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Multi-loop PI controller design based on the direct synthesis for interacting multi-time delay processes

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ABSTRACT

In this article, a new analytical method based on the direct synthesis approach is proposed for the design of a multi-loop proportional-integral (PI) controller. The proposed design method is aimed at achieving the desired closed-loop response for multiple-input, multiple-output (MIMO) processes with multiple time delays. The ideal multi-loop controller is firstly designed in terms of the relative gain and desired closed-loop transfer function. Then, the standard multi-loop PI controller is obtained by approximating the ideal multi-loop controller using the Maclaurin series expansion. The simulation study demonstrates the effectiveness of the proposed method for the design of multi-loop PI controllers. The multi-loop PI controller designed by the proposed method shows a fast, well-balanced, and robust response with the minimum integral absolute error (IAE).

1. Introduction

Multi-loop proportional-integral (PI) controllers, which are sometimes called decentralized PI controllers, have been widely utilized for processes with modest interactions for many decades, because of their many practical advantages such as their simple control structure, fewer tuning parameters, robustness against sensor/actuator failure, and easy understandability. Hence, many multi-loop design methods have been reported in the process control literature. However, most of the existing design methods are based on the extension of single-input, single-output (SISO) PI controller design methods.

The modification of the Ziegler–Nichols (Z–N) method [1] by the introduction of a detuning factor to meet the stability and performance requirements of multi-loop control systems is typical of such approaches. In the family of modified Z–N methods [2–8], the desired critical point has to be determined by identifying the critical gain and frequency and, then, the multi-loop controllers are tuned by the Z–N tuning method with a weighting factor. However, a common disadvantage of these methods is that they try to cope with the interaction effect by means of detuning, while neither dynamic nor static interactions are incorporated in the design stage.

Another widely used approach is the extension of single-loop relay tuning to the multiple-input, multiple-output (MIMO) case [9–11]. This approach is straightforward, because it directly combines single-loop relay auto-tuning and sequential tuning, wherein the multi-loop control system is tuned sequentially loop by loop, closing the *i*th loop when it has been tuned and the *j*th loop needs to be opened [9]. However, output responses are likely to deteriorate when the MIMO system has large multiple time delays, which is one of the main causes of strong dynamic interactions.

It is well known that the internal model control (IMC) method [12] is very effective for the design of IMC-PID controllers while taking into account time delays. Recently, several methods [13–15], which extend the IMC-PID method of the SISO case to the MIMO case, have been reported with promising results.

Multiple time delays often occur in complex multivariable processes such as high purity distillation columns [16], pilot plant distillation columns [17], and complex side-stream columns/stripper distillation columns [18]. This is because the complexity of dynamic interactions arises from the relation between the various input/output variables, so that the dynamics of an individual loop cannot fully cope with the closed-loop dynamic interactions. In such cases, fully cross-coupled multivariable controllers [19,20] might be mandatory to overcome the above problems. However, it is not easy to implement this type of controller in practice, despite the fact that good performances can be obtained for MIMO systems.

In this article, a simple but efficient design method for multi-loop PI controllers is presented, which exploits the process interactions for the improvement of the loop performance. The proposed method is based on the direct synthesis approach [21,22]

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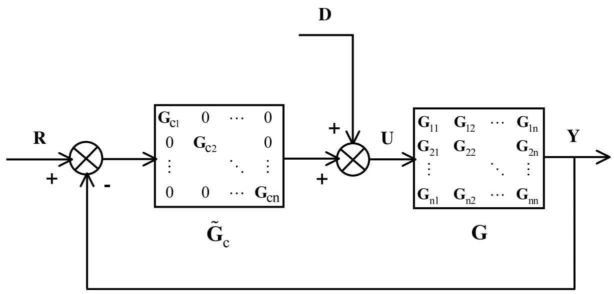

Fig. 1. Multi-loop control system.

in which the multi-loop PI controller is designed based on the desired closed-loop transfer function [13–15,19]. The resulting analytical design rule includes a frequency-dependent relative gain array (RGA) [5,23–27] that provides information on the dynamic interactions useful for estimating the controller parameters.

2. Multi-loop feedback controller design for the desired closed-loop responses

Consider an n-input and n-output open-loop multivariable process with the general transfer function matrix for stable, square, and multi-delays which is represented as the following matrix:

$$\mathbf{G}(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix} \tag{1}$$

where $g_{ij}(s) = \bar{g}_{ij}(s)\mathrm{e}^{-\theta_{ij}s}$, $i, j = 1, 2, \ldots, n$, of which $\bar{g}_{ij}(s)$ denotes the physically proper, stable, and delay-fee transfer function. θ_{ij} represents the time delay.

