<u>Assignment-6 (Goal-visualization)</u> → Deadline: April 3, 2016

To visualize the solution of a second-order differential equation.

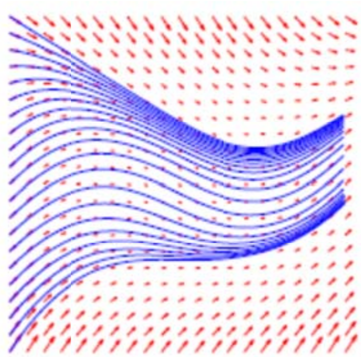
1. We have learned that the two dimensional plane defined by the variables *q* and *p* ($p = \frac{\partial L}{\partial q}$) *is* phase space (where *L* is the Lagrangian). Or position and velocity are the phase space variables.

Suppose in a certain region, the wind is blowing with velocity that is constant in time, but varies spatially according to

$$\frac{dx}{dt} = v_x = 0.2x^2 + 0.5y^2 + 20$$
$$\frac{dy}{dt} = v_y = -0.1y^3 + 0.5x^2 - 10$$

Write a matlab code that makes a quiver plot (look any matlab manual) of the wind velocity over the region "-10 to 10" for *x* and *y*. Now add "streamlines" beginning on the left edge of your plot using the streamline command (look manual)

You should be able to reproduce something as shown below (we call it a flow plot):



I. What physical insight do you get from the plot?

II. What does arrows that you produced with the quiver command show at each point?

III. What does the streamlines show (from a particle dynamics viewpoint)?

We have learned in the class, that at any point in time *t* , the coordinate [*x*(*t*), *v*(*t*)] gives the phase-space point that represents the "state" of the system, given an initial starting point, we can then trace out a curve called a *phase space trajectory. That means* we can explore the behavior of the system for a wide range of initial conditions from a phase plot.

2. Suppose we throw a ball really hard so that its acceleration is not constant, but governed by Newton's law of gravity instead:

$$\frac{d^2x}{dt} = -G\frac{M_E}{(x+R_E)^2}$$

Here **ME=6e24 kg** is the mass of the earth, **RE=6.4e6** m, is the radius of the earth, and ***G*** is the universal gravitational constant.

Write the above equation as a system of first-order equations. Then make a "quiver" plot and overlay phase-space trajectories for the following 2 situations:
(a) Throw the ball up at speeds (human) of 1 m/s, 10 m/s, and 40 m/s.
(b) Throw the ball up with a rocket, and achieve speeds of 1,000 m/s, 5,000 m/s, and 10,000 m/s.

Choose reasonable limits for the flow plot.

Additional: Make a movie by plotting the position of the ball as a dot each time the loop iterates, something like this (look manual for more details):

```
plot(x,y,'.')
axis([0 10 0 1.5])
pause(0.001)
```