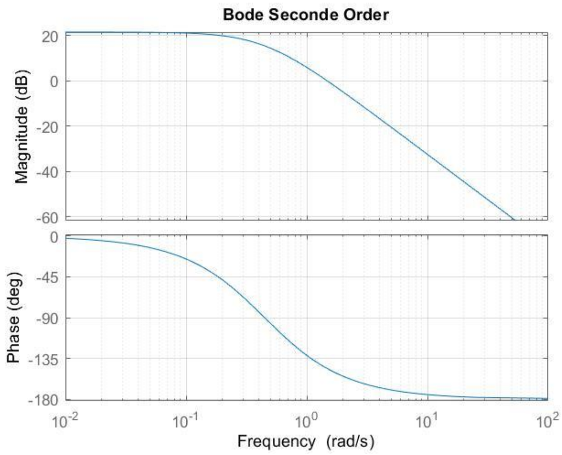


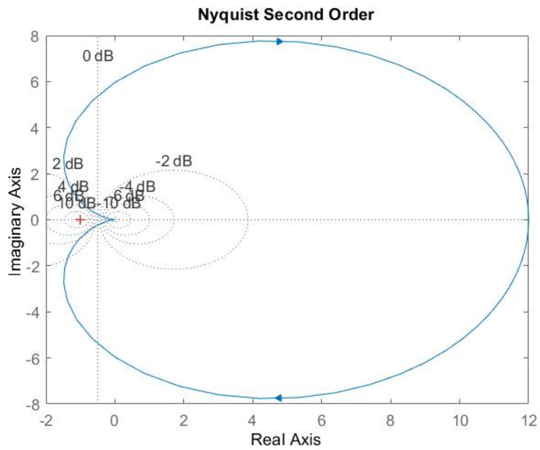
We know that multiplying the gain by the Nyquist plot causes contraction or expansion of the plot. According to the Nyquist criterion, if we vary k from 0 to infinity, the system remains stable. However, if we consider negative values of k, we find that stability is maintained for k values greater than $-\frac{1}{12}$.

In the root locus, since we are considering k>0, we can conclude that the stability condition holds and the solution is correct.

The gain margin in this system is infinite because the phase never reaches 180 degrees, which ensures that the system is inherently stable for k>0. On the other hand, the phase margin is 94.78 degrees, which indicates that the system is fully stable.

For the second-order system, the behavior is exactly the same as for the first-order system. If k ranges from 0 to infinity or is greater than $-\frac{1}{12}$, the system remains stable.





The gain margin of this system is infinite, and the phase margin is 33.75 degrees. This indicates that the system is stable, with a sufficiently large margin for both gain and phase.

For first order we have:

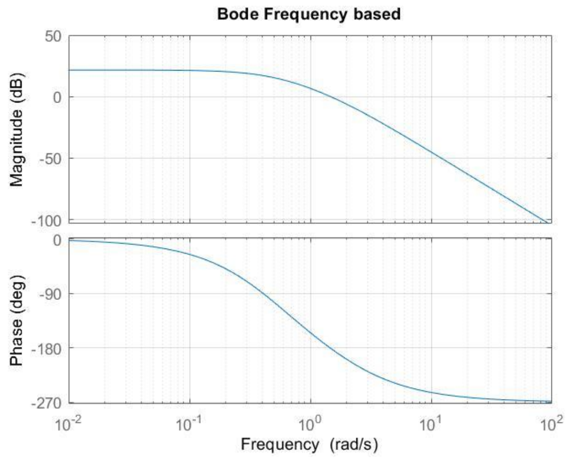
$$k > 0 \; and \; k > -\frac{1}{12}$$

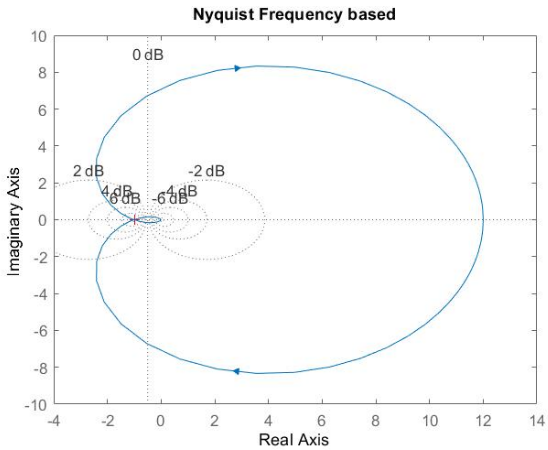
For second order we have:

$$k > 0 \; and \; k > -\frac{1}{12}$$

Note that $k > -\frac{1}{12}$, use for the negative gain.

