Exercices de math

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1.1 =>
$$(5, -12)(x, y) = (1, 0)$$

Ou

1.2 =>
$$(5-12i)(x+yi) = 1+0i$$

1.1 =>
$$(a,b)(a',b') = (aa'-bb',ab'-a'b)$$

= $(5X + 12Y,5Y - 12X)$
= $(1,0)$

$$5X + 12Y = 1$$

 $5Y - 12X = 0$ => $Y = 12X / 5$

$$5 X + \frac{12 * 12 X}{5} = 1$$

$$\frac{25 X + 144 X}{5}$$
 = 1 => 169 X / 5 = 1

$$x = 5 / 169 = y = 12 / 169$$

$$(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2})$$

$$(\frac{5}{5^2 + (-12)^2}, \frac{12}{5^2 + (-12)^2})$$

Vérification

$$(5, -12) (5/169, 12/169) = (1,0)$$

$$(4,0)(X,Y) = (1,0)$$

$$(4X,4Y) = (1,0)$$

$$4X = 1 = X = 1/4$$

$$4 Y = 0 => Y = 0$$

$$(0, -2)(X,Y) = 1 = (1, 0)$$

$$2 Y = 1$$
 => $Y = 1/2$

$$-2X = 0$$
 => $X = 0$ => $(0, \frac{1}{2})$

Vérification (au choix)

3.1.
$$\left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right) = (0,2/4) = (0,1/2)$$

3.2.
$$(0-2i)(X+Yi)=1$$

$$2Y - 2Xi = 1 + 0i$$

$$-2X = 0$$
 => $x = 0$

$$0 + 1/2 i = 1/2 i$$
 => $Y = 1/2$

$$(4/25, -4/25)(X,Y) = (1,0)$$

(1) =>
$$3/25 X + 4/25 Y = 1$$

(2) =>
$$3/25 \text{ Y} - 4/25 \text{ X} = 0$$

(2) =>
$$3/25 Y = 4/25 X$$

(2) =>
$$Y = 4/3 X$$

(1) =>
$$\frac{3}{25}X + \frac{4}{25} * \frac{4}{3}X = 1 \Rightarrow \frac{9}{75} * \frac{16X}{75} = 1$$

$$=>$$
 125 X / 75 = 1 $=>$ X = 3

La solution du 5 : (0, -1)

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3.2. Calculer les expressions suivantes :

1)
$$(2+i)^2 = 4+4i-1 = 3+4i$$

2)
$$(5-7i) - (2+4i) = 3-11i$$

3)
$$(1+3i) * 2i = 2i + 6i^2 = 2i - 6$$

4)
$$((\sqrt{6} + 5i)^2 * (2-7i) = (6 + 10\sqrt{6} - 25) (2-7i)$$

= $-38 + 133 + 20 \sqrt{6} + 70 \sqrt{6} = -38 + 70 \sqrt{6} + (133 + 2\sqrt{6}) *i$

5)
$$41*(-21)*(2+31) = (161+8)(2+3i) = 321+46-48+24i$$

= 8 (-4+71)

3.3. Calculer les expressions suivantes : (il y a juste les réponses)

1)

$$=> \frac{(1+i)(1+i)}{(1-i)*(1-i)} + \frac{(1-i)(1-i)}{(1-i)*(1-i)}$$

$$=> \frac{(1+i)^2}{2} + \frac{(1-i)^2}{2}$$

$$=> \frac{1+2i-1+1-2i-1}{2} = 0$$

2)

$$=> \frac{1+2i-1}{1-2i-1} = -\frac{2i}{2i} = -1$$

3)

$$=> \frac{i(2-5i)}{(2+5i)*(2-5i)} + \frac{i*(2+5i)}{(2-5i)*(2+5i)}$$
$$= \frac{2i+5+2i-5}{4+25} = \frac{4i}{29}$$

4)

$$=> \frac{(\sqrt{5} + i)^2}{(\sqrt{5} - i)(\sqrt{5} + i)} = \frac{5 + 2\sqrt{5} * i - 1}{5 + 1}$$

$$= \frac{4 + 2\sqrt{5} i}{6} = \frac{2 + \sqrt{5} i}{3}$$

5)

$$\frac{\cos\theta - i\sin\theta}{(\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta)} = \frac{\cos\theta - i\sin\theta}{\cos^2\theta + \sin^2\theta}$$

