spectral_clustering-2

May 21, 2022

[ ]:

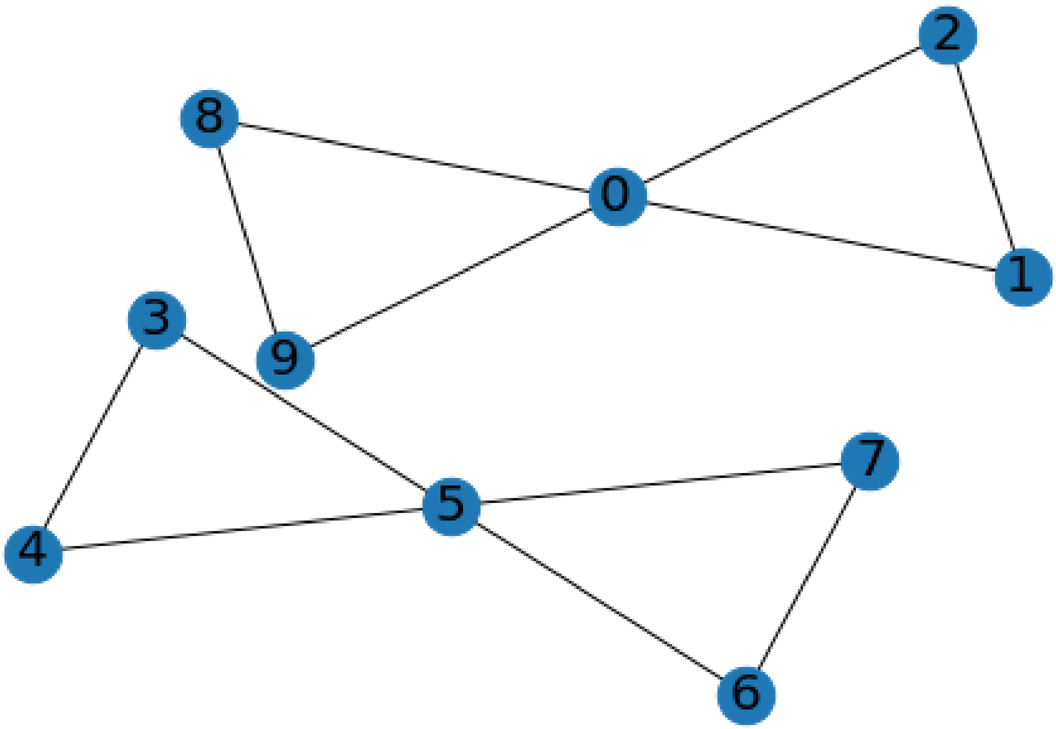
```
import numpy as np
from PIL import Image
import matplotlib.pyplot as plt
import networkx as nx
from sklearn.cluster import KMeans
```

[ ]:

```
def BuildGraph (n, adj) :
    g = nx.Graph()
    for i in range(n):
        for j in range(n):
            if (adj[i][j] != 0) :
                g.add_edge(i, j)
    return g
```

[ ]:

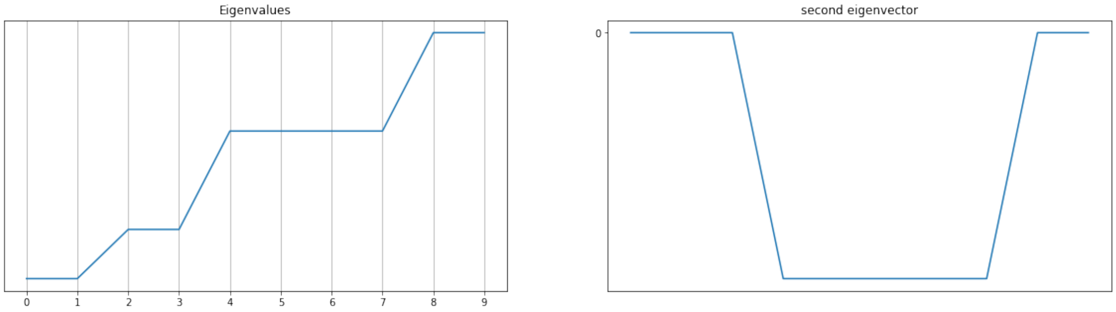
```
adj = np.array([
                [0, 1, 1, 0, 0, 0, 0, 0, 1, 1],
                [1, 0, 1, 0, 0, 0, 0, 0, 0, 0],
                [1, 1, 0, 0, 0, 0, 0, 0, 0, 0],
                [0, 0, 0, 0, 1, 1, 0, 0, 0, 0],
                [0, 0, 0, 1, 0, 1, 0, 0, 0, 0],
                [0, 0, 0, 1, 1, 0, 1, 1, 0, 0],
                [0, 0, 0, 0, 0, 1, 0, 1, 0, 0],
                [0, 0, 0, 0, 0, 1, 1, 0, 0, 0],
                [1, 0, 0, 0, 0, 0, 0, 0, 0, 1],
                [1, 0, 0, 0, 0, 0, 0, 0, 1, 0]
        ])

n = len(adj[0])
graph = BuildGraph(n, adj)
nx.draw_kamada_kawai(graph, font_size = 20, width = 1, with_labels=True,␣
 ↪node_size = 500, arrowsize = 100)
plt.show()
```