For third transfer function we have:





According to the Nyquist diagram, the range between 0 and 0.9 is also part of the stable region.

Design Controller based on Root Locus

Now, we design controllers for the three estimated systems to satisfy the following requirements:

1. Overshoot = 10%
2. Settling time (2%) = 10 seconds

First, we calculate the damping ratio (ζ) and natural frequency (ω_n) based on these two conditions to determine the desired pole and zero locations.

$$os = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$t_s = \frac{4}{\zeta\omega_n}$$

Given the 10% overshoot and the corresponding relationship, the damping ratio (ζ) is calculated as $\zeta = 0.5911$ and $\omega_n = 0.6766$. So, the desired poles are:

$$s_{1\&2} = -0.4 \mp 0.54557j$$

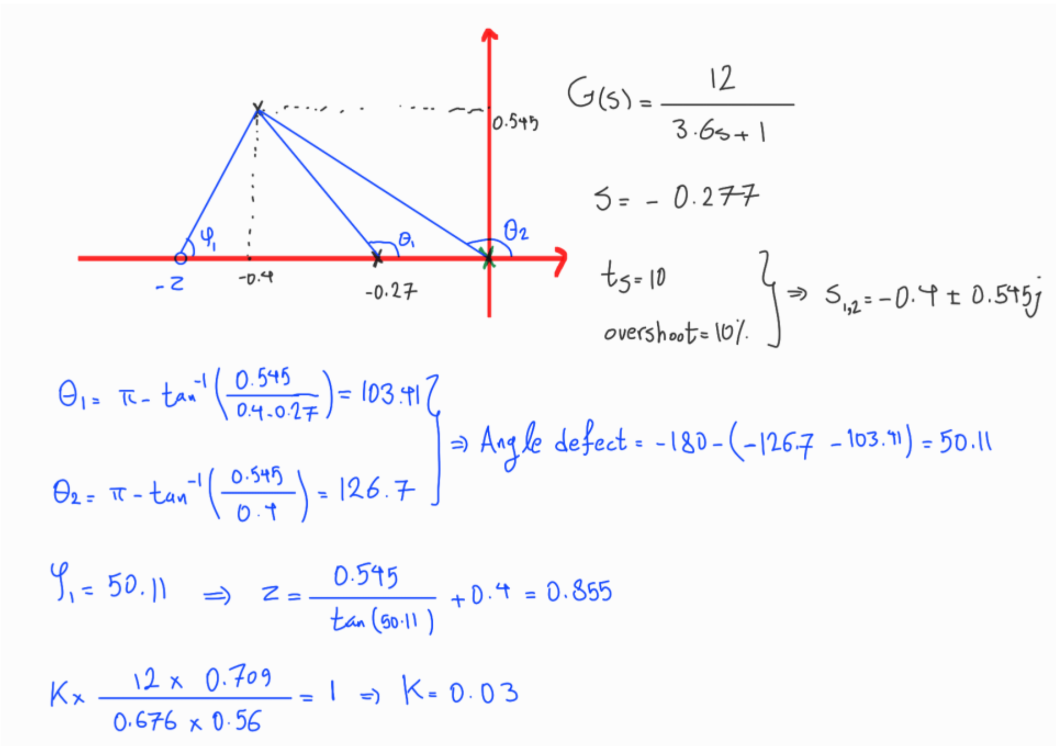
Now based on this we start to design controller base on these poles.

First order estimate

To eliminate the step response error, we first convert the system into a first-order system by placing a pole at zero. Then, we address the angle deficiency.

We know that the sum of the angles between the zero, pole, and the desired pole must be -180 degrees. Based on the calculations, the angle deficiency is 50.11 degrees. Considering this, we place a zero in the controller. The location of the zero is determined by the following calculations.

Additionally, the system's magnitude must be equal to 1, from which we calculate K.