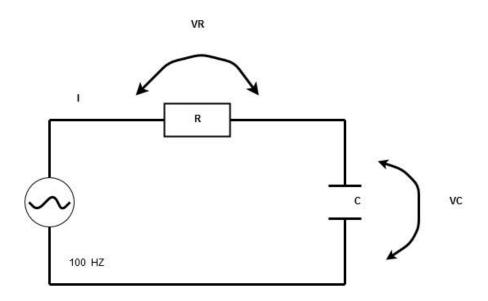
$$= \cos \theta - i \sin \theta$$

6)

$$\frac{1}{\cos\theta - i\sin\theta} = \cos\theta + i\sin\theta$$

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1) Calculer : Z ; I ; VC ; VR



$$Z = R + jx = ?$$

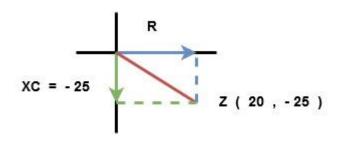
C = 64 NF

R = 20 ∩

$$Z = 20 + jx = ?$$

$$XC = 1 / JWC = \frac{1}{j 2\pi * 100 * (64 * 10^{-6})} = -j 25 \cap = 25^{-90^{\circ}}$$

$$Z = 20 + (-25j) \cap$$



$$|Z| = \sqrt{20^2 + (-25)^2} = 32 \text{ n}$$

$$-25$$

$$\theta = arctg \frac{-25}{20} = -51,34^{\circ}$$

$$z = 32^{-51,34}$$

$$U = Z * I$$

$$I = \frac{U}{Z} = \frac{10^{0^{\circ}}}{32^{-51,34^{\circ}}} = 0.3125^{51,34^{\circ}}$$

$$Vc = XC * I = 25^{-90}$$
 * 0,3125 ^{51,34}° = 7,8125 ^{-38,66}°

$$VR = R * I = 20^{0^{\circ}} * 5,25^{51,34^{\circ}}$$

Vérification:

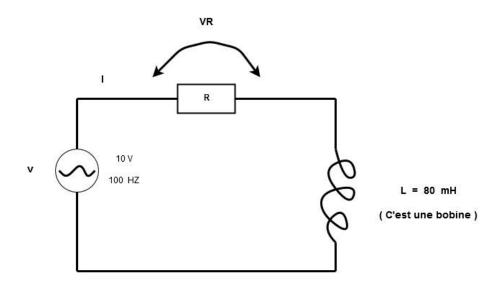
$$V = VR + VC$$

$$V = 6,25^{51,34} + 7,8125$$

VR =
$$6.25^{51.34}$$
 = $a + b j$ => $a = 6.25 * cos 51.34$

$$=>$$
 (3,9 + 4,9 j) + (6,1 - 4,9 j) = 10 + 0 j V

2) Calculer : Z ; I ; VC ; VR



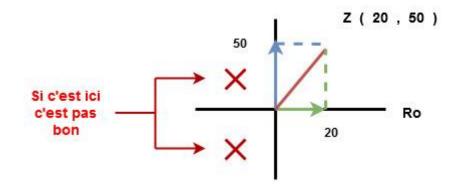
$$Z = R + j XL$$

 $XL = jWL = j*2\pi + 100 + (80*10^{-3}) = 50^{90}$ ° n
 $= j50$ n

Z = 20 + j50 = ?

$$|Z| = \sqrt{20^2 + 50^2} = 53,85$$

 $\theta = arctg \frac{50}{20} = 68,2^{\circ}$ => Z = 53,85



$$I = \frac{U}{Z} = \frac{10^{0}}{53,85^{68,2}} = 0,1875^{-68,2}$$

$$VR = R * I = 20^{0} * 0.1875^{-68.2} = 3.714^{-6.2} V$$

VL = XL * I =
$$2\pi * 100 * 3010^{-3} = 50^{30}$$
° * $0.1875^{-68.2}$ ° = $3.285^{21.8}$ ° V

Vérification:
$$V = VR + VC = 10^{0}$$
?