For the decentralized control system, n multi-loop diagonal PI controllers $\tilde{\mathbf{G}}_C(s)$ are implemented for the multivariable process, $\mathbf{G}(s)$. The controller transfer function matrix is therefore diagonal, $\tilde{\mathbf{G}}_C(s) = \mathrm{diag}\{g_{ci}\}$, $i = 1, 2, \ldots, n$. From the standard block diagram of the multi-loop feedback control as shown in Fig. 1, the closed-loop transfer function matrix between the set-points and outputs can be written as

$$\mathbf{H}(s) = \left(\mathbf{I} + \mathbf{G}(s)\tilde{\mathbf{G}}_C(s)\right)^{-1} \mathbf{G}(s)\tilde{\mathbf{G}}_C(s). \tag{2}$$

Obviously, in the MIMO case, the ideal structure of $\mathbf{H}(s)$ is a diagonal form. However, it is easily noted from Eq. (2) that $\mathbf{H}(s)$ with a diagonal structure could not be achieved by any decentralized controller $\tilde{\mathbf{G}}_C(s)$, because $\mathbf{G}(s)$ is a non-diagonal matrix. However, it is still possible to design a decentralized controller $\tilde{\mathbf{G}}_C(s)$ in such a way that only the diagonal elements of $\mathbf{H}(s)$ fulfill some desired criteria.

Let $\tilde{\mathbf{H}}(s)$ be a diagonal matrix consisting of a desired closed-loop transfer function of each loop. Then, $\tilde{\mathbf{G}}_C(s)$ to give the desired diagonal elements can be related to $\tilde{\mathbf{H}}(s)$ as

$$\begin{aligned} \tilde{\mathbf{H}}(s) &= \mathrm{diag}\left[\left(\mathbf{I} + \mathbf{G}(s)\tilde{\mathbf{G}}_C(s)\right)^{-1} \mathbf{G}(s)\tilde{\mathbf{G}}_C(s)\right] \\ &= \mathrm{diag}\left[\left(\mathbf{G}^{-1}(s)\tilde{\mathbf{G}}_C^{-1}(s) + \mathbf{I}\right)^{-1}\right]. \end{aligned} \tag{3}$$

According to Grosdidier and Morari [24], the overall closed-loop system $\mathbf{H}(s)$ is stable, if column diagonal dominance is achieved for all loops at all frequencies. Therefore, $\tilde{\mathbf{G}}_C(s)$ is usually designed for $(\mathbf{G}^{-1}(s)\tilde{\mathbf{G}}_C^{-1}(s)+\mathbf{I})^{-1}$ to achieve a diagonal dominance at all frequencies. In particular, since $\tilde{\mathbf{G}}_C(s)$ has integral terms, $(\mathbf{G}^{-1}(s)\tilde{\mathbf{G}}_C^{-1}(s) + \mathbf{I})^{-1}$ is diagonally dominant for low frequency range. Thus, the inverse of matrix can be reasonably approximated as

$$\begin{aligned} \tilde{\mathbf{H}}^{-1}(s) &= \left\{\mathrm{diag}\left[\left(\mathbf{G}^{-1}(s)\tilde{\mathbf{G}}_C^{-1}(s) + \mathbf{I}\right)^{-1}\right]\right\}^{-1} \\ &\cong \mathrm{diag}\left(\mathbf{G}^{-1}(s)\tilde{\mathbf{G}}_C^{-1}(s) + \mathbf{I}\right). \end{aligned} \tag{4}$$

The multi-loop controllers are mainly applied to processes with modest interactions. Thus, it should be noted that the proposed tuning method is also targeted for the processes with modest interactions and diagonal dominance. All subsequent derivations of the tuning rules are based on the above-mentioned assumption. Therefore, the multi-loop controller can be written by

$$\tilde{\mathbf{G}}_C(s) = \mathrm{diag}\left(\mathbf{G}^{-1}(s)\right)\left(\tilde{\mathbf{H}}^{-1}(s) - \mathbf{I}\right)^{-1}. \tag{5}$$

Note that the multi-loop controller given by Eq. (5) is not a standard PID form. The above controller consists of two parts. i.e., $\mathrm{diag}(\mathbf{G}^{-1}(s))$ and $(\tilde{\mathbf{H}}^{-1}(s) - \mathbf{I})^{-1}$.

The first part, $\mathrm{diag}(\mathbf{G}^{-1}(s))$, can be expressed as

$$\mathrm{diag}\left(\mathbf{G}^{-1}(s)\right) = \mathrm{diag}\left(\frac{\mathrm{adj}(\mathbf{G}(s))}{|\mathbf{G}(s)|}\right) = \mathrm{diag}\left\{\frac{\mathbf{G}^{ii}(s)}{|\mathbf{G}(s)|}\right\} \tag{6}$$

where $|\mathbf{G}(s)|$ is the determinant of $\mathbf{G}(s)$, the scalar $\mathbf{G}^{ij}$ denotes the cofactor corresponding to g_{ij} in $\mathbf{G}(s)$, $\mathrm{adj}(\mathbf{G}(s))$ is the adjoint of $\mathbf{G}(s)$ and thus $\mathrm{adj}(\mathbf{G}(s)) = (\mathbf{G}^{ij})^{\mathrm{T}} = (\mathbf{G}^{ji})$. Note that $\mathbf{G}^{ii}$ is the ith diagonal element of $\mathrm{adj}(\mathbf{G}(s))$.

