Group-Rubicx

ET2596 TASK-2

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Introduction:

In this report, we are going to discuss the results we have obtained from the 'software' given and answer the questions in TASK-2 of course ET2594.
Each of the questions of Task 2 is discussed in detail and according to the question needs few parameters are altered to note their effect while others are kept constant.

The reference simulation parameters are taken as:

Seed	E[Ta]	E[Ts]	Warm-up
2002	100	80	1

Arrival	Server	Length of batches	Number of arrivals per batch	Queue length
M	M	200	100	10

We initially considered the M/M/1 model with the length of batches = 200 and the number of arrivals per batch = 100, because comparatively, we got a lower Confidence-Interval (and it felt ideal as it's not too much nor too less).

Consequently, each question has its planning which is specified for each question in their respective sections, and for each question we change parameters and note their impact on other parameters.
For the first few questions instead of considering the impact on all the parameters, we have only considered the impact on Load as we feel Load gives all the crucial data necessary to understand the system at any instant, and the rest of the parameters can be understood by seeing the value of Load.

Experimental Results:

<u>Question-1</u>: Describe the impact of the number of batches on the estimation of the autocorrelation.

Autocorrelation tells us about the similarity of the data, and it is counterparts and can tell us about the randomness of the data.

In this, we have considered 4 different simulations for the number of batches keeping all the other parameters constant. Autocorrelation is done for finding the no. of batches:

System	Number of batches	Number of arrivals
M/M/1	200	100
M/M/1	1000	100
M/M/1	5000	100
M/M/1	10000	100

Table 1.1

We have calculated lag-1 autocorrelation for each of the batches

Number of batches	Lag-1 Autocorrelation
200	0.1655
1000	0.1056
5000	0.1056
10000	0.0581

Table 1.2

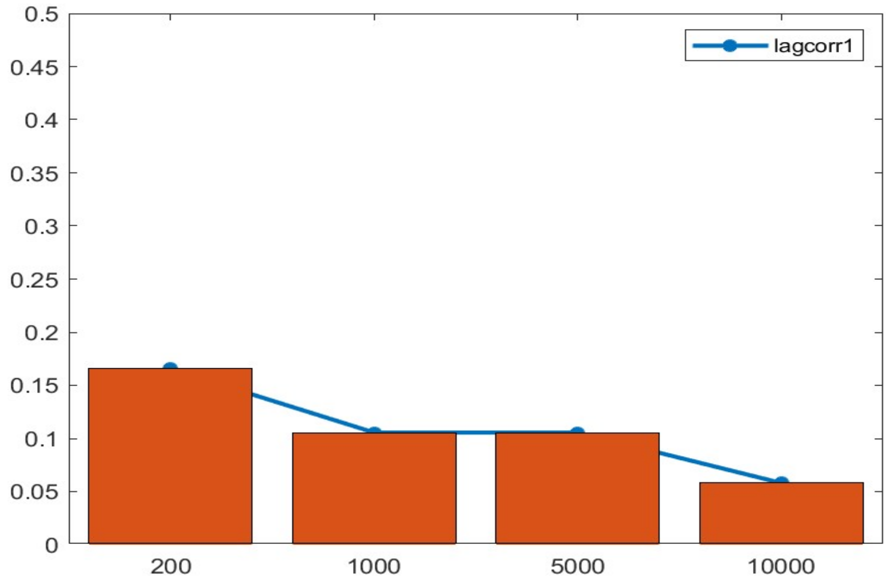


Figure 1: Autocorrelation of Load

The above Figure 1 shows the autocorrelation of load as we have considered load values to be a very important result as it talks about the condition or state of the system at that batch while other parameters like loss ratio and queuing ratio only focus on the number of people rejected in the queue or number of people in the queue. If we see the effect on load, we can know about the other parameters also.

Auto-Correlation (lag-1) values are very low showing very less similarity between the data set and its shifted self. As the number of batches increases, autocorrelation values decrease, showing more dissimilarity and randomness in the dataset. Since values are very small, one can say that varying the number of batches does not have any significant impact on autocorrelation.

<u>Question 2:</u> Describe the impact of the length of the batches on the estimation of the autocorrelation.

To determine the impact of the length of batches, load values are selected as a parameter to describe the effect of changes in length of batches for autocorrelation.