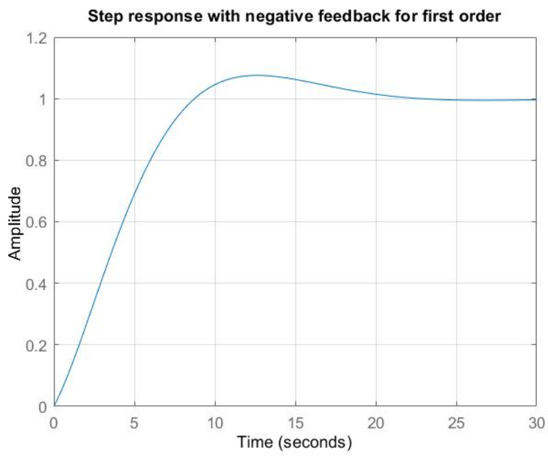


The controller is given by:

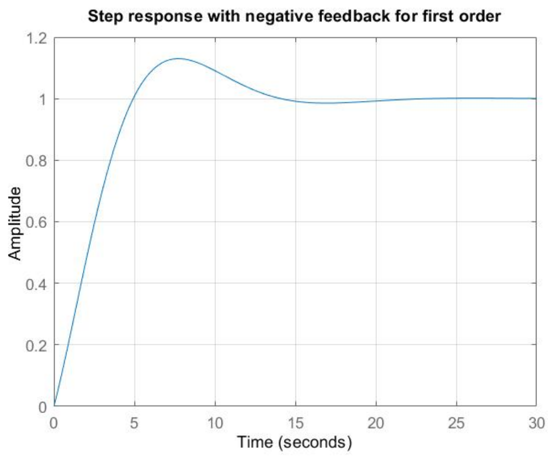
$$G_{c1}(s) = 0.03 \times \frac{s + 0.855}{s}$$

The system response with unit feedback is shown below, where the overshoot and settling time are not as desired. Therefore, some adjustments to K and Z are needed to achieve the desired outcome. Before making these adjustments, we design a controller for overshoot = 8% and $t_s = 8s$, but still, the desired result is not achieved.

Result for first design is:



As you see $os = 13\%$ and $t_s = 19.28s$. And for the second design we have:

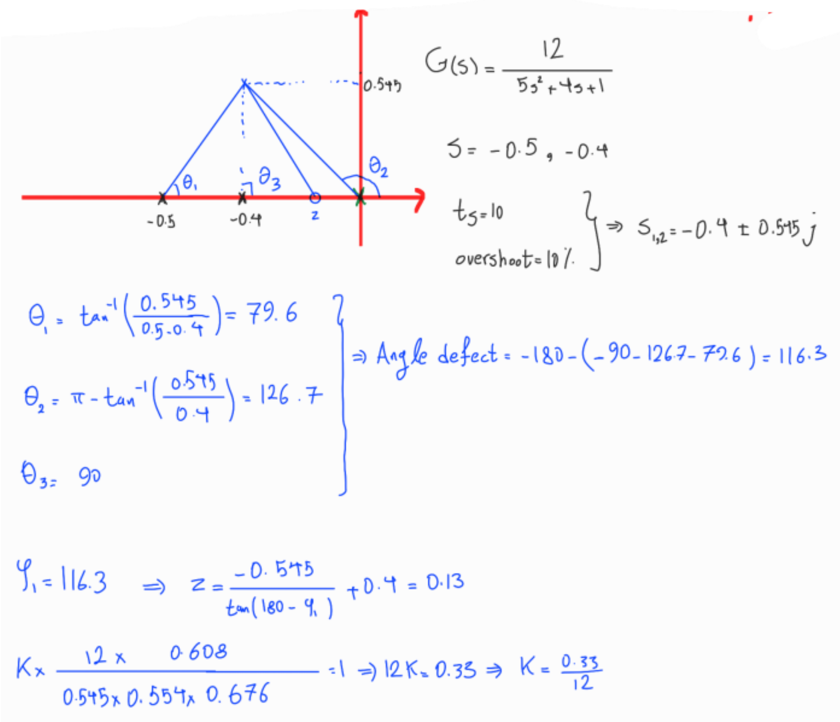


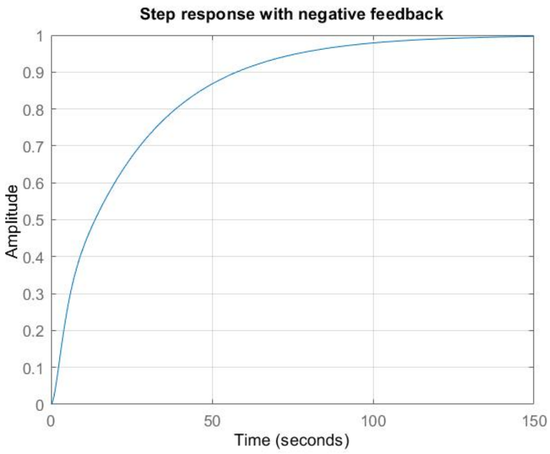
In this design we have $os = 13\%$ and $t_s = 12.828s$.

