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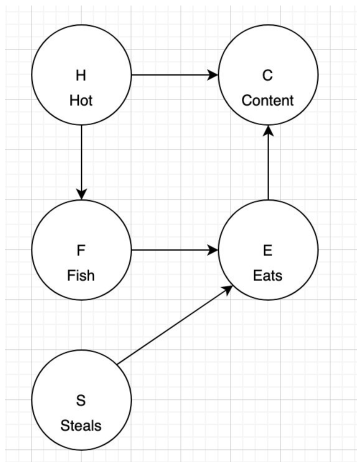
Assignment 3 - April 2
COMP 424

Questions 1:

a) Boolean variables: H, C, E, F, S



b) This Bayesian Network that displays the situation is not a polytree since it contains an undirected cycle. This is due to the fact that H effects C in two ways {H,F,E,C} and {G,C}.

c) P(F|H)

	F = 1	F = 0
H = 1	x	1-x
H = 0	y	1-y

d) P(E|F,S)

		E = 1	E = 0
S = 0	F = 0	0	1
	F = 1	1	0
S = 1	F = 0	0.5	0.5
	F = 1	1	0

e) $P(C|H) = P(C=1,H=1) / P(C=1) =$
$\sum_{F,E,S} P(C=c, F=f, S=s, H=1) / \sum_{F,E,S,C} P(C=c, F=f, S=s, H=1, C=c)$

Question 2:

a) $P(a|\neg r) = \sum_{B,T,S} P(R = 0, B = b, A = 1, S = s)$
$= \sum_{B,T,S} P(\neg r) * P(B=b) * P(T=t|\neg r,B=b,A=1) * P(A=1|B=b) * P(S=s|A=1)$
$= P(\neg r) \sum_B P(B=b) * P(A=1|B=b) * \sum_S P(S=s|A=1) * \sum_T P(T=t|\neg r,B=b,A=a)$
$= 0.8 * \mathbf{(}P(a|b)*P(b) + p(a|\neg b)*p(\neg b)\mathbf{)}$
$= 0.8 (0.7 * 0.4 + 0.2 * 0.6)$
$= \mathbf{0.32}$

b) $P(b,a) = \sum_{R,T,S} P(T=t|R=r,B=1,A=1) * P(S=s|A=1) * P(A=1|B=1) * P(R=r) * P(B=1)$
$= P(B=b) * P(A=1|B=1) \sum_R P(R=r) * \sum_T P(T=t|R=r,B=1,A=1) * \sum_S P(S=s|A=1)$
$= P(b) * P(a|b)$
$= 0.4 * 0.7$
$= \mathbf{0.28}$

c) To prune parents there must be a path from a specific node to B using node A. I.e $B \perp X|A$ where X is any node in the graph. Say $X = R \Rightarrow B \perp R|A$. This means that node **R and T can both be pruned**. Now say $X = S \Rightarrow B \perp S|A$. Thus there is a path from S to B using A as an observed node which means that **S can be pruned** as well. We notice that S is not independent from B.
We can conclude that **every node except A and B can be pruned**

d) $P(b|a) = P(a|b)*P(b) / P(a)$
** $P(a) = P(a|b)*P(b) + P(a|\neg b)*P(\neg b)$
$\Rightarrow P(b|a) = P(a|b)*P(b) / P(a|b)*P(b) + P(a|\neg b)*P(\neg b)$
$= 0.7 * 0.4 / (0.7 * 0.4 + 0.2 * 0.6)$
$= \mathbf{0.7}$

Question3:

$P(T|b=1)$ while ordering = S,A,R,T
Active factor list: $P(R), P(T|R,b,A), P(S|A), P(A|B), \delta(B,1)$

1) Eliminate S: $m_s(A) = \sum_s P(S|A) = 1$
2) Eliminate A: $m_a(b,r,T) = \sum_a P(A=a|b)P(T|R,b,A=a) * m_s(A=a) = \sum_a P(a|b)P(T|R,b,a)$
3) Eliminate R: $m_r(T,b) = \sum_r P(r)\, m_a(b,r=r,T)$
4) Eliminate T: $m_t(B) = \sum_t m_r(T,b)$

Active factor list: $\delta(B, 1), m_t(B)$
Using above steps:
$m_s(A) = \sum_s P(S|A) = 1$

$m_a(b,r,T) = \sum_a P(A|b)P(T|R,b,A)m_s(A) = \sum_a P(A|b)P(T|R,b,A)$
$= P(a|b)P(T|R,b,A) + P(\neg a|b)P(T|R,b,\neg a)$

Compute $m_a(b,r,T)$

$m_a(b,r,T) = P(a|b)P(T|R,b,a) + P(\neg a|b)P(T|R,b,\neg a)$

$$= 0.7 * \begin{bmatrix} 0.98 & 0.5 \\ 0.02 & 0.5 \end{bmatrix} + 0.3* \begin{bmatrix} 0.88 & 0.4 \\ 0.12 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.95 & 0.47 \\ 0.0.5 & 0.53 \end{bmatrix}$$