We have considered 7 different lengths of batches

System	Number of batches	Number of Arrivals
M/M/1	200	100
M/M/1	200	3000
M/M/1	200	5000
M/M/1	200	7000
M/M/1	200	10000
M/M/1	200	15000
M/M/1	200	20000

Table 2.1

The respective lag-1 autocorrelation of all the arrivals is:

Number of Arrivals	Lag-1 Autocorrelation
100	0.1654
3000	-0.0627
5000	0.0846
7000	0.1197
10000	0.1350
15000	0.097
20000	0.049

Table 2.2

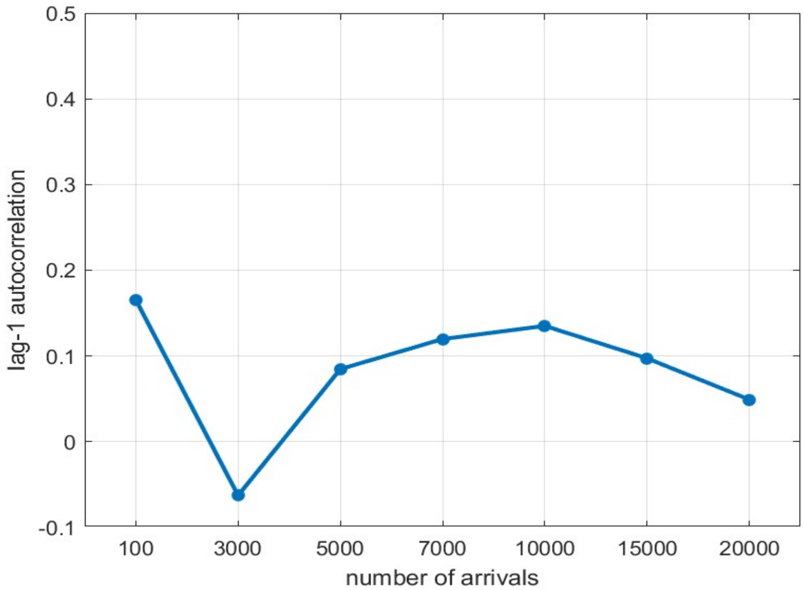


Figure 2

From Figure 2 we can conclude that the values of lag-1 autocorrelation are varying slightly and for the number of arrivals = 3000 we get a negative value but since then it tends to increase up to a certain point before decreasing again. But, these values are small and are not changing drastically. Hence, we can say that there is not much effect of the increase in the number of arrivals/length of batches on autocorrelation.

<u>Question-3:</u> Describe the impact of the number of batches on the size of the confidence intervals.

To give a better analysis we have simulated 7 different batches to know their impact on the size of the confidence interval (90% CI, 95% CI & 99% CI).

System	Number of batches	Number of arrivals
M/M/1	50	100
M/M/1	200	100
M/M/1	500	100
M/M/1	1000	100
M/M/1	2500	100
M/M/1	5000	100
M/M/1	10000	100

Table 3.1

Their responding confidence interval values are:

Number of batches	90% CI	95% CI	99% CI
50	0.049	0.059	0.078
200	0.024	0.028	0.0377
500	0.014	0.0166	0.0219
1000	0.0098	0.0117	0.0155
2500	0.0062	0.0073	0.0096
5000	0.0043	0.0051	0.0067
10000	0.0031	0.0036	0.0048

