Introduction & Applications

Defination :

Linear Algebra is branch of mathematics which deals with linear equation and their representation through vectors and matrices .

Some main topic and definition involved in linear algebra are :

1) Basis Vector
2) Linear Combination
3) Span and Linearly Dependent & Independent Vectors
4) Matrix
5) Determinant
6) Eigen Value & Eigen vector

Here all abstract view of the definition are given :

1) **Basis Vector** : They are the unit vector in x,y and z direction .

   a. i is unit vector on x-axis .
   b. j is unit vector on y-axis .
   c. k is unit vector on z-axis .

   here i ,j , k are called basis vector in XYZ co-ordinate system .

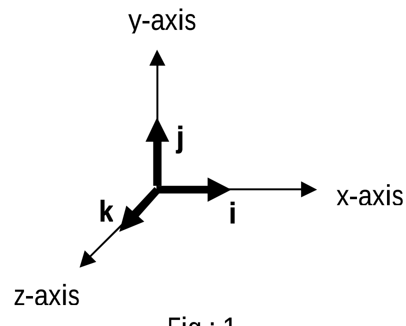

Fig : 1

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Co-ordinates of i vector

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Co-ordinates of j vector

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Co-ordinates of k vector

2) **Linear Combination :** Whenever we scale the unit vector and add them up we get the linear combination of those two vectors .

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- For example :

$$w = 3u + 2v$$

Here w is called the linear combination of u and v and the scalers are 2 & 3 .

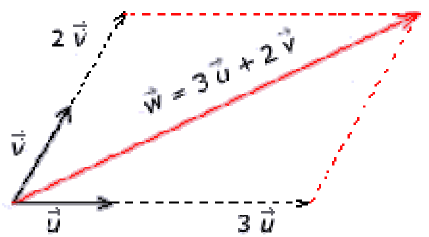

Fig : 2

w is linear combination of u and v

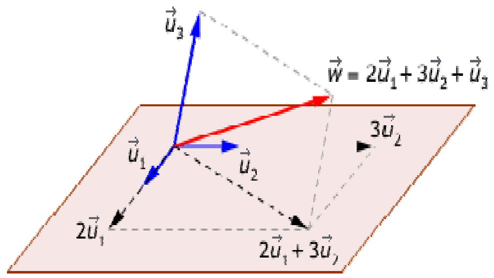

Fig : 3

w is linear combination of u1, u2 & u3

3) Span and linear dependence :

The set of all possible vector that can be reach in vector space with the linear combination of the original vectors(basis vector) is called span of the original vector(basis vector)

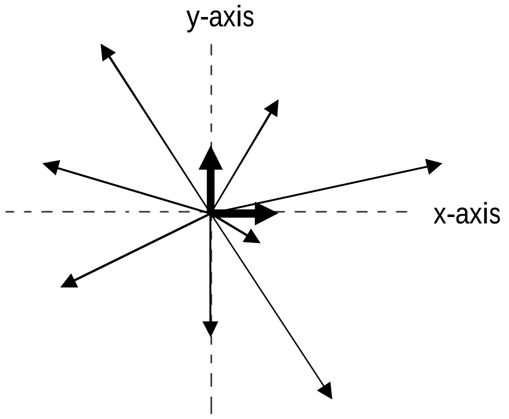

Fig : 4

The figure shows the span of 2D basis vector

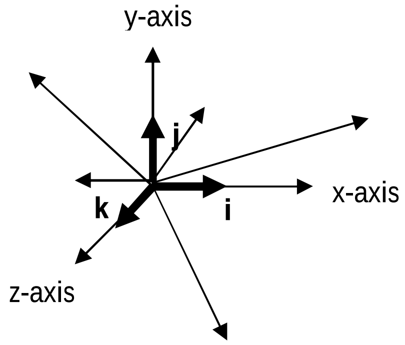

Fig : 5

The figure shows the span of 3D basis vectors

Linearly Dependent Vectors

If we remove one of the vector from the linear combination of two vector and still if the span of that combination remains same than that vectors are linearly dependent vectors .

The span does not changes because they are scalar multiple of each other

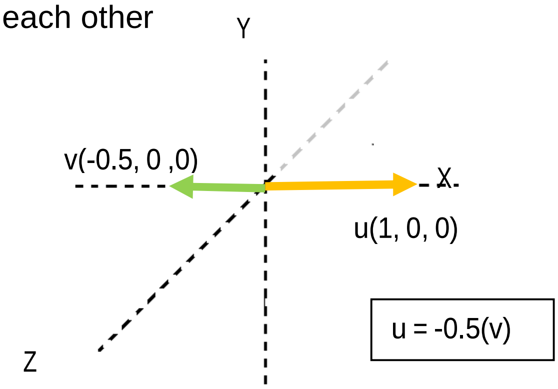

Fig : 6

Linearly Independent vectors

If we remove one of the vector from the linear combination of two vector and if the span of that combination changes than that vectors are linearly independent vectors .

