Problem 1:-

Equation of a Position of an object moving horizontally with the Constant velocity V (=50 m/s):

Euler Method:-

$$V = \frac{dX}{dt} = \frac{(X(t+dt) - X(t))}{dt}$$

$$X(t+dt) = X(t) + V * dt$$

Exact Method:-

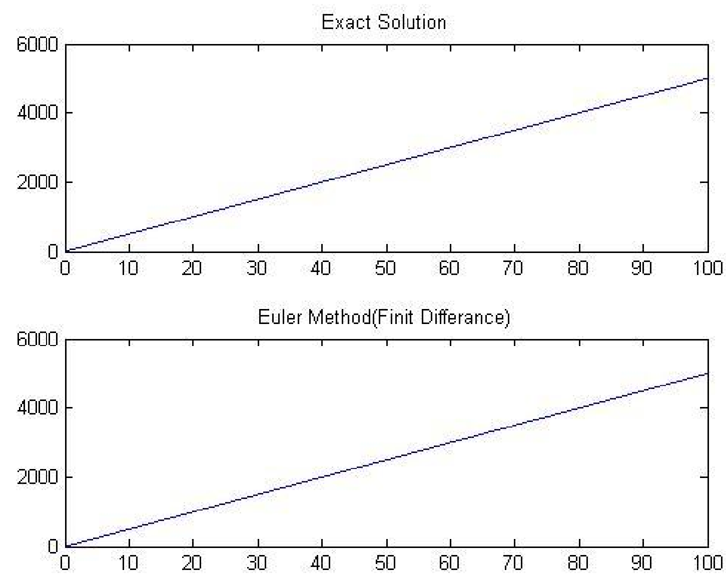
$$V = \frac{dX}{dt}$$

$$\int_{x0=0}^{x} dx = V \int_{t0=0}^{t} dt$$

$$X = X_0 + Vt$$

Code in MATLAB:-

```
% Lab 2 --> Question 1
%201401449
clear;
close all;
v=50;
del_t = .1;
total_time = 100;
niter=total_time/del_t;
init_position=0;
time=zeros(niter,1);
position = zeros(niter,1);
positionX = zeros(niter,1);
time(1)=0;
position(1)=init_position;
positionX(1)=init_position;
for step=1:niter-1
   position(step+1) = v*time(step);
   time(step+1)=time(step)+del_t;
   positionX(step+1) = positionX(1) + v*time(step+1);
end
```

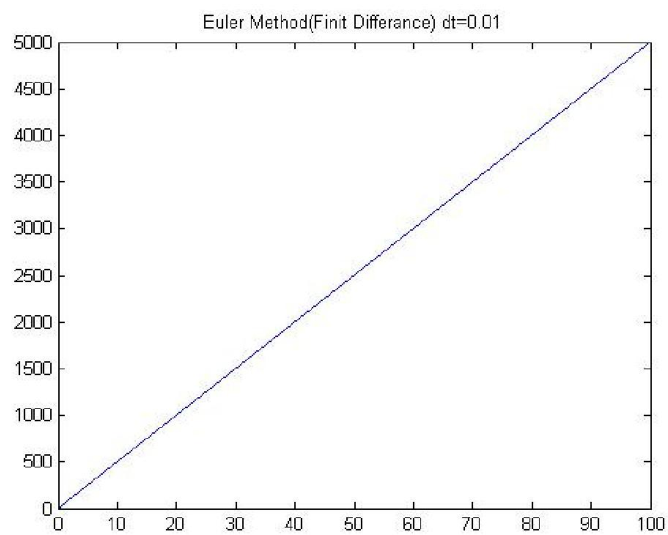


```
subplot(2,1,2)
plot(time,position)
title('Euler Method(Finit Differance)')
subplot(2,1,1)
plot(time,positionX)
title('Exact Solution')
```

Observation:-

We get the same output on computation and solving equation.

If we change step time to 0.01 Then we get O/p:-



Observation:-

As we can see if we change step time, There is no much effect on the velocity Vs time Graph.

Problem 2:- (Parachute Problem)

Equation for the velocity - $\frac{dV}{dt}$ = a – bV (a & b are Constants.)