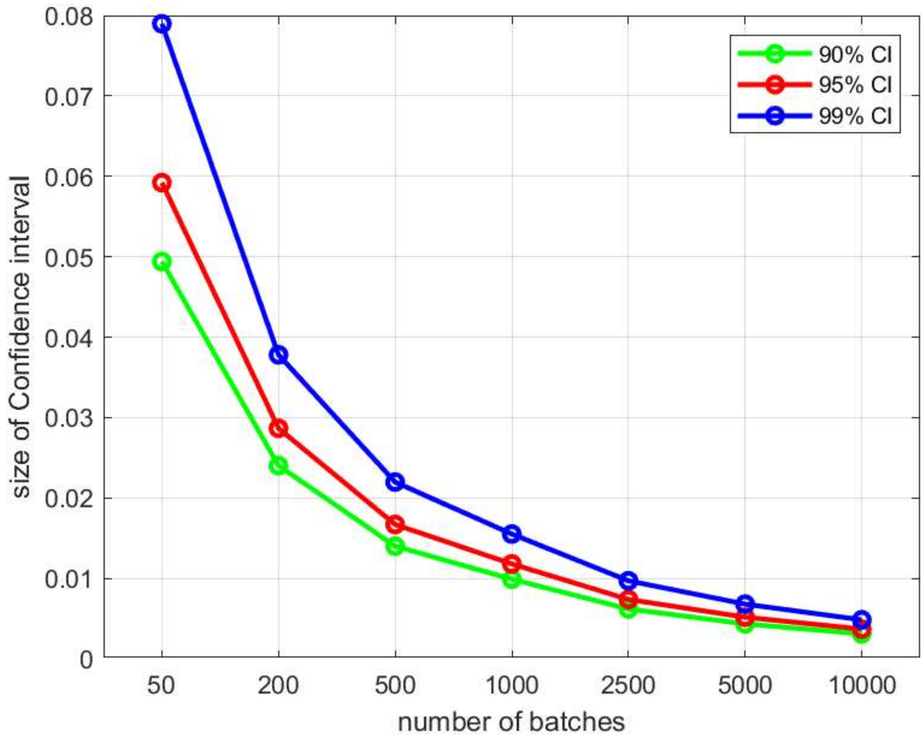


Figure 3

From above Figure 3 we can see confidence interval for all batches at three different CI sizes and it can be concluded that increase in the length of batches decreases the confidence interval for all values.

We already know that formula of CI is

$$CI = \bar{x} \pm z\frac{s}{\sqrt{n}}$$

Where 'n' represents the sample size and as the 'n' increases the value of interval will decrease which is exactly shown in the Figure 3.

Question 4: Describe the impact of the length of the batches on the size of the confidence intervals.

For a better understanding of the effect of batch length on the confidence interval of mean load values in each batch, we have simulated 6 different lengths of batches and have calculated for 3 different intervals those are: 90% CI, 95% CI, and 99% CI.

System	Number of batches	length of batches
M/M/1	200	100
M/M/1	200	500
M/M/1	200	1000
M/M/1	200	3000
M/M/1	200	10000
M/M/1	200	20000

Table 4.1

After simulation there corresponding Confidence interval values are:

length of batches	90% CI	95% CI	99% CI
100	0.024	0.028	0.037
500	0.011	0.013	0.017
1000	0.0074	0.0088	0.011
3000	0.0041	0.0050	0.0066
10000	0.0023	0.0027	0.0036
20000	0.0018	0.0021	0.0028

Table 4.2

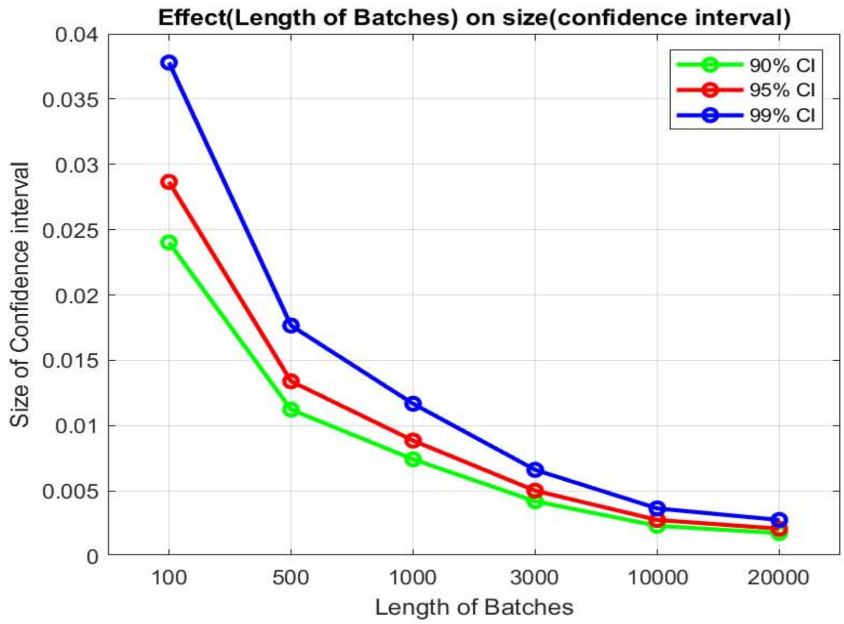


Figure 4

From above Figure 4, we can conclude that although the values of the confidence intervals are very small as the length increases there is a gradual decrease in the interval of the confidence interval.

This is validated by the property of confidence interval when the sample size is increased, i.e., when that happens, we say that standard error decreases and hence there is less variation in samples which results in decreasing the confidence interval.

<u>Question 5:</u> Discuss the impact of the average service time on load, loss ratio, and queuing ratio.

