

Discrete II Notes

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Set Theory Review

Sets are collections of unique elements.

Complement

Everything that's not in the set. Denoted by A' , A^C , or \overline{A}

Intersection

An operation that takes two sets, and returns the common elements between them. Denoted by $A \cap B$

Union

An operation that takes two sets, and returns all the elements that are in A , B , and $A \cap B$. Denoted by $A \cup B$.

De Morgan's Laws

$$(A \cup B)^C = A^C \cap B^C$$

$$(A \cap B)^C = A^C \cup B^C$$

Disjoint Sets

Disjoint sets or **Mutually Exclusive** sets, are sets that have no elements in common. More formally: $(A \cap B) = \emptyset$

Intro to Probability

Sample Space of an experiment is the set of all possible outcomes of that experiment. An **event** is any collection (subset) of outcomes contained in the sample space S . An event is said to be **simple** if it consists of exactly one outcome and **compound** if it consists of more than one outcome.

Counting

For an ordered pair defined by (x, y) where x can be selected in n_1 ways, and y can be selected in n_2 ways, the number of pairs is $n_1 n_2$. Can be extended to k dimensions. This is known as the **Multiplication rule**.

Permutations

For k selections made **with replacement** on n distinct elements, there are n^k possible outcomes.

Without replacement however, there are n options for the first selection, $n - 1$ choices for the next selection, and $n - k + 1$ choice(s) for the k^{th} selection. This yeilds.

$${}_n P_k = n(n-1)(n-2) \dots (n-k+1)$$

Combinations

Given n distinct objects, the number of **unordered** subsets of size k is given by ${}_n C_k$, or $\binom{n}{k}$ (n choose k).

$${}_n C_k = \frac{n!}{(n-k)!(k!)}$$

Overcounting with Groups

For n distinct objects being devided into k groups, $\binom{n}{k}$ over counts by a factor of $k!$. To account for that, we do the following:

$$\frac{\binom{n}{k}}{k!}$$

Bose Einstein

For counting ways to separate n indistinguishable objects into k groups. We can use the following:

$$\binom{n-1+k}{k}$$

Analogy: For splitting n peices of candies between k kids, add $k - 1$ placeholder candies. $\binom{n-1+k}{k}$ represents all the ways the dividers can be placed, such that the kids get the candy.

Counting methods summary

| | With order | Without Order |
|---------------------|---------------------|--------------------|
| With replacement | n^k | $\binom{n-1+k}{k}$ |
| Without Replacement | $\frac{n!}{(n-k)!}$ | $\binom{n}{k}$ |

Set inclusion/exclusion principal

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Conditional Probability

$P(A|B)$ represents the probability that A happens, given that B *already* happened.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Bayes' Theorem

Sometimes we don't have some of those probabilities. In this situation, we can use Bayes' Theorem:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Law of total probability

For an A, B where $P(B) \neq 0$ and $P(B) \neq 1$

$$P(A) = P(A|B) \cdot P(B) + P(A|B^C) \cdot P(B^C)$$