Problem Chosen	2022	Team Control Number
E	MCM/ICM Summary Sheet	2200973

The Optimal Strategies of Forest Management

Summary Sheet

Forest is the largest carbon pool in the terrestrial ecosystem, which plays a very important and unique role in reducing the concentration of greenhouse gases in the atmosphere and slowing down global warming. The expansion of forest cover is an important mitigation measure that is economically feasible and less costly in the future. To study the carbon sequestration capacity of forests and its economic value, we established two models: Model I, the best carbon sequestration rate model based on the Logistic Growth Model and BEF; Model II is the best economic benefit model based on the Logistic Growth Model.

In Model I, we solve the problem of how to predict the future amount of carbon sequestration in forests and their products. To predict the amount of carbon sequestration in the forest, we first predict the future growth and development of the forest according to the Logistic Growth Model and obtain the change curve of the number of timber trees in the forest with time. And then calculate that specific stand volume of the forest in combination with the species of the tree which is in the forest. Then we calculate the total biomass in the forest according to the BEF of different forests and calculate the specific carbon content in the forest. According to the curve of forest carbon sequestration over time, we obtained the forest size under the maximum carbon sequestration rate.

In Model II, we solve the problem of how to determine the best time and amount of logging in the case of considering economic benefits. After considering many factors such as inflation coefficient, logger's salary, logging efficiency, timber market unit price, and so on, we calculate the best logging time and the best number of loggers according to the Logistic growth model and the method of differential equation. We obtained the forest size under the best economic benefit of the forest was determined.

In addition, we also predict the carbon sequestration of forests after 100 years and propose the best strategy for cutting down trees. And wrote a news report to educate the public about forest management.

Keywords: BEF[1];Differential Equation;Logistic Growth Model

Contents

1 Introduction

1.1 Problem Background

Carbon sequestration plays an important role in improving the current human living environment. To mitigate the effects of climate change, we need to take action to reduce the content of greenhouse gases such as carbon dioxide in the atmosphere. Forest carbon sequestration is the most economical and environmental protection way at present.

Thanks for the gift of nature! At present, there are still large areas of natural forests on earth to help us absorb carbon dioxide and produce oxygen. Unfortunately, with the continuous improvement of the human industrial level, the existing forest carbon sequestration capacity is worrying. We must and have to plant trees artificially. Because of its unique ecological, scientific, and economic values, we do not recommend cutting down or destructive management of natural forests. Plantation has many factors, such as single tree species, high value of forest products, easy management, and so on, which can be reasonably cut down or planted under certain conditions. Therefore, in this paper, we mainly discuss plantations. All the time starting points in the article are the completion points of the plantation.

1.2 Restatement of the Problem

Considering the background information and restricted conditions identified in the problem statement, the following problems are needed to be solved:

‡ **Problem 1** Develop a carbon sequestration model to determine how muchcarbon dioxide a forest and its products can be expected to sequester over time.

‡ **Problem 2** Forest management strategies considering economic benefits.

‡ **Problem 3** What kind of trees will be cut down.

‡ **Problem 4** How much carbon dioxide will this forest and its products sequester over 100 years.

1.3 Our Work

In Model I, we build a model to simulate the growth process of a tree and the relationship between trees and total carbon sequestration in the forest. We mainly use the logistic growth prevention model and learn from the BEF factor method to lay a good foundation for the establishment and evaluation of the later model and the proposal of Solutions.

In Model II, we mainly consider the economic value of forests, build a mathematical model from the timing and quantity of deforestation, accurately calculate the optimal timing and quantity of deforestation, and simulate the economic benefits that forests can obtain under this condition. At this time, we noticed that we only determined the number of felling, so we constructed different mathematical models to discuss which trees should be felled and which trees should be retained, and gave specific practical schemes.

In the next part(6.1.1), we calculate how much carbon dioxide forests and forest products will absorb in 100 years in our plan. At the same time, we also give reasonable strategies to transition the forest management plan from the existing schedule to the new schedule.

At the end of our articles, we wrote a non-technical newspaper article, which roughly described our research ideas and research results, and explained the truth to the public and the local community in a pertinent tone, so that the local community can approve and adopt our plan.

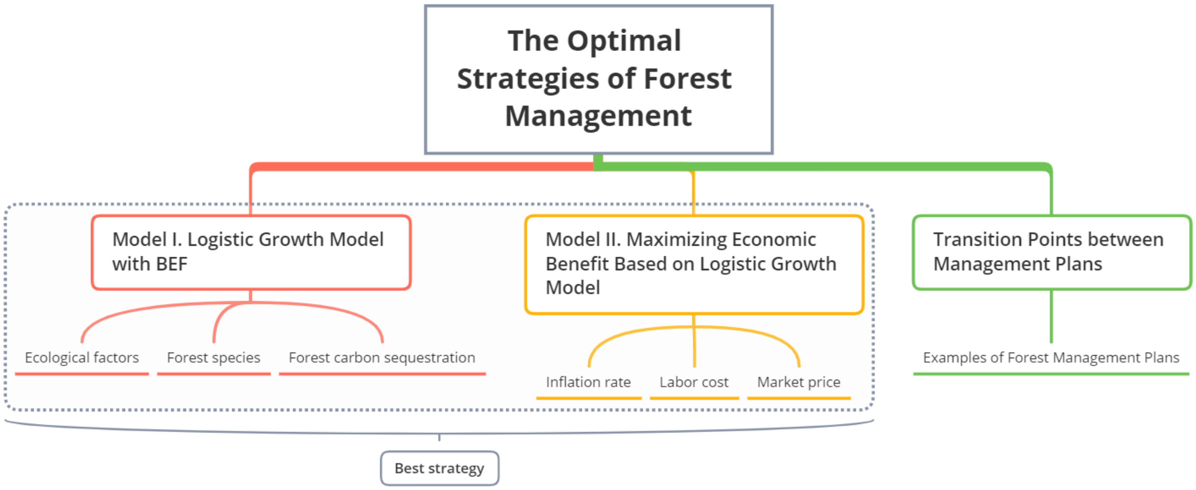


Figure 1: The Structure of Our Paper

2 Assumptions, Justifications and Reasons

†**Assumption 1:** We assume that the stand biomass in the forest is only provided by mature trees.
Justification: The young trees are small in volume and carbon sequestration, and contribute less to stand biomass; Over mature trees are vulnerable to death or collapse from a variety of causes. Therefore, it is reasonable to think that the stand biomass in the forest is only provided by mate trees.