To know about the impact of load, loss ratio and queuing ratio we have varied the service times and the 4 service times considered are:

System	E[Ta]	E[Ts]
M/M/1	100	25
M/M/1	100	40
M/M/1	100	60
M/M/1	100	80

Table 5.1

Then later mean of all the values was calculated to know the average values at the corresponding time the results obtained are as follows:

E[Ts]	Load	Loss Ratio	Queueing Ratio
25	0.241	0	0.250
40	0.393	0	0.400
60	0.604	0.0012	0.608
80	0.758	0.01665	0.787

Table 5.2

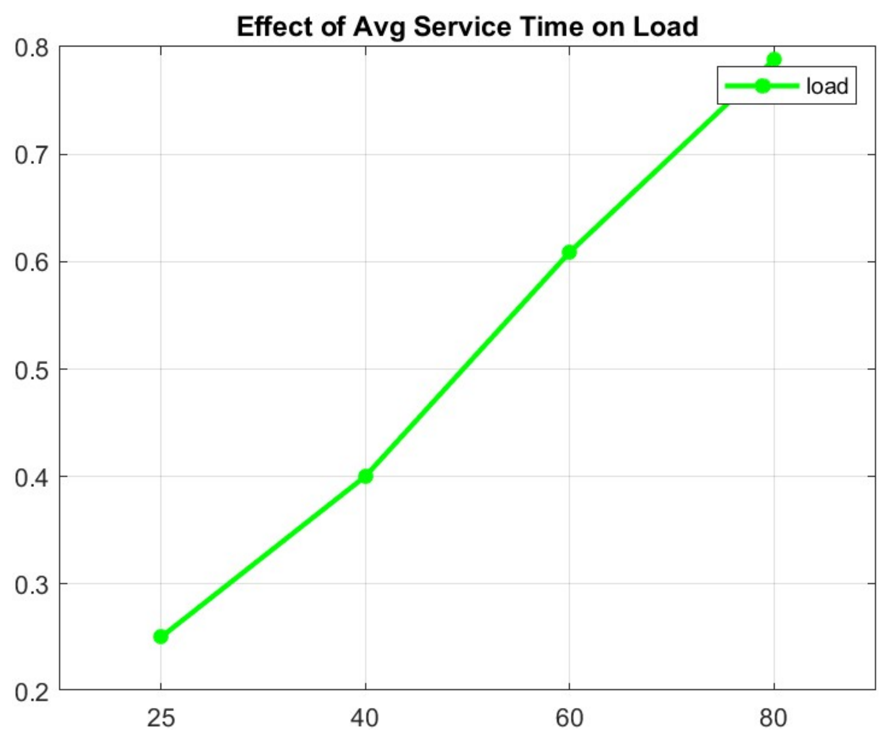


Figure 5.1

Figure 5.2

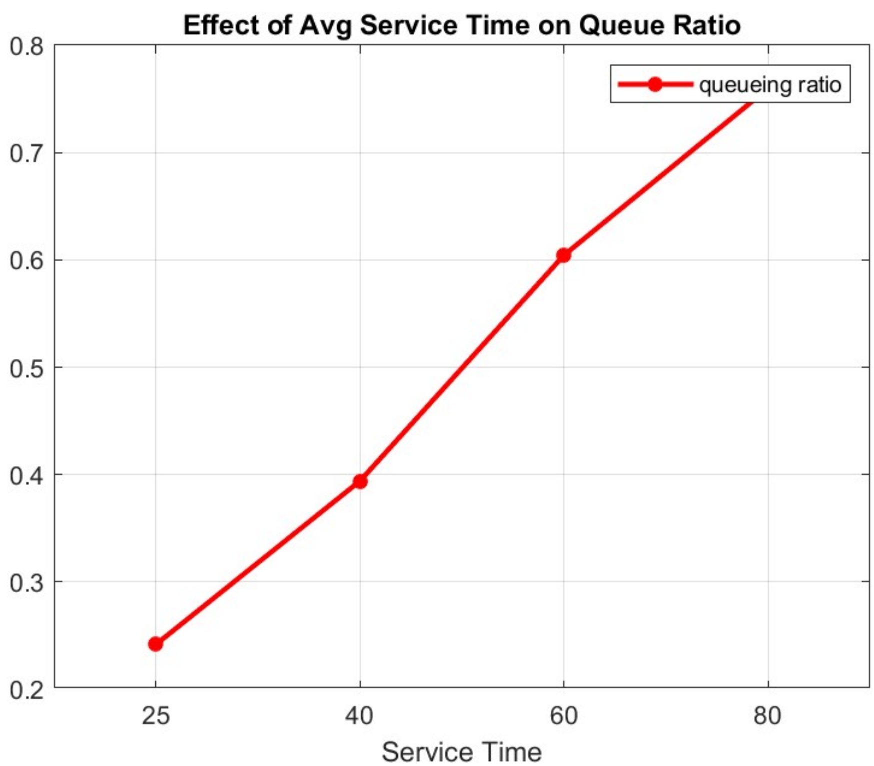


Figure 5.3

From Figure 5.1, Figure 5.2, and Figure 5.3 we can say that as the service time increases all the parameters increase gradually i. when the service time increases time spent by the person getting service increases as well as time spent in the queue also increases. When the service time is low there are more chances of the queue becoming empty early such that the incoming person is not rejected since the queue is full but and hence loss ratio is very less almost nil when the service time is less but when the service is time is more people can be rejected only because the queue was full. This can be seen from the simulation shown in class[1].

<u>Question 6:</u> Discuss the impact of the average service time on the size of the confidence intervals.