According to Bristol [25], the diagonal element of the frequency-dependent RGA for $\mathbf{G}(s)$ is calculated by

$$\Lambda_{ii}(s) = g_{ii}(s)\frac{\mathbf{G}^{ii}(s)}{|\mathbf{G}(s)|}. \tag{7}$$

Hence, Eq. (6) can be rewritten as follows:

$$\mathrm{diag}\left(\mathbf{G}^{-1}(s)\right) = \mathrm{diag}\left\{\frac{\Lambda_{ii}(s)}{g_{ii}(s)}\right\}. \tag{8}$$

Furthermore, $(\tilde{\mathbf{H}}^{-1}(s) - \mathbf{I})^{-1}$ can be expressed as

$$\left(\tilde{\mathbf{H}}^{-1}(s) - \mathbf{I}\right)^{-1} = \mathrm{diag}\left\{\frac{h_{ii}(s)}{1 - h_{ii}(s)}\right\} \tag{9}$$

where h_{ii} is the diagonal element of $\tilde{\mathbf{H}}(s)$ and corresponds to the desired closed-loop transfer function of each loop.
Substituting Eqs. (8) and (9) into Eq. (5), the multi-loop controller can be rewritten as

$$\tilde{\mathbf{G}}_C(s) = \mathrm{diag}\{g_{ci}(s)\} = \mathrm{diag}\left\{\Lambda_{ii}(s)g_{ii}^{-1}(s)\left(\frac{h_{ii}(s)}{1 - h_{ii}(s)}\right)\right\}. \tag{10}$$

According to the IMC theory [12], under the assumption of stable and causal $\Lambda_{ii}(s)$, the desired closed-loop transfer function $h_{ii}(s)$ of the ith loop is chosen as

$$h_{ii}(s) = \frac{\mathrm{e}^{-\theta_{ii}s}}{(\lambda_i s + 1)^{r_i}} \prod_{k=1}^{q_i} \frac{z_k - s}{z_k^* + s} \tag{11}$$

where θ_{ii}, z_k, and z_k^* denote the time delay, the RHP zeros, and the corresponding complex conjugate of RHP zeros of the ith diagonal element of the process transfer function matrix, respectively. q_i is the number of the RHP zeros. The IMC filter time constant, λ_i, which is also equivalent to the closed-loop time constant, is an adjustable

parameter controlling the tradeoffs between the performance and robustness. r_i is the relative order of the numerator and denominator in $g_{ii}(s)$.
Substituting Eq. (11) into Eq. (10), the multi-loop controller of the ith loop can be rewritten by

$$g_{ci}(s) = \Lambda_{ii}(s) g_{ii}^{-1}(s) \left(\frac{e^{-\theta_{ii}s} \prod_{k=1}^{q_i} \frac{z_k - s}{z_k^* + s}}{(\lambda_i s + 1)^{r_i} - e^{-\theta_{ii}s} \prod_{k=1}^{q_i} \frac{z_k - s}{z_k^* + s}} \right). \tag{12}$$

Note that in Eq. (12), the non-minimum portion of $g_{ii}(s)$ is cancelled out by the time delay and RHP zero z_k in the numerator, and thus the controller has neither causality nor stability problems.

3. Reduction to the multi-loop PI controller

The resulting multi-loop controller given in Eq. (12) does not have a standard PI controller form. To obtain the PI controller that approximates the multi-loop controller given in Eq. (12) most closely, the Maclaurin series expansion based approach [28] is used as follows:
Since the multi-loop feedback controller has the integral term for offset free, $g_{ci}(s)$ can be rewritten as

$$g_{ci}(s) = s^{-1} p_i(s) \tag{13}$$

where

$$p_i(s) = s\Lambda_{ii}(s) g_{ii}^{-1}(s) \left(\frac{e^{-\theta_{ii}s} \prod_{k=1}^{q_i} \frac{z_k - s}{z_k^* + s}}{(\lambda_i s + 1)^{r_i} - e^{-\theta_{ii}s} \prod_{k=1}^{q_i} \frac{z_k - s}{z_k^* + s}} \right). \tag{14}$$

The rational approximation form of Eq. (13) can be found by expanding $g_{ci}(s)$ in a Maclaurin series.

$$g_{ci}(s) = \frac{1}{s} \left[p_i(0) + s p_i'(0) + \cdots \right]. \tag{15}$$

The first two terms of the above equation can be constituted as the standard PI controller given by

$$g_{ci}(s) = \frac{1}{s} \left(K_{Ii} + s K_{Ci} \right) \tag{16}$$

where K_{Ii} and K_{Ci} correspond to the integral and proportional terms of the standard PI controller, respectively.
Finally, the proposed PI controller parameters can be found by

$$K_{Ci} = p_i'(0) \tag{17}$$

$$K_{Ii} = p_i(0). \tag{18}$$

4. Example of two-input, two-output (TITO) case

Two-input, two-output (TITO) multi-delay processes are one of the most frequently encountered multivariable processes in process industry. A large number of previous studies focused on designing multi-loop control system of TITO processes. In this section, TITO multi-delay processes with first-order plus delay time (FOPDT) dynamics are considered. The multi-loop feedback controller can be derived from Eq. (12) as

$$g_{ci}(s) = \Lambda_{ii}(s) \frac{(T_{ii}s + 1)}{K_{ii}} \left(\frac{1}{(\lambda_i s + 1) - e^{-\theta_{ii}s}} \right) \tag{19}$$

where K_{ii} and T_{ii} denote the gain and time constant of g_{ii}, respectively. The order of the IMC filter is selected as 1 in order for the controller to be realizable.

