Discrete II Homework 4

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1a

100 choose 50, then adjust for overcounting

$$\frac{\binom{100}{50}}{2}$$

1b

$$\frac{\binom{100}{10}\binom{90}{10}\cdots\binom{10}{10}}{10!}$$

1c

$$100 \cdot \frac{\binom{99}{33}\binom{66}{33}\binom{33}{33}}{3!}$$

2

$$2 \cdot \frac{\binom{20}{2}\binom{18}{2}\cdots\binom{2}{2}}{10!}$$

3

$$\frac{\binom{10}{2}\binom{5}{1}\binom{8}{2}\binom{4}{1}\binom{6}{2}\binom{3}{1}\binom{4}{2}\binom{2}{1}\binom{2}{2}\binom{1}{1}}{5!}$$

4a

Let's pretend for a moment that the balls are identical. To our 10 balls, let's add 6 placeholders. The location of these placeholders will dictate into which bins the balls preceding the placeholder will go. What are all the possible locations of these placeholders?

$$\binom{16}{6}$$

Now, if the balls weren't identical, the combinations would be some multiple off this. Once the balls are distrubted, they can be rearanged 6! times. So:

$$\binom{16}{6}6!$$

4b

$$\binom{16}{6}$$

4c

4d

$$\frac{6}{\binom{16}{6}}$$

5

$$\binom{52+10-1}{10}$$

6a

$$\frac{\binom{13\cdot 3}{5}}{\binom{52}{5}} \cdot \frac{1}{4}$$

6b

$$\frac{\binom{13\cdot 3}{5}}{\binom{52}{5}} + \frac{\binom{13\cdot 2}{5}}{\binom{52}{5}} + \frac{\binom{13\cdot 1}{5}}{\binom{52}{5}}$$

7a

$$1 - \frac{\binom{20}{5}}{\binom{32}{5}}$$

7b

$$1 - (\frac{\binom{17}{5}}{\binom{32}{5}} + \frac{\binom{23}{5}}{\binom{32}{5}} + \frac{\binom{24}{5}}{\binom{32}{5}})$$