We know that the relationship between service time and load is that they are directly proportional and hence for better understanding we have only used load as a reference.

We have simulated for 4 different service times:

System	E[Ta]	E[Ts]
M/M/1	100	25
M/M/1	100	40
M/M/1	100	60
M/M/1	100	80

Table 6.1

The obtained confidence intervals of service time are:

E[Ts]	90% CI	95% CI	99% CI
25	0.0086	0.010	0.0136
40	0.0133	0.015	0.0209
60	0.0197	0.023	0.031
80	0.02401	0.0286	0.0378

Table 6.2

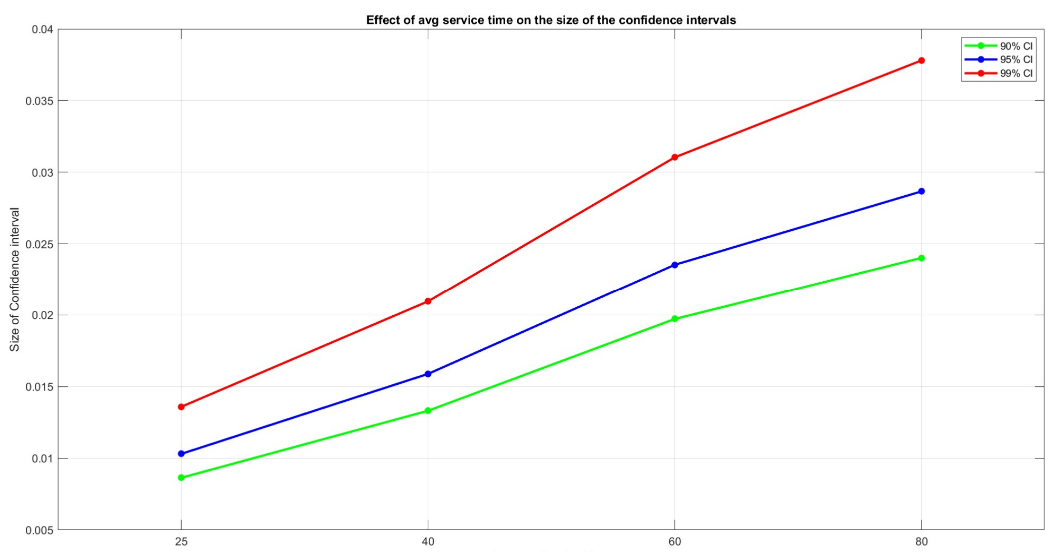


Figure 6

From above Figure 6, we can say that as the service time increases then the load increases, and hence the confidence interval size of the load also increases. We can conclude that for less service time we will get more precision. This shows that the accuracy of the dataset tends to deviate more with more variance. With increase in size of confidence interval of load, the randomness in data increases.

<u>Question 7:</u> Comment on the impact of the size of the loss ratio on the absolute sizes of the confidence intervals.

As we know that loss ratio is the number of customers rejected when they are trying to enter the system. The only reason people will be rejected is when the queue size is filled such that there is no space for the next arriving customer.