The diagonal element of the frequency-dependent RGA is calculated by

$$\Lambda_{ii}(s) = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}} \frac{(T_{11}s+1)(T_{22}s+1)}{(T_{12}s+1)(T_{21}s+1)} e^{-\theta_{ei}s}} \tag{20}$$

where the effective delay θ_{ei} is defined by $\theta_{ei} = \theta_{12} + \theta_{21} - \theta_{11} - \theta_{22}$.
Substituting Eq. (20) into Eq. (19), an analytical tuning rule of the multi-loop PI controller can be obtained by using Eqs. (17) and (18) as follows:

$$K_{Ci} = \frac{\Lambda_{ii}(0)}{2K_{ii}(\lambda_i + \theta_{ii})^2} \times \left\{ \theta_{ii}^2 + 2\Lambda_{ii}(0)(\lambda_i + \theta_{ii}) \left[K_{ei}(T_{ei} - \theta_{ei}) + T_{ii} \right] \right\} \tag{21}$$

$$K_{Ii} = \frac{\Lambda_{ii}(0)}{K_{ii}(\lambda_i + \theta_{ii})} \tag{22}$$

where K_{ei} denotes the interaction quotient [29] defined by $K_{ei} = \frac{K_{12}K_{21}}{K_{11}K_{22}}$. The effective time constant T_{ei} is defined by $T_{ei} = T_{jj} - T_{ij} - T_{ji}, j \neq i$. It is noted that $\Lambda_{ii}(0)$ corresponds to the diagonal element of the steady-state RGA proposed by Bristol [25].

5. Multi-loop control system performance and robustness analysis

5.1. Integral absolute error index

To evaluate the closed-loop performance, the integral absolute error (IAE) criterion is considered, which is defined as

$$IAE = \int_0^T |e(t)| \, dt \tag{23}$$

where T is a finite time, which is chosen for the integral approach steady-state value.

5.2. Total variation (TV)

To evaluate the magnitude of the manipulated input usage, the total up and down movement of the control signal is considered as

$$TV = \sum_{k=1}^{T} |u(k+1) - u(k)|. \tag{24}$$

TV is a good measure of the smoothness of controller output and should be small [30].

5.3. Robust stability analysis

The robustness of a control system is one of the most important issues in any controller design, because the dynamics of real plants usually have many sources of uncertainty, which cause poor performance or even instability in the control systems. In this study, a well-known method for robust stability [30–32] is introduced for a fair comparison with other existing controller design methods.
The robust stability can be examined under output multiplication uncertainty. For a multi-delay process with an output multiplicative uncertainty of Δ_0, the upper bound of the robust stability can be written as

$$\gamma = \bar{\sigma}(\Delta_0) < 1/\bar{\sigma}\left[\left(I + \mathbf{G}(j\omega)\tilde{\mathbf{G}}_c(j\omega) \right)^{-1} \mathbf{G}(j\omega)\tilde{\mathbf{G}}_c(j\omega) \right] < \underline{\sigma}\left[I + \left(\mathbf{G}(j\omega)\tilde{\mathbf{G}}_c(j\omega) \right)^{-1} \right], \quad \forall\omega \geq 0 \tag{25}$$

where $\mathbf{G}(j\omega)\tilde{\mathbf{G}}_c(j\omega)$ is invertible.

Table 1
Controller parameters and resulting performance indices for the WB column.

Tuning method	Loop	K_{ci}	τ_{Ii}	λ_i	γ	Set-point		Disturbance	
						IAE	TV	IAE	TV
Proposed	1	0.75	10.07	1.11	0.47	22.12	2.50	137.92	8.03
	2	−0.08	7.98	7.11					
SAT	1	0.87	3.25	–	0.33	24.60	4.24	136.46	11.58
	2	−0.09	10.40	–					
Lee et al.	1	0.24	8.36	4.55	0.47	25.87	1.17	165.75	7.75
	2	−0.10	7.46	4.55					
Ho et al.	1	0.57	20.70	–	0.47	29.74	2.12	188.42	8.12
	2	−0.11	12.88	–					
Grosdider and Morari	1	0.74	17.20	–	0.47	31.74	2.62	210.24	8.31
	2	−0.10	15.90	–					

IAE: Total sum of each loop's IAE_i; TV: Total sum of each loop's TV_i.

To insure a fair comparison, the degree of robust stability will be held at the same level for all of the design methods being compared. In the simulation study, the proposed multi-loop PI controller is tuned by adjusting the closed-loop time constant, λ_i, so that the γ value of the proposed control system is kept the same as or larger than those of the other methods.

6. Simulation study

In this section, three examples are considered to demonstrate the performance of the proposed method in comparison with those of other well-known methods.

