Problem 1:- (Hypothetical Solar System)

Units:-

Radius (R) = 1 AU (Astronomical Unit)

Velocity (V) = 2π

Time Period (T) = 1 Years

Keplers 3rd Law:- (We will Prove this in next case)

$$\frac{T^2}{R^3} = 1\ (\frac{Year^2}{AU^3})\ \rightarrow \text{Constant}$$

According to Kaplers law: - The Path of Planet is Elliptical around the Sun

General Equation of Ellipse:-

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Another representation is;

x = a sin (ωt) & y = a cos (ωt)

Here, $\omega = \frac{2\pi}{T} \rightarrow \frac{T^2}{R^3} = 1$ (According to keplers 3rd law)

So we have,

$T^2 = R^3 \rightarrow T = R^{\frac{3}{2}}$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{R^{\frac{3}{2}}}$ ___________1.

By putting the ω value in the formula of x & y,

x = a sin $\left(\frac{2\pi}{R^{\frac{3}{2}}}\ t\right)$ & y = a cos $\left(\frac{2\pi}{R^{\frac{3}{2}}}\ t\right)$

$V_x = \frac{dx}{dt} \rightarrow V_x = \frac{2\pi}{R^{\frac{3}{2}}} . \left(a \sin\left(\frac{2\pi}{R^{\frac{3}{2}}}\ t\right)\right)$

$$\frac{d^2x}{dt^2} = \left(\frac{2\pi}{R^{\frac{3}{2}}}\right)^2 . \left(a \sin\left(\frac{2\pi}{R^{\frac{3}{2}}} t\right)\right)$$

$$\frac{d^2x}{dt^2} = -\frac{4\pi^2}{R^3} x$$

Euler – Cromer Method:- (Applying the same for Y- Co-ordinate)

$$V_{x,i+1} = V_{x,i} - \frac{4\pi^2}{R^3} x.dt$$

$$V_{y,i+1} = V_{y,i} - \frac{4\pi^2}{R^3} y.dt$$

So we get the formula For the Velocity at (x,y).

From this Formula we will get the trajectory of projectile object.

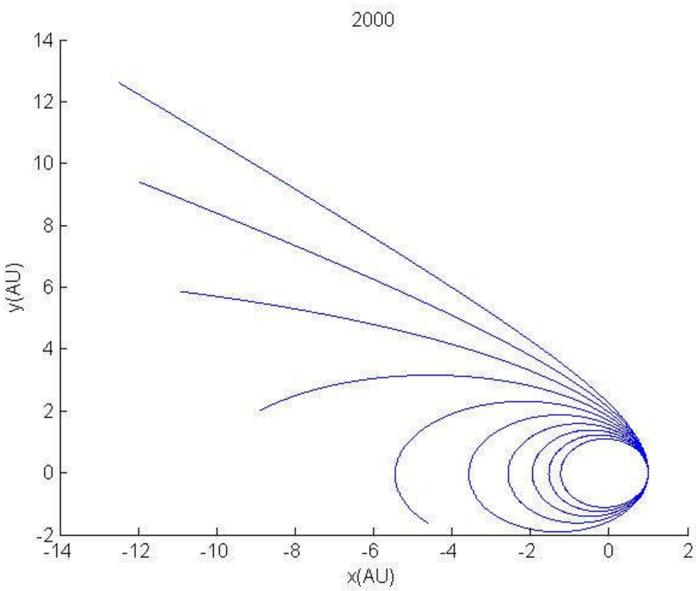
MATLAB Code:-

```
% 201401449
% Planetary orbit using Euler Cromer Method
% Problem 1 - Plot of Orbit for Differant initial Velocity

clear all;
close all;

n=8000;
dt=0.002;

for i=1:10
    xlabel('x(Astronomical Unit)');
    ylabel('y(Astronomical Unit)');
    hold on;
    v_x = zeros(n,1);
    v_x(1) =0;
    x = zeros(n,1);
    x(1) =1;
    v_y = zeros(n,1);
    v_y(1) = (2*pi) + ((i/10)*pi);
    y = zeros(n,1);
    y(1) = 0;
    for j=2:n
        r = sqrt(x(j-1)^2 + y(j-1)^2);
        v_x(j) = v_x(j-1) - ((4*pi^2*x(j-1)*dt) / r^3 );
        v_y(j) = v_y(j-1) - ((4*pi^2*y(j-1)*dt) / r^3 );
        x(j) = x(j-1) + (v_x(j)*dt);
        y(j) = y(j-1) + (v_y(j)*dt);
    end
    plot(x,y);
    xlabel('x(AU)');
```