†**Assumption 2:** We assume that all forests develop normally without biological invasion and natural disasters.
Reason: Biological invasion and natural disasters will greatly change the state of forests, which are difficult to predict.

†**Assumption 3:** We assume that there is no forest desertification or negative growth caused by human factors.
Reason: Human activities are difficult to predict.

3 Notations and Definitions

3.1 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations Used in This Paper

Symbol	Description
t	Time elapsed since the forest was formed
$x(t)$	Variation function of the number of mature trees per unit area with time
x_m	Maximum number of mature trees per unit area
r	Innate rate of increase [1] [2]
x_0	Initial number of wood
V_a	Volume of a single mature tree
C_a	Carbon sequestration
C'_a	Carbon fixation rate
S	Acreage of forest

3.2 Definitions

BEF:

Defined as the ratio of all stand biomass to growing stock volume.[1]

Mature trees

Mature trees can be made into various products. Usually, we cut down mature trees and over mature trees.

4 Model I. Logistic Growth Model with BEF

To determine the best forest management model to achieve the best carbon sequestration effect, we carried out the following work.

4.1 Differential Equation Modeling

In the real environment, it can be known from the basic knowledge of biology that the population presents a logistic growth pattern under the condition of limited resources. The trees in the forest constitute the basic population, the tree growth space is limited, and the total amount of inorganic salts is limited. Therefore, the growth in the number of mature trees conforms to the logistic growth model[2], [3].

[1]This value is affected by many factors, such as the location of the forest, light, precipitation, temperature and so on. Or call it as *environmental factors.*

[2]this is an example of a footnote.

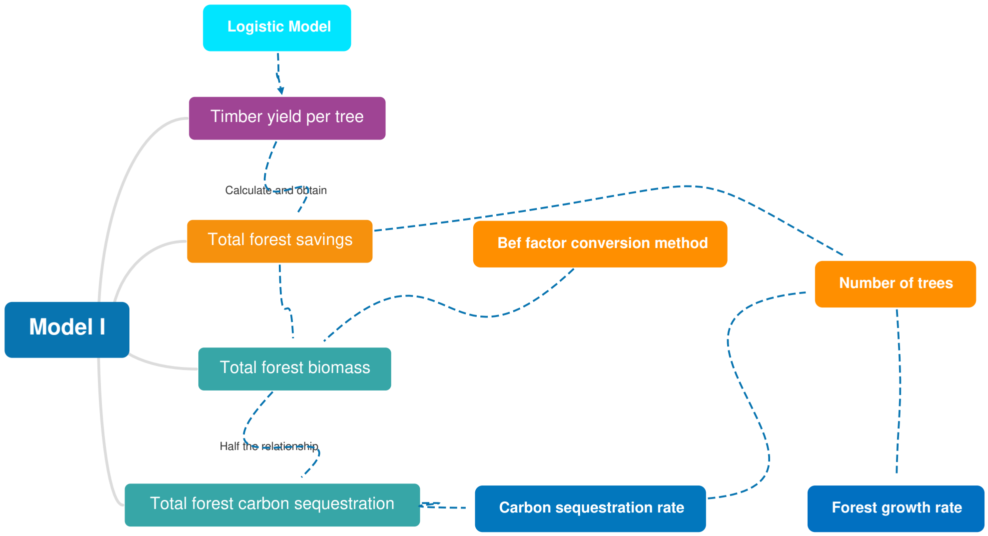


Figure 2: Mind map of Model I

According to the Logistic Growth Model, we can get the following equation:

$$\begin{cases} x'(t) = rx\left(1 - \dfrac{x}{x_m}\right) \\ x(0) = x_0 \end{cases} \tag{1}$$

Integrating the differential equations (1), we can get the following changes of mature trees per unit area with time($x(t)$):

$$x(t) = \frac{x_m}{1 + (\frac{x_m}{x_0} - 1)e^{-rt}} \tag{2}$$

$$x'(t) = \frac{rx_m\left(\frac{x_m}{x_0} - 1\right)e^{-rt}}{\left(\left(\frac{x_m}{x_0} - 1\right)e^{-rt} + 1\right)^2} \tag{3}$$

Therefore, the total forest stand biomass(X_v) is following:

$$X_v = SV_a x(t) \tag{4}$$

4.2 BEF (Biomass Expansion Factor) Method

According to the relevant forestry data, the total biomass(Y_b) in the forest can be calculated by the BEF factor[1]:

$$Y_b = BEF \times X_v \tag{5}$$

$$BEF = a + \frac{b}{X_v} \tag{6}$$

Typically the carbon stock is 0.5 times the biomass. Therefore, at time t, the forest carbon sequestration(C_a) is:

$$C_a = Y_b/2 = \frac{aSV_a x_m}{2\left(\frac{x_m}{x_0} - 1\right)e^{-rt} + 2} + \frac{b}{2} \tag{7}$$

After derivation of the Function (7), it can be obtained that the forest carbon sequestration rate(C_a') is as follows:

$$C_a' = \frac{2arSV_a x_m \left(\frac{x_m}{x_0} - 1\right)e^{-rt}}{\left(2\left(\frac{x_m}{x_0} - 1\right)e^{-rt} + 2\right)^2} \tag{8}$$