Example 1 (*Wood and Berry (WB) Distillation Column*). Wood and Berry [33] introduced the following transfer function model of a pilot-scale distillation column, which consists of an eight-tray plus reboiler separating methanol and water,

$$G(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}. \tag{26}$$

The WB column in Eq. (26) is one of the most representative TITO process models widely used for evaluating the performance of the multi-loop controllers. The performance of the proposed method was compared with those by the existing design methods such as the sequential auto-tuning (SAT) [9], Lee et al. [14], Grosdidier and Morari's [24], and Ho et al.'s [34] methods. In order to ensure a fair comparison, the robust stability is examined for all of the comparative design methods by using Eq. (25). The proposed controller was tuned to have $\gamma = 0.47$, so that the robust level is the same as those of Ho et al., Grodidier et al., and Lee et al., and higher than that of SAT ($\gamma = 0.33$). The sequential unit step changes in the set-point were made to the 1st and 2nd loops, respectively. The sequential unit step changes in the disturbance were also made to the 1st and 2nd loops, respectively. For the design of the proposed controller, the order of the IMC filter, r_i, was set to 1 for all of the loops.

The resulting performance indices and controller parameters are listed in Table 1. The closed-loop responses to the set-point and disturbance changes are shown in Figs. 2 and 3, respectively. The controller output (manipulated variable) responses are also shown in Fig. 4. It is apparent from the table and figures that the proposed controller provides superior performance for both the set-point tracking and disturbance rejection.

The robustness of the controller is evaluated by inserting a perturbation uncertainty of ±10% in the process gain, time constant, and time delay into the actual process, simultaneously, whereas the controller settings are those provided for the nominal process. The simulation results of the model mismatch for various tuning methods are given in Table 2. The proposed method shows better robust performance for both the set-point tracking and disturbance rejection when compared with Lee et al.'s [14], Grosdidier and Morari's [24], and Ho et al.'s [34] methods. For the disturbance rejection, the SAT has the similar robust performance in comparison with those of the proposed method.

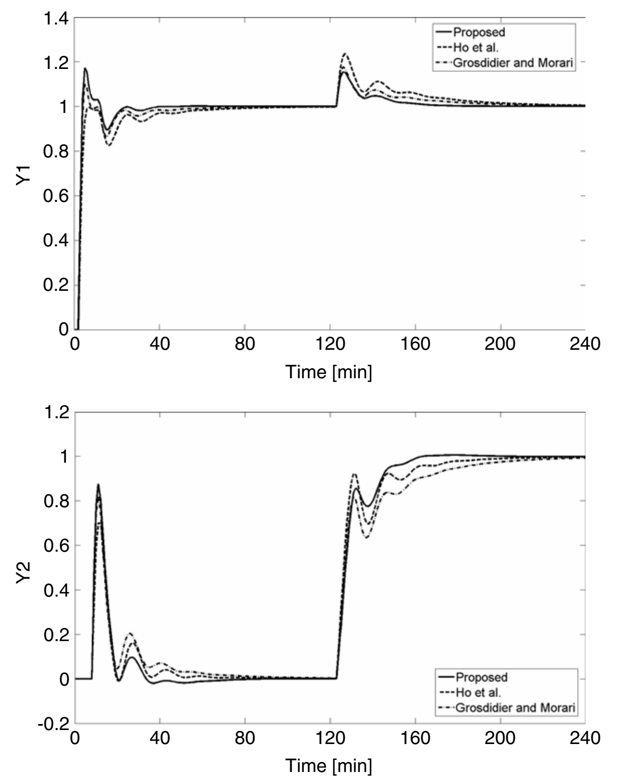

Fig. 2. Closed-loop responses to the sequential unit step changes in the set-point for the WB column.

Example 2 (*Industrial-Scale Polymerization (ISP) Reactor*). The transfer function matrix for an ISP reactor system was introduced by Chien et al. [35] as follows:

$$\mathbf{G}(s) = \begin{bmatrix} \dfrac{22.89e^{-0.2s}}{4.572s+1} & \dfrac{-11.64e^{-0.4s}}{1.807s+1} \\ \dfrac{4.689e^{-0.2s}}{2.174s+1} & \dfrac{5.8e^{-0.4s}}{1.801s+1} \end{bmatrix}. \tag{27}$$

The steady-state RGA of the ISP reactor is $\Lambda_{11}(0) = 0.7087 < 1$, which indicates that the closed-loop gain is greater than the open-loop gain. The ISP reactor system does not exhibit open-loop diagonal dominance. Chien et al. [35] previously demonstrated the

Table 2
Robust analysis for the WB column under $\pm 10\%$ parametric uncertainty in all parameters.

	WB (+10%)				WB (−10%)			
	Set-point		Disturbance		Set-point		Disturbance	
	IAE	TV	IAE	TV	IAE	TV	IAE	TV
Proposed	22.48	2.67	140.20	8.72	23.19	2.32	136.10	7.47
SAT	25.02	5.24	140.20	13.10	24.55	3.84	133.92	10.25
Lee et al.	29.22	1.41	185.08	9.19	25.35	1.01	161.66	6.78
Ho et al.	27.77	2.34	190.55	9.05	32.79	1.97	187.43	7.42
Grosdider and Morari	29.81	2.81	211.28	8.97	35.73	2.44	209.98	7.68

Table 3
Controller parameters and resulting performance indices for the ISP reactor.