Take a=10 & b=1

Euler Method (Finite Difference):-

$$\frac{dV}{dt} = a - bV$$

$$\frac{V(t+dt) - V(t)}{dt} = a - bV$$

$$V(t+dt) = V(t) + ((a\text{-}bV).dt) \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \quad 1.$$

Exact Method:-

$$\frac{dV}{dt} = a - bV$$

$$\int_{V0}^{V} \frac{dV}{a-bV} = \int_{t0=0}^{t} dt$$

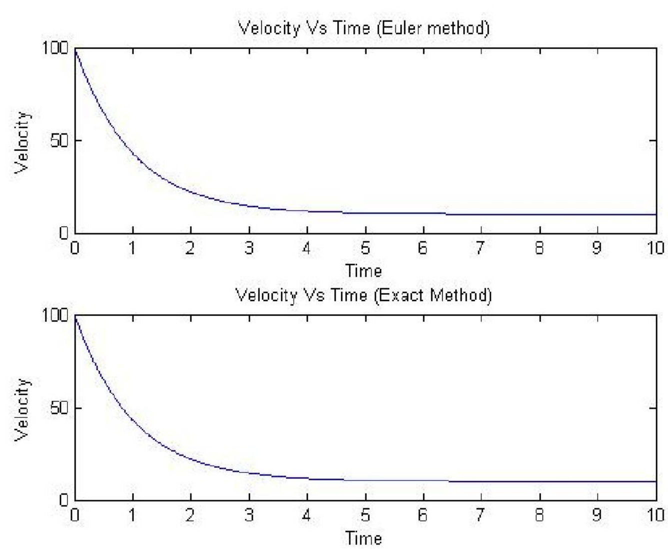
$$\log \frac{|a-bV|}{|a-bV0|} = -bt$$

$$a - bV = e^{-bt}.(a\text{-}bV0)$$

$$V = \frac{a-(e^{-bt}.(a-bV0))}{b} \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \quad 2.$$

MATLAB code:-

```
% Lab 2 --> Question 2
clear;
close all;
a=10;
b=1;
dt = .01;
total_time = 10;
niter=total_time/dt;
init_velocity=100;
time=zeros(niter,1);
velocity = zeros(niter,1);
velocityX = zeros(niter,1);
```



```
time(1)=0;
velocity(1)=init_velocity;
velocityX(1)=init_velocity;
for step=1:niter-1
   velocity(step+1) = velocity(step)+(a-(b*velocity(step)))*dt ;
   time(step+1)= time(step)+dt;
   velocityX(step+1) = 0-(((exp(-b*time(step+1)))*(a-b*velocityX(1))) - a)/b;
end
subplot(2,1,1)
plot(time,velocity)
subplot(2,1,2)
plot(time,velocityX)
% Saturation Velocity = 10 m/s
```

Hear in Code, velocityX is velocity measured by Formula Number 2. & velocity is velocity measured by Euler's Finite difference method (i.e. Formula Number 1.).

Answer:-

The terminal Velocity for the given problem And Given values of Constants (a=10 & b=1) is 10m/s.

The Formula for the Terminal Velocity is:-

$$\frac{dV}{dt} = 0$$

$$a - b.Vt = 0$$

$$Vt = \frac{a}{b}$$

Problem 3:- (Population Growth)

Rate of change of Population: - $\frac{dN}{dt}$ = aN – bN^2 (a & b are Constants.)

Euler Method (Finite Difference):-

$$\frac{N(t+dt)-N(t)}{dt} = aN - bN^2$$

$$N(t+dt) = N(t) + (dt \, . \, (aN(t) - bN(t)^2))$$

By Solving:-

$$\frac{dN}{dt} = aN - bN^2 = N\,(a-bN)$$

$$\frac{dN}{dt} = 0, \text{ When } N=0 \text{ or } N = \frac{a}{b}$$

Population will Increase or Decrease up to N =0 Or N = $\frac{a}{b}$ because after that rate of change of population will become Zero.