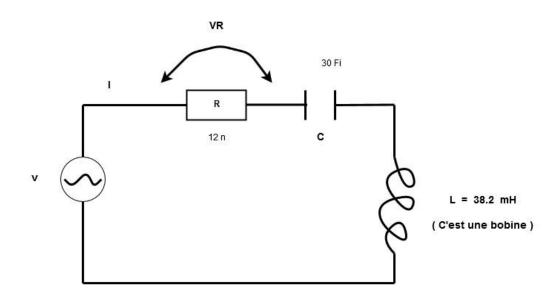
$$VR = 3,714^{-68,2} + 9,285^{21,8}$$
°

$$3,714 \cos (-68,2) = 1,38$$
 | $9,285 \cos (21,8) = 8,62$

$$3,714 \sin (-68,2) = -3,45$$
 | $9,285 \sin (21,8) = 3,45$

$$(1,34 + 8,62j) + (8,62 + 3,45j) = 10 V$$
 OK

3) Calculer: Z; I; VC; VR; VL



$$Z = R + j (XL + XC)$$

$$XL = jXL = j 2 \pi * 100 * 38,2 * 10^{-3} = 24 j n = 24^{90} n$$

$$xc = \frac{1}{jXL} = \frac{1}{2\pi 100 * 80*10^{-6}} = 20^{-90} n = -20^{90} n$$

$$Z = 12 + j(24(-20)) = 12 + 4j n$$

$$|Z| = \sqrt{12^2 + 4^2}$$

$$\rho = arcyg \left(\frac{4}{12}\right) = 18,43^{\circ}$$

$$Z = 12,65^{18,43}$$
°

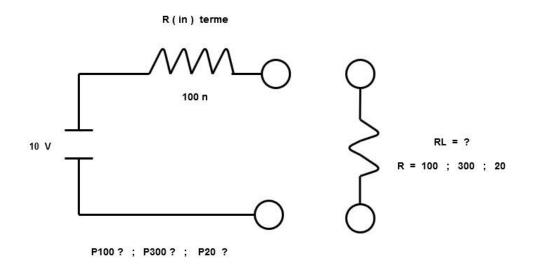
$$I = \frac{U}{Z} = \frac{10^{0^{\circ}}}{12,65^{18,43^{\circ}}} = 0,79^{-18,43^{\circ}}$$

VL =
$$XL*I$$
 = -20^{90} ° * $0.79^{-18,43}$ ° = $-15.8^{71,57}$ °

VC = XC * I =
$$-20^{90}$$
° * $0.79^{18.43}$ ° = $-15.8^{7.57}$ °

$$VR = R * I = 9,49^{-18,43}$$
° V

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Pour RL = 100 n

Rtot = Rin + RL = 100 + 100 = 200 N

$$\rho \ reor = Rin * I^2 = 100 * 0.05^2 = 0.25 W$$

 $\rho \ RL = Rl * I^2 = 100 * 0.05^2 = 0.25 W$

Pour RL = 300 n

Rtot = Rin + RL = 300 + 100 = 400 n

$$I = \frac{10}{400} = 0,025 A$$

$$\rho rin = 100 * 0,025^2 = 0,0625 W$$

$$\rho resit = 300 * 0,025^2 = 0,1879 W$$

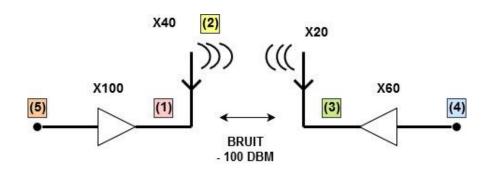
Pour RL = 20 n

$$I = \frac{10}{120} = 0.083 A$$

$$\rho \ rin = Rin * I^2 = 100 * 0,083^2 = 0,69 W$$

$$\rho RL = RL * I^2 = 20 * 0.083^2 = 0.138 W$$

MATH exercices en rapport avec la page inconnue (Aprés 25)



Calculer: (Avec en (3) on a 0,1 mW)

(1) Calculer les différentes p en dBm / dB

$$(X 40)$$
 => $10 \log 40$ = 16 dB

Source =>
$$(1 \text{ mW}) = 10 \log \frac{1}{1} = 0 \text{ dBm}$$

Réception =>
$$(X 20)$$
 => $10 \log 20$ = $13 dB$

$$(X 60) = 10 \log 60 = 17.8 dB$$

Puissance reçue =>
$$(0.1 \text{ mW})$$
 => $10 \log \frac{0.1}{1}$ = - 10 dB

(2) Calculer la puissance d'émission (à l'antenne)