Tuning method	Loop	K_{ci}	τ_{Ii}	λ_i	γ	Set-point		Disturbance	
						IAE	TV	IAE	TV
Proposed	1	0.43	3.95	0.09	0.57	4.47	1.99	24.13	7.80
	2	0.13	1.18	0.69					
Lee et al.	1	0.51	6.52	0.20	0.57	4.62	2.41	31.09	7.62
	2	0.19	2.61	1.25					
BLT	1	0.21	2.26	–	0.57	6.64	1.27	43.69	6.79
	2	0.18	4.25	–					
Chien et al.	1	0.26	1.42	–	0.41	6.86	1.86	26.74	8.88
	2	0.16	1.77	–					
Shen and Yu	1	0.46	1.50	–	0.47	7.41	2.68	32.36	8.62
	2	0.18	4.45	–					

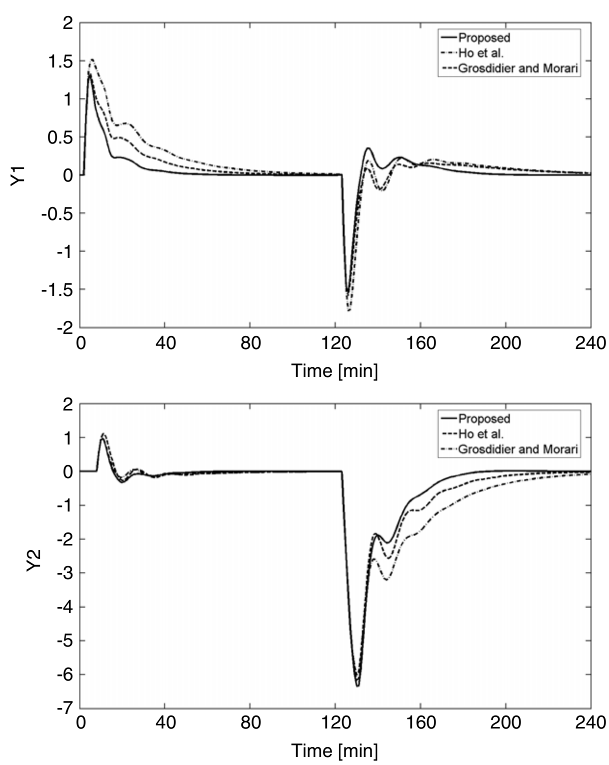

Fig. 3. Closed-loop responses to the sequential unit step changes in the disturbance for the WB column.

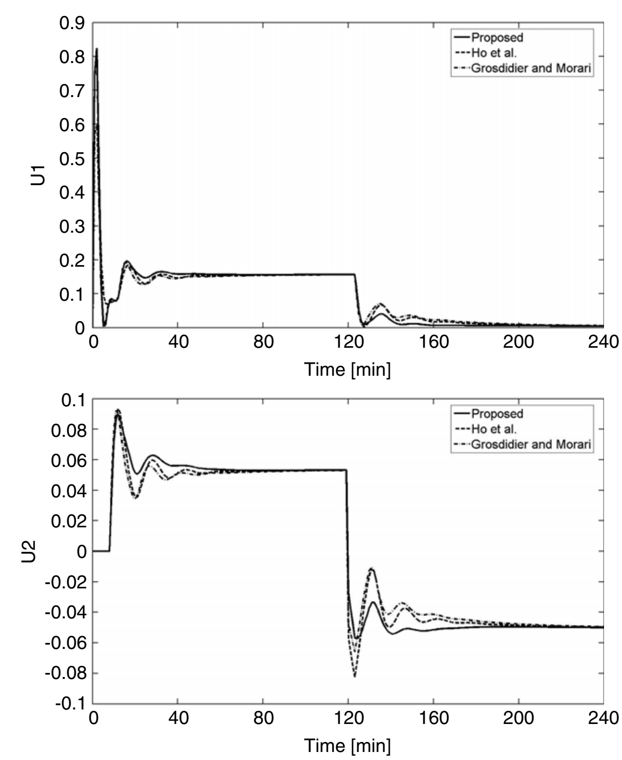

Fig. 4. Controller output responses to the sequential unit step changes in the set-point for the WB column.

superiority of their method over the BLT [3] method and that of Shen and Yu [10]. In this simulation study, the proposed method is compared with these methods as well as Lee et al.'s [14] method. For the proposed method and that of Lee et al. [14], the λ_i values are adjusted to obtain $\gamma = 0.57$, in order to have a same robustness level as that of the BLT method and to have a higher robustness level than those of the Chien et al.'s [35] and Shen and Yu's [10] methods. It is noted that the BLT method is not based on the λ tuning method and we, therefore, used their values without adjusting the γ value.