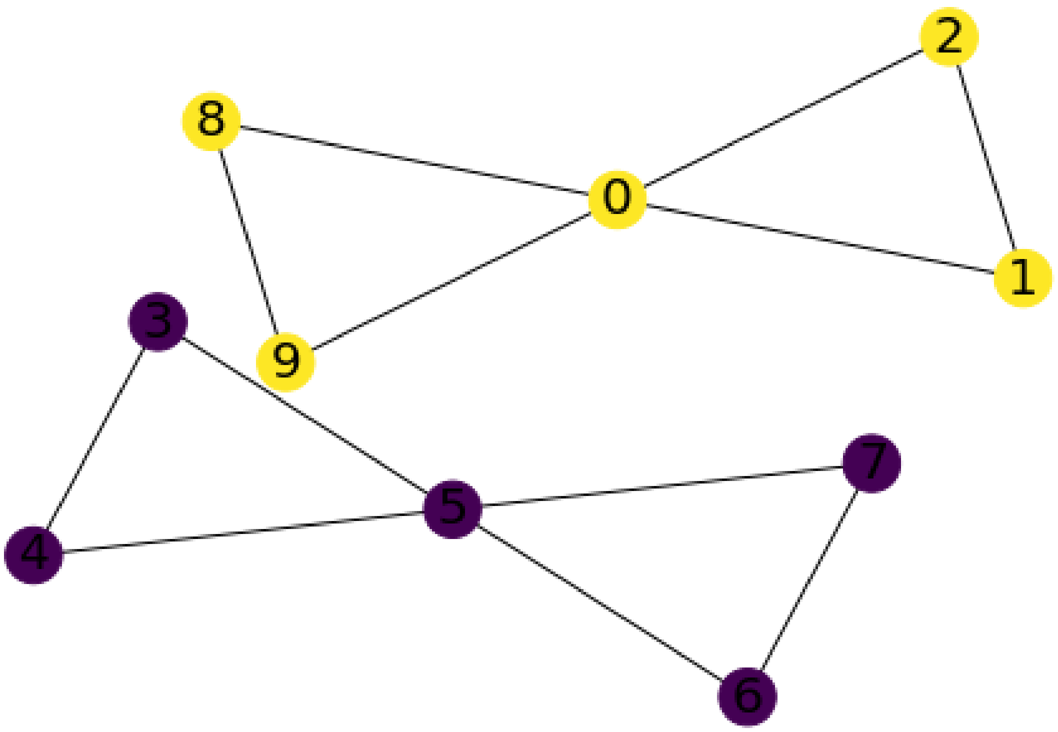
```
[ ]: deg = np.diag(adj.sum(axis=1))
     laplacian = deg - adj

     eigenvalues, eigenvectors = np.linalg.eig(laplacian)

     print("Laplacian: ")
     print (laplacian)

     fig, axs = plt.subplots(1, 2, figsize=(20, 5), subplot_kw=dict(xticks=[],␣
      ↪yticks=[]))
     eigenvectors = eigenvectors[:, np.argsort(eigenvalues)]
     eigenvalues = eigenvalues[np.argsort(eigenvalues)]
     axs[0].plot(eigenvalues)
     axs[0].set_xticks(range(len(eigenvalues)))
     axs[0].grid()
     axs[0].set_title("Eigenvalues")

     axs[1].plot(eigenvectors[:,1])
     axs[1].set_yticks([0])
     axs[1].set_title("second eigenvector")
     plt.show()

     print ("Fiedler value(second eigenvelue): ", eigenvalues[1])
```

```
kmeans = KMeans(n_clusters=2)
kmeans.fit(eigenvectors[:,1:2])
colors = kmeans.labels_

print ("Fiedler vector(second eigenvector): ", np.round(eigenvectors[:,1], 3))
print ("Cluster centers: \n", kmeans.cluster_centers_)
print ("Cluster labels: ", colors)

nx.draw_kamada_kawai(graph, font_size = 20, width = 1, with_labels=True,␣
 ↪node_size = 500, arrowsize = 100, node_color=colors, nodelist=range(0, n))
plt.show()
```

```
Laplacian:
[[ 4 -1 -1  0  0  0  0  0 -1 -1]
 [-1  2 -1  0  0  0  0  0  0  0]
 [-1 -1  2  0  0  0  0  0  0  0]
 [ 0  0  0  2 -1 -1  0  0  0  0]
 [ 0  0  0 -1  2 -1  0  0  0  0]
 [ 0  0  0 -1 -1  4 -1 -1  0  0]
 [ 0  0  0  0  0 -1  2 -1  0  0]
 [ 0  0  0  0  0 -1 -1  2  0  0]
 [-1  0  0  0  0  0  0  0  2 -1]
 [-1  0  0  0  0  0  0  0 -1  2]]
```

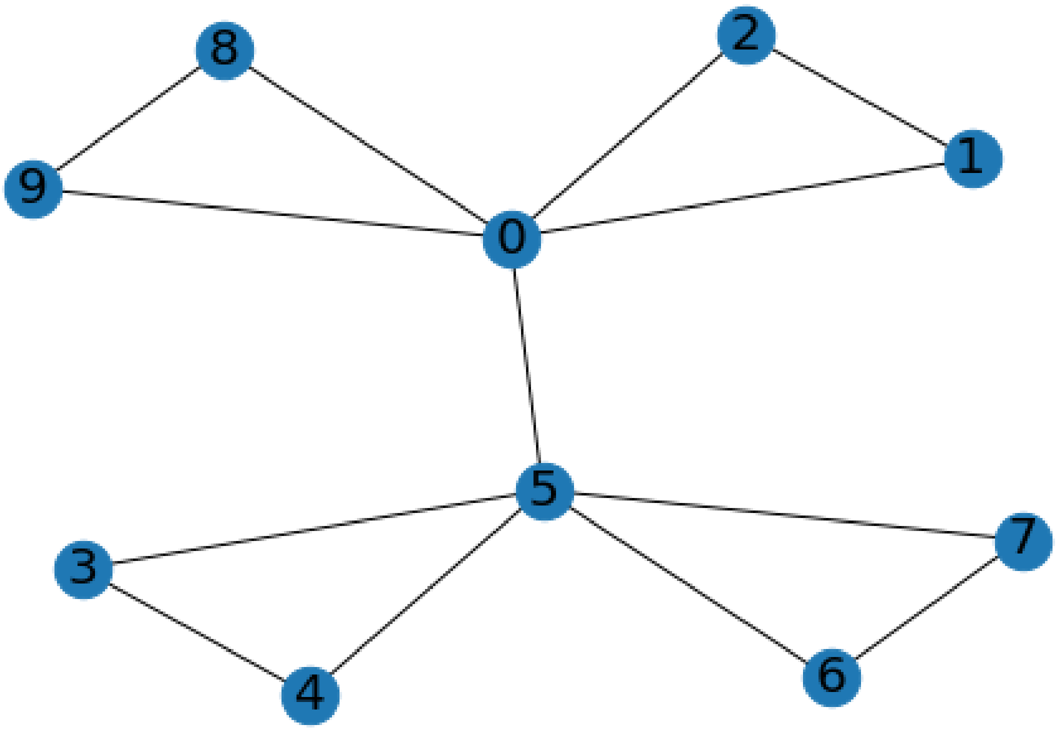


```
Fiedler value(second eigenvelue):  3.3134615995519985e-16
Fiedler vector(second eigenvector):  [ 0.      0.     -0.     -0.447 -0.447 -0.447
-0.447 -0.447  0.     -0.    ]
Cluster centers:
 [[-0.4472136]
 [ 0.       ]]
Cluster labels:  [1 1 1 0 0 0 0 0 1 1]
```



[ ]:

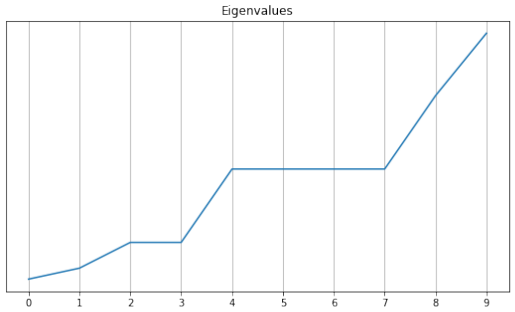
```
adj = np.array([
                [0, 1, 1, 0, 0, 1, 0, 0, 1, 1],
                [1, 0, 1, 0, 0, 0, 0, 0, 0, 0],
                [1, 1, 0, 0, 0, 0, 0, 0, 0, 0],
                [0, 0, 0, 0, 1, 1, 0, 0, 0, 0],
                [0, 0, 0, 1, 0, 1, 0, 0, 0, 0],
                [1, 0, 0, 1, 1, 0, 1, 1, 0, 0],
                [0, 0, 0, 0, 0, 1, 0, 1, 0, 0],
                [0, 0, 0, 0, 0, 1, 1, 0, 0, 0],
                [1, 0, 0, 0, 0, 0, 0, 0, 0, 1],
                [1, 0, 0, 0, 0, 0, 0, 0, 1, 0]
        ])
n = len(adj[0])
graph = BuildGraph(n, adj)
nx.draw_kamada_kawai(graph, font_size = 20, width = 1, with_labels=True,␣
 ↪node_size = 500, arrowsize = 100)
plt.show()
```

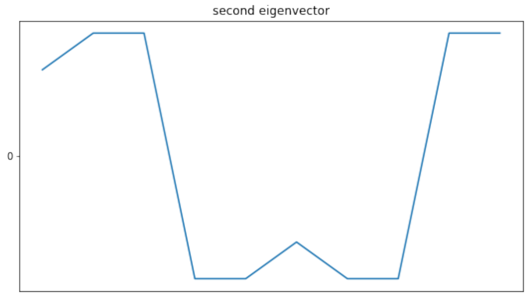


[ ]:

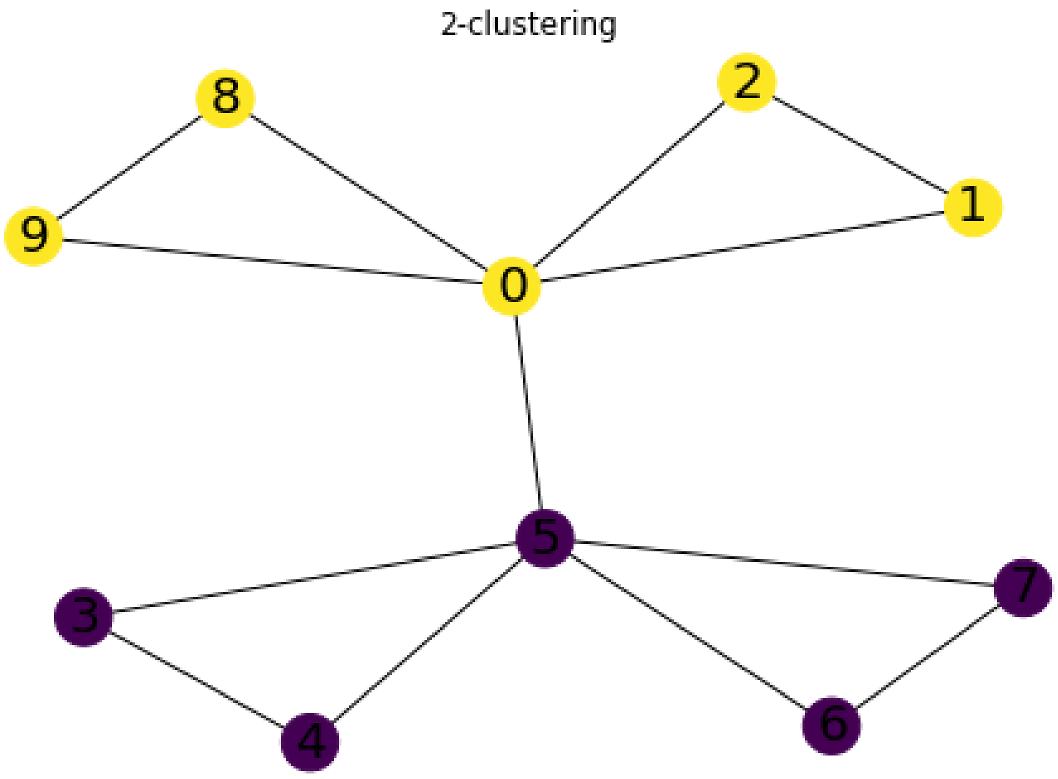
```
deg = np.diag(adj.sum(axis=1))
laplacian = deg - adj

eigenvalues, eigenvectors = np.linalg.eig(laplacian)

fig, axs = plt.subplots(1, 2, figsize=(20, 5), subplot_kw=dict(xticks=[], yticks=[]))
eigenvectors = eigenvectors[:, np.argsort(eigenvalues)]
eigenvalues = eigenvalues[np.argsort(eigenvalues)]
axs[0].plot(eigenvalues)
axs[0].set_xticks(range(len(eigenvalues)))
axs[0].grid()
axs[0].set_title("Eigenvalues")

axs[1].plot(eigenvectors[:,1])
axs[1].set_yticks([0])
axs[1].set_title("second eigenvector")
plt.show()

print ("Fiedler value(second eigenvelue): ", eigenvalues[1])

kmeans = KMeans(n_clusters=2)
kmeans.fit(eigenvectors[:,1:2])
```