4.3 Analysis of Equation

Logistic growth model points out that when the forest grows to a certain extent, its growth rate will decrease significantly.[4] The carbon sequestration rate will also decrease. To ensure that the forest carbon sequestration rate always reaches the highest point, forest managers need to cut down trees regularly to expect the forest carbon sequestration rate to always maintain the maximum value. According to Function (8), when the forest carbon sequestration rate reaches the maximum, The value of t is

$$t_m = \frac{\ln\left(\frac{x_m - x_0}{x_0}\right) + 2i\pi c_1}{r}, c_1 \in \mathbb{Z} \tag{9}$$

Function (8) has realistic meaning and can not be an imaginary number , so $c_1 = 0$. Obtaining:

$$t_m = \frac{\ln\left(\frac{x_m - x_0}{x_0}\right)}{r} \tag{10}$$

Bring this result into (3) to get the number of trees per unit area at this time:

$$x(t_m) = \frac{x_m}{2}, (Steady\ state\ value) \tag{11}$$

t_m is brought into the growth rate formula, and it is obtained that at this time, the growth rate of trees is:

$$\frac{rx_m}{4} \tag{12}$$

4.4 Analysis of Model I

When the carbon sequestration rate reaches the maximum, the number of forest trees is half of the maximum(x_m). This is in line with the conclusion of the population growth curve in the ecology textbook[3]. From this point, it can be judged that the model is in line with objective facts. To demonstrate the effect of this model, the following will bring in the example data for simulation calculation.

We selected fir forest as the research object, and the relevant data were consulted as follows(Table 2). Bringing the above data into Function (7) and Function (8), we get the Figure 3 and Figure 4.

[1]These data are related to the specific size and location of the forest. Different types of forests have different values.

Table 2: Data of *Cunninghamia lanceolata* (Lamb.) Hook.[5]

Symbol	Data	Unit
V_a	137.375	m^3
x_m	0.333	$plants/m^2$
r	0.25	s^{-1}

Symbol	Data[1]	Unit
S	10000000	m^2
x_0	0.05	$plants/m^2$

Table 3: Parameters used to calculate biomass expansion factor (BEF)[1]

Forest type	$a(Mg/m^3)$	$b(Mg)$	N	R^2
Abies and Picea	0.4642	47.4990	13	0.98
Betula	1.0687	10.2370	9	0.70
Casuarina	0.7441	3.2377	10	0.95
Cunninghamia lanceolata	0.3999	22.5410	56	0.95
Cypress	0.6129	46.1451	11	0.96

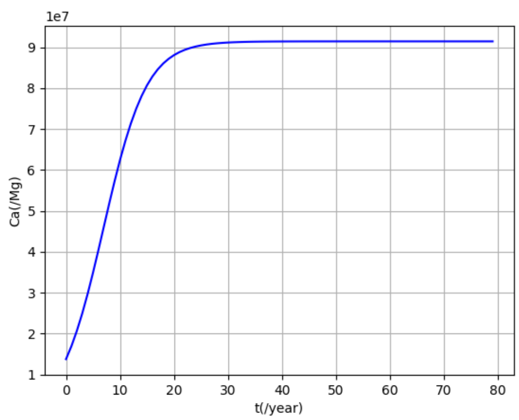


Figure 3: Image of C_a

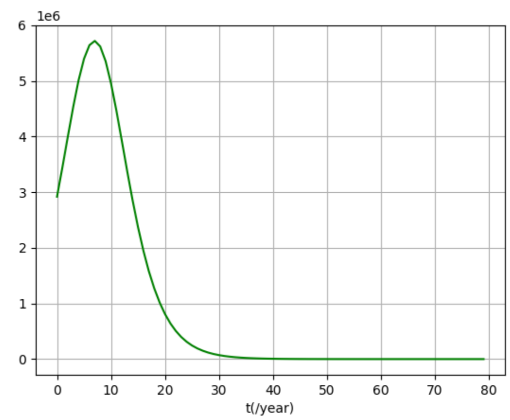


Figure 4: Image of C_a'

From the function figure, we can get: Fir forests cannot grow infinitely. After about 25 years, the fir forest reaches the limit allowed by the environmental carrying capacity. During this process, the carbon sequestration rate first increases and then slowly returns to zero.

To keep this forest at its maximum carbon sequestration rate, the model recommends that deforestation begin gradually after about 7 years. According to the model, the rate at which the loggers cut down the forest should be consistent with the growth rate of the forest. The value of it is given by Function 3.

4.5 Conclusion

Under the premise of maximizing carbon sequestration efficiency, the forest size should be kept at half of the maximum capacity allowed by the environment. The felled trees will be turned into forest products, and the carbon in them will be preserved longer. The graph of carbon content in the forest is shown in Figure 5. Forest carbon sequestration rate is shown in Figure 6.

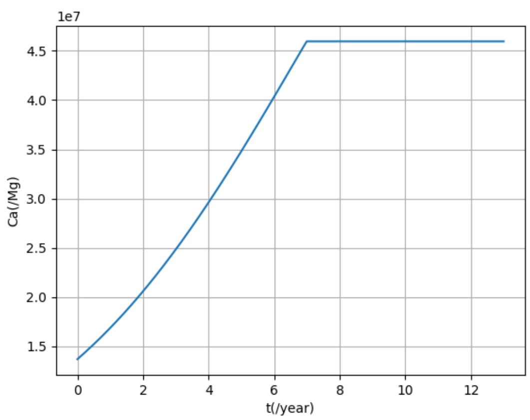


Figure 5: Carbon Content

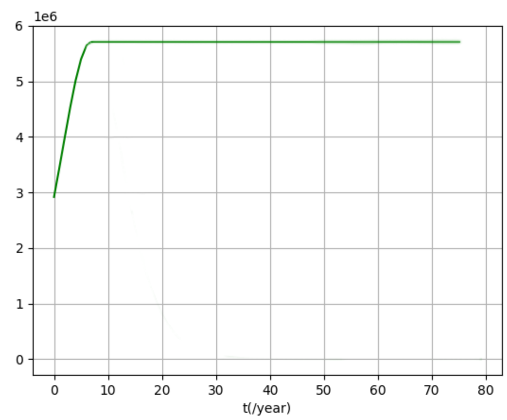


Figure 6: Carbon Sequestration Rate

The comparison between the carbon sequestration of this scheme and the forest carbon sequestration under natural growth is shown in the Figure 7.

