TOC - Assignment 2

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Section :- B

PART - A

1. Given,

$$L = \{ w^\wedge w^R \mid w \text{ is in } (0+1)^* \}$$

To find whether CFL or not.

$w^R \rightarrow$ reverse of string w.
$w^\wedge \rightarrow$ string w with bits flipped.

$\Rightarrow$ Let $w = (0101)^n$
$w^\wedge = (1010)^n$
$w^R = (1010)^n$

Now,

$$L = \{ (1010)^{2n} \mid n > 0 \}$$

~~Let~~ Case - I :- v and y have different symbols

$\Rightarrow$ We use pumping lemma to prove whether a language is in CFL or not.

So,

Let p (pumping length) $= 3$

$\therefore \quad S = (1010)^6$

$= \underbrace{10101010}_{u}\,\underbrace{10}_{v}\,\underbrace{1010}_{x}\,\underbrace{10}_{y}\,\underbrace{1010101010}_{z}$

Now, According to pumping Lemma
Lets assume
$i = 2$
$\therefore \quad s = uv^i x y^i z \quad , \quad |vy| \leq p$

$s = uv^2 xy^2 z$
$= 1010101\ 01\ 01\ 0101010\ 1\ 1\ 010010\ 0$
This clearly does not belong to L.
$s \notin L$.

$\therefore$ The Language L is not a CFL.

2. Given,
$L = \{ w^R w \mid w \text{ is in } (0+1)^* \}$

$\Rightarrow$ Let, $w = (0101)^n$
$w^R = (1010)^n$

$L = \{ (1010)^n (0101)^n \mid n > 0 \}$

Case - I :- v and y have different symbols.

We use Pumping Lemma to prove whether a language is in CFL or not.
So,
Let pumping length $p = 3$

$s = (1010)^3 (0101)^3$
$= \underbrace{101010101}_{u}\underbrace{01}_{v}\underbrace{0\ 01}_{x}\underbrace{01}_{y}\underbrace{0101010101}_{z}$

Now, according to pumping Lemma

Lets assume, $i = 2$

$S = uv^i xy^i z$, $|vy| \leq p$

$S = 1010101010\ 10100100\ 001010101$

This clearly does not belong to L,

$S \notin L$.

$\therefore$ The Language is not a CFL.

PART - B

1. Given Language,

$L = \{ w \mid w \text{ has equal number of } 0\text{'s and } 1\text{'s} \}$

⇒ To construct PDA,

The 6-tuple for PDA is

$P = (\{q_0, q_1, q_2\}, \{0,1\}, \{0,1,Z_0\}, \delta, q_0, \{q_2\})$

$\{q_0, q_1, q_2\}$ → set of states

$\{0,1\}$ → input alphabets

$\{0,1,Z_0\}$ → stack alphabets

δ → transition function

q_0 → start state.

$\{q_2\}$ → Final state (or) accept state.

