Determinant

- **Determinant :** The factor by which the linear transformation changes the area or volume(either scale or squeeze it) is called the determinant of that matrix .
    - The determinant of a matrix *A* is denoted det(*A*) or |*A*|.

- **Determinant of 2D matrix** :

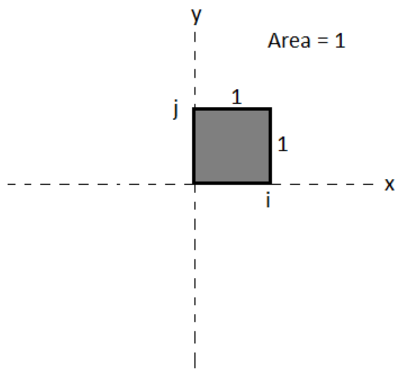


Fig : 1

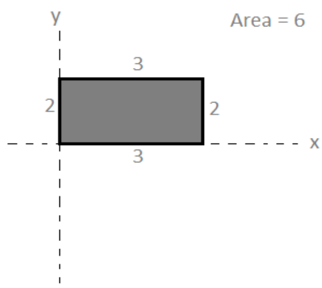


Fig : 2

$$\det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1(1) - 0(0) = 1$$

$$\det \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} = 3(2) - 0(0) = 6$$

As shown in above figure the determinant of the matrix of fig-2 is 3 which says that the area of the transformed matrix .

- **Similarly for 3D matrix** :

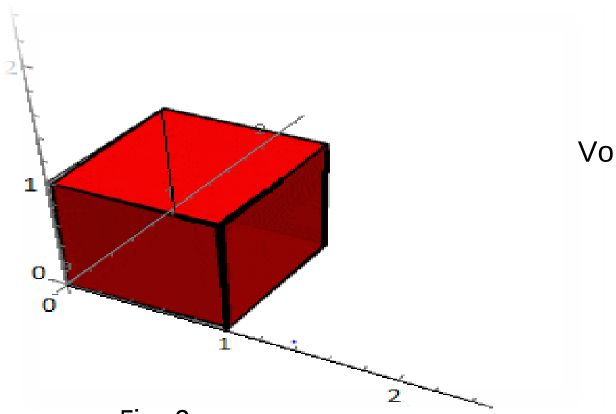
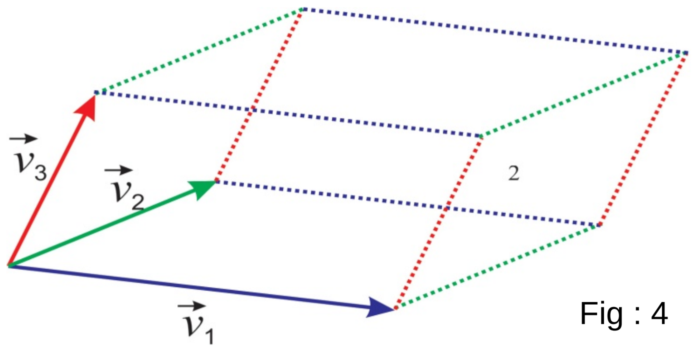


Fig : 3

$$\text{Volume} = \det \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix} = 1*1*1 = 1$$



$$V = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$$

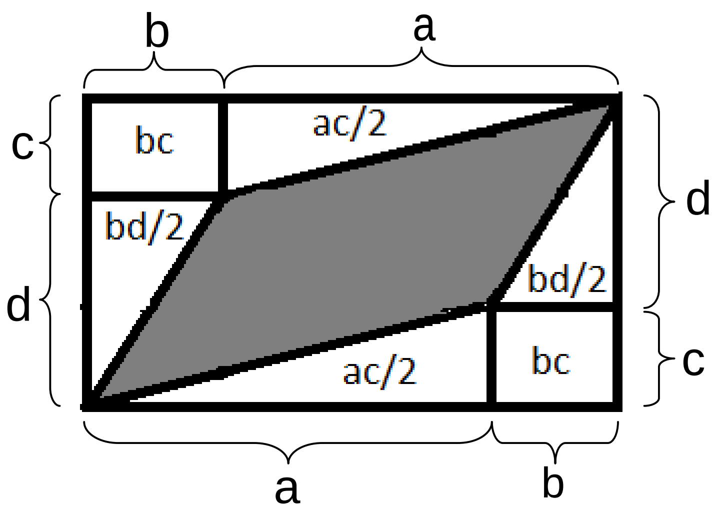
Fig : 4

The volume formed by 3 vectors (parallelepiped) can be calculated by finding its determinant as follows :

$$V = x_1(y_2*z_3 – y_3*z_2) – x_2(y_1*z_3 –y_3*z_1) + x_3(y_1*z_2 – y_2*z_1)$$

- **Derivation of the formula for 2D matrix :**

Fig :5



$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a+b)(c+d) - ac - bd - 2bc = \underline{\boldsymbol{ad - bc}}$$

Here value of **‘ad’** tells how much the area is scaled or squeeze, while the value of **‘bd’** tells how much the area is stretched or squeeze in the diagonal direction .

- **Determinant to solve linear equation :**

Consider the following case of linear equation :

$3x + 2y - z = 2$

$2x - y - 3z = 13$

$x + 3y - 2z = 1$

Sol^n :

Step 1) Calculate the determinant of the matrix formed by putting the coefficients of x,y and z in matrix .

$$D = \det\begin{bmatrix} 3 & 2 & -1 \\ 2 & -1 & -3 \\ 1 & 3 & -2 \end{bmatrix} = 3(2 - (-9)) - 2(-4 - (-3)) + 1(-6 - 1)$$

$$D = 28$$

Step 2) Now find the value of Dx , Dy, Dz by replacing the constant value by corresponding coefficents of x , y and z values .

$$D_x = \det\overbrace{\begin{bmatrix} 2 & 2 & -1 \\ 13 & -1 & -3 \\ 1 & 3 & -2 \end{bmatrix}}^{\text{Constants replacing x coefficents}} = 2(2 - (-9)) - 13(-4 - (-3)) + 1(-6 - 1)$$

$$D_x = 28$$

$$D_y = \det \overbrace{\begin{bmatrix} 3 & 2 & -1 \\ 2 & 13 & -3 \\ 1 & 1 & -2 \end{bmatrix}}^{\text{Constants replacing y coefficents}} = 3(26 - (-3)) \; - 2(-4 - (-1)) + 1(-6 - (-13))$$

$$D_y = -56$$

$$D_z = \det \overbrace{\begin{bmatrix} 3 & 2 & 2 \\ 2 & -1 & 13 \\ 1 & 3 & 1 \end{bmatrix}}^{\text{Constants replacing z coefficents}} = 3(-1 - 39) \; - 2(2-6) + 1(26 - (-2))$$

$$D_z = -84$$

Step 3) Now calculate the solution of x,y and z by dividing D with all the D_x ,D_y and D_z respectively .

$x = D_x/D = 28/28 = 1 \Rightarrow x=28$

$y = D_y/D = -56/28 = -2 \Rightarrow y=-2$

$z = D_z/D = -84/28 = -3 \Rightarrow z=-3$

Note :

- The system is dependent if all the D_x , D_y ,D_z have a value equal to 0 .
- The system is inconsistent if at least one of the D_x ,D_y ,D_z , has a value not equal to 0 and the denominator D has a value equal to 0.

Refrence

Fig 3 : https://i.stack.imgur.com/4LrnA.gif

Fig 4:
https://chem.libretexts.org/@api/deki/files/141561/3dvolume.jpg?revision=1&size=bestfit&width=647&height=237