So, we experimentally adjust the zero location and gain to achieve the desired outcome. Based on the result below, by increasing K and shifting the zero to the right, we can control the system to meet the desired specifications.

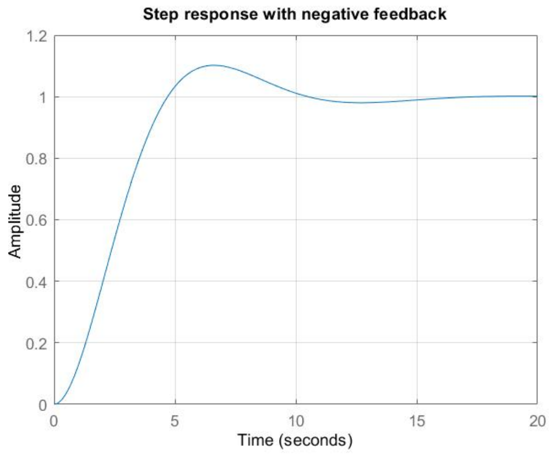
Second order estimate

Now, we proceed with designing the controller for the higher-order system. Based on the previous reasoning, we place a pole at zero to eliminate the step response error. Then, we calculate the angle deficiency and determine the zero location and controller gain.





As before, the negative feedback system response does not fully meet our requirements. Therefore, we experimentally adjust the parameters until we achieve the desired outcome. The result is:



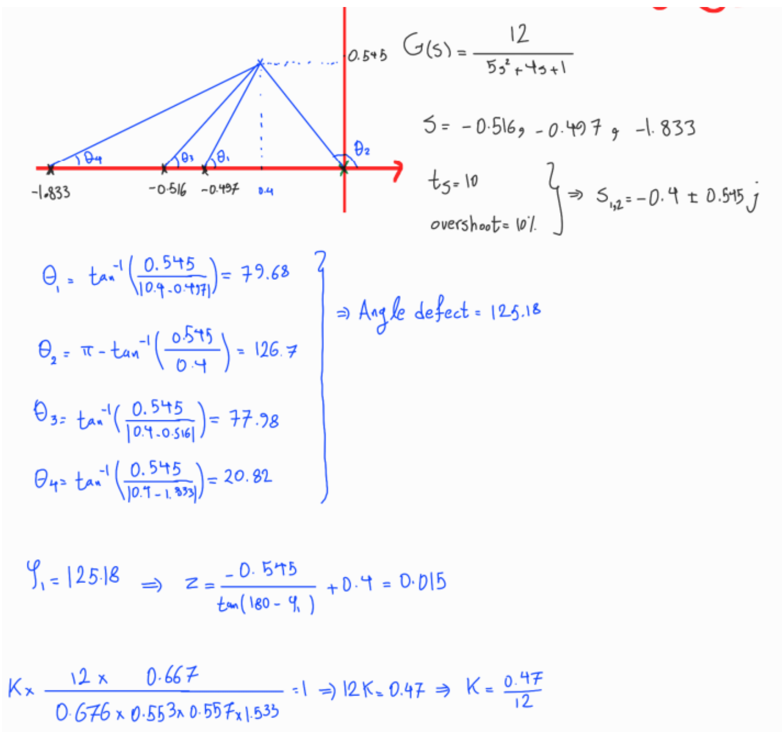
The $os = 10.17\%$ and $t_s = 9.682$.