The transition table

$\delta(a_0, \varepsilon, \varepsilon) = (a_1, z_0)$

$\delta(a_1, 0, z_0) = (a_2, 0z_0)$

$\delta(a_1, 1, z_0) = (a_3, 1z_0)$

$\delta(a_2, 0, 0) = (a_2, 00)$

$\delta(a_2, 1, 0) = (a_2, \varepsilon)$

$\delta(a_2, \varepsilon, z_0) = (a_1, z_0)$

$\delta(a_2, \varepsilon, z_0) = (a_4, \varepsilon)$

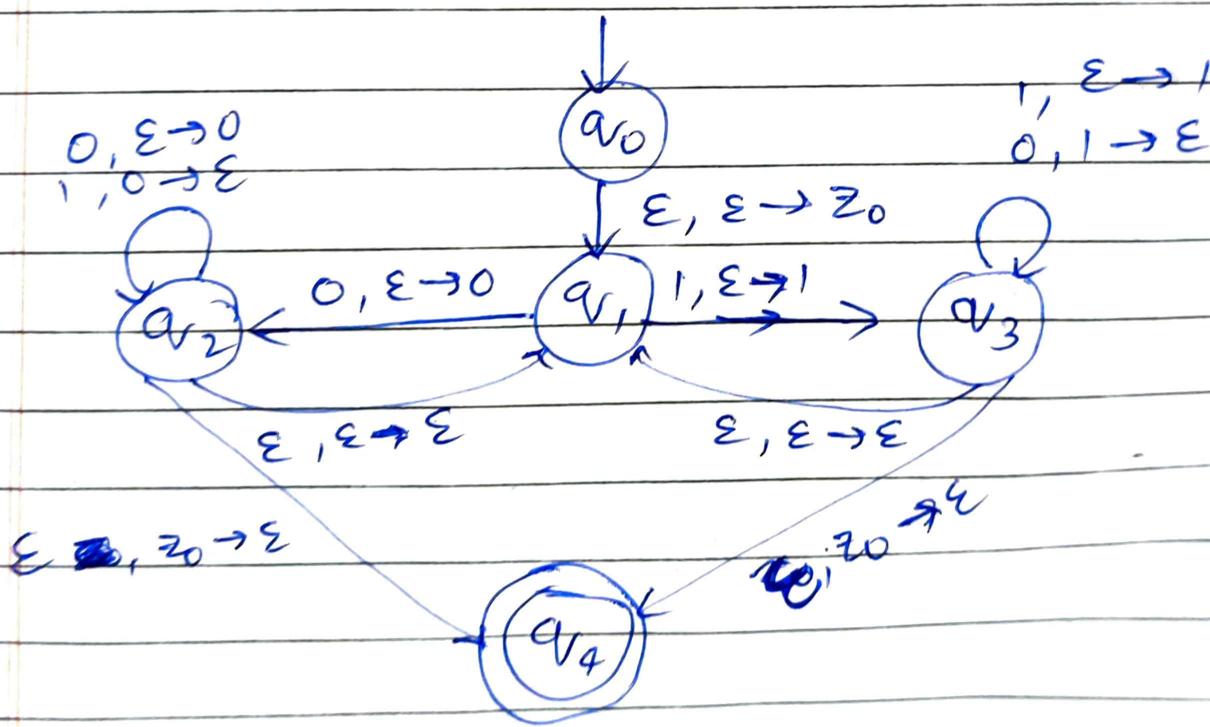
$\delta(a_3, 1, 1) = (a_3, 11)$

$\delta(a_3, 0, 1) = (a_3, \varepsilon)$

$\delta(a_3, \varepsilon, z_0) = (a_1, z_0)$

$\delta(a_3, \varepsilon, z_0) = (a_4, \varepsilon)$

The transition diagram



2. Given Language,

$L = \{ w \mid w \text{ has equal number of 0's and 1's} \}$

$\Rightarrow$ the 7-tuple of the turing machine is

$P = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$.

$P = (\{q_0, q_1, q_2, q_3, q_4, q_{accept}\}, \{0, 1\}, \{0, 1, B\}, q_0,$

q_{accept}, reject on halt.)

