Eigen Value And Eigen Vector

Eigen Value and Eigen Vector : The vector v which do not change its direction on applying the transformation T is called the eigen vector of that transformation T. On applying the transformation the eigen vector only changes its size by λ which is either scaled or squeeze .This condition can be written as :

$$T(v) = \lambda v$$

λ can be positive , negative ,zero or complex number depending on the transformation applied .

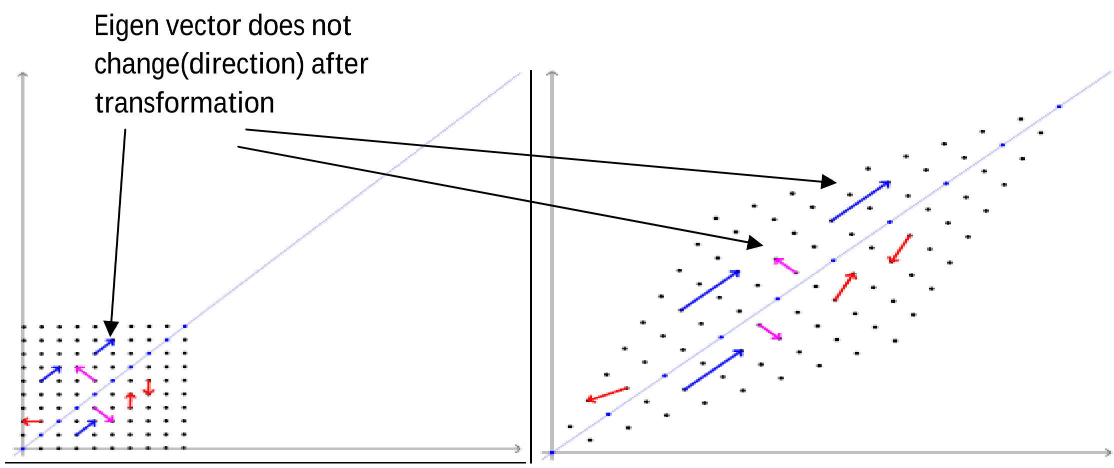

Fig : 1

Before transformation

Fig : 2

After transformation

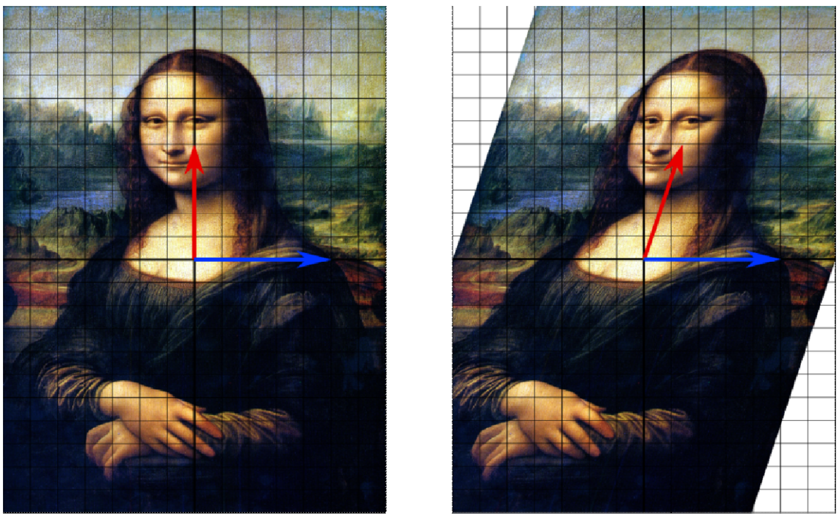

Fig : 3

In this figure the shear transformation is applied and as we can see in the figure the blue vector is not changing the direction and the size is also constant (λ=1) which means it is the eigen vector of this shear transformation and eigen value is 1.

Eigen value and Eigen vector of matrix :

From above definition of eigen value and eigen vector we know that the eigen vector never change its direction but scale (in +ve or –ve direction)only .

Consider the following example of vector :

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad y = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

In this case $\lambda = 2$.

These vectors are said to be scalar multiples or parallel or collinear, if there is a scalar λ such that $x\lambda = y$.

Lets take a matrix A having n*n dimenstion and linear transformation is performed on it

$$\begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

$$Av = w$$

If it occurs than v and w are scalar multiples of each other, that is if

$$W = \lambda v$$

than we can write as

$$Av = \lambda v$$

$$(A - \lambda I)v = 0 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

Where I is the n*n Identity matrix and 0 represents the zero vector

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- **Calculating the eigen value and eigen vector of 2D matrix**

Consider the following 2D matrix

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Now placing the value of A in the eigen equation (1) we get

$$| A - \lambda I | = \text{Det} \begin{bmatrix} 2 - \lambda & 1 \\ 1 & 2 - \lambda \end{bmatrix} = 0$$

$$(2 - \lambda)(2 - \lambda) - 1 = 0$$

$$3 - 4\lambda + \lambda^2 = 0$$

The roots of the above equation are $\lambda = 1$ and $\lambda = 3$, which are the two eigenvalues of *A*.

In this example, the eigenvectors are any nonzero scalar multiples of

$$v_{\lambda = 1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad v_{\lambda = 3} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

If the elements of the matrix *A* are all real numbers, then the coefficients of the of the equations will also be real numbers, but the eigenvalues formed by them may still imaginary parts.

- **Calculating the eigen value and eigen vector of 3D matrix**

Consider the following 3D matrix :

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 9 \end{bmatrix}$$

Now placing the value of A in the eigen equation (1) we get

$$| A - \lambda I | = \begin{bmatrix} 2 - \lambda & 0 & 0 \\ 0 & 3 - \lambda & 4 \\ 0 & 4 & 9 - \lambda \end{bmatrix} = 0$$

$$(2 - \lambda)((3 - \lambda)(9 - \lambda) - 16) = 0$$

$$-\lambda^3 + 14\lambda^2 - 35\lambda + 22 = 0$$

The roots of above equation are 2, 1 and 11 which are the eigen value of A and their corresponding eigen vector are [1, 0, 0] , [0, -2, 1] and [0, 1, 2] respectively .

Refrence

Fig 1: https://en.wikipedia.org/wiki/Eigenvalues_and_eigenvectors#/media/File:Eigenvectors.gif

Fig 2: https://en.wikipedia.org/wiki/Eigenvalues_and_eigenvectors#/media/File:Eigenvectors.gif

Fig 3: https://en.wikipedia.org/wiki/Eigenvalues_and_eigenvectors#/media/File:Mona_Lisa_eigenvector_grid.png