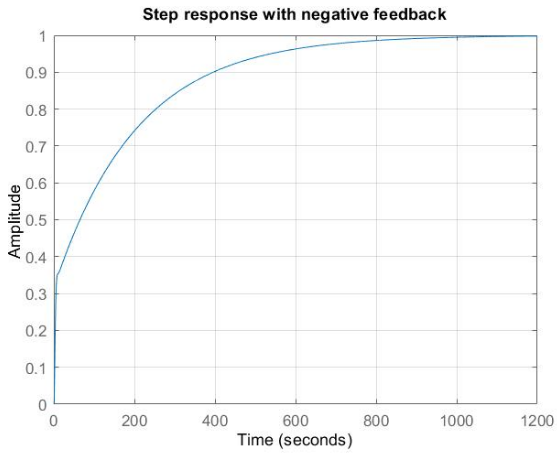
So, second controller is

$$G_{c2}(s) = 0.13 \times \frac{s + 0.3}{s}$$

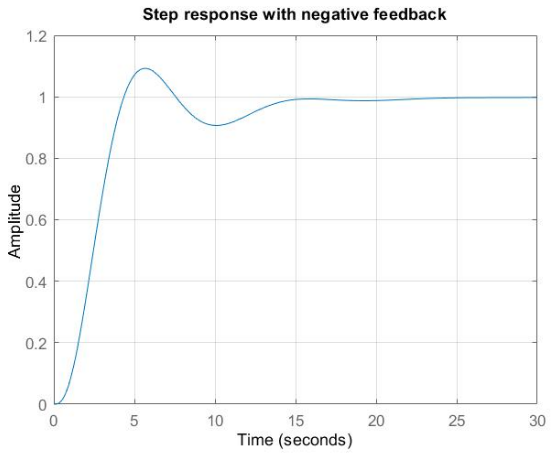
Bode based estimate

Finally, we design the controller for the last system. Based on the calculations (as shown below), the final result still does not fully meet our desired specifications.





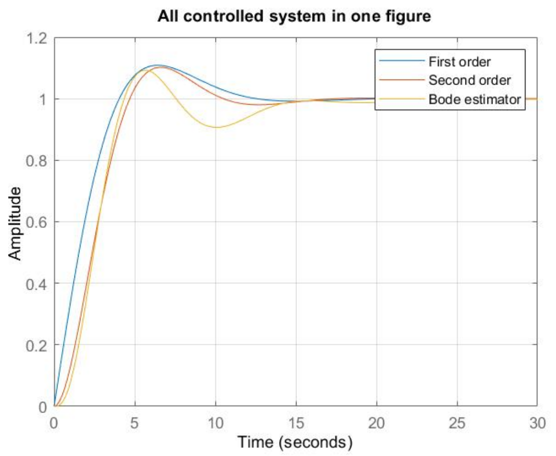
Additionally, we solve the problem experimentally to achieve the desired response. We have:



$$G_{c3}(s) = 0.15 \times \frac{s + 0.2}{s}$$

The $os = 9.24\%$ and $t_s = 13.853$.

The results of all systems with their respective controllers are as follows:



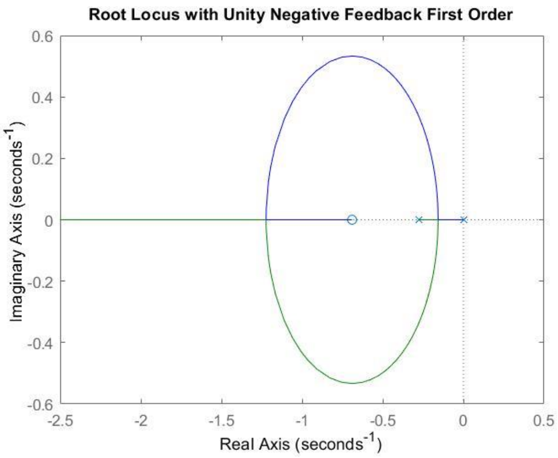
Their transient and steady-state characteristics are also shown below.

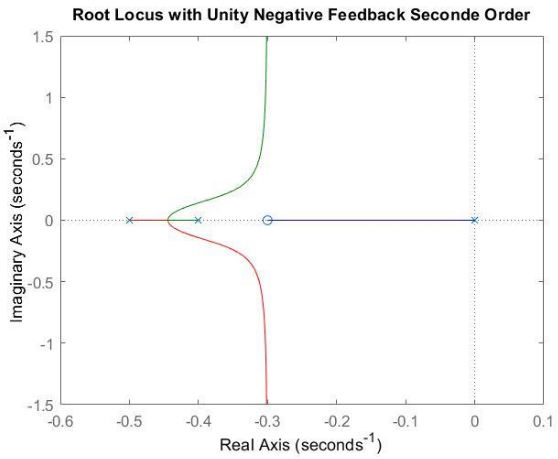
System	t_s	os %	t_r
First order	10.93	10.84	3
Second order	9.68	10.17	3.15
Bode based	13.82	9.24	2.69

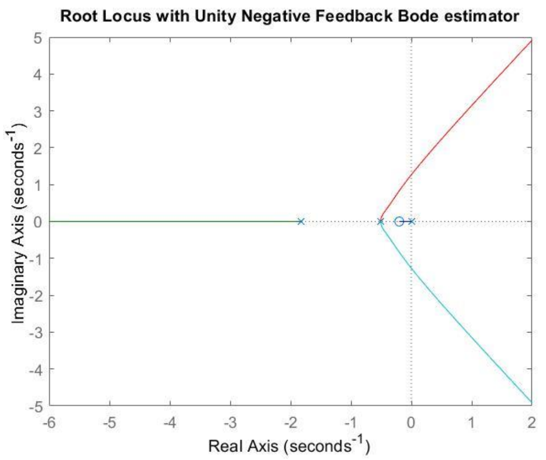
For the first- and second-order systems, due to their simplicity, we were able to design effective controllers. However, for the third-order system, we had to find a good trade-off, and the results are presented in the table.