The transition table:-

$\delta(q_0, x) = (q_0, x, R)$

$\delta(q_0, 1) = (q_1, x, R)$

$\delta(q_1, 1) = (q_1, 1, R)$

$\delta(q_1, x) = (q_1, x, R)$

$\delta(q_1, 0) = (q_2, x, L)$

$\delta(q_2, 1) = (q_2, 1, L)$

$\delta(q_2, 0) = (q_2, 0, L)$

$\delta(q_2, x) = (q_2, x, L)$

$\delta(q_2, B) = (q_0, B, R)$

$\delta(q_0, 0) = (q_3, x, R)$

$\delta(q_3, 0) = (q_3, 0, R)$

$\delta(q_3, x) = (q_3, x, R)$

$\delta(q_3, 1) = (q_4, x, L)$

$\delta(q_4, 0) = (q_4, 0, L)$

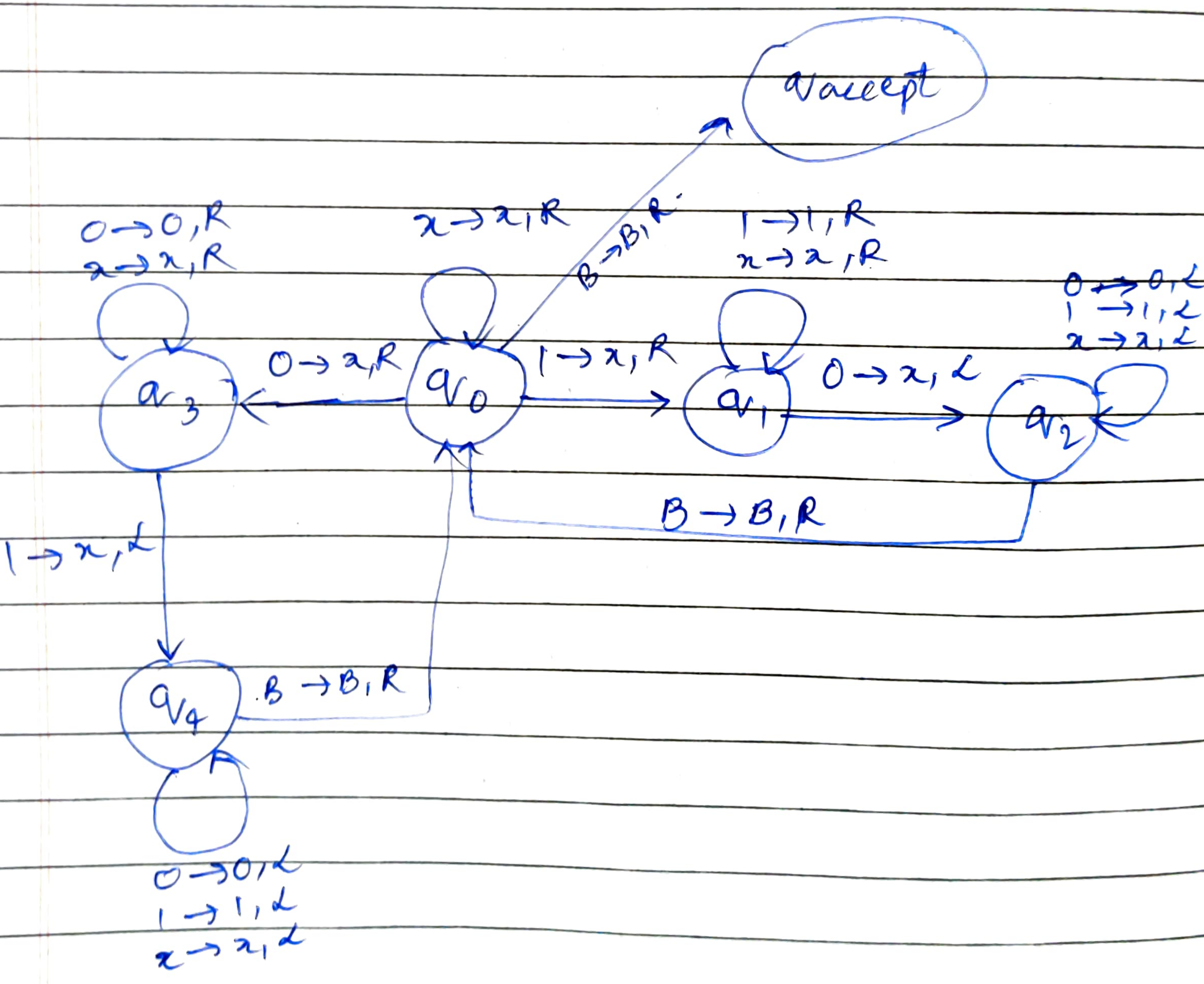
$\delta(q_4, 1) = (q_4, 1, L)$

$\delta(q_4, x) = (q_4, x, L)$

$\delta(q_4, B) = (q_0, B, R)$

$\delta(q_0, B) = (q_{accept}, B, R)$

The transition diagram :-



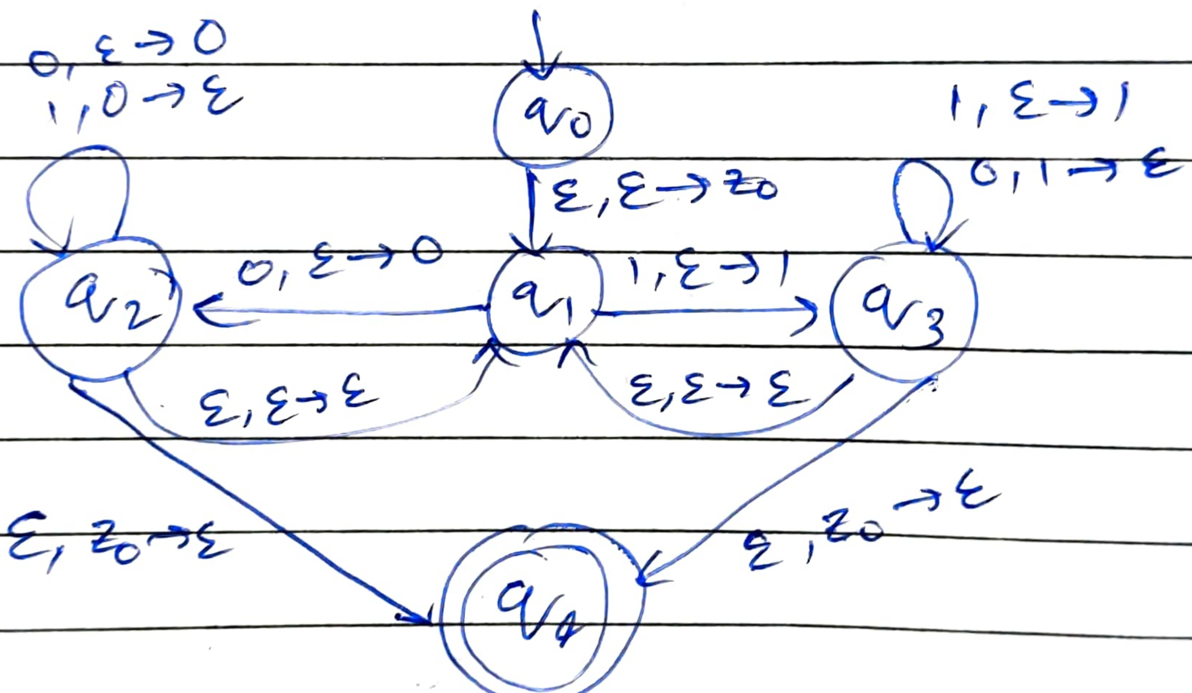
3. The last digit of roll number is 7

$w = 0111$

(i) $w^{\wedge}w = 10000111$

(ii) $ww^{\wedge} = 01111000$

The PDA is :-



Execution trace for string input (i)

$(q_0, 10000111, \varepsilon)$

$\downarrow$

$(q_1, 10000111, Z_0)$

$\downarrow$

$(q_3, 0000111, 1Z_0)$

$\downarrow$

$(q_3, 000111, Z_0)$

$\downarrow$

$(q_1, 000111, Z_0)$

$\downarrow$

$(q_2, 00111, 0Z_0)$

$\downarrow$

$(q_2, 0111, 00Z_0)$

$\downarrow$