En W => P source * As * Aa =
$$1 * 10^{-3} * 100 * 40 = 4 W$$

En dBm => P source (dBm) + As (dB) + Aa(dB) = ?
$$0 + 20 + 16 = 36 \text{ dBm}$$

Verif => 4 W = 36 dBm ? =>
$$10 \log \frac{4000}{1}$$
 = 36dBm
$$10^{\frac{36}{10}} = 10^{3,6} = 4000 \text{ mW}$$

(3) Calculer la puissance au niveau du récepteur

En W =>
$$10^{-4} (W) * 20 * 60 = 0.12 W$$

Vérification:

(1) =>
$$10 \log \frac{120}{1}$$
 (en mW) = 20,8 dBm

(2) =>
$$10^{\frac{20.8}{10}}$$
 = $10^{\frac{2.08}{1}}$ = 120 mW

(4) Calculer les pertes entre l'émetteur et le récepteur au ni veau des antennes en dBm et en W

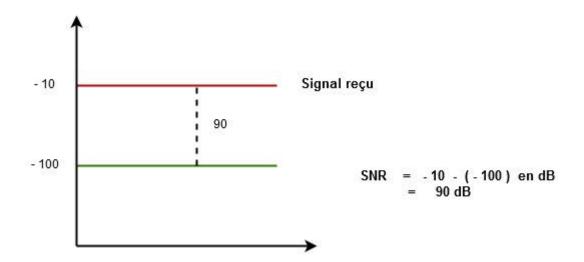
En W =>
$$\frac{1*10^{-4}}{4}$$
 = $25*10^{-6}$ W

Vérification:

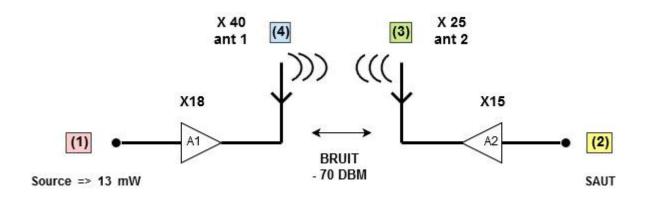
(1) =>
$$10 \log 25 \times 10^{-6} = 20.8 \text{ dB}$$

(2) =>
$$10^{\frac{-46}{10}}$$
 = $25 * 10^{-6}$

(5) Calculer le rapport signal sur bruit en $d\mathcal{B}$ et en \mathcal{W}



Version 2:



(A) Calculer les différents gains en dB

+ Puissance du signal en (1) et (2) en dB

$$A 1 = 10 \log 18 = 12,55 dB$$

$$A 2 = 10 \log 15 = 11,76 dB$$

Ant 1 =
$$10 \log 40 = 16,02 dB$$

(1) =
$$10 \log \frac{12}{1}$$
 = $10,79 \text{ dBm}$

(B) Calculer la puissance d'emission (4) => en W et dBw

En W =>
$$(12 * 10^{-3}) * 18 * 40 = 8,64 W$$

Vérification :

$$\Rightarrow$$
 10 log (8,64 * 10⁻³) = 39,36 dBm

$$\Rightarrow$$
 $10^{\frac{39,39}{10}} = 8689 \text{ dBm}$

(C) Calculer la puissance de réception (2 et 3) => en W et dBw

(4) =>
$$\frac{39,39 \ dBm}{8,64 \ W}$$
 => Pertes => (3) => 39,39 - 70 = -30,61 dBm

$$= 10^{\frac{-30,61}{10}} = 0.87 * 10^{-3} mW = 0.87 UW$$

(3)
$$\Rightarrow \frac{8,64}{1*10^7} = 0.86 \text{ UW}$$

Perte * -70 dB =>
$$10^{\frac{-70}{10}}$$
 = 1 * 10^{-7}

(2) =>
$$0.864 * 10^{-3}$$
 mW * $25 * 15$ = 0.3265 mW $0.864 * 10^{-3}$ W * $25 * 15$ = $0.3265 * 10^{-3}$ mW EndBm => $-30.61 + 13.98 + 11.76 = -4.87$ dBm