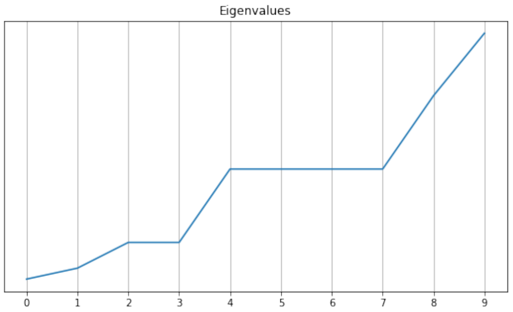
```python
colors = kmeans.labels_

print ("Fiedler vector(second eigenvector): ", np.round(eigenvectors[:,1], 3))
print ("2-Cluster centers: \n", kmeans.cluster_centers_)
print ("2-Cluster labels: ", colors)

nx.draw_kamada_kawai(graph, font_size = 20, width = 1, with_labels=True, node_size = 500, arrowsize = 100, node_color=colors, nodelist=range(0,n))
plt.title("2-clustering")
plt.show()

fig, axs = plt.subplots(1, 2, figsize=(20, 5), subplot_kw=dict(xticks=[], yticks=[]))
eigenvectors = eigenvectors[:, np.argsort(eigenvalues)]
eigenvalues = eigenvalues[np.argsort(eigenvalues)]
axs[0].plot(eigenvalues)
axs[0].set_xticks(range(len(eigenvalues)))
axs[0].grid()
axs[0].set_title("Eigenvalues")

axs[1].plot(eigenvectors[:,1])
axs[1].plot(eigenvectors[:,2])
axs[1].plot(eigenvectors[:,3])

axs[1].set_title("second eigenvector")
axs[1].set_yticks([0])
plt.show()

print ("First* 3 eigenvectors:")
print (np.round(eigenvectors[:,1], 3))
print (np.round(eigenvectors[:,2], 3))
print (np.round(eigenvectors[:,3], 3))

kmeans = KMeans(n_clusters=4)
kmeans.fit(eigenvectors[:,1:4])
colors = kmeans.labels_

print ("Cluster centers: \n", kmeans.cluster_centers_)
print ("Cluster labels: ", colors)

nx.draw_kamada_kawai(graph, font_size = 20, width = 1, with_labels=True, node_size = 500, arrowsize = 100, node_color=colors, nodelist=range(0,n))
plt.title("Optimal clustering")
plt.show()
```

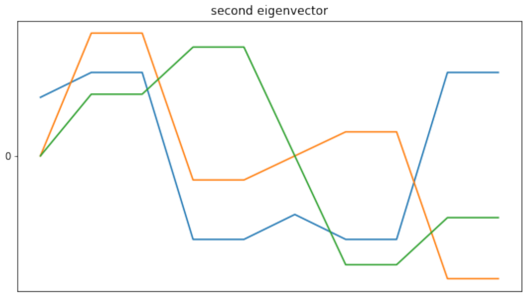




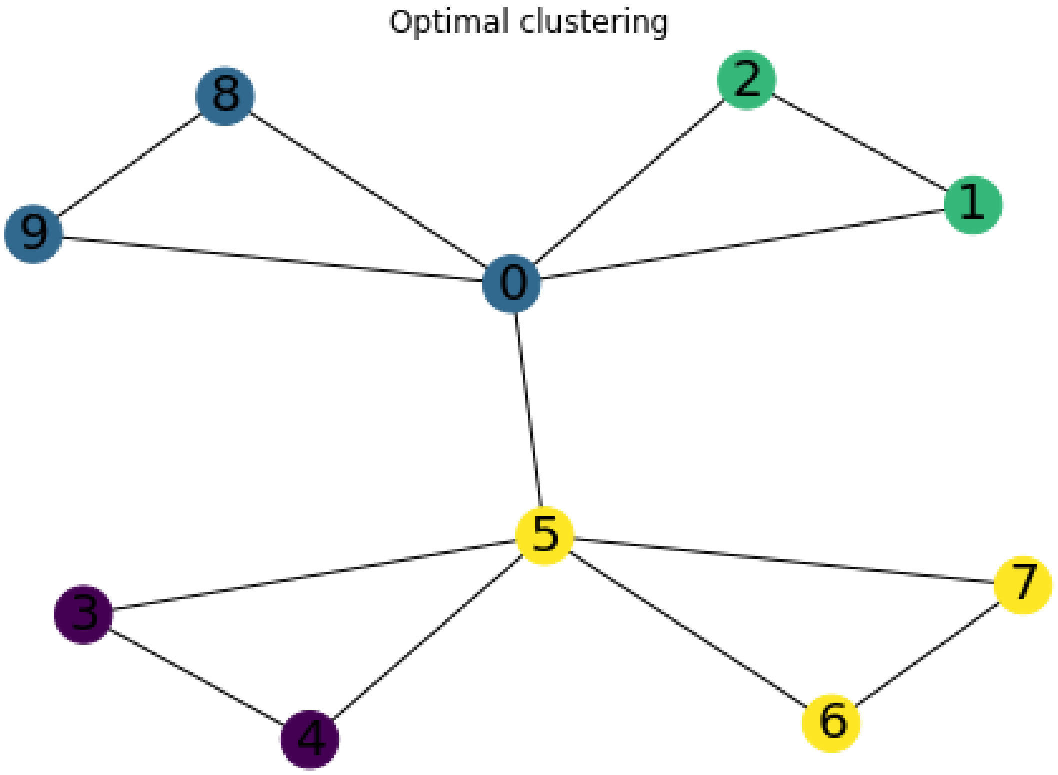
```
Fiedler value(second eigenvelue):  0.29843788128357546
Fiedler vector(second eigenvector):  [ 0.234  0.334  0.334 -0.334 -0.334 -0.234
-0.334 -0.334  0.334  0.334]
2-Cluster centers:
 [[-0.31370984]
 [ 0.31370984]]
2-Cluster labels:  [1 1 1 0 0 0 0 0 1 1]
```







```
First* 3 eigenvectors:
[ 0.234  0.334  0.334 -0.334 -0.334 -0.234 -0.334 -0.334  0.334  0.334]
[-0.     0.491  0.491 -0.096 -0.096  0.     0.096  0.096 -0.491 -0.491]
[-0.     0.247  0.247  0.435  0.435 -0.    -0.435 -0.435 -0.247 -0.247]
Cluster centers:
 [[-0.33362299 -0.09621369  0.43490268]
 [ 0.30043441 -0.32710374 -0.16446501]
 [ 0.33362299  0.49065561  0.24669751]
 [-0.30043441  0.06414246 -0.28993512]]
Cluster labels:  [1 2 2 0 0 3 3 3 1 1]
```