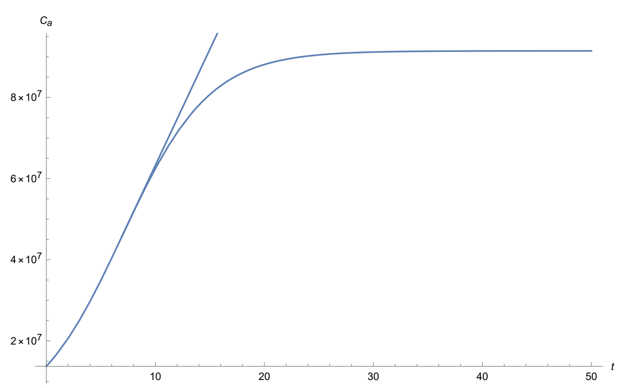


Figure 7: Carbon sequestration in forests and their products

5 Model II. Maximizing Economic Benefit Based on Logistic Growth Model

The forest management plan that is best for carbon sequestration is not necessarily the one that is best for society given the other ways that forests are valued. The following model will ensure that the Forest Manager gets the *maximum benefit*.

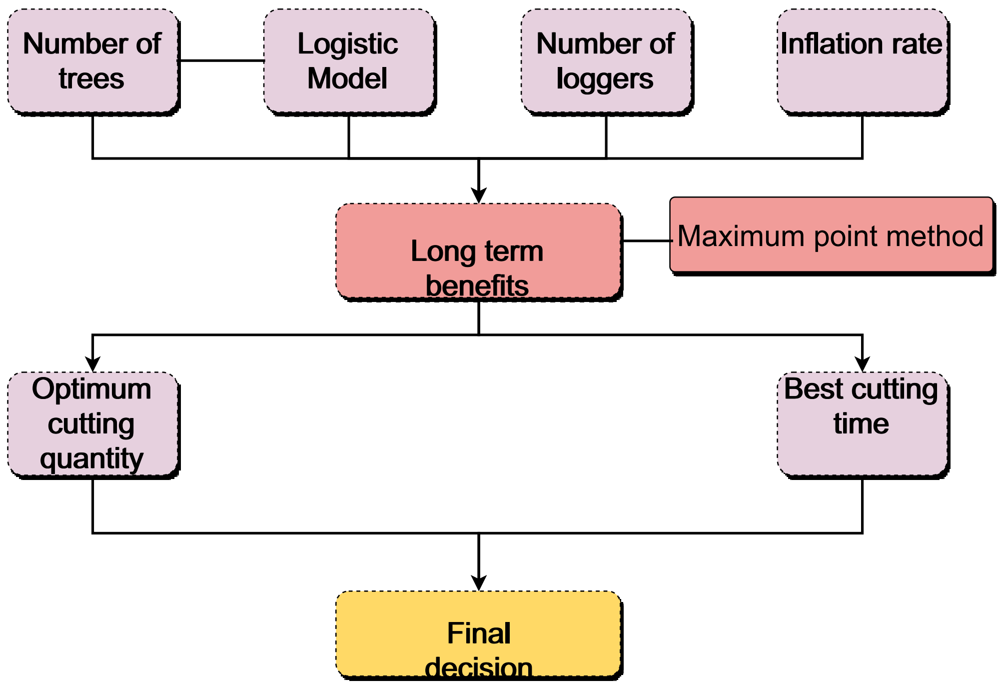


Figure 8: Mind map of Model II

5.1 Model Building

It is assumed that the natural growth of the number of trees $x(t)$ in the forest follows the logistic law, and the amount of felling per unit time is directly proportional to the number of loggers $u(t)$ and the number of trees $x(t)$, which meets the following requirements:

$$x'(t) = f(x) - h(u,x) \tag{13}$$
$$f(x) = rx(1 - \frac{x}{x_m}) \tag{14}$$
$$h(u,x) = qu(t)x(t) \tag{15}$$

q is the cutting volume per unit time of each lumberjack.
Number of trees at initial time:

$$x(0) = \frac{x_m}{k}, k \gg 1 \tag{16}$$

$x(0)$ is very small, at time $0 \leqslant t \leqslant \tau$ no felling in it. When $t > \tau$, the number of post fellers remains constant U, i.e.:

$$u(t) = \begin{cases} 0, 0 \leqslant t \leqslant \tau \\ U, t > \tau \end{cases} \tag{17}$$

τ, U is the undetermined parameter. $x(t)$ remains stable when $t > \tau$.

The selling unit price of timber is p, the unit time cost of each lumberjack is c, and the inflation rate (discount factor) is δ. Under this assumption, the profit per unit time is: $e^{-\delta t}$.
The objective function should be the long-term benefit with $u(t)$ as the control function, as follows:

$$\begin{aligned} J[u(t)] &= \int_0^\infty e^{-\delta t}[ph(u(t), x(t)) - cu(t)]dt \\ &= \int_0^\infty e^{-\delta t}[pqx(t) - c]u(t)dt \end{aligned} \tag{18}$$

$$x'(t) = rx(1 - \frac{x}{x_m}) - qu(t)x \tag{19}$$

When $0 \leqslant t \leqslant \tau$, $x(t)$ can be solved by (19) under condition (16).
When $t > \tau$, $u(t) = U$, let $x'(t) = 0$. Combined with (19), the constant $x(t)$ can be solved, as following:

$$x(t) = \begin{cases} \dfrac{x_m}{1 + (k-1)e^{-rt}}, 0 \leqslant t \leqslant \tau \\ x_m(1 - \dfrac{qU}{r}) \quad , t > \tau \end{cases} \tag{20}$$

