

# Discrete II Notes

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## Set Theory Review

Sets are collections of unique elements.

### Complement

Everything that's not in the set. Denoted by  $A'$ ,  $A^C$ , or  $\overline{A}$

### Intersection

An operation that takes two sets, and returns the common elements between them. Denoted by  $A \cap B$

### Union

An operation that takes two sets, and returns all the elements that are in  $A$ ,  $B$ , and  $A \cap B$ . Denoted by  $A \cup B$ .

### De Morgan's Laws

$$(A \cup B)^C = A^C \cap B^C$$

$$(A \cap B)^C = A^C \cup B^C$$

### Disjoint Sets

**Disjoint** sets or **Mutually Exclusive** sets, are sets that have no elements in common. More formally:  $(A \cap B) = \emptyset$

## Intro to Probability

**Sample Space** of an experiment is the set of all possible outcomes of that experiment. An **event** is any collection (subset) of outcomes contained in the sample space  $S$ . An event is said to be **simple** if it consists of exactly one outcome and **compound** if it consists of more than one outcome.

## Counting

For an ordered pair defined by  $(x, y)$  where  $x$  can be selected in  $n_1$  ways, and  $y$  can be selected in  $n_2$  ways, the number of pairs is  $n_1 n_2$ . Can be extended to  $k$  dimensions. This is known as the **Multiplication rule**.

## Permutations

For  $k$  selections made **with replacement** on  $n$  distinct elements, there are  $n^k$  possible outcomes.

**Without replacement** however, there are  $n$  options for the first selection,  $n - 1$  choices for the next selection, and  $n - k + 1$  choice(s) for the  $k^{th}$  selection. This yields.

$${}_n P_k = n(n - 1)(n - 2) \dots (n - k + 1)$$

## Combinations

Given  $n$  distinct objects, the number of **unordered** subsets of size  $k$  is given by  ${}_n C_k$ , or  $\binom{n}{k}$  ( $n$  choose  $k$ ).

$${}_n C_k = \frac{n!}{(n - k)!(k!)}$$

## Overcounting

### Overcounting with Groups

### Bose Einstein