We have simulated 4 different service times to know their impact on the size of the confidence interval:

System	**K(Queue)**	**E[Ta]**	**E[Ts]**
M/M/1	10	100	25
M/M/1	10	100	40
M/M/1	10	100	60
M/M/1	10	100	80

Table7.1

The results obtained are for 3 different absolute confidence intervals:

E[Ts]	**90%**	**95%**	**99%**
25	[-0.0043 0.0043]	[-0.005 0.005]	[-0.0067 0.0067]
40	[-0.0066 0.0066]	[-0.0079 0.0079]	[-0.010 0.010]
60	[-0.0098 0.0098]	[-0.011 0.011]	[-0.0155 0.0155]
80	[-0.012 0.012]	[-0.014 0.014]	[-0.0188 0.0188]

Table 7.2

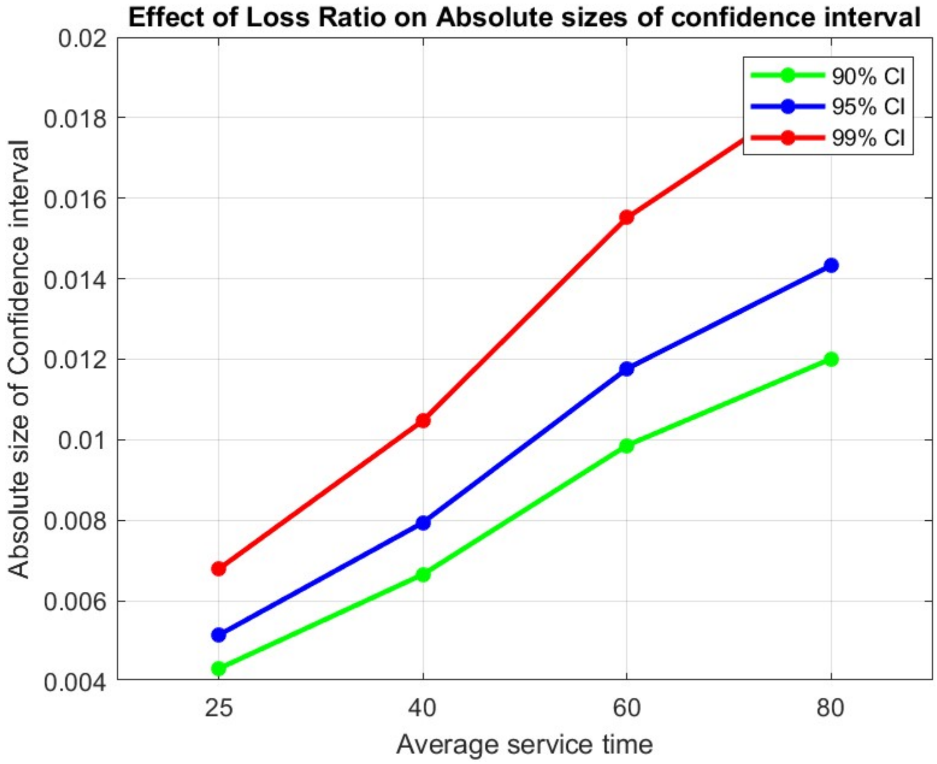


Figure 7

From above Figure 7, we can say that as the service time increases the absolute confidence interval increases indicating that there are more chances of loss since, the customers coming will have to spend more time in the queue and few of the customers can even be rejected if the queue is full, which can be seen from the simulation shown in class [1].

<u>Question 8:</u> Comment on the impact of the size of the load or queuing ratio on the absolute sizes of the confidence intervals.

To understand the impact of absolute sizes of the confidence interval we can formulate two different simulations i.e., one is to check its impact on the loss ratio and the second one is to check its impact on queuing ratio.

We have changed the service time to know the impact of the following two parameters, 4 different service times are considered as follows:

System	**K(Queue)**	**E[Ta]**	**E[Ts]**
M/M/1	10	100	25
M/M/1	10	100	40
M/M/1	10	100	60
M/M/1	10	100	80

Table 8.1

The following results have been obtained upon the simulation:

1. Impact of the absolute confidence interval on load by varying service time:

Service-time	**90% CI**	**95% Ci**	**99%CI**
25	[-0.0043 0.0043]	[-0.0051 0.0051]	[-0.0067 0.0067]
40	[-0.0066 0.0066]	[-0.0079 0.0079]	[-0.0104 0.0104]
60	[-0.0098 0.0098]	[-0.0117 0.0117]	[-0.0155 0.0155]
80	[-0.0120 0.0120]	[-0.0143 0.0143]	[-0.0188 0.0188]