$x(t)$ in $t = \tau$ is continuous, i.e.:

$$\tau = \frac{1}{r}\ln[(k-1)(\frac{r}{qU} - 1)] \tag{21}$$

τ, Only one quantity in U is independent. As U is independent, Only one quantity in τ and U is independent. Taking U to be independent, then $\tau = \tau(U)$, combined with (17)(18)(20), there have:

$$\begin{aligned} F(U) &= \int_0^\infty Ue^{-\tau t}[pqx_m(1 - \frac{qU}{r}) - c]dt \\ &= \frac{pqx_mU}{\delta}e^{-\delta\tau(U)}(1 - \frac{qU}{r} - b) \\ b &= \frac{c}{pqx_m} \end{aligned} \tag{22}$$

It is observed that x_m is the maximum number of trees, and b is the lower bound of the cost price ratio. Obviously, $b < 1$, otherwise the cost is higher than the selling price. From (22), it can be seen that the condition of benefit $F(U) > 0$ is $\frac{1-qU}{r-b} > 0$, i.e.:

$$0 < U < \frac{r(1-b)}{q} \tag{23}$$

By different methods, the maximum value point of $F(U)$ under the condition of (23) is:

$$U^* = \frac{r}{4q}\left[3 - b + \frac{\delta}{r} - \sqrt{(1 + b - \frac{\delta}{r})^2 + \frac{8b\delta}{r}}\right] \tag{24}$$

Combined with (21)(24), $\tau^* = \tau(U^*)$.
So the U^* and τ^* are the best number and time for felling.

5.2 Difference between Model I and Model II

Typically, inflation is very low. To facilitate the analysis of horses, here we set δ to 0. At this time, Equation (24) is simplified to the following form:

$$U_0 = \frac{r}{2q}(1-b) \tag{25}$$

Plug U_0 into Equation (20), we get the following data:

$$x(t) = \frac{c}{2pq} + \frac{x_m}{2}, (when : t > \tau, Steady\ state\ value) \tag{26}$$

This is the steady-state value of forest size after deforestation begins. Compared with Model I

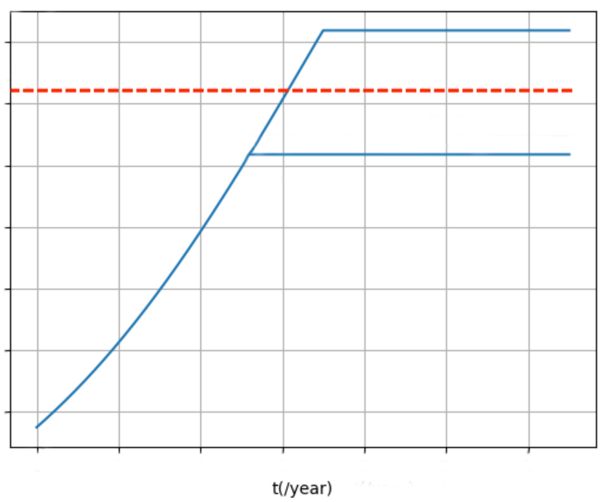


Figure 9: Comparison of Forest Size in Model I and Model II

$$x(t_m) = \frac{x_m}{2}$$

This value increases a part of the coefficient.We call it the economic impact factor and denote it as B.

$$B = \frac{c}{2pq}$$

The economic impact factor(B) and the cost-to-price ratio(b) together determinethe forest manager's management strategy for the forest, that is, the size of the forest to be controlled.

For different forests: affected by factors such as forest location, forest climate, and local economic development level, the final steady-state size of the forest will change to varying degrees. However, in order to balance the carbon sequestration rate and economic benefits, the final steady-state size of the forest should be kept between

$$\frac{x_m}{2}$$

and

$$\frac{c}{2pq} + \frac{x_m}{2}$$

According to the model analysis, under normal circumstances, the forest must be cut down. If and only if the forest management agency is incapable of managing the forest, it may lead to the forest not being cut down.

5.3 Felled or Not

As mentioned above, we know the amount and timing of deforestation. Now let's discuss what kind of tree is felled under the conditions of time τ. It is considered that the following two situations occur in the felled trees:

1. The trees are in poor health. If they are not cut down, the trees will absorb nutrients and affect the growth of other trees.

2. Trees already have high economic value.

Firstly, let's discuss point 1. Assuming that the health of trees is related to time t and environmental condition x, i.e.: $f(t, x)$. It is considered that $x = 0$ represents that the environment has no impact on the trees, and $f(0, 0) = 1$ represents that the trees are in a completely healthy state. When $f(t, x) = 0$, it is considered that the trees are completely dead. We find that $f(t, 0)$ is only related to time or age. According to common sense, the image of $f(t, 0)$ is roughly as follows(Figure 10): Assuming that t

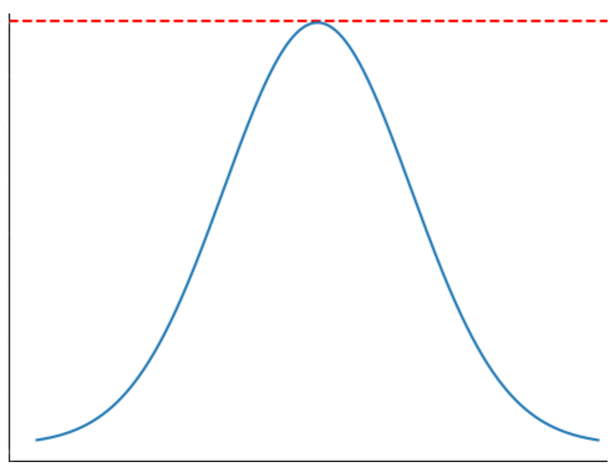


Figure 10: $f(x, t)$

and x of all trees in a forest are known, when $f(t, x) \leqslant 0.1$, the trees are no longer suitable for survival and must be cut down. There are N trees in total, and n trees are conforming to the above formula. It can be seen from the above that qU^* trees need to be cut down at this time.

