Matrices

Matrix : A matrix is a collection of m rows and n columns in m*n matrix

$$
\begin{array}{cc}
 & \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{array} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}
\end{array}
$$

Fig :1

- **Addition of matrix :**

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

- **Scalar Multiplication :**

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow 2A = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

- **Transpostion :**

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix} \quad \rightarrow \quad A^T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- **Matrix Multiplication :**

Multiplication of two matrices is possible if number of column of matrix on left is equal to the number of rows of right matrix. If A is an 3*3 matrix and B is an 3*1 matrix, then the result of matrix multiplication AB is the 3*1 matrix .

$$A = \overset{3 * 3}{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}} \qquad B = \overset{3 * 1}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

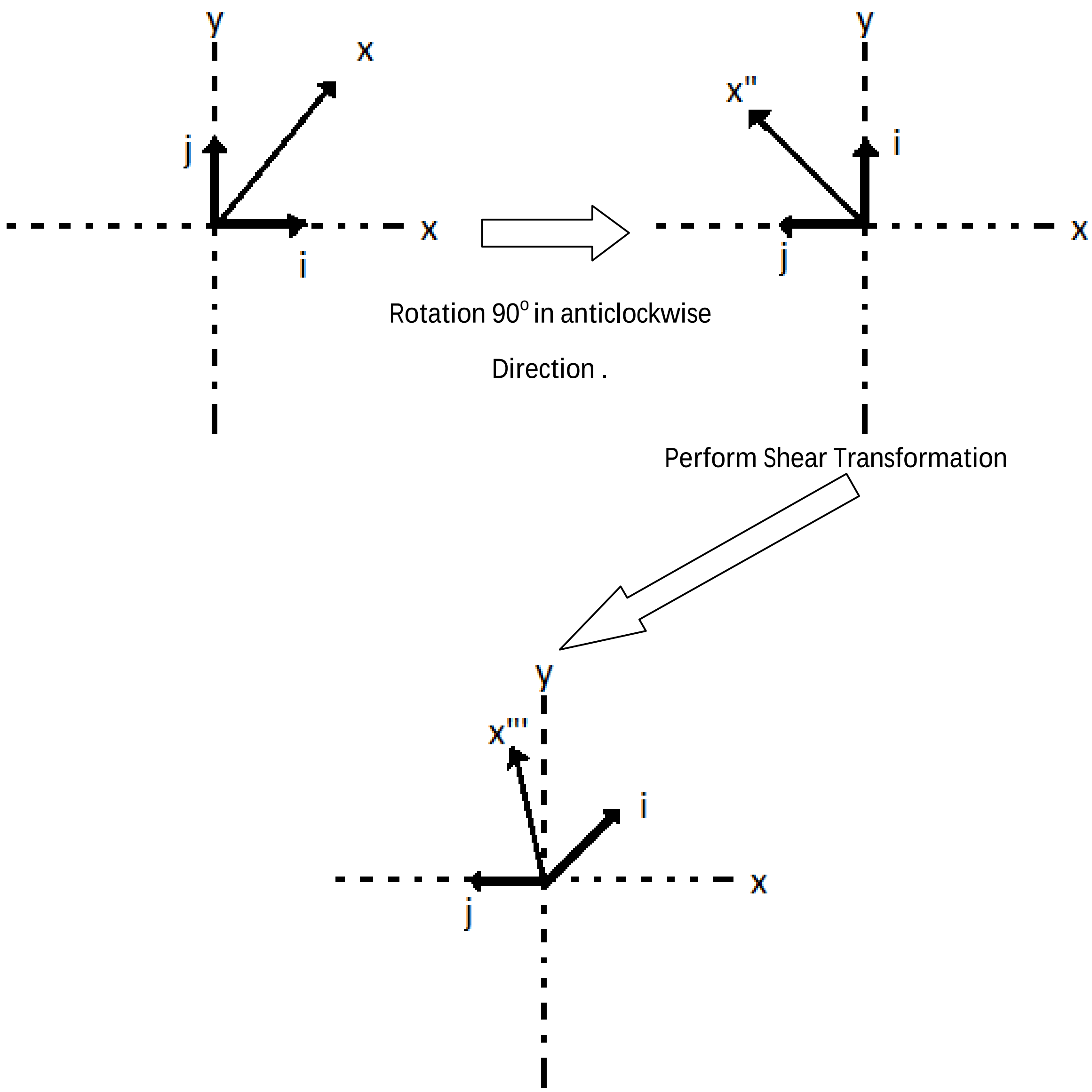
$$AB = \begin{bmatrix} 1(1) + 2(1) + 3(1) \\ 1(0) + 2(0) + 3(1) \\ 1(0) + 2(0) + 3(1) \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 \\ 0 \\ 3 \end{bmatrix}$$

What really happen during matrix multiplication is shown in the below figure .

Suppose we want to perform some transformation one after the other , Lets say :

1) Rotation in 90 degree in anticlockwise direction .
2) Than applying shear transformation .



$$\underbrace{\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}}_{\text{Shear}} \underbrace{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}_{\text{Rotation } (-90^{o})} = \underbrace{\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}}_{\text{Composition}}$$

Read from left to right

- **Matrix Dot Product**

$$\begin{bmatrix} a & b \end{bmatrix} \bullet \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \bullet \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \bullet \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{bmatrix}$$

Fig : 2

- **Inverse of the matrix**

$$\mathsf{A} \equiv \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

$$\mathsf{A}^{-1} = \frac{1}{|\mathsf{A}|} \begin{bmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}.$$

Fig : 3

- **Some properties of matrices :**

Let suppose A, B and C are m*n matrices and c and d are scaler than properties of matrices are given below :

1. Commutative Property of Addition : $A + B = B + A$
2. Associative Property of Addition : $A + (B + C) = (A + B) + C$
3. Associative Property of Multiplication : $cd(A) = c(d(A))$
4. Multiplicative Property : $IA = A$
5. Distributive Property : $c(A + B) = cA + cB$
6. Distributive Property : $(c + d)A = cA + cB$

Refrences

Fig 1 : https://en.wikipedia.org/wiki/File:Matris.png

Fig 3 : https://i.stack.imgur.com/ChJRa.gif