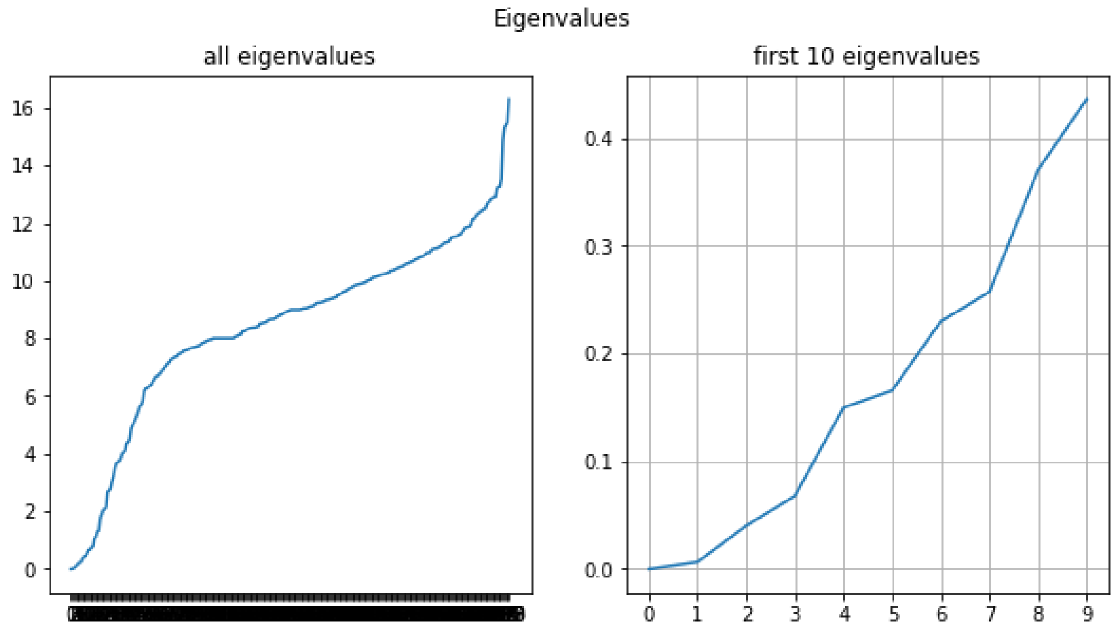
[ ]:

```
def spectral_cluster(n, adj, k= 2):
        graph = BuildGraph(n, adj)

        deg = np.diag(adj.sum(axis=1))
        laplacian = deg - adj

        eigenvalues, eigenvectors = np.linalg.eigh(laplacian)

        eigenvectors = eigenvectors[:, np.argsort(eigenvalues)]
        eigenvalues = eigenvalues[np.argsort(eigenvalues)]

        fig, axs = plt.subplots(1, 2, figsize=(10, 5))

        axs[0].plot(eigenvalues)
        axs[0].set_xticks(range(len(eigenvalues)))
        axs[0].set_title("all eigenvalues")
        axs[1].plot(eigenvalues[:10])
        axs[1].set_xticks(range(10))
        axs[1].grid()
        axs[1].set_title("first 10 eigenvalues")
        fig.suptitle("Eigenvalues")
        plt.show()

        kmeans = KMeans(n_clusters=k)
        kmeans.fit(eigenvectors[:,1:k])
        colors = kmeans.labels_

        return colors
```

[ ]:

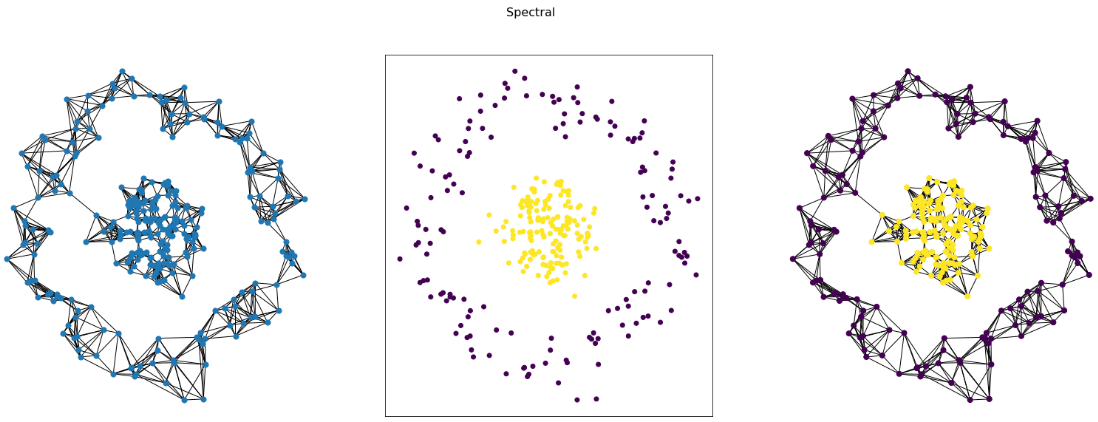
```
from sklearn.datasets import make_circles
from sklearn.neighbors import kneighbors_graph

X, labels = make_circles(n_samples=300, noise=0.12, factor=.2)
adj = kneighbors_graph(X, n_neighbors=7, ).toarray()
n = len(adj)
for i in range(n):
        for j in range(i, n):
                if (adj[i][j] != adj[j][i]):
                        adj[i][j] = 1
                        adj[j][i] = 1
graph = BuildGraph(n, adj)

colors = spectral_cluster(n, adj, k=2)

from sklearn.metrics import classification_report
print("Spectral clustering accuracy:\n", classification_report(labels, colors))
```

```
fig, axs = plt.subplots(1, 3, figsize=(27, 9), subplot_kw=dict(xticks=[], yticks=[]))
nx.draw(graph, font_size = 20, width = 1, with_labels=False, node_size = 50, arrowsize = 100, nodelist=range(0,n), pos= X, ax= axs[0])
axs[1].scatter(X[:,0], X[:,1], c=colors)
nx.draw(graph, font_size = 20, width = 1, with_labels=False, node_size = 50, arrowsize = 100, nodelist=range(0,n), pos= X, node_color=colors, ax = axs[2])
fig.suptitle("Spectral", fontsize=16)
plt.show()

kmeans = KMeans(n_clusters=2)
kmeans.fit(X)
colors = kmeans.labels_

print("K-means clustering accuracy:\n", classification_report(labels, colors))

fig, axs = plt.subplots(1, 3, figsize=(30, 10), subplot_kw=dict(xticks=[], yticks=[]))
axs[0].scatter(X[:,0], X[:,1])
axs[1].scatter(X[:,0], X[:,1], c=colors)
nx.draw(graph, font_size = 20, width = 1, with_labels=False, node_size = 50, arrowsize = 100, nodelist=range(0,n), pos= X, node_color=colors, ax = axs[2])
fig.suptitle("K-means", fontsize=16)
plt.show()
```