Root Locus of Controlled System

The Root Locus plot results for each system are shown below:





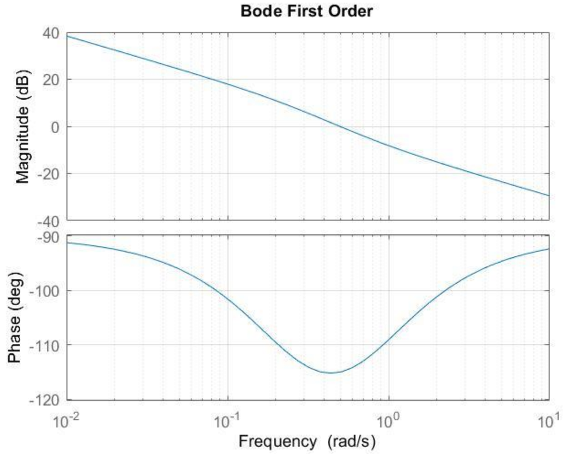


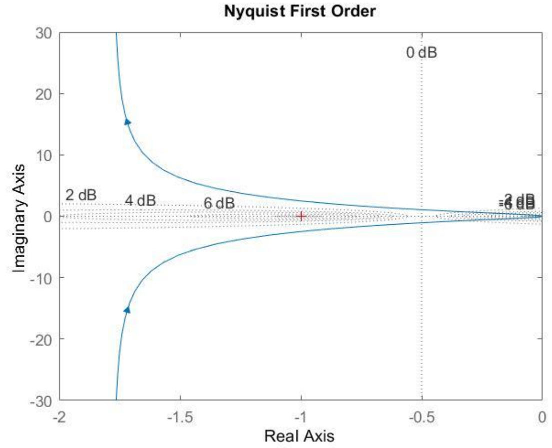
The first two systems are stable for all positive values of K, but the third system must be within the following range to remain stable.

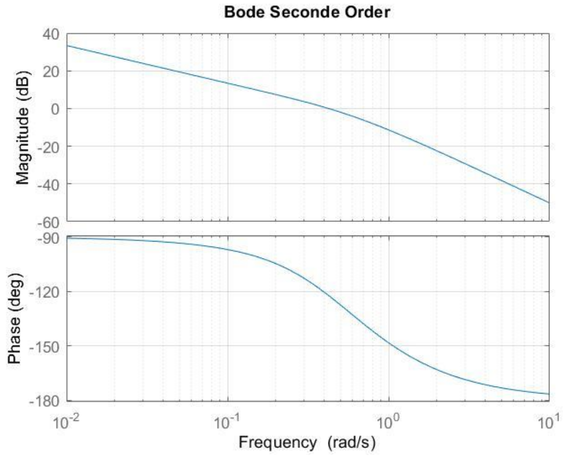
$$4.8 > k > 0$$

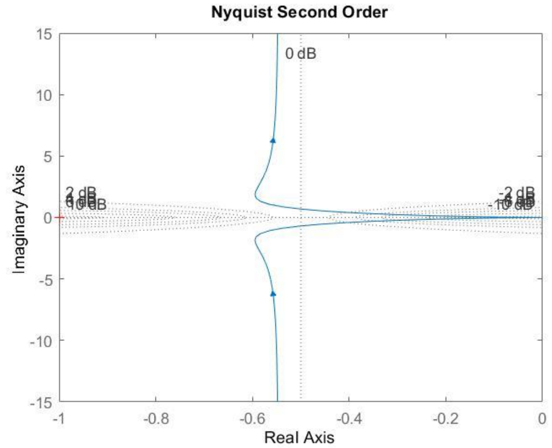
Bode & Nyquist

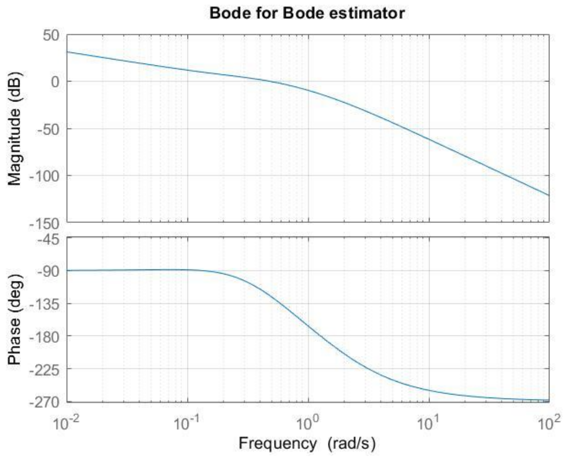
Additionally, the Bode and Nyquist plots for each system are shown below, with each labeled accordingly in the images.