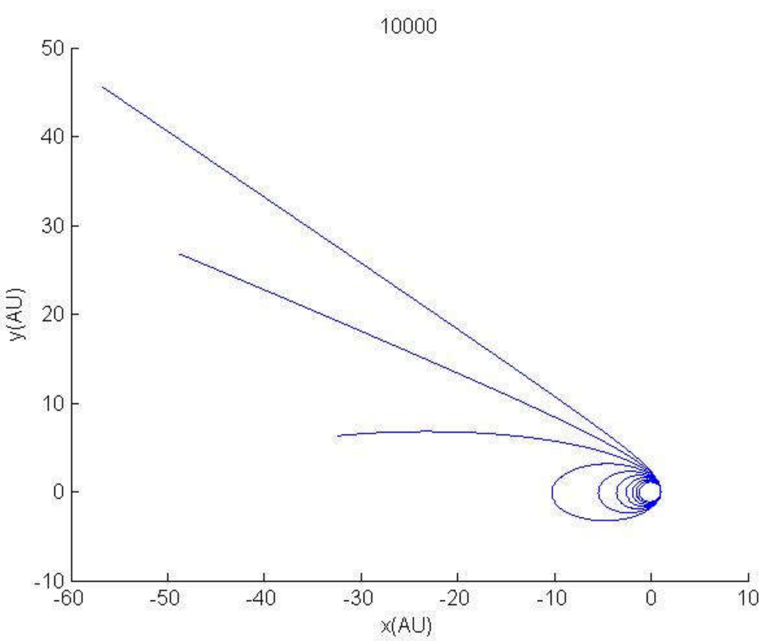
```
    ylabel('y(AU)');
    title(n);
    pause(0.05);
end
```

Output:-

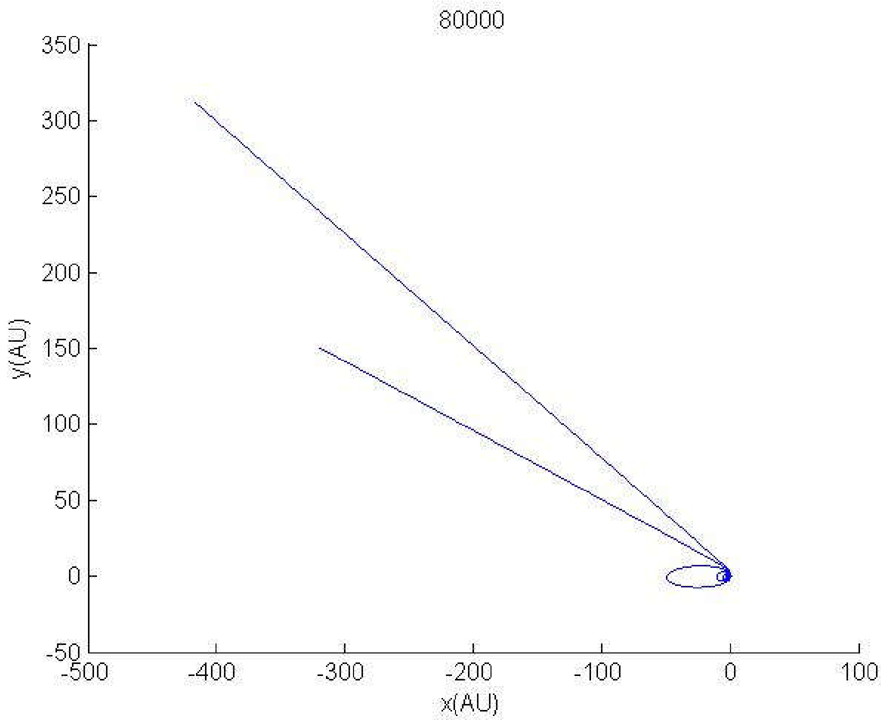
1) n = 2000



2) n = 10000



3) n = 80000



Observation:-

As we can see From the Output that the Time taken by object to return on the earth will increase as increase in initial projection velocity up to some extent that limit is escape velocity.