MALTAB Code:-

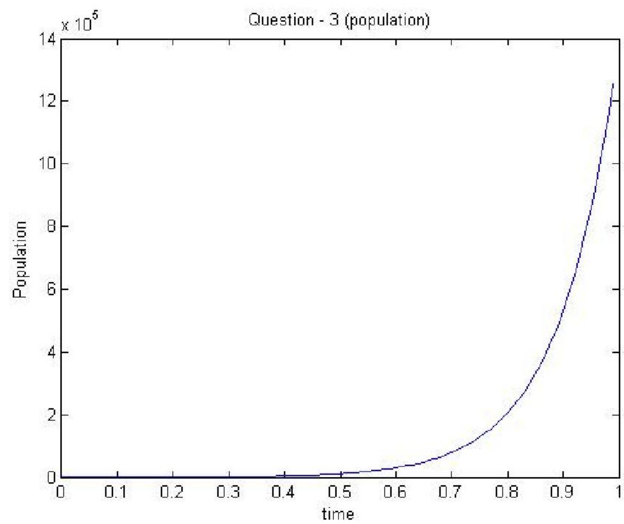
```
clear;
close all;
a = input('Enter a');
b = input('Enter b');
dt = .01;
total_time = 1;
niter=total_time/dt;
init_population=100;

time=zeros(niter,1);
population = zeros(niter,1);

time(1)=0;
population(1)=init_population;

for step=1:niter-1
   population(step+1) = population(step) + (population(step)*(a - (b*population(step)))*dt) ;
   time(step+1)=time(step)+dt;
end
```

```
plot(time,population)
title('Question - 3 (population)')
xlabel('time')
ylabel('Population')
```

Answers :-

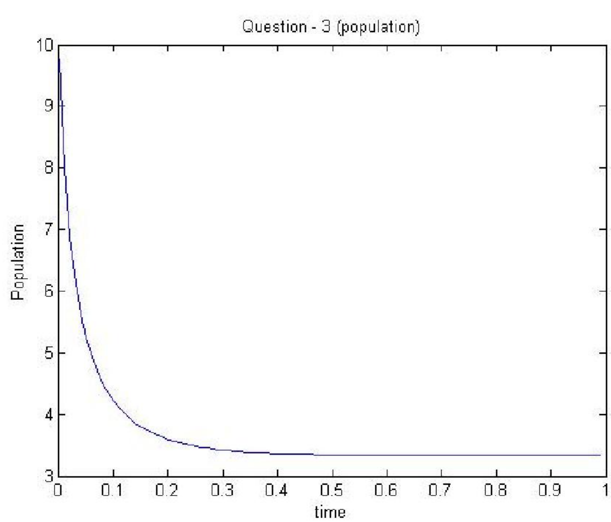
1) a=10 & b=0

   Population tends to infinity (= $\frac{a}{b}$ where b=0)



2) a=10 & b=3

   Population tends to 3.33 (= $\frac{a}{b}$) or 0 (depends on initial Population)



Problem 4:- Bicycle Problem

CASE 1:- (Without Friction)

$$\frac{dV}{dt} = \frac{F}{m} \quad \& \quad E = \frac{1}{2}(mv^2)$$

$$P = \frac{dE}{dt} = mv\frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{P}{mv} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} 1.$$

CASE 2:- (Considering Air-Drag)

Work done by Dragged Air = $\frac{1}{2}m_{air}v^2$

m_{air} = (Density of Air) * (Frontal Area of Object) * V * dt

In Ideal Case – Frontal Area of Object (A) = 0.33

Density of air (p_{air}) = 1.225

Energy that rider needs to give against air-drag = - Work done by the dragged air

$$= -\frac{1}{2}p_{air}.A.V.dt.V^2 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_} 2.$$

$$\text{Energy (K.E)} = F_{drag}\, V.dt = -\frac{1}{2}p_{air}.A.V.dt.\, V^2$$

$$F_{drag} = -\frac{1}{2}p_{air}.A.V^2 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} 3.$$