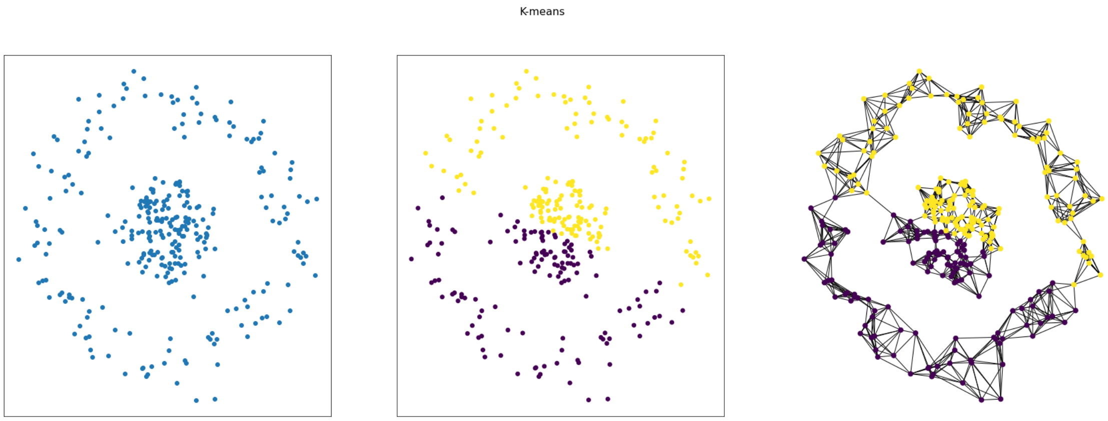


```
Spectral clustering accuracy:
```

```
              precision    recall  f1-score   support

           0       1.00      0.99      1.00       150
           1       0.99      1.00      1.00       150

    accuracy                           1.00       300
   macro avg       1.00      1.00      1.00       300
weighted avg       1.00      1.00      1.00       300
```



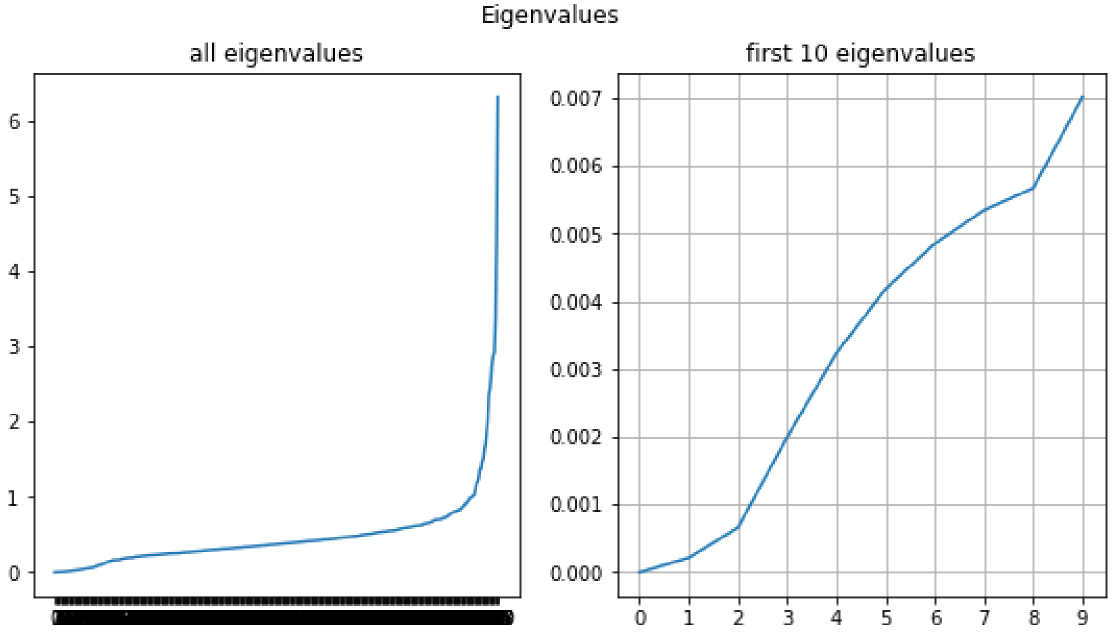
```
K-means clustering accuracy:
              precision    recall  f1-score   support

           0       0.53      0.47      0.50       150
           1       0.52      0.59      0.55       150

    accuracy                           0.53       300
   macro avg       0.53      0.53      0.52       300
weighted avg       0.53      0.53      0.52       300
```



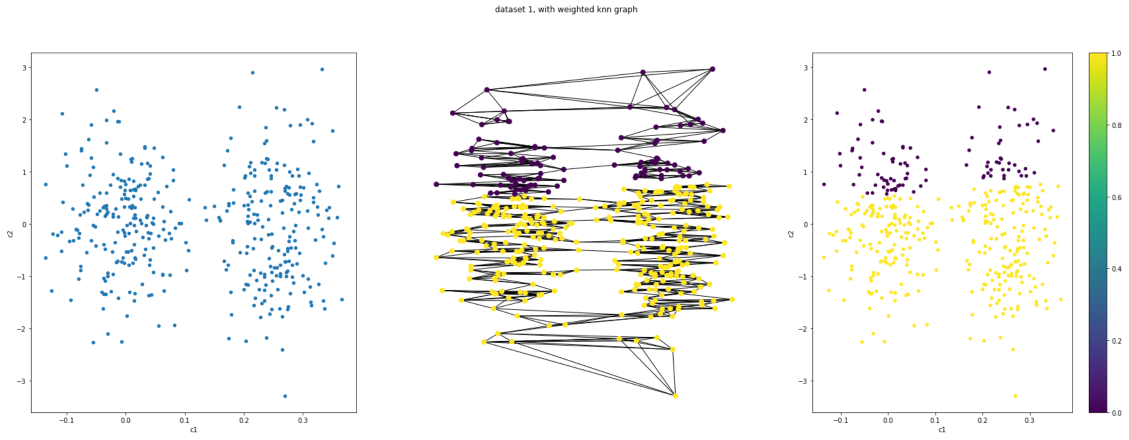
[ ]:

```
import pandas as pd
df1 = pd.read_csv("./first_clustering_dataset.csv", names=['c1', 'c2'])
df2 = pd.read_csv("./second_clustering_dataset.csv", names=['c1', 'c2'])
df3 = pd.read_csv("./third_clustering_dataset.csv", names=['c1', 'c2'])

from sklearn.neighbors import kneighbors_graph

adj1 = kneighbors_graph(df1, n_neighbors=5, ).toarray()
n = len(adj1)
for i in range(n):
    for j in range(i, n):
        if (adj1[i][j] != adj1[j][i]):
            adj1[i][j] = 1
            adj1[j][i] = 1
graph1 = BuildGraph(len(adj1), adj1)

adj1_2 = kneighbors_graph(df1, n_neighbors=5, mode='distance').toarray()
n = len(adj1_2)
for i in range(n):
    for j in range(i, n):
        if (adj1_2[i][j] != adj1_2[j][i]):
            adj1_2[i][j] = max(adj1_2[i][j], adj1_2[j][i])
            adj1_2[j][i] = adj1_2[i][j]
graph1_2 = BuildGraph(len(adj1_2), adj1_2)

adj2 = kneighbors_graph(df2, n_neighbors=7, ).toarray()
n = len(adj2)
for i in range(n):
    for j in range(i, n):
        if (adj2[i][j] != adj2[j][i]):
            adj2[i][j] = 1
            adj2[j][i] = 1
graph2 = BuildGraph(len(adj2), adj2)


adj3 = kneighbors_graph(df3, n_neighbors=7, ).toarray()
n = len(adj3)
for i in range(n):
    for j in range(i, n):
        if (adj3[i][j] != adj3[j][i]):
            adj3[i][j] = 1
            adj3[j][i] = 1
graph3 = BuildGraph(len(adj3), adj3)
```