If $n \geqslant qU^*$, trees with a smaller f value will be cut first. If $n \leqslant qU^*$, $qU^* - n$ trees will still be cut down. Then discuss 2. As mentioned above, the selling unit price of wood is p, we assume that p is fluctuating, and we think that p is related to the timber volume V of trees. That is, $p(V)$, and the functional relationship that $p(V)$ should meet is roughly as follows: (Figure 11)

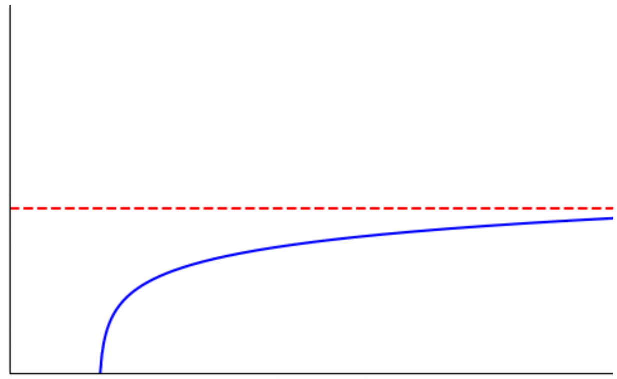


Figure 11: $p(v)$

Note: if the volume of trees is too small, it is obvious that no merchant is willing to buy them; No matter how large the volume of a single tree is, its selling unit price must have a threshold.

In this way, we can select $qU^* - n$ trees with large $p(V)$ value among $N - n$ trees.

6 Transition Points between Management Plans

In the above two decision-making plans and the "balanced plans", there are only two management plans:

1. Conservation and cultivation: Make the forest develop rapidly.
2. Maintaining the size of forest stable(ie: cutting down at the rate of forest growth).

According to the analysis process above, the *Transition Points* of these two management plans is when the forest size reaches the desired state. Such as:$\frac{x_m}{2}$ or $\frac{c}{2pq} + \frac{x_m}{2}$.

In addition, we provide a brand new thinking direction (Inheriting the idea of Model II):

Start timing from 0, and set the original time t_0.

It can be seen from the above that the best time to cut down is τ^*. Now we discuss the felling time. The felling time of a single tree by a woodcutter should be directly proportional to the strength of the number and inversely proportional to the physical strength of the woodcutter. The physical strength of the woodcutter decreases with time. When the woodcutter has sufficient physical strength, the workload per unit time is large and the physical strength decreases rapidly, so the image of the woodcutter's physical strength function is roughly as Figure 12.

Our simulation is:

$$g(t) = e^{-at}$$

a is related to personal health status. Assuming that the average strength of trees is b and b remains unchanged during logging if the time spent by loggers cutting a single tree is negligible compared with the whole logging project when the loggers start a certain logging function at time t:

$$T(t) = k\frac{b}{e^{-at}}$$

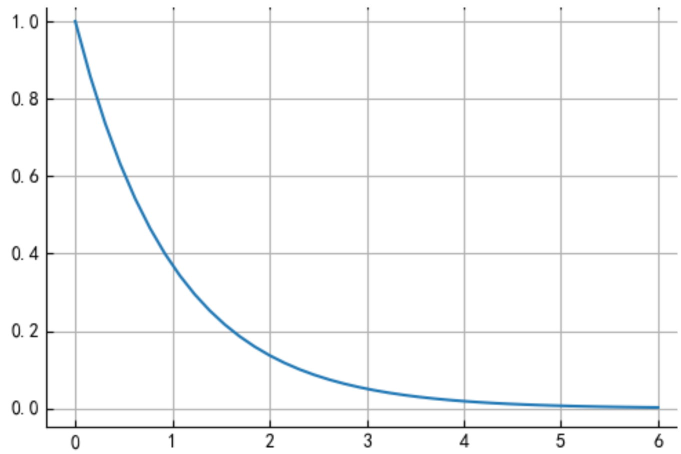


Figure 12: Woodcutter's Physical Strength

Then there are:

$$T(0) = kb = t_1,\ T(kb) = \frac{kb}{e^{-akb}} = t_2,\ T(t_2) = t_3 \dots$$

Total time:

$$T = t_1 + t_2 + t_3 + \dots$$

Assuming that each woodcutter cuts down d trees, there are:

$$T = t_1 + t_2 + t_3 + \cdots + t_d\ d = t_{d-1},\ \tau^* + T = t_0 + 10$$

Therefore, the scheme we give is that the economic investment in this forest needs to be increased to the original scheme $\dfrac{\tau^* + T}{t_0}$ times. The reason for it is that the new scheme may have small fluctuations due to uncertain factors.

6.1 Examples of Forest Management Plans

6.1.1 Carbon dioxide absorption in 100 years

Here, we still use the data from *Analysis of Model I* Bringing the data into Function (7), we get:

$$\begin{aligned} C_a =& Y_b/2 = \frac{aSV_a x_m}{2\left(\frac{x_m}{x_0} - 1\right)e^{-rt} + 2} + \frac{b}{2} \\ =& 11.2705 + \frac{1.82938 \times 10^8}{11.32e^{-0.25t} + 2} \end{aligned} \tag{27}$$

Derive Function (27):

$$C_a' = \frac{5.17714 \times 10^8 e^{-0.25t}}{\left(11.32e^{-0.25t} + 2\right)^2}$$

The extreme point of this function is:

$$\text{Maximize}\left[\frac{5.17714\times10^{8}e^{-0.25t}}{\left(11.32e^{-0.25t}+2\right)^{2}},\{t\}\right]$$
$$\Rightarrow$$
$$\{5.7168\times10^{6},\{t\to6.9337\}\}$$

Therefore, when t is 6.93 (i.e.:6.93 years), the carbon sequestration rate reaches the maximum. Deforestation at this time (keeping the forest size unchanged), the forest carbon sequestration rate will always remain at the maximum rate. At this time, the forest has sequestered carbon of 4.57133×10^{7}.