Assuming the Planetary motion is **Circular the Orbit** of all 7 planets will be,

Velocity of each planet will remains Constant that is,

There are Two forces acting on Planet

1) Centripetal Force
2) Force of attraction between Sun and planet
   Both are equal in Magnitude & Opposite in direction.

$$\frac{GmMs}{R^2} = \frac{mv^2}{R}$$

$$v = \sqrt{\frac{GMs}{R}} \;\&\; v = \frac{2\pi r}{T}$$

$$\frac{4\pi^2 R^2}{T^2} = \frac{GMs}{r} \rightarrow \frac{T^2}{R^3} = \frac{4\pi^2}{GMs}$$

$$\frac{T^2}{R^3} \rightarrow Constant$$

Angular Velocity of any planet is,

$$\omega = \frac{2\pi}{T}$$

From the above derivation We can put the value of T in ω,

$$\omega = \frac{2\pi}{T} \rightarrow \omega = \frac{2\pi}{\sqrt{\frac{R^3 4\pi^2}{GMs}}} \rightarrow \omega = \frac{\sqrt{GMs}}{R^{\frac{2}{3}}}$$

General Formula for the Circle (Special case of the Ellipse):-

$x^2 + y^2$ = Constant

x = a $\sin(\omega t)$ & y = a $\cos(\omega t)$

By the Derivation we get the following equations:-

$$\frac{d^2x}{dt^2} = -\frac{GMs}{R^3}x$$

Euler – Cromer Method:- (Applying the same for Y- Co-ordinate)

$$V_{x,i+1} = V_{x,i} - \frac{GMs}{R^3} x.dt$$

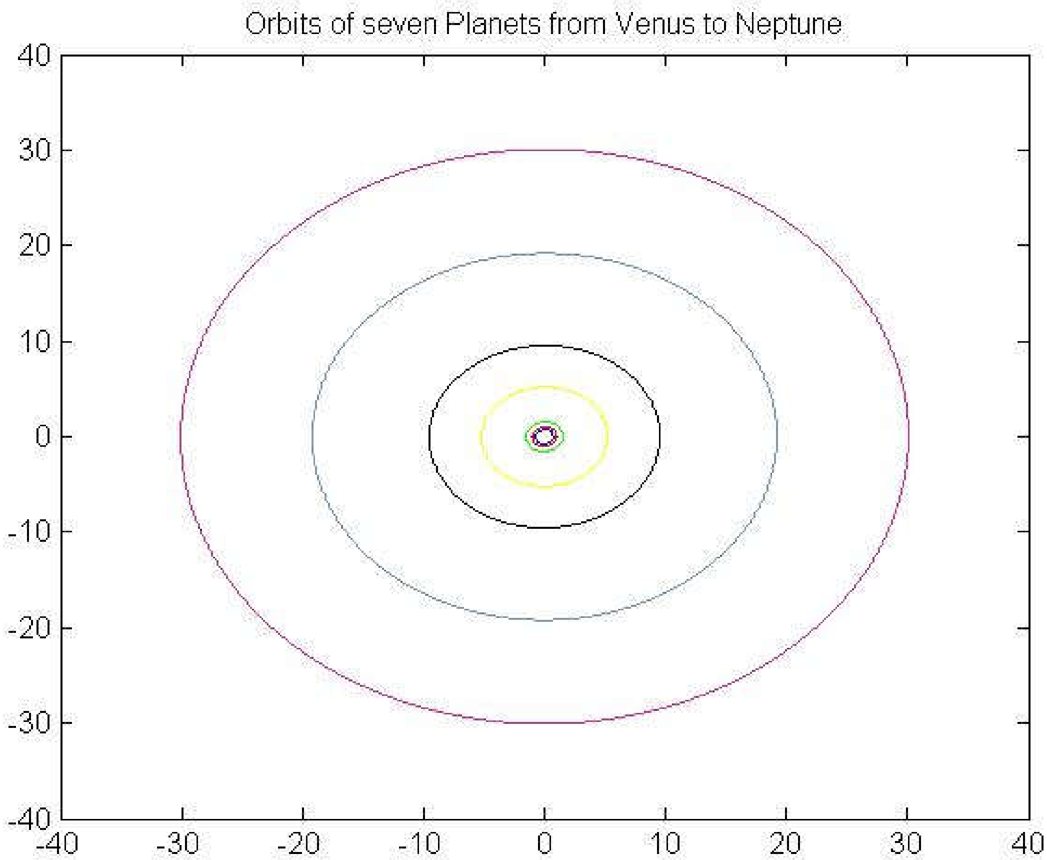
$$V_{y,i+1} = V_{y,i} - \frac{GMs}{R^3} y.dt$$