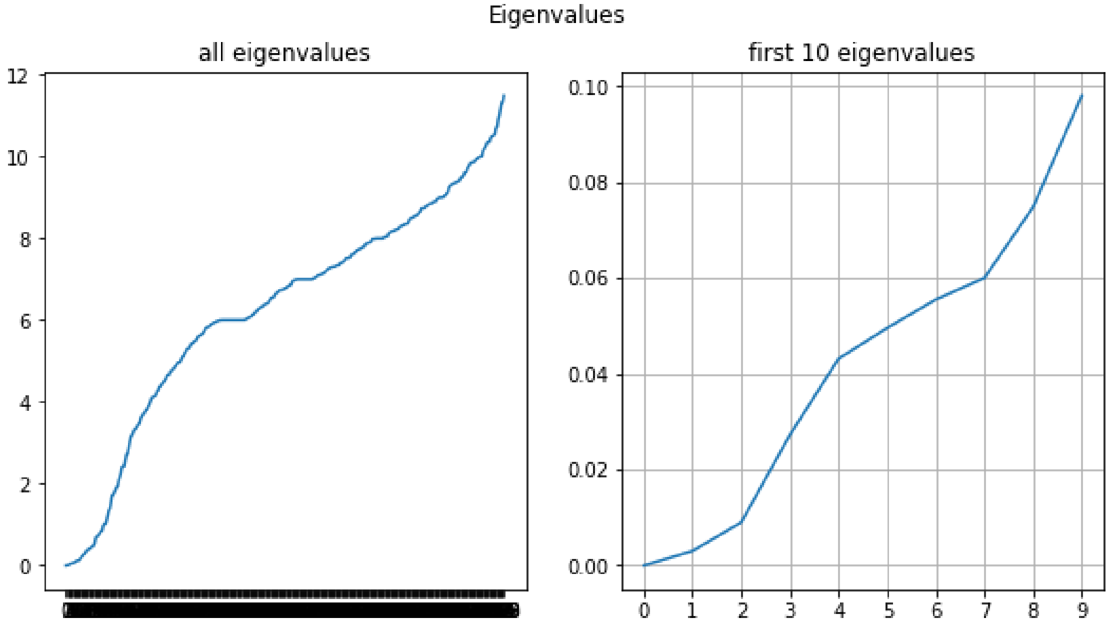
```
[ ]: colors1_2 = spectral_cluster(len(adj1_2), adj1_2, 2)

     fig, axs = plt.subplots(1, 3, figsize=(30, 10))
     df1.plot.scatter(ax= axs[0], x = 'c1', y = 'c2')
     nx.draw(graph1_2, font_size = 20, width = 1, with_labels=False, node_size = 50,␣
      ↪arrowsize = 100, nodelist=range(0,len(adj1_2)), pos= df1.to_numpy(),␣
      ↪node_color=colors1_2, ax = axs[1])
     df1.plot.scatter(ax= axs[2], x = 'c1', y = 'c2', c=colors1_2,␣
      ↪colormap='viridis')
     fig.suptitle("dataset 1, with weighted knn graph")
```



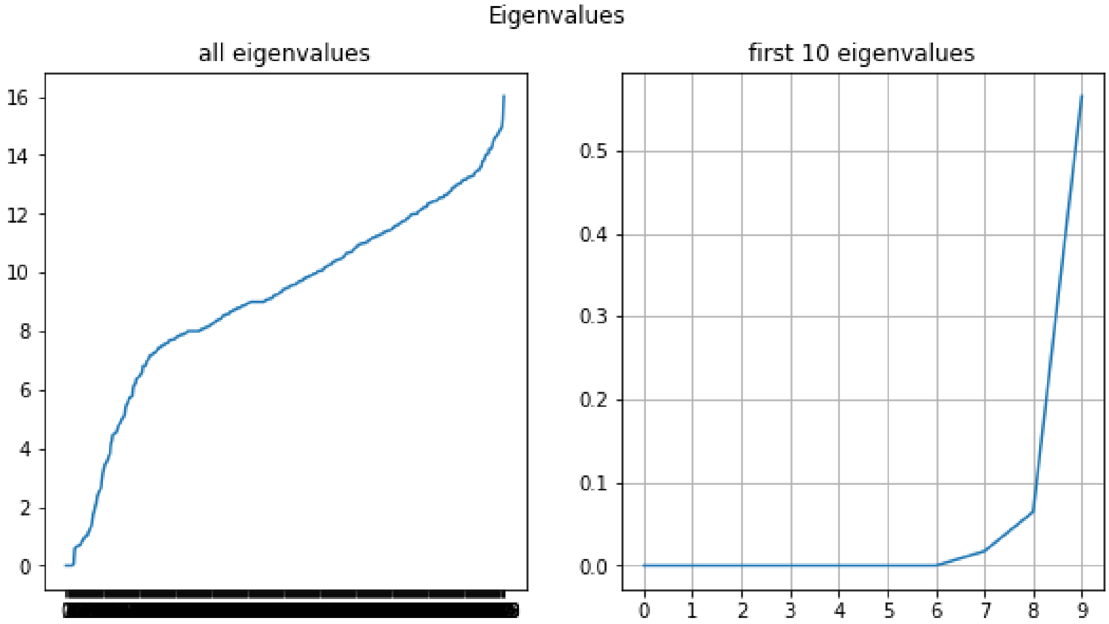
```
[ ]: Text(0.5, 0.98, 'dataset 1, with weighted knn graph')
```



