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Tutorial 6

1) a) $P(H) = \lambda$
$P(T) = 1-\lambda$ $P(\text{first head at } k+1) = (1-\lambda)^k \cdot \lambda$

b) Let M be no. of tosses required to get first head,
Let $S = E[M]$ as tosses are independent & eqn. is additive,
$S = \lambda \times 1 + (1-\lambda)(S+1)$
$= \lambda + S + 1 - \lambda S - \lambda$
$\therefore \lambda S = 1$ $\therefore S = 1/\lambda$

2) $X \rightarrow$ random var.
a) $var(X) = E[X - E(X)^2]$
to prove : $var(X) = E[X^2] - (E[X])^2$
now, we have,
$var(X) = E[(X - E[X])^2]$
$= E[X^2 - 2X E[X] + E[X]^2]$
$= E[X^2] - 2E[X E[X]] + E[X]^2$
$= E[X^2] - 2E[X]^2 + E[X]^2$
$= E[X^2] - (E[X])^2$ hence proved.

b) $E[X] = 0$, $E[X^2] = 1$
i) $var(X) = E[X^2] - (E[X])^2 = 1 - 0^2 = 1$

ii) $Y = a + bX$
$E[Y^2] = E[(a+bX)^2] = E[a^2 + 2abX + b^2X^2]$
$= E[X] \cdot 2ab + a^2 + b^2 E[X^2]$
$= a^2 + 2ab(0) + b^2$
$= a^2 + b^2$ //

$E[Y] = E[a + bX] = a + b\,E[X]$
$= a + b(0) = a$

$\therefore var(Y) = -E[X]^2 + E[X^2]$
$= a^2 + b^2 - a^2$
$= b^2$ //

3) A ⇒ Aku predicts given horse is winning horse
~A ⇒ " " " " is not " "

B ⇒ event that given horse wins
~B ⇒ " " " " does not win

a) given a horse, probability it wins
$P(B) = P(B, A) + P(B \cap \sim A)$
$= P(B/A) . P(A) + P(B/\sim A) . P(\sim A)$
$= 0.99 \times 10^{-5} + (1 - 0.99999)(1 - 10^{-5})$
$= 1.99 \times 10^{-5}$

b) probability that Aku predicts black beauty is winning

$$P(A/B) = \frac{P(A,B)}{P(B)} = \frac{P(A/B) . P(A)}{P(B)} = \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}$$
$= 0.497$