#4 (Total 30 Points)

[1]

The inverse $(\boldsymbol{X}'\boldsymbol{X})^{-1}$ must be calculated. The Gauß algorithm is by far too complicated; therefore, we calculate the ***determinant*** and the ***adjoint***.

$$(\boldsymbol{X}'\boldsymbol{X})^{-\mathbf{1}} = \frac{1}{\det(\boldsymbol{X}'\boldsymbol{X})} * Adj(\boldsymbol{X}'\boldsymbol{X})$$

In this exercise, we ***start*** with the ***determinant*** given the matrix multiplication $(\boldsymbol{X}'\boldsymbol{X})$:

$$(\boldsymbol{X}'\boldsymbol{X}) = \begin{bmatrix} 6 & 8 & 9 \\ 10 & 5 & 7 \\ 8 & 5 & 6 \end{bmatrix}$$

This means

$$\det(\boldsymbol{X}'\boldsymbol{X}) = 6 * \begin{vmatrix} 5 & 7 \\ 5 & 6 \end{vmatrix} - 8 * \begin{vmatrix} 10 & 7 \\ 8 & 6 \end{vmatrix} + 9 * \begin{vmatrix} 10 & 5 \\ 8 & 5 \end{vmatrix} = (6 * 5 * 7 - 6 * 7 * 5) - (\dots) + (\dots) = 28$$

The value equals a scalar.

[2]

In our case, we compute the cofactor matrix of $(\boldsymbol{X}'\boldsymbol{X})$:

$$\boldsymbol{C} = \begin{bmatrix} \begin{vmatrix} 5 & 7 \\ 5 & 6 \end{vmatrix} & -\begin{vmatrix} 10 & 7 \\ 8 & 6 \end{vmatrix} & \begin{vmatrix} 10 & 5 \\ 8 & 5 \end{vmatrix} \\ -\begin{vmatrix} 8 & 9 \\ 5 & 6 \end{vmatrix} & \begin{vmatrix} 6 & 9 \\ 8 & 6 \end{vmatrix} & -\begin{vmatrix} 6 & 8 \\ 8 & 5 \end{vmatrix} \\ \begin{vmatrix} 8 & 9 \\ 5 & 7 \end{vmatrix} & -\begin{vmatrix} 6 & 9 \\ 10 & 7 \end{vmatrix} & \begin{vmatrix} 6 & 8 \\ 10 & 5 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} -5 & -4 & 10 \\ -3 & -36 & 34 \\ 11 & 48 & -50 \end{bmatrix}$$

[3]

The adjoint corresponds with the transposed cofactor matrix.

$$\boldsymbol{C}' = \begin{bmatrix} -5 & -3 & 1 \\ -4 & -36 & 48 \\ 10 & 34 & -50 \end{bmatrix} = Adj(\boldsymbol{X}'\boldsymbol{X})$$

Bringing all together leads to the following result:

$$(\boldsymbol{X}'\boldsymbol{X})^{-\mathbf{1}} = \frac{1}{\det(\boldsymbol{X}'\boldsymbol{X})} * Adj(\boldsymbol{X}'\boldsymbol{X}) = \frac{1}{28} * \begin{bmatrix} -5 & -3 & 1 \\ -4 & -36 & 48 \\ 10 & 34 & -50 \end{bmatrix} = \begin{bmatrix} -0{,}18 & -0{,}11 & 0{,}39 \\ -0{,}14 & -1{,}29 & 1{,}71 \\ 0{,}36 & 1{,}21 & -1{,}79 \end{bmatrix}$$

[4]

Finally, we can compute the **b**-vector:

$$\Rightarrow \boldsymbol{b} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y} = \frac{1}{28} * \begin{bmatrix} -5 & -3 & 1 \\ -4 & -36 & 48 \\ 10 & 34 & -50 \end{bmatrix} * \begin{bmatrix} 120 \\ 150 \\ 160 \end{bmatrix} = \begin{bmatrix} 25{,}36 \\ 64{,}29 \\ -60{,}71 \end{bmatrix}$$