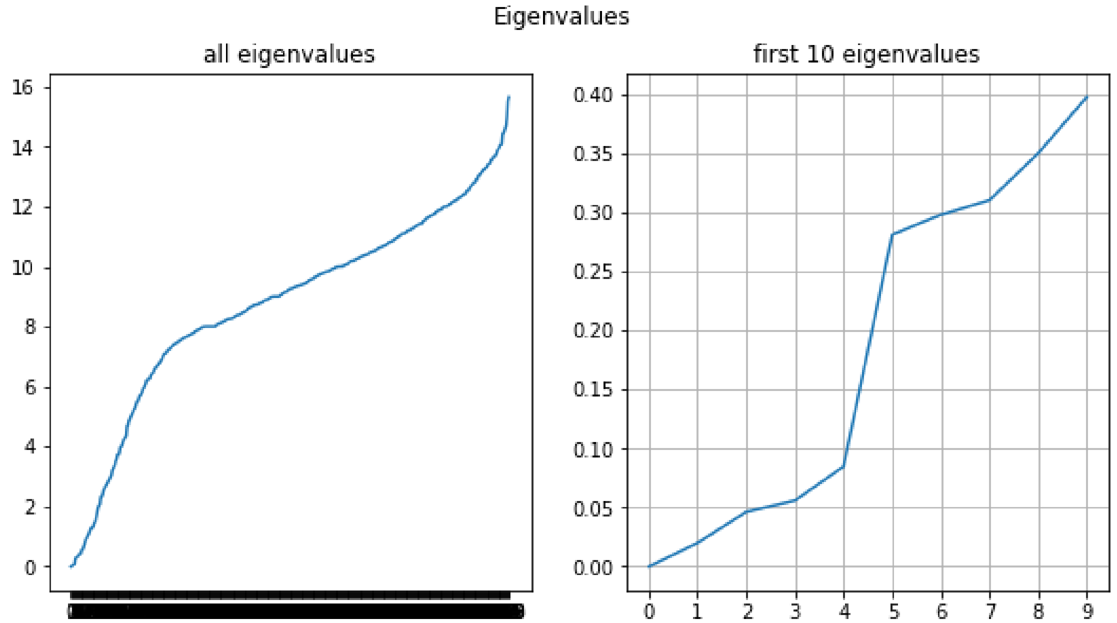
```
[ ]: colors1 = spectral_cluster(len(adj1), adj1, 3)
```



```
[ ]: colors2 = spectral_cluster(len(adj2), adj2, 9)
```

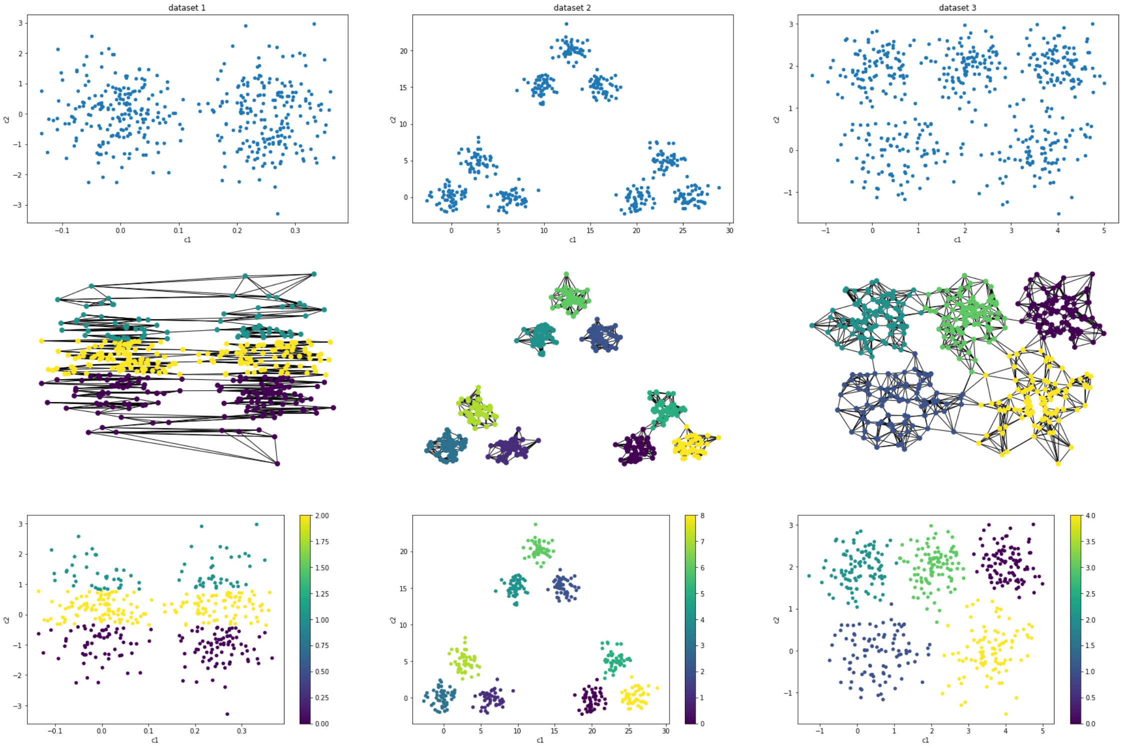


[ ]:
```
colors3 = spectral_cluster(len(adj3), adj3, 5)
```



[ ]:
```
fig, axs = plt.subplots(3, 3, figsize=(30, 20))
df1.plot.scatter(ax= axs[0][0], x = 'c1', y = 'c2')
df2.plot.scatter(ax= axs[0][1], x = 'c1', y = 'c2')
df3.plot.scatter(ax= axs[0][2], x = 'c1', y = 'c2')
axs[0][0].set_title("dataset 1")
axs[0][1].set_title("dataset 2")
axs[0][2].set_title("dataset 3")
nx.draw(graph1, font_size = 20, width = 1, with_labels=False, node_size = 50,␣
 ↪arrowsize = 100, nodelist=range(0,len(adj1)), pos= df1.to_numpy(),␣
 ↪node_color=colors1, ax = axs[1][0])
nx.draw(graph2, font_size = 20, width = 1, with_labels=False, node_size = 50,␣
 ↪arrowsize = 100, nodelist=range(0,len(adj2)), pos= df2.to_numpy(),␣
 ↪node_color=colors2, ax = axs[1][1])
nx.draw(graph3, font_size = 20, width = 1, with_labels=False, node_size = 50,␣
 ↪arrowsize = 100, nodelist=range(0,len(adj3)), pos= df3.to_numpy(),␣
 ↪node_color=colors3, ax = axs[1][2])
df1.plot.scatter(ax= axs[2][0], x = 'c1', y = 'c2', c=colors1,␣
 ↪colormap='viridis')
df2.plot.scatter(ax= axs[2][1], x = 'c1', y = 'c2', c=colors2,␣
 ↪colormap='viridis')
df3.plot.scatter(ax= axs[2][2], x = 'c1', y = 'c2', c=colors3,␣
 ↪colormap='viridis')
```

[ ]:
```
<AxesSubplot:xlabel='c1', ylabel='c2'>
```


dataset 1
dataset 2
dataset 3
c1
c2
−0.1
0.0
0.1
0.2
0.3
0
5
10
15
20
25
30
−1
1
2
3
4
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
6
7
8