Vérification :

=>
$$10^{\frac{-4,87}{10}}$$
 = 0,3265 mW
=> $10 \log 0,3265$ = -4,87 dBm

(D) Calculer la perte entre (1) et (2)

$$\frac{0,3265}{12} = 2,72 * 10^{-2}$$

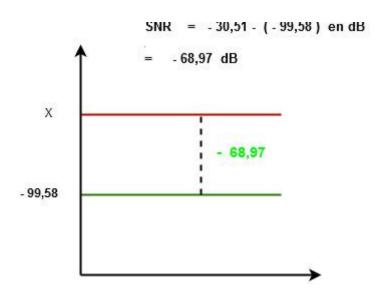
Vérification :

$$\Rightarrow$$
 10 log 2,72 * 10⁻² = -15,65 dB

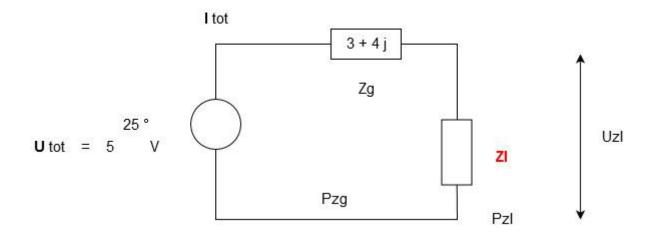
(D) SNR

Bruit = $1.1 * 10^{-10}$ mW => $10 log 1.1 * 10^{-10}$

Bruit = -99,58 dBm



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Exercice 1 : $Zl = 3 + 2j \cap$

A) Z tot

Z tot =
$$(3+4j) + (3+2j) = 6+6j \cap$$

=> $|Z| = \sqrt{6^2 + 6^2} = 8,48$
 $\rho = \arctan \frac{6}{6} = 45^{\circ}$
Z tot = $8,48^{45^{\circ}} \cap$

B) I tot

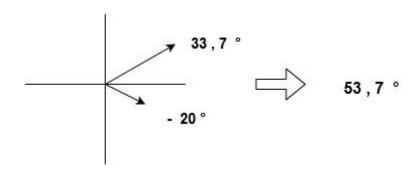
I tot =
$$\frac{U}{Z \text{ tot}}$$
 = $\frac{5^{25^{\circ}}}{8.48^{45^{\circ}}}$ = 0,59^{-20°} A

C) Uzl

Uzl = Zl * I tot =
$$(3+2j)$$
 * $0,59^{-20}$ °
=> $|Z| = \sqrt{3^2 + 2^2} = 3,6$
 $\rho = \arctan \frac{2}{3} = 33,7$ ° => $Zl = 3,6$ $33,7$ ° \cap Uzl = $3,6$ $33,7$ ° \vee V

D) Pzl

 $Pzl = Uzl * I tot cos \theta$



$$Pzl = 2,124 * 0,59 \cos 53,7 = 1,042 W$$

E) Pzg

Pzg = Uzg * I tot * cos
$$\theta$$

=> Uzg = (3+4j) * 0,59⁻²⁰ = 2,95
=> |Zg| = $\sqrt{3^2 + 4^2}$ = 5
 ρ = arct $\frac{4}{3}$ = 53°
Pzg = 2,95 * 0,59 * cos [33 - (-20°)] = 1,047 W

F) P tot

$$P \text{ tot} = Pzg + Pzl = 2,089 W$$

Verification:

P tot = U tot * I tot * Cos
$$\theta$$

 θ = 25° - (-20) = 45°
P tot = 2,086 W

Exercice 2 : $Zl = 4 \cap$

A) Z tot

Z tot =
$$(3+4j) + (4) = 7+4j \cap$$

=> $|Z| = \sqrt{7^2 + 4^2} = 8,062$
 $\rho = \arctan \frac{7}{4} = 29,75^{\circ}$
Z tot = $8,062^{29,75^{\circ}} \cap$

B) I tot

I tot =
$$\frac{U}{Z \text{ tot}}$$
 = $\frac{5^{25^{\circ}}}{8.062^{\circ}}$ = 0,62^{-4,75} A