The resulting multi-loop PI controllers by the proposed and other methods are tabulated in Table 3. For a sequential unit step change in the set-points at $t = 0$ and at $t = 15$, Fig. 5 compares the closed-loop time responses afforded by the proposed method with those given by Chien et al.'s [35] and Shen and Yu's [10] methods. The proposed controller shows a superior response with a faster settling time and less overshoot over the other methods. The sequential unit step changes in the disturbance were also made to the 1st and 2nd loops, respectively. The values of the performance indices in Table 3 confirm the superior performance

Table 4
Robust analysis for the ISP reactor under $\pm$10% parametric uncertainty in all parameters.

	ISP (+10%)				ISP (−10%)			
	Set-point		Disturbance		Set-point		Disturbance	
	IAE	TV	IAE	TV	IAE	TV	IAE	TV
Proposed	3.95	1.94	25.5	8.48	3.23	2.67	23.55	7.20
Lee et al.	3.55	2.44	31.43	8.23	3.79	2.12	30.79	7.11
BLT	5.62	1.34	44.29	7.29	5.86	1.16	43.11	6.36
Chien et al.	5.48	1.98	29.15	10.00	4.28	1.52	24.90	7.95
Shen and Yu	5.04	2.64	32.96	9.45	5.26	2.22	31.93	7.94

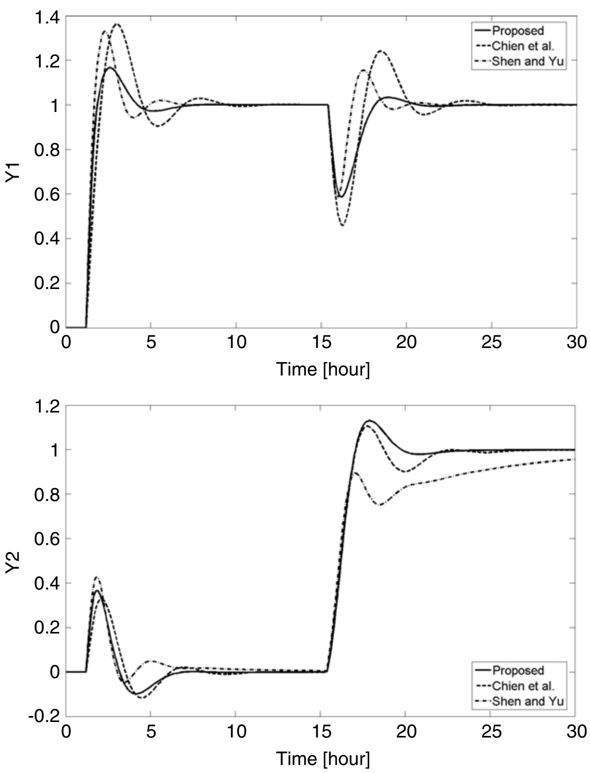

Fig. 5. Closed-loop responses to the sequential unit step changes in the set-point for the ISP reactor.

of the proposed controller for the load changes as well as the set-point changes.

For the robustness study, the controller is investigated by inserting a perturbation uncertainty of $\pm$10% in all three parameters, simultaneously. As shown in Table 4, the controller settings of the proposed method provide superior robust performance both for the set-point and disturbance changes.

Example 3 (*Ogunnaike and Ray (OR) Column*). A well-known multi-product distillation column for the separation of a binary ethanol–water mixture modeled experimentally in Ogunnaike et al. [17] is considered. The open-loop transfer function matrix is given by

$$G(s) = \begin{bmatrix} \dfrac{0.66e^{-2.6s}}{6.7s+1} & \dfrac{-0.61e^{-3.5s}}{8.64s+1} & \dfrac{-0.0049e^{-s}}{9.06s+1} \\ \dfrac{1.11e^{-6.5s}}{3.25s+1} & \dfrac{-2.36e^{-3s}}{5s+1} & \dfrac{-0.01e^{-1.2s}}{7.09s+1} \\ \dfrac{-34.68e^{-9.2s}}{8.15s+1} & \dfrac{46.2e^{-9.4s}}{10.9s+1} & \dfrac{0.87(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix}. \quad (28)$$

In the simulation study, the proposed multi-loop PI controller is compared with those of the BLT [3], Halevi et al.'s [11], Chien et al.'s [35], and Lee et al.'s [14] methods. For both the proposed method and Lee et al. [14] method, the adjustable parameters λ_i are chosen to obtain $\gamma = 0.035$, in order to ensure the same or higher robustness as those of the other comparative methods. In the simulation, the magnitudes of the sequential step changes in the set-points of loops 1, 2 and 3 were 1, 5, and 20, respectively. Fig. 6 compares the closed-loop responses by the proposed controller and those by the BLT [3] method and that of Halevi et al. [11]. In the figure, one can see that the proposed controller has the faster rising and settling responses over the others.

