Question 1: Understanding and Simulating Lognormal Distribution

The distribution of earnings in a small society of 10,000 individuals follows a lognormal distribution due to multiplicative growth over time.

Tasks:

1. Initialize a population of 10,000 individuals, each with an initial earning of $1.
2. Simulate the growth process for 20 periods:
    -   Each year, earnings grow or shrink by a factor of:
        -   1.10 (10% increase),
        -   1.00 (no change),
        -   0.90 (10% decrease), with equal probabilities.
3. Record the final earnings of all individuals after 20 periods.
4. Plot the histogram of the final earnings to visualize the distribution.
5. Estimate the parameters of the lognormal distribution (point estimation) from the simulated data.
6. Overlay the estimated lognormal distribution curve on the histogram.
7. Plot the fitted lognormal distribution separately.

Question 2: Multivariate Analysis - Earnings and Wealth

In addition to earnings, families in this society accumulate wealth over time. Wealth is correlated with earnings, as a fraction of earnings is saved annually.

Tasks:

1. Assume each family saves 20% of their annual earnings and spends the rest.
2. Simulate the wealth accumulation process over the same 20 periods:
$$\text{Wealth}_{t+1} = 0.20 \times \text{Earnings}_{t+1} + \text{Wealth}_t$$
3. Plot the joint distribution of final earnings and wealth using a contour plot.
4. Compute the Pearson correlation coefficient between final earnings and wealth.
5. Analyze the wealth distribution for families with low earnings (e.g., bottom 20%).