#3 (Total 15 Points)

(1)

M/G/k-model

with

- M = multiple server
- G = unspecified probability distribution
- k = k server

$$P_j = \frac{(\lambda/\mu)^j/j!}{\sum_{j=0}^{k} \left(\lambda/\mu\right)^j/j!}$$

$$\lambda = 42$$

$$\mu = 20$$

j	$(\boldsymbol{\lambda}/\boldsymbol{\mu})^{\boldsymbol{j}}/\boldsymbol{j}!$
0	$(42/20)^0/0! = 1{,}0000$
1	2,1000
2	2,2050
3	1,5435
Σ	**6,8485**

j	$\boldsymbol{P_j}$
0	1/6,8485 = 0,1460
1	2,1000/6,8485 = 0,3066
2	2,2050/6,8485 = 0,3220
3	1,5435/6,8485 = 0,2254

(2)

The most important probability value is P_k, which is the probability that all k servers are busy, and arrivals are blocked.

⇨ Here: k = 3 with $P_3 = 0{,}2254$

(3)

The average number of units in the system

$$L = \frac{\lambda}{\mu} * (1 - P_k) = \frac{42}{20} * (1 - 0{,}2254) = 1{,}6267$$

(4)

We examine a k = 4 server model ($\lambda = 50$, $\mu = 20$)

j	$(\boldsymbol{\lambda}/\boldsymbol{\mu})^{\boldsymbol{j}}/\boldsymbol{j}!$	$\boldsymbol{P_j}$
0	1,0000	0,0921
1	2,500	0,2303
2	3,1250	0,2878
3	2,6042	0,2399
4	1,6276	0,1499
Σ	**10,8568**	

With 0,1499 < 0,2254 we meet management expectation, so k = 4 is enough.