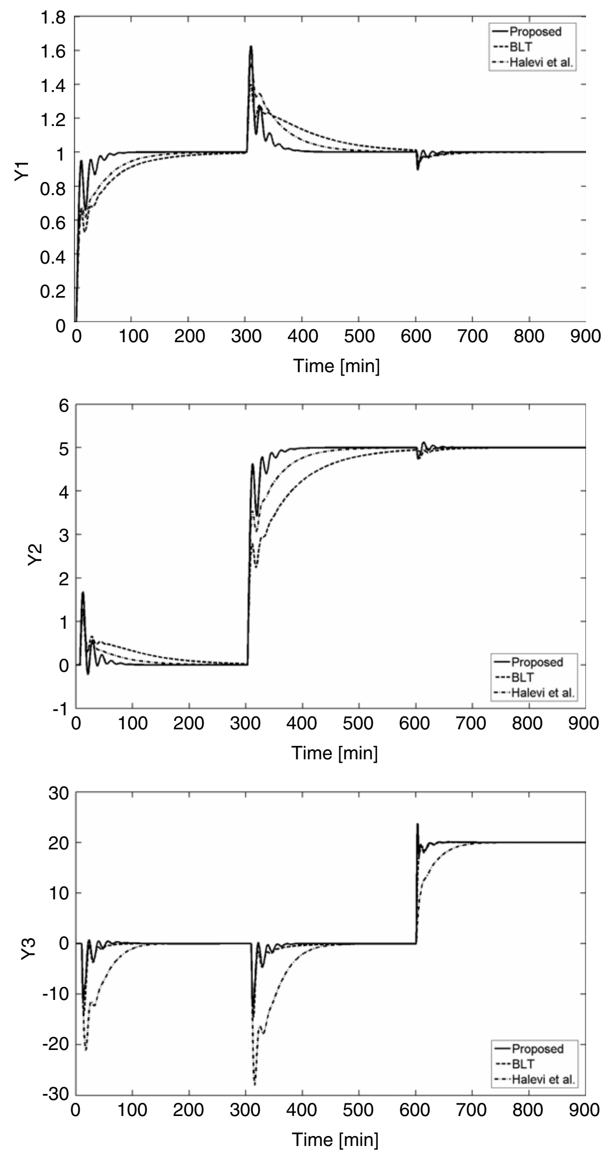

Fig. 6. Closed-loop responses to the sequential step changes in the set-point for the OR column.

Table 5
Controller parameters and resulting performance indices for the OR column.

Tuning method	Loop	K_{ci}	τ_{Ii}	λ_i	γ	Set-point		Disturbance	
						IAE	TV	IAE	TV
Proposed	1	1.57	5.96	8.85					
	2	−0.31	4.81	8.85	0.035	195.58	275.25	146.06	292.95
	3	6.10	9.60	1.65					
Lee et al.	1	1.06	3.59	7.42					
	2	−0.22	2.87	7.42	0.035	206.20	244.09	196.42	295.32
	3	5.08	7.42	1.55					
Chien et al.	1	1.08	4.25	–					
	2	−0.23	3.32	–	0.026	266.12	197.64	265.06	237.23
	3	2.78	5.24	–					
BLT	1	1.51	16.40	–					
	2	−0.30	18.00	–	0.035	361.70	133.86	283.85	153.82
	3	2.63	6.61	–					
Halevi et al.	1	1.25	10.50	–					
	2	−0.34	10.50	–	0.035	982.75	85.52	473.74	72.13
	3	0.92	10.50	–					

Table 6
Robust analysis for the OR column under ±10% parametric uncertainty in all parameters.

	OR (+10%)				OR (−10%)			
	Set-point		Disturbance		Set-point		Disturbance	
	IAE	TV	IAE	TV	IAE	TV	IAE	TV
Proposed	239.45	263.30	193.97	246.87	222.64	216.10	177.47	191.56
Lee et al.	391.73	367.30	319.16	326.77	274.51	249.98	218.12	213.63
Chien et al.	429.18	272.18	377.90	249.81	336.91	203.47	288.18	185.41
BLT	320.45	153.73	393.24	165.87	354.80	146.15	400.12	157.24
Halevi et al.	864.81	90.67	763.37	93.40	1009.8	90.43	756.55	87.21

Disturbance rejection performance was also evaluated by introducing the sequential unit step changes in the disturbance into loops 1, 2, and 3. The resulting PI controller parameters together with the performance indices calculated using the above-mentioned methods are summarized in Table 5. The closed-loop response by the proposed controller shows the smallest total IAE among all the comparative methods with the same or higher robustness level than the others.

To demonstrate the robust performance of the proposed method, the simulation study was also done by inserting a perturbation uncertainty of ±10% in all three process parameters. The simulation results for the plant-model mismatch are tabulated in Table 6. As seen from the table, it is obvious that the proposed controller affords superior performance consistently.

7. Conclusions

In this article, an analytical design method of the multi-loop PI controller is proposed for multi-delay processes. The proposed method is straightforward and easy to implement in the multi-loop control systems. The robustness and performance can be efficiently compromised by adjusting a single parameter, i.e., the closed-loop time constant. For a fair comparison, the maximum upper bound in the output multiplicative uncertainty for robust stability was utilized. The time-domain simulation demonstrates the superior performance of the proposed controller with a fast and well-balanced closed-loop time response for both the set-point and load changes. The robustness study was also conducted by inserting a perturbation uncertainty of ±10% in all three process parameters. The simulation results showed that the proposed method afforded the superior robust performance in the plant-model mismatch case.

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