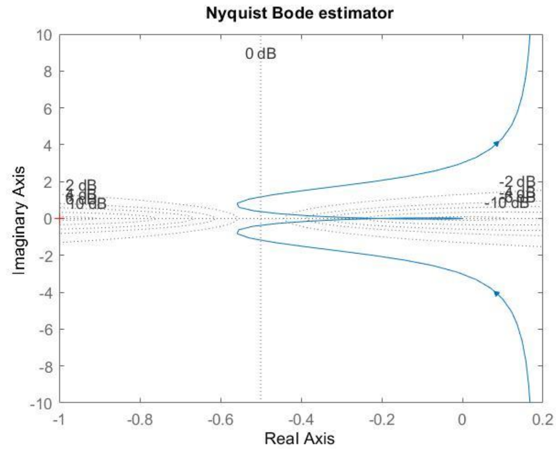






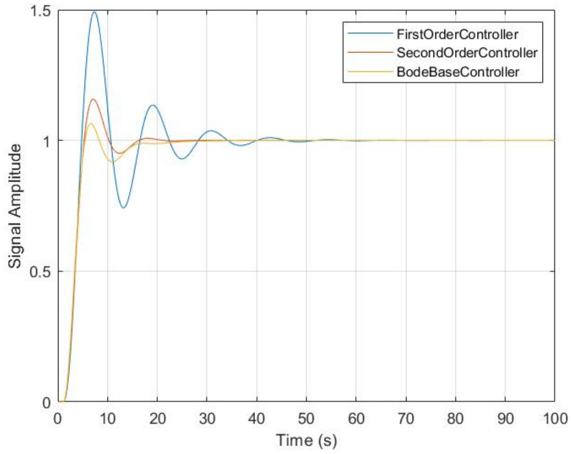






Black Box controller

Now, the designed controllers are connected to the black box and placed in unit feedback. The results are as follows:



According to the diagram obtained from the three controllers, the best controller for the black box is the one designed for the second-order system. This is because it is closest to the desired overshoot, and its t_s is also closer compared to the others.

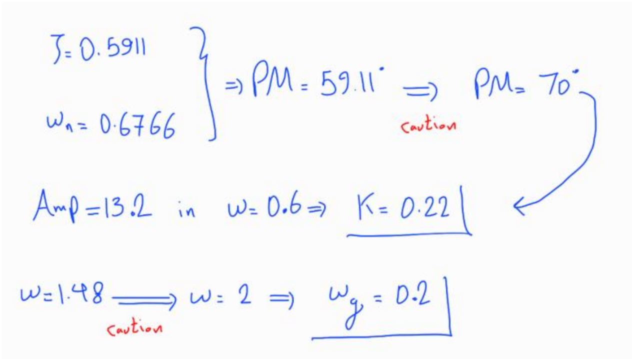
So, we choose this system ($G_2(s)$) for the last part.

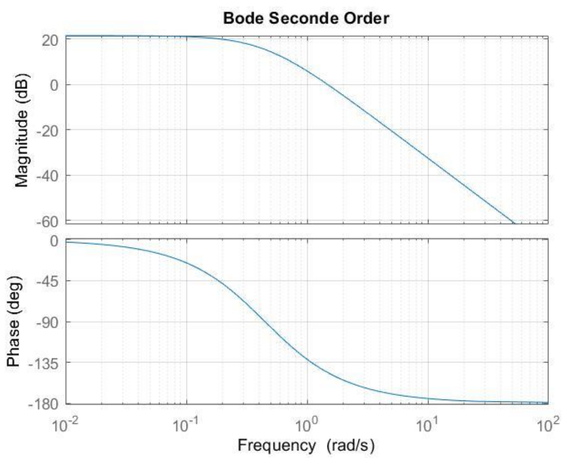
Design Frequency based Controller

In this section, we estimate the best system that, with the designed controller, was able to achieve the closest desired response.