MATLAB Code:-

```
%Quetion 1
%Information of all 7 planet

clear all;
close all;
v0x=0;
G=4*pi*pi;
Ms=1;
R=[ 0.732, 1, 1.524, 5.203, 9.537, 19.191,30.069 ];
tinfinite=200;
M=[ 2.4478383e-6, 3.04043263333e-6, 0.3227151e-6, 954.79194e-6, 285.8860e-6,
43.66244e-6, 51.51389e-6 ];
timep=zeros(length(M),1);
const=zeros(length(M),1);
for stp=1:1:7
    Me= M(stp);
    v0y=sqrt(G*Ms/R(stp));
    tstart=0;
    dt=0.0001;
    t=tstart:dt:tinfinite;
    C = {'b','r','g','y','k',[.5 .6 .7],[.8 .2 .6]};
    u=zeros(length(t),4);
    u(1,1)=v0x;
    u(1,2)=v0y;
    u(1,3)=R(stp);
    u(1,4)=0;
    pq=0;
    for step=2:1:length(t)
        u(step,1)=u(step-1,1)+dt*(-G*Ms*u(step-1,3)/(R(stp)*R(stp)*R(stp)));
        u(step,2)=u(step-1,2)+dt*(-G*Ms*u(step-1,4)/(R(stp)*R(stp)*R(stp)));
        u(step,3)=u(step-1,3)+dt*u(step,1);
        u(step,4)=u(step-1,4)+dt*u(step,2);
        if t(step)>0.5
            if u(step-1,4)<0&&u(step,4)>0
                   timep(stp)=t(step)
                   pq=step;
                   const(stp)=(timep(stp)*timep(stp))/(R(stp)*R(stp)*R(stp))
                   break;
            end
        end
    end
    vx=u(1:pq,1);
    vy=u(1:pq,2);
    x=u(1:pq,3);
    y=u(1:pq,4);
```

```
    plot(x,y,'color',C{stp});
    hold on;
    title('Orbits of seven Planets from Venus to Neptune')
end
```

Output:-



Observation:-

As we get the information about all the Seven planets:-

Planets	Mass(in kg)	Radius (in AU)	a (Length of Major axis/2) (in AU)	Time period (in AU)	Eccentricity	Ratio T^2/a^3
Earth	6*10^24	1	1	0.99	0.0974	1.000
Mars	0.64*10^24	1.881	1.52	1.87	0.0756	1.002
Venus	4.86*10^24	0.952	0.72	0.61	0.10	0.9969
Jupiter	1998*10^24	11.209	5.1989	11.83	0.0456	0.9959
Uranus	86.81*10^24	4.007	19.1565	83.73	0.0436	0.9972
Saturn	568.34*10^24	9.449	9.544	29.44	0.0469	0.9969
Neptune	102.413*10^24	3.883	30.05	164.21	0.015	0.9937

As we can see from the output and the table that the value of $\frac{T^2}{R^3}$ will remain constant for all the planets. (Third Law of Kepler).