$(q_2, 111, 000Z_0)$

$\downarrow$

$(q_2, 11, 00Z_0)$

$\downarrow$

$(q_2, 1, 0Z_0)$

$\downarrow$

(q_2, ε, Z_0)

$\downarrow$

$(q_4, \varepsilon, \varepsilon)$ $\Rightarrow$ accepted.

Execution trace for input string (ii)

$(q_0, 0111000, \epsilon)$
$\downarrow$
$(q_1, 0111000, z_0)$
$\downarrow$
$(q_2, 111000, 0z_0)$
$\downarrow$
$(q_2, 11000, z_0)$
$\downarrow$
$(q_1, 111000, z_0)$
$\downarrow$
$(q_3, 11000, 1z_0)$
$\downarrow$
$(q_3, 1000, 11z_0)$
$\downarrow$
$(q_3, 000, 111z_0)$
$\downarrow$
$(q_3, 00, 11z_0)$
$\downarrow$
$(q_3, 0, 1z_0)$
$\downarrow$
(q_3, ϵ, z_0)
$\downarrow$
$(q_4, \epsilon, \epsilon) \Rightarrow$ accepted

4. The last digit of roll number is 7.

$w = 0111$

(i) $w^{\wedge} w = 10000111$

(ii) $w w^{\wedge} = 01111000$

Execution trace for input string (i).

$B q_0 10000111 \rightarrow$

$B x q_1 0000111 \rightarrow$

$B x q_2 x 000111 \leftarrow$

$B q_2 x x 000111 \leftarrow$

$B q_2 x 000111 \rightarrow$

$B x x q_1 000111 \rightarrow$

$B x x x q_3 00111 \rightarrow$

$B x x x 0 q_3 0111 \rightarrow$

$B x x x 00 q_3 111 \rightarrow$

$B x x x 00 q_4 x 11 \leftarrow$

Looping and going directly to the start.