First, we analyze the transformation of the desired requirements into the frequency domain, determining the phase margin and cutoff frequency. Then, we analyze the Bode plot of the estimated system. As observed, we use 0.48 Hz as the cutoff frequency.

So, we have to apply K on the system to reduce the amplitude of that frequency. For precaution, we consider slightly larger values during calculations to achieve the precise required system and adequately address the dominant pole-zero issues.

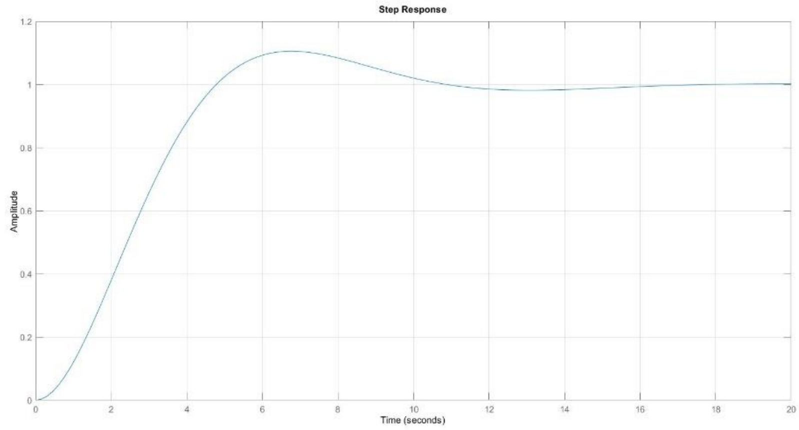




The result of the calculation is:

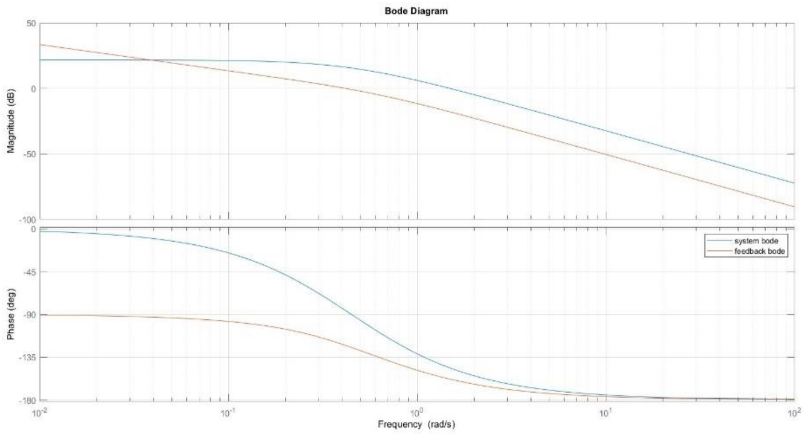
$$G_{fc} = 0.125 \times \frac{s + 0.31}{s}$$

The negative feedback result is:

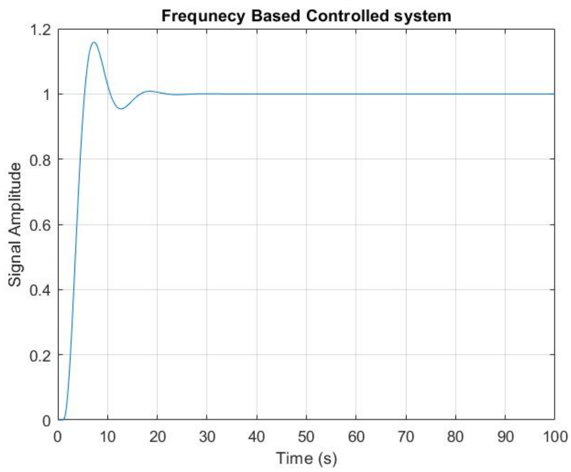


The $os = 10.52\%$ and $t_s = 10.002$s.

Additionally, by comparing the Bode plots before and after applying the controller, we can understand how the stated modifications affect the plot and, consequently, the system's step response.



Now, we connect the controller to the black box to see how well it brings us closer to our desired response.



As we can see, both the overshoot and the settling time have changed.

$$t_s = 15s \;\&\; os = 15\%$$

Conclusion

As we observed in this project, system estimation can be performed using various methods. However, it still presents many challenges because, without precise knowledge of the system's poles and zeros, designing a controller while accurately considering the influence of each zero and pole on the output is difficult. Nevertheless, we have seen that it is possible to achieve the closest possible response.