C) Uzl

Uzl = Zl * I tot =
$$(3+2j)$$
 * $0,59^{-20}$ °
=> $|Z| = \sqrt{3^2 + 2^2} = 3,6$
 $\rho = \arctan \frac{2}{3} = 33,7$ ° => $Zl = 3,6$ $33,7$ ° \cap $Uzl = 3,6$ $33,7$ ° \cap V

D) Pzl = Uzl * I tot *
$$\cos \theta$$

$$Pzl = 2,48 * 0,62 \cos 0 = 1,5376 W$$

E) Pzg = Uzg * I tot *
$$\cos \theta$$

=> Uzg =
$$(3+4j) * 0,62^{-4,75}$$

=> $|\mathbf{Zg}| = \sqrt{3^2+4^2} = 5$
 $\rho = 53,13^{\circ}$

$$Pzg = 3,1 * 0,62 * \cos [48,38 - (-4,75°)] = 1,153 W$$

(Attention les rouges sont pas égaux ou presque donc pas la bonne impédance)

F) P tot

$$P tot = Pzg + Pzl = 2,6909 W$$

Verification:

$$P \; tot \quad = \quad U \; tot \quad * \quad I \; tot \quad * \quad Cos \quad \theta$$

Exercice $3 : Zl = 4 \cap$

A) Z tot

Z tot =
$$(3+4j) + (10j) = 3+14j \cap$$

=> $|Z| = \sqrt{3^2 + 14^2} = 14,32$
 $\rho = \arctan \frac{14}{3} = 78^{\circ}$

Z tot =
$$14,32^{78}$$
 $^{\circ}$ \cap

B) I tot

I tot =
$$\frac{U}{Z \text{ tot}}$$
 = $\frac{5^{25^{\circ}}}{14,32^{\circ}}$ = 0, 35⁻⁵³ A

C) Uzl

Uzl = Zl * I tot =
$$(10j)$$
 * 0.35^{-53} °

Uzl = 10^{90} * 0.35^{-53} = 3.5^{37} ° V

D) Pzl = Uzl * I tot *
$$\cos \theta$$

$$Pzl = 3,5 * 0,35 \cos [37 - (-53)]$$

 $Pzl = 3,5 * 0,35 \cos 90^{\circ} = 0 W$

E) Pzg = Uzg * Itot *
$$\cos \theta$$

=>
$$Uzg = 5^{53,13} * 0,35^{-53} 1,75^{0,13}$$
°

$$Pzg = 1,75 * 0,35 * \cos [0,13 - (-53°)] = 0,3675 W$$

(Attention les rouges sont pas égaux ou presque donc pas la bonne impédance)

F) P tot

$$P \text{ tot} = Pzg + Pzl = 0 + 0,3675 = 0,3675 W$$

Verification: Ptot =
$$5 * 0.35 * Cos 78^{\circ}$$

Exercice 4 : $Zl = 3 - 4j \cap$

A) Z tot

$$Z tot = (3+4j) + (3-4j) = 6 \cap$$

B) I tot

I tot =
$$\frac{U}{Z \text{ tot}}$$
 = $\frac{5^{25^{\circ}}}{6}$ = 0, 83 ²⁵ ° A

C) Uzl

Uzl = Zl * I tot =
$$(3-4 \text{ j})$$
 * 0 , 83^{25} °

$$Uzl = 5^{53} * 0.83^{25} = 4.15^{-28}$$
 V

D) Pzl = Uzl * I tot *
$$\cos \theta$$

$$Pzl = 4,5 * 0,83 \cos [25 - (-28,13)]$$

$$Pzl = 3,5 * 0,35 \cos 53,13 \circ = 2,066 W$$

E) Pzg = Uzg * Itot *
$$\cos \theta$$

$$=>$$
 Uzg = $5^{53,13} * 0,83^{25} = 4,15^{78,13}$

$$Pzg = 4,15 * 0,83 * \cos [78,13 - (-25°)] = 2,066 W$$

(Attention les rouges sont pas égaux ou presque donc pas la bonne impédance)

F) P tot

$$P \text{ tot} = Pzg + Pzl = 4,13 W$$

Verification: Ptot = 5 * 0.83 * Cos 0° = 4.132 W