$B q_0 x x x 00 x 11 \rightarrow$

Looping and going till reading 0.

$B x x x q_0 00 x 11 \rightarrow$

$B x x x x q_3 0 x 11 \rightarrow$

$B x x x x 0 q_3 x 11 \rightarrow$

$B x x x x 0 x q_3 11 \rightarrow$

$B x x x x 0 x q_4 x 1 \leftarrow$

Looping ~~and~~ to the start.

$B q_4 x x x x 0 x x 1 \rightarrow$

$B q_0 x x x x 0 x x 1 \rightarrow$

~~$B x x x x q_0$~~

$B\ x\,x\,x\,x\ q_0\ 0\ x\,x\ 1 \rightarrow$

$B\ x\,x\,x\,x\,x\ q_3\ x\,x\ 1 \rightarrow$

~~B After looping.~~

~~$B\ q_3\ x\,x\,x$~~

$B\ x\,x\,x\,x\,x\,x\ q_3\ x\ 1 \rightarrow$

$B\ x\,x\,x\,x\,x\,x\,x\ q_3\ 1 \rightarrow$

$B\ x\,x\,x\,x\,x\,x\,x\ q_4\ x \leftarrow$

Looping to the start.

$B\ q_4\ x\,x\,x\,x\,x\,x\,x\,x \rightarrow$

$B\ q_0\ x\,x\,x\,x\,x\,x\,x\,x\ B \rightarrow$

Looping till the end since all are x.

$B\ x\,x\,x\,x\,x\,x\,x\,x\ q_0\ B \rightarrow$

Since q_0 reads B the string is accepted.

Execution trace for input string (ii).

$B\, q_0\, 0\, 1111\, 000 \rightarrow$

$B\, x\, q_3\, 1111\, 000 \rightarrow$

$B\, x\, q_4\, x\, 111\, 000 \leftarrow$

$B\, q_4\, x\, x\, 111\, 000 \rightarrow$

$B\, q_0\, x\, x\, 111\, 000 \rightarrow$

$B\, x\, x\, q_0\, 111\, 000 \rightarrow$

$B\, x\, x\, x\, q_1\, 11\, 000 \rightarrow$

$B\, x\, x\, x\, 11\, q_1\, 000 \rightarrow$

$B\, x\, x\, x\, 11\, q_2\, x00 \rightarrow$

Looping till the start.

$B\, q_2\, x\, x\, x\, 11\, x\, 00 \rightarrow$

$B\, q_0\, x\, x\, x\, 11\, x\, 00 \rightarrow$

$B\, x\, x\, x\, q_0\, 11\, x\, 00 \rightarrow$

$B\, x\, x\, x\, x\, q_1\, 1\, x\, 00 \rightarrow$

$B\, x\, x\, x\, x\, 1\, q_1\, x\, 00 \rightarrow$

$B\, x\, x\, x\, x\, 1\, x\, q_1\, 00 \rightarrow$

$B\, x\, x\, x\, x\, 1\, x\, q_2\, x\, 0 \leftarrow$

Looping till the start.

$B\, q_2\, x\, x\, x\, x\, 1\, x\, x\, 0 \rightarrow$

$B\, q_0\, x\, x\, x\, x\, 1\, x\, x\, 0 \rightarrow$

$B\, x\, x\, x\, x\, q_0\, 1\, x\, x\, 0 \rightarrow$

$B\, x\, x\, x\, x\, x\, q_1\, x\, x\, 0 \rightarrow$

$B\, x\, x\, x\, x\, x\, x\, x\, q_1\, 0 \rightarrow$

$B\, x\, x\, x\, x\, x\, x\, x\, q_2\, x \leftarrow$

Looping till the start.

$B\, q_2\, x\, x\, x\, x\, x\, x\, x\, x\, B \rightarrow$

$B\, q_0\, x\, x\, x\, x\, x\, x\, x\, x\, B \rightarrow$

Looping till end since all are x.

$B\, x\, x\, x\, x\, x\, x\, x\, x\, q_0\, B \rightarrow$

Since q_0 reads B the string is accepted.