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Tutorial 4

Independent Component Analysis

- Mixing statistically independent sources now variance of mixture

$$= var(x) = \langle (x - \langle x \rangle)^2 \rangle$$

$$= \langle x^2 \rangle - (\langle x \rangle)^2$$

$$= \langle (\sum_i w_i s_i)^2 \rangle - (\langle \sum_i w_i s_i \rangle)^2$$

$$= \langle (\sum_i w_i s_i)(\sum_j w_j s_j) \rangle - (\sum_i w_i \langle s_i \rangle)(\sum_j w_j s_j)$$

$$= \langle \sum_{i,j} w_i w_j s_i s_j \rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$$

$$= \sum_i w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle) +$$

$$\sum_{i,j \; i \neq j} w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle)$$

$$= \sum_i w_i^2 (\langle s_i s_j \rangle - \langle s_i \rangle^2) + \sum_{i,j \; i \neq j} w_i w_j (\langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle)$$

s_i & s_j are statistically independent for $i \neq j$

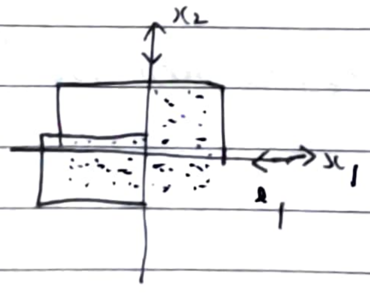
$$\Rightarrow \langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle = 0$$

and $var(s_i) = 1$

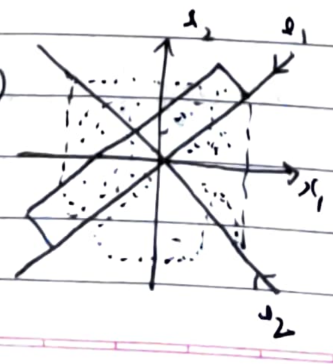
$\therefore$ $var(x) = \sum_i w_i^2$ to guarantee that mixture has unit variance, $var(x) \overset{!}{=} 1$

$\therefore \sum_i w_i^2 = 1$ ← The following constraint has to be imposed on the weight w_i for the mixture to have unit variance.

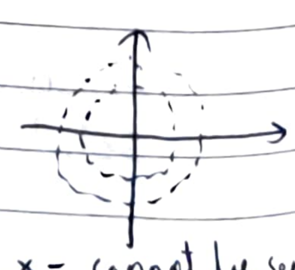
Ex.2. a)



b)



c)



x - cannot be separated into independent components