Euler Method (Finite Difference):-

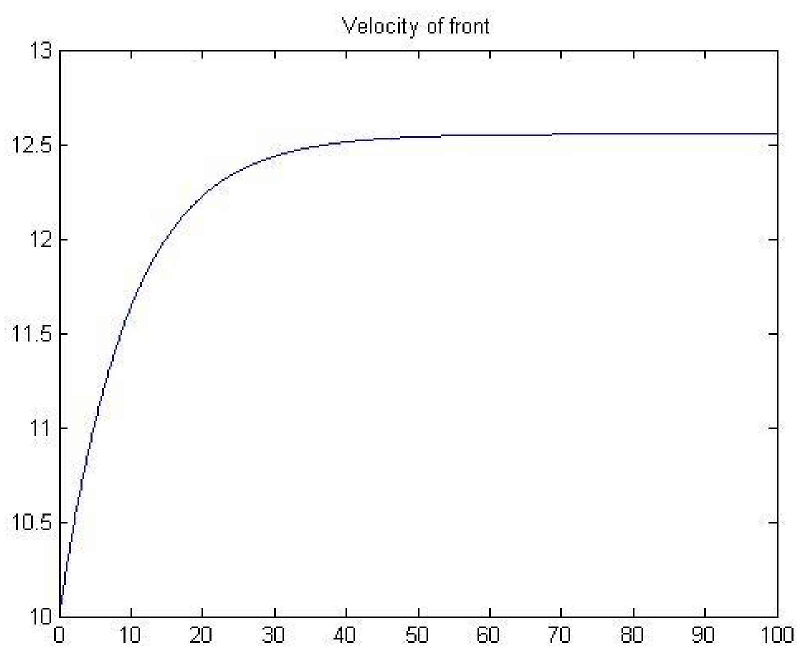
$$\frac{dV}{dt} = \frac{P}{mv} - \frac{\frac{1}{2}\text{pair}.A.V^2}{m}$$

$$\frac{V(t+dt)-V(t)}{dt} = \frac{P}{mv} - \frac{\frac{1}{2}\text{pair}.A.V^2}{m}$$

$$V(t+dt) = V(t) + dt * \left(\frac{P}{mv} - \frac{\frac{1}{2}\text{pair}.A.V^2}{m}\right) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} 4.$$

MATLAB code:-

```
% 201401449
clear all;
total_time=100;
init_vel=10;
dt=.1;
niter=total_time/dt;
time=zeros(niter,1);
speedr=zeros(niter,1);
time(1)=0;
speedr(1)=init_vel;
mass=75;
power=400;
constant=.5;
density=1.225;
area=.33;
for step=1:niter-1
  speedr(step+1)=speedr(step)+power*dt/(mass*speedr(step))-
dt*constant*density*area*speedr(step)*speedr(step)/(mass);
  time(step+1)=time(step)+dt;
end
plot(time,speedr)
title('Velocity of front');
```



PART-A:-

1) Investigate the effect of rider's power, mass and frontal area on the ultimate velocity.

Effect of power:-

Power	Time takes to obtain Terminal Velocity	Terminal Velocity
400	66.5	12.55
200	121	9.965
100	142	7.909
50	163	6.277

Terminal Velocity α Power

Time takes to obtain Terminal Velocity α $\frac{1}{\text{Power}}$

Effect of Mass:-

Mass	Time takes to obtain Terminal Velocity	Terminal Velocity
75	66.5	12.55
150	133.2	12.55
300	266.7	12.55

Time takes to obtain Terminal Velocity α Mass

No effect of Mass of a rider on ultimate velocity.

Effect of Frontal Area:-

Area	Time takes to obtain Terminal Velocity	Terminal Velocity
0.33	66.4	12.55
0.66	61.1	9.965
1.32	35.2	7.909
2.64	20.2	6.277