Compute $m_r(b,T)$

$$m_r(b,T) = P(r)m_a(b,r,T) = 0.2* \begin{bmatrix} 0.95 \\ 0.05 \end{bmatrix} + 0.8* \begin{bmatrix} 0.47 \\ 0.53 \end{bmatrix} = \begin{bmatrix} 0.566 \\ 0.434 \end{bmatrix}$$

Compute MAP

$m_t(b) = m_r(b,T)$

$P(T=1|b=1) = m_r(T=1) =$ **0.566 ⇒ MAX**

$P(T=0|b=1) = m_r(T=0) = 0.434$

Given that the max is $P(T=1|b=1) > P(T=0|b=1)$. **We conclude that T = 1 is MAP**

Question 4:

a)

	MLE	Laplace	MAP
θ_a	**60/144**	**61/146**	**0**
$\theta_{b\|a}$	**33/60**	**34/62**	**1**
$\theta_{b\|\neg a}$	**57/84**	**58/86**	**1**
$\theta_{c\|a}$	**50/60**	**51/62**	**1**
$\theta_{c\|\neg a}$	**62/84**	**63/86**	**1**
$\theta_{d\|b,c}$	**20/65**	**21/67**	**0**
$\theta_{d\|\neg b,\neg c}$	**4/7**	**5/9**	**1**
$\theta_{d\|\neg b,c}$	**26/47**	**27/49**	**1**
$\theta_{d\|b,\neg c}$	**11/25**	**12/27**	**0**

b)

i) E-step

$W_{b=0,\ d=0}$= P(B=0, D=0| a,c)
= P(a,¬b, c, ¬d) / P(a,c)
= $\theta_a\ \theta_{\neg b|a}\ \theta_{c|a}\ \theta_{\neg d|\neg b,c}$ / $\theta_a\ \theta_{c|a}$
= $\theta_{\neg b|a}\ \theta_{\neg d|\neg b,c}$
= 27/60 * 21/ 47
= **0.201**

$w_{b=0,\ d=1}$= $\theta_{\neg b|a}\ \theta_{d|\neg b,c}$
= 0.45 * 26/47
= **0.2489**

$w_{b=1,\ d=0}$= P(B=1,D=0|a,c)
= P(a,b, c, ¬d) / P(a,c)
= $\theta_a\ \theta_{b|a}\ \theta_{c|a}\ \theta_{\neg d|b,c}$ / $\theta_a\ \theta_{c|a}$
= $\theta_{b|a}\ \theta_{\neg d|b,c}$
= 33/60 * 0.69
= **0.3808**

$w_{b=1,\ d=1}$= $\theta_{b|a}\ \theta_{d|b,c}$
= 33/60 * 20/65
= **0.169**

ii)

Θ_a = 60+2 / 144 + 2 = 62/146

$\Theta_{b|a}$ = 34 + $w_{b=1,\ d=0}$ + $w_{b=1,\ d=1}$/62
= 34.55 / 62

$\Theta_{b|\neg a}$ = 57/84

$\Theta_{c|a}$ = 51/62

$\Theta_{c|\neg a}$ = 62/84

$\Theta_{d|b,c}$ = 20+$w_{b=1,\ d=1}$+$w_{b=0,\ d=1}$/65+$w_{b=1,d=0}$+$w_{b=1,d=1}$
= 20.418 / 65.55

$\Theta_{d|\neg b,\neg c}$ = 4/7

$\Theta_{d|\neg b,c}$ = 26+$w_{b=0,\ d=1}$+$w_{b=1,\ d=1}$/47+$W_{b=0,\ d=0}$+$w_{b=0,d=1}$
= 26.4179 / 47.4499

$\Theta_{d|b,\neg c}$ = 11/25

iii) Second E-Step

$W_{b=0,\ d=0} = \theta_a\ \theta_{\neg b|a}\ \theta_{c|a}\ \theta_{\neg d|\neg b,c}\ /\ \theta_a\ \theta_{c|a}$
$= \theta_{\neg b|a}\ \theta_{\neg d|\neg b,c}$
$= (1\text{-}34.55\ /\ 62) * (1\text{-}26.449\ /\ 47.4499)$
$=$ **0.196**

$w_{b=0,\ d=1} = \theta_{\neg b|a}\ \theta_{d|\neg b,c}$
$= (1\text{-}34.55\ /\ 62) * 26.449\ /\ 47.4499$
$=$ **0.2467**

$w_{b=1,\ d=0} = \theta_{b|a}\ \theta_{\neg d|b,c}$
$= 34.55\ /\ 62 * (1\text{-}20.418\ /\ 65.55)$
$=$ **0.3837**

$w_{b=1,\ d=1} = \theta_{b|a}\ \theta_{d|b,c}$
$= 34.55\ /\ 62 * 20.418\ /\ 65.55$
$=$ **0.1735**