Than span changes here because they are not scalar multiple of each other

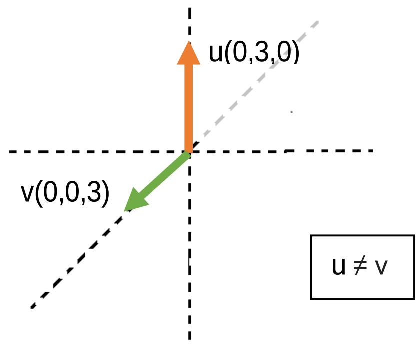

Fig : 7

4) **Matrix :** A matrix is collection of m rows and n column in and m*n matrix .

$$\begin{array}{c c} & \begin{matrix} 1 & 2 & \cdots & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{matrix} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \end{array}$$

Fig : 8

5) Determinant : The factor by which the linear transformation changes the area (scale or squeeze) is called the determinant of that transformation .

For 2D matrix the determinant tells the amount of change in area .

For 3D matrix the determinant tells the amount of change in Volume .

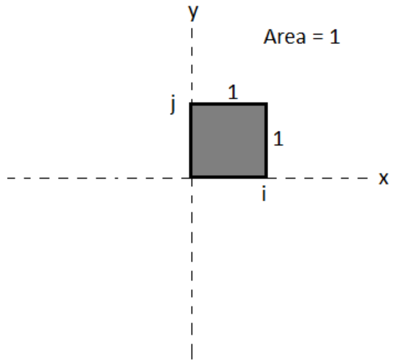

Fig : 9

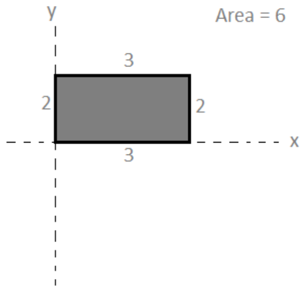

Fig : 10

Determinant tells the area(2D) or volume(3D) formed by the vectors .

$$\det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1(1) - 0(0) = 1 \qquad \det\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} = 3(2) - 0(0) = 6$$

6) Eigen Value and Eigen Vector : The vector v which do not change its direction on applying the transformation T is called the eigen vector of that transformation T. On applying the transformation the eigen vector only changes its size by λ which is either scaled or squeeze

This condition can be written as the equation

$$T(v) = \lambda v$$

λ can be positive , negative ,zero or complex number depending on the transformation applied .

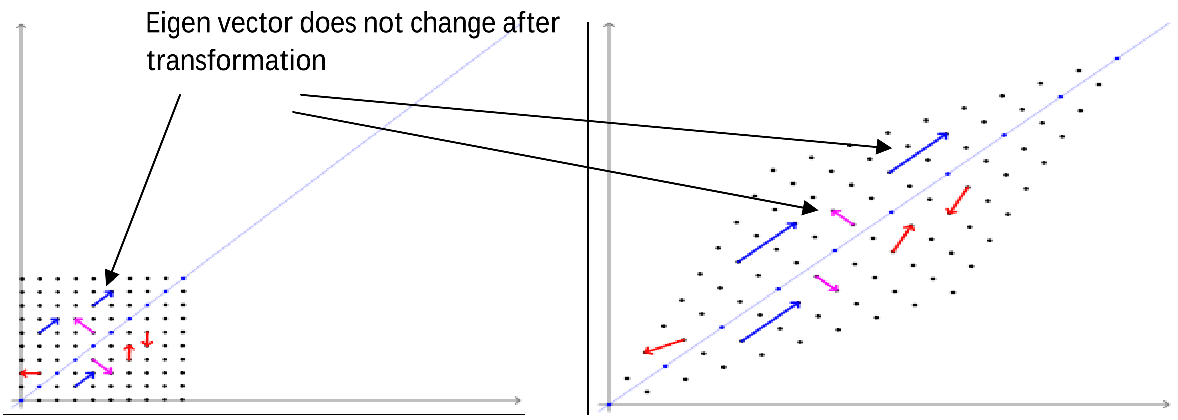

Fig : 11

Before transformation

Fig : 12

After transformation

As shown by the above figure , there are three different vectors

1) Blue Vector : This vector are the eigen vector as they do not change the direction on applying transformation but they change their size (scaled or squeeze) by λ
2) Red Vector : This vector are not eigen vector as on applying the transformation they change their direction.
3) Pink Vector : This vector are the eigen vector as they do not change the direction on applying transformation but they change their size (scaled or squeeze) by λ .

Refrences

Fig 2 : https://www.lemat.unican.es/lemat/proyecto_lemat/geometria/nivel2/img/geoma203.gif

Fig 3: https://www.lemat.unican.es/lemat/proyecto_lemat/geometria/nivel2/img/geoma204.gif

Fig 8: https://en.wikipedia.org/wiki/File:Matris.png

Fig 11: https://en.wikipedia.org/wiki/Eigenvalues_and_eigenvectors#/media/File:Eigenvectors.gif

Fig 12: https://en.wikipedia.org/wiki/Eigenvalues_and_eigenvectors#/media/File:Eigenvectors.gif