Problem 2:- Orbits

Three types of orbit:-

1) Circular
2) Elliptical
3) Parabolic (if extended Hyperbolic)

For **Circular Motion**:- (Special case of elliptical Motion)

There are two forces acting on Planet

1) Centripetal Force
2) Gravitational Force of attraction between 2 Bodies
   Both are equal in Magnitude & Opposite in direction.

$$\frac{GmMs}{R^2} = \frac{mv^2}{R}$$

$$v = \sqrt{\frac{GMs}{R}}$$

Where Ms → Orbited Body

R → distance between 2 bodies

For **parabolic** (Return to its initial condition at ∞ Time):-

Kinetic Energy = Potential Energy

$\frac{1}{2}mv^2 = \frac{GMm}{R} \rightarrow V_P = \sqrt{\frac{2GM}{R}}$

Velocity > V_P → Hyperbolic Path (Never return to its initial Condition)

For **Elliptical Motion**:-

In this type of Rotational Motion the orbit can be determine by,

e → essentricity

a → Minor Axis

ea → The distance between focus & Centre of ellipse

Area swept out in time dt → dA

Area of the Triangle:- which has the base = R.dθ & hight = R

$$dA = \frac{1}{2} R^2 d\theta \;\rightarrow\; \frac{dA}{dt} = \frac{1}{2} R^2 \frac{d\theta}{dt} \;\rightarrow \frac{dA}{dt} = \frac{1}{2} R^2 \omega$$

As we know th formula of Angular Momentum:-

$$L = R \text{ X } P$$

$$L = R.(m\omega R) \;\rightarrow\; L = m\omega R^2$$

$$\frac{dA}{dt} = \frac{L}{2m} \rightarrow \text{Constant}$$

From the Above derivation proves the keplers Second law of Equel Area in Equel Time.

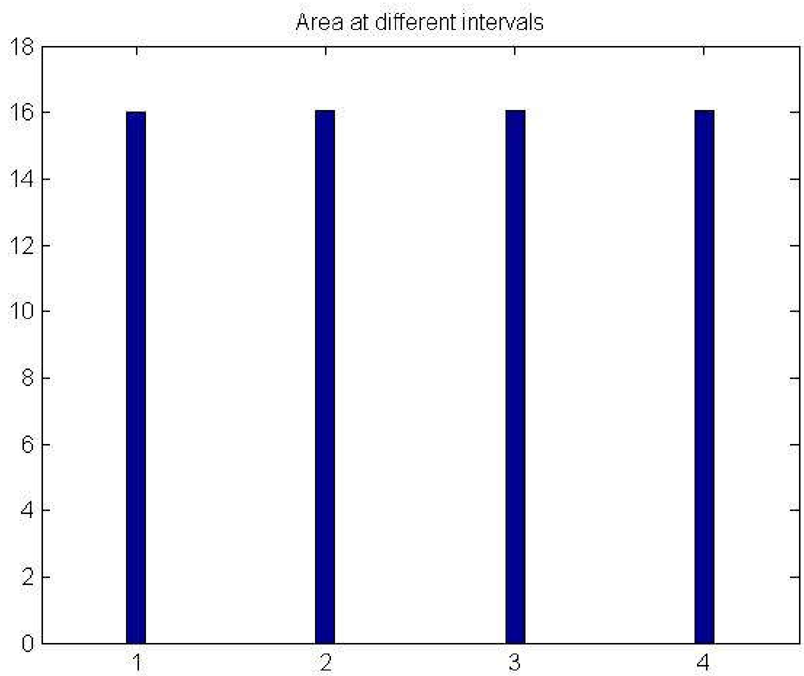
MATLAB Code:-

```
clear all;
close all;

G=4*pi*pi;
Ms=1;
R=1;
v0y=0;
v0x=sqrt(2*G*Ms/R)-1;


tinfinite=200;
Me=3.04e-6;

tstart=0;
dt=0.01;
t=tstart:dt:16;

u=zeros(length(t),4);

u(1,1)=v0y;
u(1,2)=v0x;
u(1,3)=R;
u(1,4)=0;

ax=zeros(length(t),1);
ay=zeros(length(t),1);
```

```
ax(1)=-G*Ms/(R*R);
area=zeros(length(t),1);
ay(1)=0;

flag=0;
for i=2:1:length(t)
        R=sqrt((u(i-1,3)^2)+(u(i-1,4)^2));
        u(i,1)=u(i-1,1)+dt*(-G*Ms*u(i-1,3)/(R*R*R));
        u(i,2)=u(i-1,2)+dt*(-G*Ms*u(i-1,4)/(R*R*R));
        u(i,3)=u(i-1,3)+dt*u(i,1);
        u(i,4)=u(i-1,4)+dt*u(i,2);
        a=-(G*Ms/(R*R));
        dx=abs(u(i,3)-u(i-1,3));
        dy=abs(u(i,4)-u(i-1,4));
        area(i)=dx*dy/2;
        dr=sqrt(dx^2+dy^2);
        area(i)=R*dr/2;

            ax(i)=a*(u(i,3)/R);
            ay(i)=a*(u(i,4)/R);

        if u(i-1,4)<0&&u(i,4)>0&&flag==0
                   tp=(i);
                   flag=1;
        end
end

vx=u(:,1);
vy=u(:,2);
x=u(:,3);
y=u(:,4);
plot(x,y);
str = sprintf('Eliptical Orbit for v = %f in AUs',v0x);
title(str)
figure;
quiver(x,y,vx,vy)
title('Velocity quiver')
KE=Me.*(vx.^2+vy.^2)/2;
PE=-G*Me./sqrt(x.^2+y.^2);
figure;
plot(t,KE,'r')
hold on;
plot(t,PE,'b')
hold on;
plot(t,(KE+PE),'black')
title('Red represents Kinetic, Blue represents Potential and black represents
Total Energy')
figure;
Ax=ax(1:tp);
Ay=ay(1:tp);
quiver(x(1:tp),y(1:tp),Ax,Ay);
title('Acceleration quiver')
figure;
tm=1:1:round(length(t)/tp);
ar=zeros(length(tm),1);
for stp=1:1:round(length(t)/tp)
```