After that, using a typical linear prediction model, the prediction function is:

$$\hat{C}_a = 5.7168\times10^{6}(t-6.93)+4.57133\times10^{7}(Mg)$$

Thus, after 100 years, the amount of carbon sequestered by forests and their products is $5.77776\times10^{8}Mg$

6.1.2 Selection of Forest Management Plan

The specific choice of forest management scheme is closely related to local economic conditions. If the local economy is dominated by forests, the forest management plan is more inclined to Model II; if the local economic development has nothing to do with forests, we recommend the Model I plan.

According to the situation of this forest, the forest management plan of protection and cultivation should be adopted in the current situation. Because the current forest size has not reached the scale corresponding to the maximum carbon sequestration rate. After about 7 years, the forest scale has reached the corresponding maximum carbon sequestration rate. At this time, some overripe and mature trees should be cut down according to the felling principle in order to maintain the maximum carbon sequestration rate.

Time(year)	**Forest Management**
$t<6.93$	Conservation and Cultivation
$t>6.93$	Deforestation at the same rate as forest growth, in order to maintain forest size

7 Model Evaluation

7.1 Strengths

- There are many factors affecting plantation growth, but all populations fit the Logistic Growth Model when well managed. Both Model I and Model II are plans made directly based on the Logistic Growth Model and have high reliability.

- In Model II, we comprehensively consider various factors including inflation rate (δ), lumberjack wages(c), and the market price of lumber(p). Model coverage is comprehensive.

7.2 Weaknesses

- For natural forests, the variables required for prediction are difficult to measure directly.

8 Conclusion

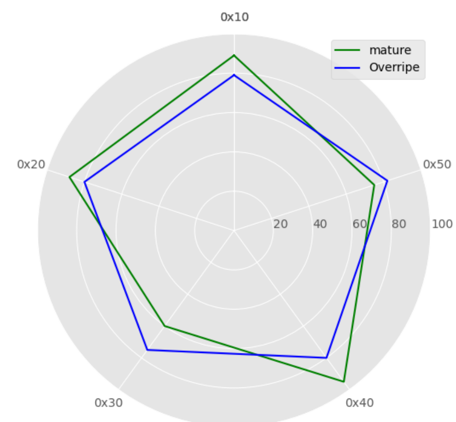
1. We predicted the forest growth law through the Logistic Growth Model, and calculated the actual carbon sequestration and carbon sequestration rate of the forest in combination with the BEF.
2. Through differential equations, we obtain the logging scale and the number of loggers on the premise of ensuring the best economic benefits.
3. By comparing Model I with Model I, we developed a concrete plan for determining forest size.
4. By analysing trees at different growth stages, we derive specific felling strategies
5. We use the above two models to predict the amount of forest carbon sequestration in 100 years.

The Forest Said: I Can't Eat Anymore!

—Newspaper Article

The traditional view is that we can never cut down trees for any reason. Our research found that this is not the case. No matter what kind of trees, no matter how they grow, they will age or even die. It is undeniable that the greatest function, or purpose, of planting trees is to protect the environment, such as wind prevention, sand fixation, air purification, and so on. Our research shows that when a tree reaches a certain age, its direct value will be greatly reduced. In addition, it will continue to absorb nutrients from the soil. In this way, it will affect the growth of other trees. Although from a scientific point of view, nutrients will be returned to the soil after the tree dies, and there is not much loss of nutrients in general, But we think this process is too long, especially for tall trees, which requires a lot of decomposers to work for a long cycle. In other words, in this process, the residual value of the tree has not been fully utilized. Although it has no great impact on the overall environment, we always hope for better results. If we can make better use of the economic value of the tree, more funds will be invested in the construction of the local ecological environment. We think this process is beneficial to the tree population from the perspective of results. Firstly, the carbon sequestration we are most concerned about has only a small reduction in this process, which is completely negligible from a mathematical point of view. Cutting down trees does not affect the most important carbon sequestration function of the tree population. Secondly, we can provide a rigorous and reasonable forest management plan and analyze the most reasonable cutting quantity and cutting time with data, We can even elaborate on which trees are retained and which trees are cut down. Finally, we can put forward efficient forest regeneration schemes to protect the environment of the local government to the greatest extent. In general, we believe that logging should not be prohibited. At least in terms of forest management, appropriate and planned logging can bring more benefits to forests.

The comparison between mature trees and over mature trees is shown in the figure below:



(Where 0x10 represents the carbon fixation capacity, 0x20 represents the wind and sand fixation capacity, 0x30 represents the occupation of natural resources, 0x40 represents the economic value, and 0x50 represents the maintenance consumption fund.)

References

[1] J. Fang, A. Chen, C. Peng, S. Zhao, and L. Ci, "Changes in forest biomass carbon storage in china between 1949 and 1998," *Science*, vol. 292, no. 5525, pp. 2320–2322, 2001. DOI: 10.1126/science.1058629.

[2] F. Jingyun, L. Guohua, and X. U. Songling, "Biomass and net production of forest vegetation in china," *ACTA ECOLOGICA SINICA*, 1996.

[3] J. L. Chapman and M. J. Reiss, "Ecology : Principles and applications," *W.b.saunders Company Philadelphia Pa*, 2009.

[4] H. Lieth and R. H. Whittaker, "Primary roductivity of the boisphere," *New York: Springer-Verlag*, 1975.

[5] S. Ouyang, K. Xiao, Z. Zhao, *et al.*, "Stand transpiration estimates from recalibrated parameters for the granier equation in a chinese fir (cunninghamia lanceolata) plantation in southern china," *Forests*, vol. 9, no. 4, p. 162, 2018.

