Discrete II Homework 8

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1. X = First dice, Z = sum of dice. X and Z are not indpendent events:

$$P(X = 4) = \frac{2}{6}$$
$$P(Z = 4) = \frac{5}{\binom{6}{2}}$$
$$P(X = 4, Z = 4) = 0$$

$$P(X = 4) \cdot P(Z = 4) \neq P(X = 4, Z = 4)$$

   Expected value for X is:

$$E(X) = \frac{1}{6} \cdot (2 * 1 + 2 + 3 + 4 * 2) = 2.5$$
$$E(Z) = 2 \cdot E(X) = 5$$

2. Probability distribution for X and Y:

	0	1
0	$\frac{1}{2} \cdot p$	$\frac{1}{2} \cdot (1-p)$
1	$\frac{1}{2} \cdot p$	$\frac{1}{2} \cdot (1-p)$

$$P(same) = \frac{1}{2} \cdot p + \frac{1}{2} \cdot (1-p) = \frac{1}{2}$$
$$P(different) = 1 - P(same) = \frac{1}{2}$$

   X and XOR will always be independent, Y will only be independent if $p = 0.5$.

3. Sample space for X is $\{4, 5, 6, 7\}$.

$$P(X = 4) = \frac{2}{2^4} = \frac{1}{8}$$
$$P(X = 5) = 2 \cdot \binom{4}{3} \cdot \frac{1}{2}^5 = \frac{1}{4}$$
$$P(X = 6) = 2 \cdot \binom{6}{3} \cdot \frac{1}{2}^6 = \frac{5}{16}$$
$$P(X = 7) = 2 \cdot \binom{6}{3} \cdot \frac{1}{2}^7 = \frac{5}{32}$$

$$E(X) = \frac{1}{8} \cdot 4 + \frac{1}{4} \cdot 5 + \frac{5}{16} \cdot 6 + \frac{5}{32} \cdot 7$$

4.