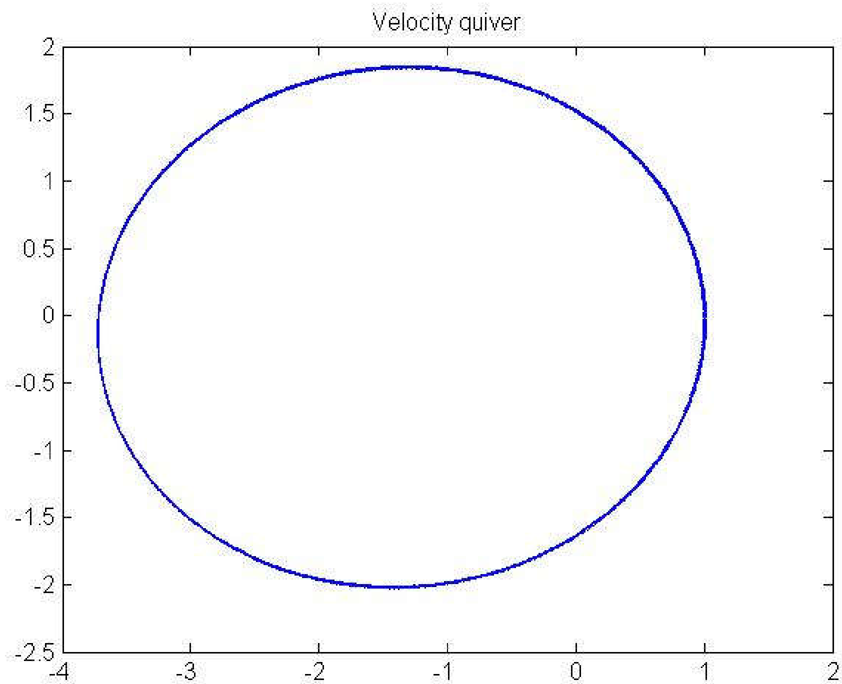
```
    for i=round((stp-1)*tp+1):1:round(stp*tp)
        ar(stp)=ar(stp)+area(i);
    end
end
bar(tm,ar,0.1)
title('Area at different intervals')
ylim([0 18])
```