Table 8.2

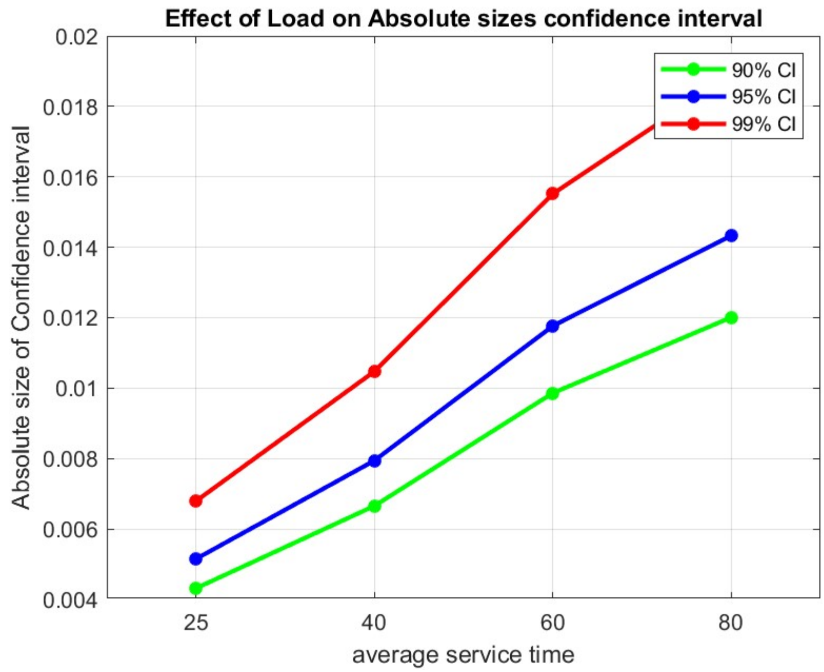


Figure 8.1

Figure 8.1 tells us about how the load is affected by the increase in service time and as that increases the absolute confidence interval also increases.

2. Impact of the absolute confidence interval on queueing ratio by varying service time:

Service-time	**90% CI**	**95% Ci**	**99%CI**
25	[-0.0063 0.0063]	[-0.0075 0.0075]	[-0.0099 0.0099]
40	[-0.0081 0.0081]	[-0.0097 0.0097]	[-0.0128 0.0128]
60	[-0.0105 0.0105]	[-0.0125 0.0125]	[-0.0165 0.0165]
80	[-0.0115 0.0115]	[-0.0137 0.0137]	[-0.0181 0.0181]

Table 8.3

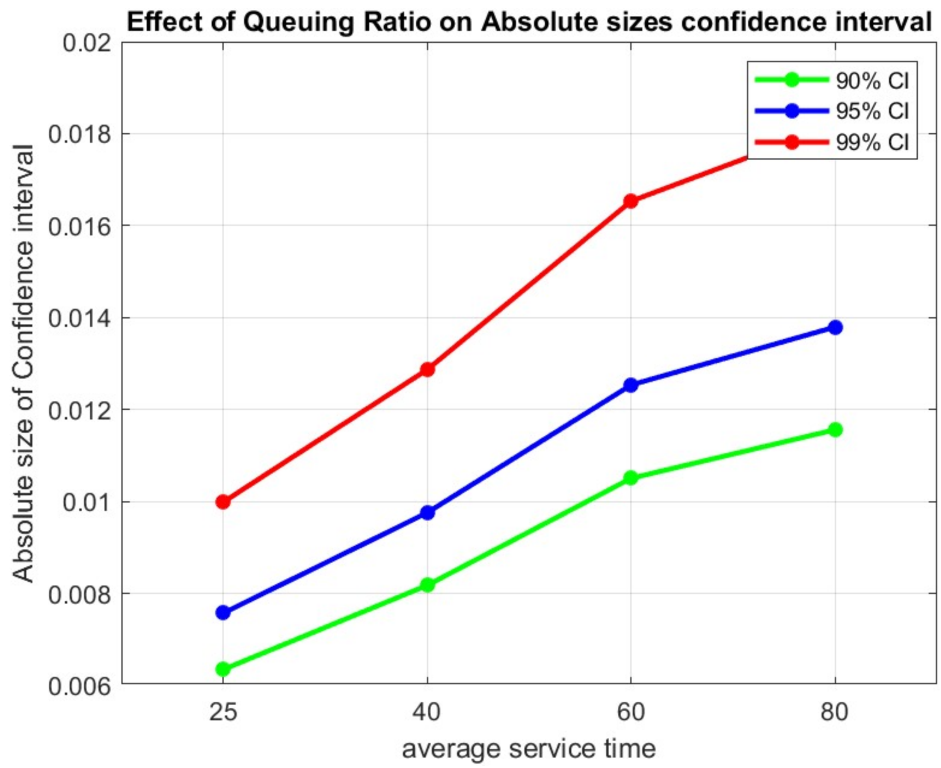


Figure 8.2

A trend similar to figure 8.1 is seen in figure 8.2 i.e., there is an increase in the size of the absolute confidence interval as the service time increases i.e., we can say that there is a delay for people in the queue when service time is more.