Terminal Velocity α $\frac{1}{\text{Area}}$

Time takes to obtain Terminal Velocity α Area

2) Effective frontal area is about 30% less than the rider at the front

After reaching at Terminal Velocity $\frac{dV}{dt}$ =0

So We get ,

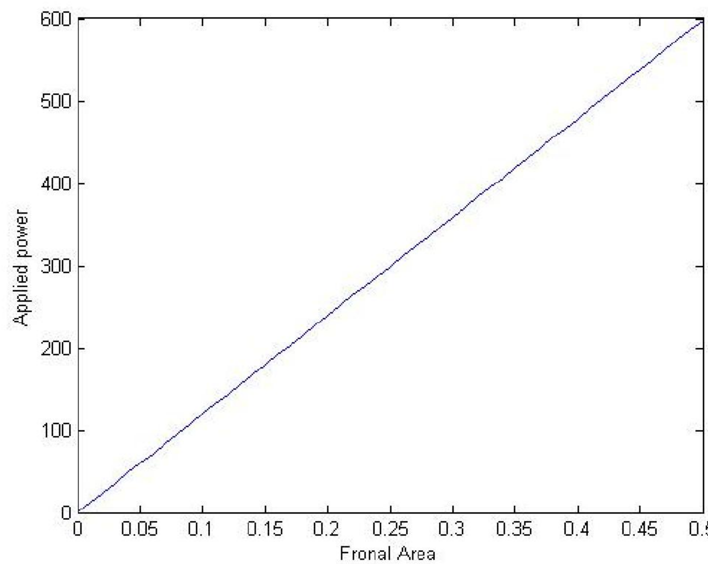
$$\frac{P}{mv} = \frac{\frac{1}{2}\text{pair.A.V}^2}{m}$$

$$\text{P} = \frac{1}{2}\ \text{pair. A. V}^3$$

So After reaching at terminal Velocity V remains same So the Power will be directly proporstional to Frontal Area.

MATLAB code:-

```
%201401449
clear all?
area = 0:.01:.5;
npoints=51;
V = 12.5;
power=zeros(npoints,1);
air_density=1.225;
const=0.5;
for step=1:npoints
power(step)=const*air_density*area(step
)*V*V*V;
end
f = figure;
plot(area,power)
xlabel('Fronal Area');
ylabel('Applied power');
```



Observation:-

As we can see in the Output Graph that Power is proportional to Frontal area. So if the rider at the middle have effective frontal area 30% less than the rider at Front then Power expend by rider at the middle will be 30% less than the Power expend by rider at the Front.

PART-B:-

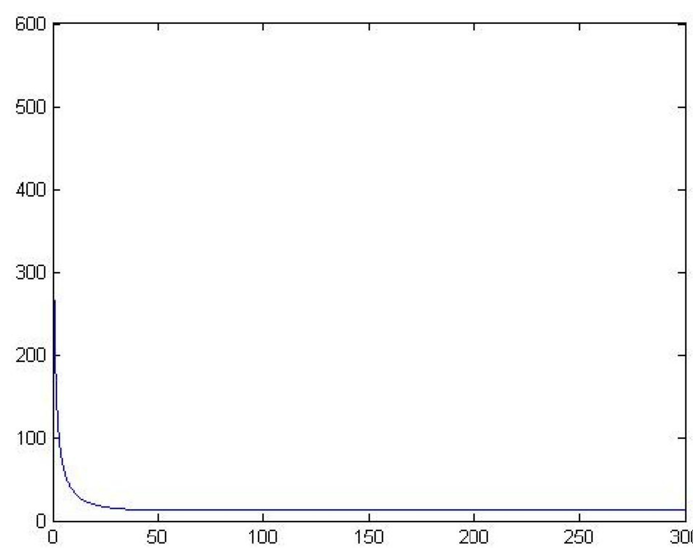
In PART A we a formula of acceleration in which there is a term where we are dividing it with the velocity But if we will take initial velocity as Zero then at time Zero the acceleration becomes infinite so Velocity also jump directly to infinity. (This causes the graph Blank)

We have taken power as constant (Power = Force * velocity) but if our assumption is true then at velocity equals to Zero then Force becomes infinity, which is not possible. So we must have some positive initial velocity.