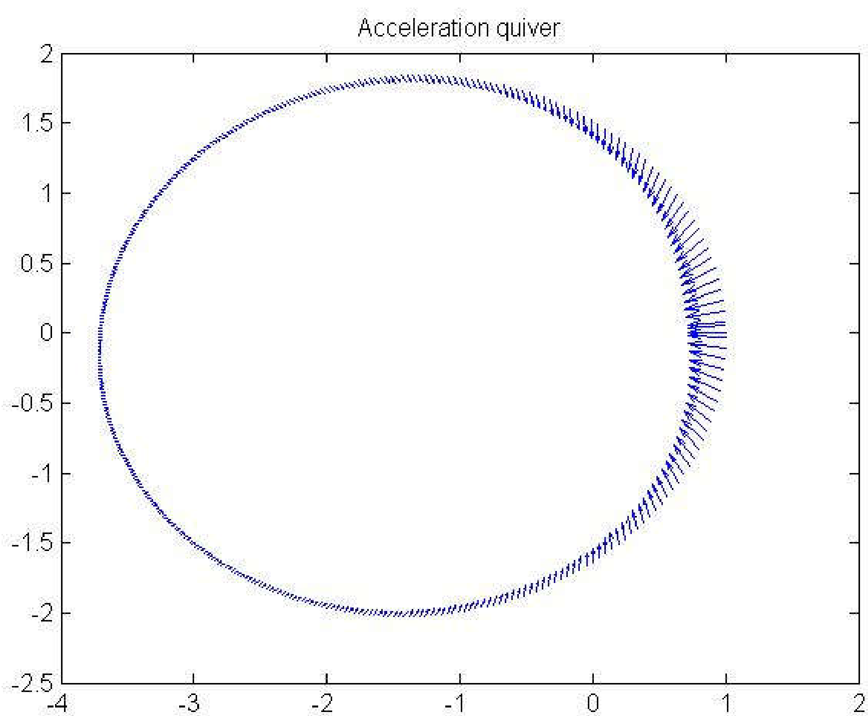
Output:-

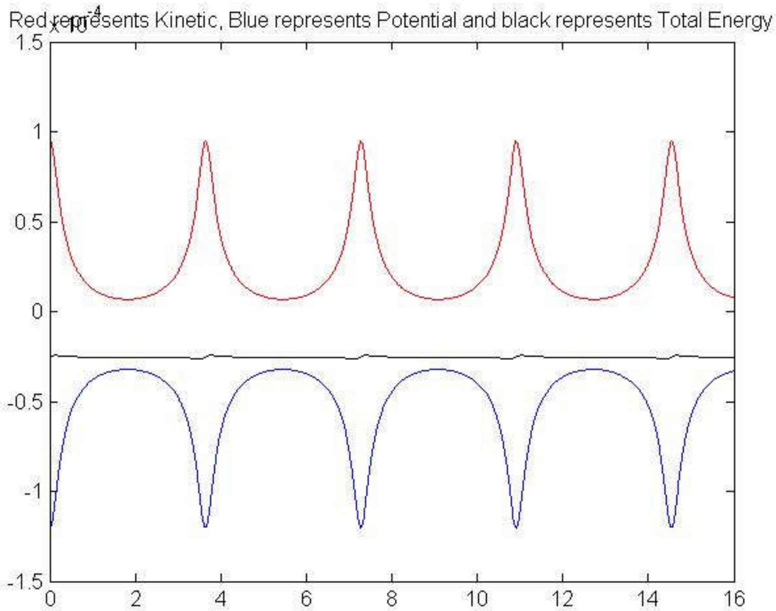


Observation:-

From the above output we can say that the Area swept by the particle moving on the elliptical orbit with centered orbited body is same at any instance (As we proved theoretically above) – the Keplers Second Law


Velocity quiver
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-4
-3
-2
-1
0
1
2


Acceleration quiver
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-4
-3
-2
-1
0
1
2



Observation:-

From the above shown figure we can say that the Total Energy will remain Constant for whole rotation motion of particle (Energy Conservation).