Homework 2

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Section 7
pm619

1.7.29

$$\begin{pmatrix} 1 & -1 & -1 & 0 \\ -1 & 0 & -4 & 1 \\ -1 & 1 & 1 & -2 \\ 2 & -1 & 3 & 1 \end{pmatrix}$$

linearly dependant?

$$R_1 + R_2 \to R_2$$

$$R_1 + R_3 \to R_3$$

$$-2R_1 + R_4 \to R_4$$

$$\begin{pmatrix} 1 & -1 & -1 & 0 \\ 0 & -1 & -5 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 1 & 5 & 1 \end{pmatrix}$$

$$R_2 + R_4 \to R_4$$

$$\begin{pmatrix} 1 & -1 & -1 & 0 \\ 0 & -1 & -5 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

Yes, this set of vectors is very linearly dependant.

1.7.41

$$\begin{pmatrix} -2 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & -3 & r \end{pmatrix}$$

$$2R_3 + R_1 \to R_1$$

$$\begin{pmatrix} 0 & -5 & -1+2r \\ 0 & 1 & 1 \\ 1 & -3 & r \end{pmatrix}$$

$$-5R_2 \to R_2$$

$$\begin{pmatrix} 0 & -5 & -1+2r \\ 0 & -5 & -5 \\ 1 & -3 & r \end{pmatrix}$$

$$-5x_2 = -1 + 2r$$

$$-5x_2 = -5$$

$$-1 + 2r = -5$$

if

$$r = -2$$

then this set of vectors will be linearly dependant

1.7.57

Find the general solution:

$$\begin{pmatrix} -1 & 0 & 2 & -5 & 1 & -1 \\ 1 & 0 & -1 & 3 & -1 & 2 \\ 1 & 0 & 1 & -1 & 1 & 4 \end{pmatrix}$$

$$R_1 + R_2 \to R_2$$

$$R_1 + R_3 \to R_3$$

$$\begin{pmatrix} -1 & 0 & 2 & -5 & 1 & -1 \\ 0 & 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 3 & -4 & 2 & 3 \end{pmatrix}$$

$$-2R_2 + R_1 \to R_1$$

$$-3R_2 + R_3 \to R_3$$

$$\begin{pmatrix} -1 & 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 2 & 0 \end{pmatrix}$$

$$\frac{1}{2}R_3 \to R_3$$

$$\begin{pmatrix} -1 & 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$-x_1 - x_4 + x_5 - 3x_6 = 0$$

$$x_3 - 2x_4 + x_6 = 0$$

$$x_4 + x_5 = 0$$

$$x_1 = -x_4 + x_5 - 3x_6$$

$$x_2 = x_2$$

$$x_3 = 2x_4 - x_5$$

$$x_4 = -x_5$$

$$x_5 = x_5$$

$$x_6 = x_6$$

General Solution:

$$x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} 1 \\ 0 \\ -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + x_6 \begin{pmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

2.3.13

$$(AB^T)^-1$$

$$(B^T)^{-1}A^{-1}$$

$$(B^{-1})^T A^{-1}$$

$$(B^{-1})^T = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 0 & 4 \\ 3 & -2 & 1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$(B^{-1})^T A^{-1} = \begin{pmatrix} 3 & 7 & 9 \\ 4 & 4 & 4 \\ 3 & 7 & 8 \end{pmatrix}$$

2.3.19

Inverse of:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\frac{1}{4}R_2 \to R_2$$

Inverse:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2.3.22

Inverse of:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$-2R_4 + R_2 \to R_2$$

Inverse:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2.3.29

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & -3 & 1 \\ 0 & 4 & 5 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & -1 & 2 \\ 2 & -3 & 1 \\ -10 & 19 & 0 \end{pmatrix}$$

Find E where

$$EA = B$$

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{pmatrix}$$

2.3.31

This is Row interchange between R_2 and R_3, so E is:

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2.4.9

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 \\ 2 & 3 & 4 & 0 & 0 & 1 \end{array}\right)$$

$$-2R_1 + R_2 \to R_2$$

$$-2R_1 + R_3 \to R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 1 \end{array}\right)$$

$$-R_3 + R_2 \to R_2$$

$$-R_3 + R_1 \to R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 3 & 0 & -1 \\ 0 & 0 & -3 & 0 & 1 & -1 \\ 0 & 1 & 0 & -2 & 0 & 1 \end{array}\right)$$

$$R_2 \leftrightarrow R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 3 & 0 & -1 \\ 0 & 1 & 0 & -2 & 0 & 1 \\ 0 & 0 & -3 & 0 & 1 & -1 \end{array}\right)$$

$$\frac{-1}{3} R_3 \to R_3$$

$$-2R_2 + R_1 \to R_1$$

Inverse:

$$\frac{1}{3}\begin{pmatrix} -7 & -6 & 8 \\ 2 & 0 & 1 \\ 3 & 3 & -3 \end{pmatrix}$$

2.4.29.a

$$\begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 2 & 3 & -1 & -5 \end{pmatrix}$$

$$2R_1 + R_3 \to R_3$$

$$\begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 3 & -5 & -3 \end{pmatrix}$$

$$-3R_2 + R_3 \to R_3$$

$$\begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -8 & 0 \end{pmatrix}$$

$$\frac{-1}{8} R_3 \to R_3$$

$$\begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$-R_3 + R_2 \to R_2$$

$$-2R_3 + R_1 \to R_1$$

$$\begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$-R_1 \to R_1$$

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

2.4.29.b

If we do all the above row operations from 2.4.29.a to the I_3 matrix, we will get the following matrix for P

$$P = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -3 & 1 \end{pmatrix}$$