PART-C:-

MATLAB Code:-

```
%201401449
clear all?
tot_time=300;
init_val=1e-4;
dt=0.01;
npoints=tot_time/dt;

time=zeros(npoints,1);
speed=zeros(npoints,1);
speed(1)=init_val;

mass=75;
power=400;
area=.33;
air_density=1.225;
const=0.5;
time(1)=0;

for step=1:npoints-1
speed(step+1)=speed(step)+power*dt/(mass*speed(step))-
dt*const*area*air_density*speed(step)*speed(step)/mass;
time(step+1)=time(step)+dt;
end

figure;
plot(time,speed);
```

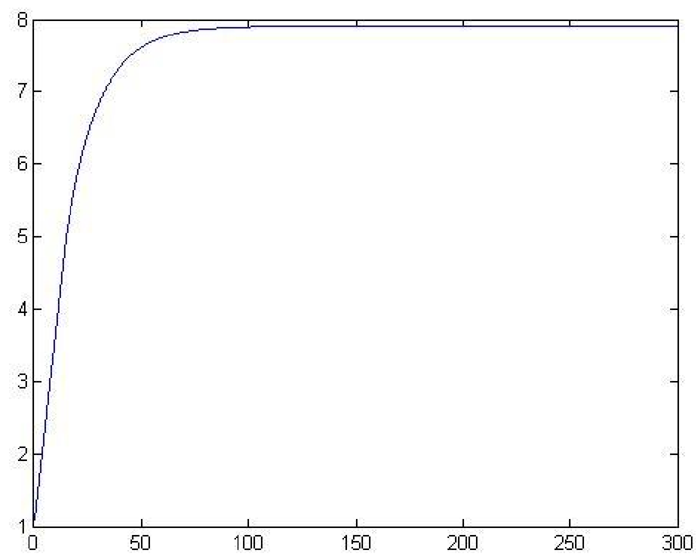


Observation:-

At lower velocity if we assume power is constant then force would be very large and also in equation which we used in part A power term dominates air drag term. So at the first instant velocity is going to be very high. As velocity becomes large air drag term again dominates the power term and velocity is going to be decrease and end ups to terminal velocity.

PART-D:-

MATLAB Code:-



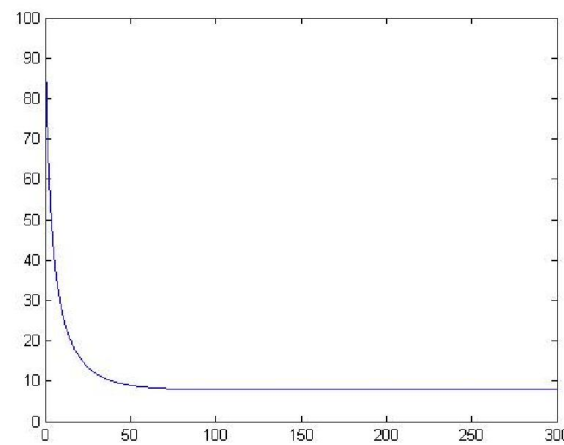

```
%201401449
clear all?
tot_time=300;
init_val=100;
dt=0.01;
npoints=tot_time/dt;
time=zeros(npoints,1);
speed=zeros(npoints,1);
speed(1)=init_val;
time(1) = 0;
turn_v = 5;
mass=75;
area=.33;
f = 20;
air_density=1.225;
const=0.5;
power = f * turn_v;
for step=1:npoints-1
  if(speed(step)<=turn_v)
    speed(step+1) = speed(step) + (f/mass)*dt;
  else
    speed(step+1)=speed(step)+power*dt/(mass*speed(step))-
dt*const*area*air_density*speed(step)*speed(step)/mass;
  end
  time(step+1)=time(step)+dt;
end
figure;
plot(time,speed);
```

Observation:-

As we can notice from the graph that initially, up to velocity is less than the v_turn (=5m/s) it increases constantly because force is constant. After it reaches to v_turn (=5m/s) the Power becomes constant and force applied will change so It gets terminal Velocity.

1) If we change the Initial Velocity then there will be no effect on terminal velocity.

O/p at Initial Velocity = 100



Other parameters like Frontal area, Mass & Power will affect the graph as mentioned in Part A (1) because for any initial velocity it has to come to the region of Constant power & For constant Power region we have seen all the effects in Part A (1).
