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Tutorial 5
(Evaluation & Measurement of hypothesis testing)

1) $H_0 : P = 0.7$
$H_1 : P \neq 0.7$
Level of significance $= \alpha = 0.1$
Test Statistic: Binomial val. with $p = 0.7$, $n = 15$
$X = 8$ & $np_0 = 15 \times 0.7 = 10.5$
$\therefore p = 2P(X \leq 8 \text{ when } p = 0.7)$
$= 2 \sum_{x=0}^{8} b(x; 15, 0.7)$

$= 2 \times 0.1311$ (from binomial prob. table)
$= 0.2622$
$\because p > 0.1$ i.e. $P > \alpha$
$\therefore$ We do not reject H_0. So, there is insufficient evidences to doubt the builder's claim.

2) $H_0 : P = 0.6$
$H_1 : P > 0.6$
$\alpha = 0.05$ given, $x = 70$, $n = 100$, $P = 0.6$

$$Z = \frac{x - nP_0}{\sqrt{nP_0 q_0}} = \frac{70 - 100 \times 0.6}{\sqrt{100 \times 0.6 \times 0.4}} = 2.04$$

$P = P(z > 2.04) = 0.0207$ (from table)

as $P < \alpha$, reject H_0 & conclude that new drug is superior

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3) let P_1 be proportion of Mumbai voters
P_2 be proportion of surrounding area residents

$\hat{P}_1 = \frac{120}{200} = 0.6$, $\hat{P}_2 = \frac{240}{500} = 0.48$, $\hat{P}_p = \frac{120+240}{200+500} = 0.514$

$\alpha = 0.05$

Hypothesis: $H_0 : P_1 \le P_2$
$H_1 : P_1 > P_2$

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}_p(1-\hat{P}_p)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.6 - 0.48}{\sqrt{0.514(1-0.514)\left(\frac{1}{200}+\frac{1}{500}\right)}}$$

$\therefore Z = 2.869 \Rightarrow P(Z > 2.869) = 0.0044$

now as $P < \alpha$, reject H_0 and conclude that the proportion of Mumbai voters favoring the proposal is higher than proportion of surrounding area voters.

4) a) $H_0 : p = 0.2$, the critical region is in right tail
$H_1 : p > 0.2$

b) $H_0 : \mu = 3$, the critical region is in both tails
$H_1 : \mu \neq 3$

c) $H_0 : p \geq 0.15$, the critical region is in left tail.
$H_1 : p < 0.15$

d) $H_0 : \mu = 500$, the critical region is in right tail.
$H_1 : \mu > 500$

e) $H_0 : \mu = 15$, the critical region is in both tails.
$H_1 : \mu \neq 15$

5) Let M_1 = population mean "robustness" - laptops - company A
M_2 = population mean "robustness" - laptops - company B

$H_0 : M_1 = M_2$, $\alpha = 0.05$
$H_1 : M_1 \neq M_2$

$$\bar{x}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_{1i} = \frac{9.3 + 8.8 + 6.8 + 8.7 + 8.5 + 6.7 + 8 + 6.5 + 9.2 + 7}{10}$$

$$= 7.95$$

$$\bar{x}_2 = \frac{1}{n_2}\sum_{i=1}^{n_2} x_{2i} = \frac{11 + 9.8 + 9.9 + 10.2 + 10.1 + 9.7 + 11 + 11.1 + 10.2 + 9.6}{10}$$

$$= 10.26$$

$$s_1^2 = \frac{1}{n_1 - 1}\left(\sum_{i=1}^{n_1} x_{1i}^2 - n_1\bar{x}_1^2\right) = \frac{10.865}{9} = 1.207$$

$$s_2^2 = \frac{1}{n_2 - 1}\left(\sum_{i=1}^{n_2} x_{2i}^2 - n_2\bar{x}_2^2\right) = \frac{2.924}{9} = 0.325$$

as sample variances are very different, we cannot assume population variances equal, so use the "unpooled t-test"

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1} + \left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2} + \left(\frac{s_2^2}{n_2}\right)^2} = \frac{\frac{1.207}{10} + \frac{0.325}{10}}{\frac{1}{9}\left(\frac{1.207}{10}\right)^2 + \frac{1}{9}\left(\frac{0.325}{10}\right)^2}$$

$\therefore \nu = 10.30 \approx 10$

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The test statistic used to test hypothesis is: ~~T~~

$$T = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

which under the null hypothesis, follows approximately t-distribution with $v = 10$ degrees of freedom. Under null hypothesis, $(\mu_1 - \mu_2) = 0$

$\therefore$ value of $T = \frac{7.95 - 10.26}{\sqrt{\frac{1.207}{10} + \frac{0.325}{10}}} = -5.90$

Since, the test is two-sided, the value of test is the doubled area under density curve of t-distribution with ($v = 10$), right of the absolute value of test statistic

$|t| = |-5.9| = 5.9$ i.e. p-value $= 2P(T \geq |t|)$
$= 2P(T \geq 5.9)$

$t_{0.0005}(10) = 4.587$ & since $|t| = 5.9$ is even greater than $P(T \geq 5.9) < 0.0005$ so,
p-value < 0.001

as $p < \alpha$, ~~accept~~ reject null hypothesis, conclude that the mean "robustness" of laptop is not same for both companies