A Program Source Code

A.1 Python Code for Figure 3

```
import numpy as np
import matplotlib.pyplot as plt
x=np.arange(0,80,1)
y=0.3999*1e7*137.375*0.333/(2*(0.333/0.05-1)*np.exp(-0.25*x)+2)+11.27
plt.figure()
#plt.style.use('ggplot')
plt.plot(x,y,linestyle='-',color='blue')
plt.xlabel('t(/year)')
plt.ylabel('Ca(/Mg)')
plt.grid()
plt.show()
```

A.2 Python Code for Figure 4

```
import numpy as np
import matplotlib.pyplot as plt
x=np.arange(0,80,1)
y=2*0.3999*0.25*1e7*137.375*0.333*(0.333/0.05-1)*np.exp(-0.25*x)/(2*(0.333/0.
                                                        05-1)*np.exp(-0.25*x)+2)/(2*(0.333/0
                                                        .05-1)*np.exp(-0.25*x)+2)
plt.figure()
plt.xlabel('t(/year)')
plt.plot(x,y,linestyle='-',color='green')
plt.grid()
plt.show()
```

B Parameters used to calculate biomass expansion factor (BEF)

Forest type	$a(Mg/m^3)$	$b(Mg)$	N	R^2
Abies and Picea	0.4642	47.4990	13	0.98
Betula	1.0687	10.2370	9	0.70
Casuarina	0.7441	3.2377	10	0.95
Cunninghamia lanceolata	0.3999	22.5410	56	0.95
Cypress	0.6129	46.1451	11	0.96
Deciduous oaks	1.1453	8.5473	12	0.98
Eucalyptus	0.8873	4.5539	20	0.80
Larix	0.6096	33.8060	34	0.82
Lucidophyllous forests	1.0357	8.0591	17	0.89
Mixed conifer and deciduous forests	0.8136	18.4660	10	0.99
Mixed deciduous andSassafras	0.6255	91.0013	19	0.86
Nonmerchantable woods	0.7564	8.3103	11	0.98
Pinus armandii	0.5856	18.7435	9	0.91
P. koraiensis	0.5185	18.2200	17	0.90
P. massoniana, P. yunnanensis	0.5101	1.0451	12	0.92
P. sylvestris var. mongolica	1.0945	2.0040	11	0.98
P. tabulaefomis	0.7554	5.0928	82	0.96
Other pines and conifer forests	0.5168	33.2378	16	0.94
Populus	0.4754	30.6034	10	0.87
Tsuga, Cryptomeria, Keteleeria	0.4158	41.3318	21	0.89
Tropical forests	0.7975	0.4204	18	0.87

C Effects of Different Growth Rates on Carbon Sequestration

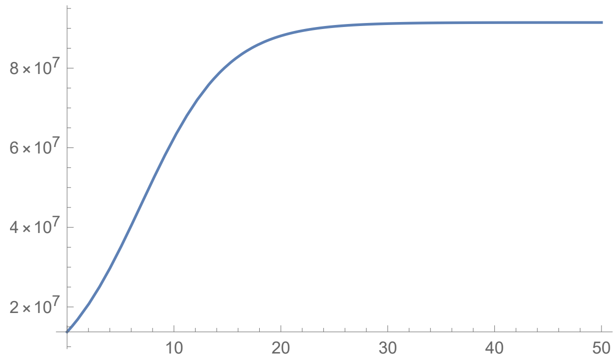


Figure 13: $r = 0.25$

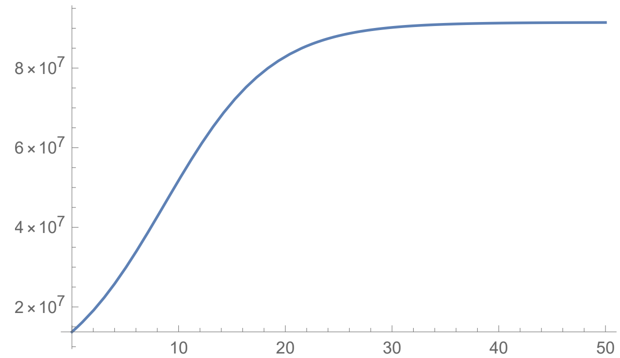


Figure 14: $r = 0.20$

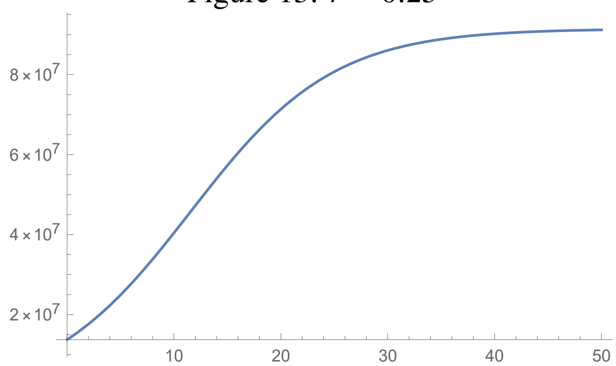


Figure 15: $r = 0.15$

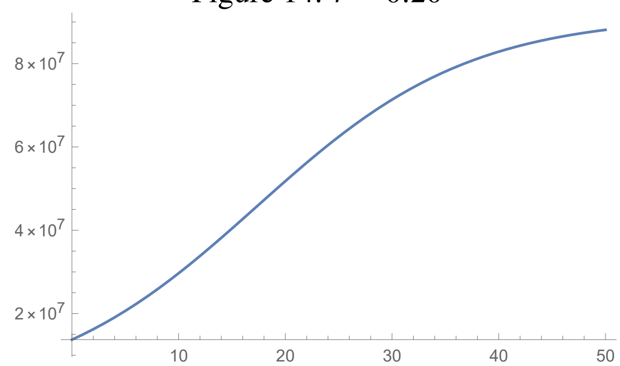


Figure 16: $r = 0.10$