<u>Question-9:</u> Comment on the impact of the size of the loss ratio on the relative sizes of the confidence intervals.

Similar simulations were made to know the effect of the loss ratio on relative sizes of the confidence interval i.e., we changed the service time and tried to find its impact.

To calculate the relative half size of the confidence interval the formula is:

$$\frac{CI_{size}}{\mu} \text{ x } 100$$

We have changed the service time to know the impact of falling two parameters, 4 different service times are considered as follows:

System	K(Queue)	E[Ta]	E[Ts]
M/M/1	10	100	25
M/M/1	10	100	40
M/M/1	10	100	60
M/M/1	10	100	80

The results obtained by changing above parameters are:

Service-time	90% CI	95% CI	99%CI
25	0	0	0
40	0	0	0
60	1.024	1.223	1.613
80	0.4656	0.5557	0.7329

Table 9.1

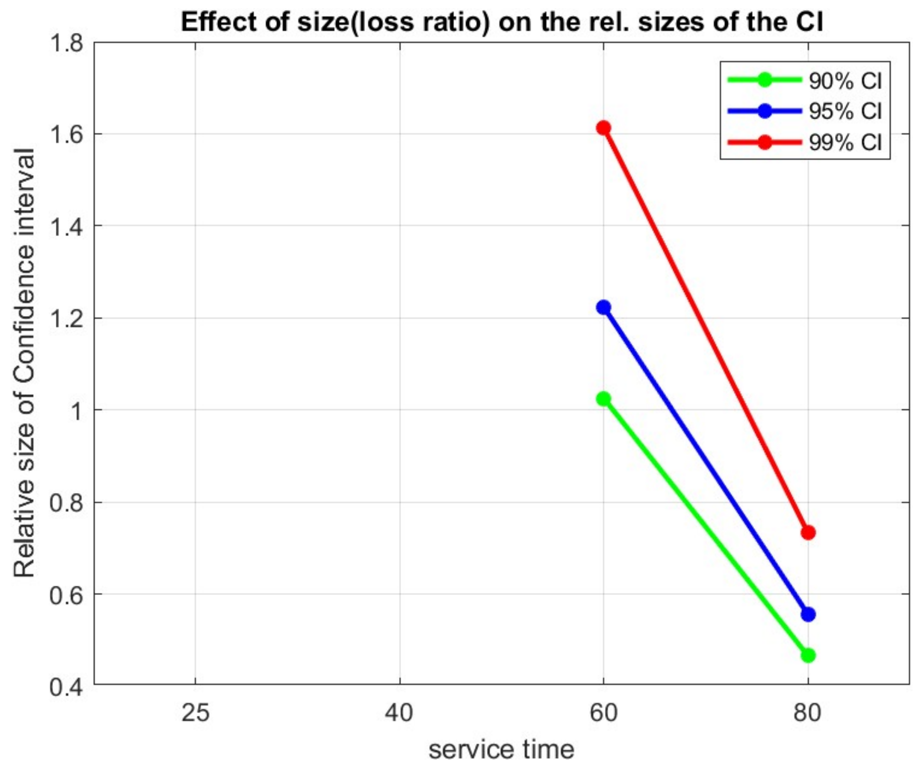


Figure 9

The above figure tells us about the relative size of CI and comparing results from Q7 we can say that when service time increases the absolute size of the confidence interval increases while the relative size of the confidence interval decreases and this is validated by the formula since we are dividing the size of CI by mean. If service is more the loss ratio is also more due to more waiting time, and when divided by its mean which is a higher value we get a comparatively lower value for a relative confidence interval.

<u>Question-10:</u> Comment on the impact of the size of the load or queuing ratio on the relative sizes of the confidence intervals.

Like Q8 we have carried out two different simulations by varying service time and noting their impact on each load and queuing ratio.

1. Impact of the relative half-size confidence interval on load by varying service time:

Service-time	90% CI	95% Ci	99%CI
25	[-0.0172 0.0172]	[-0.0205 0.0205]	[-0.0271 0.0271]
40	[-0.0166 0.0166]	[-0.0198 0.0198]	[-0.0261 0.0261]
60	[-0.0161 0.0161]	[-0.0193 0.0193]	[-0.0254 0.0254]
80	[-0.0152 0.0152]	[-0.0181 0.0181]	[-0.0239 0.0239]

Table 10.1

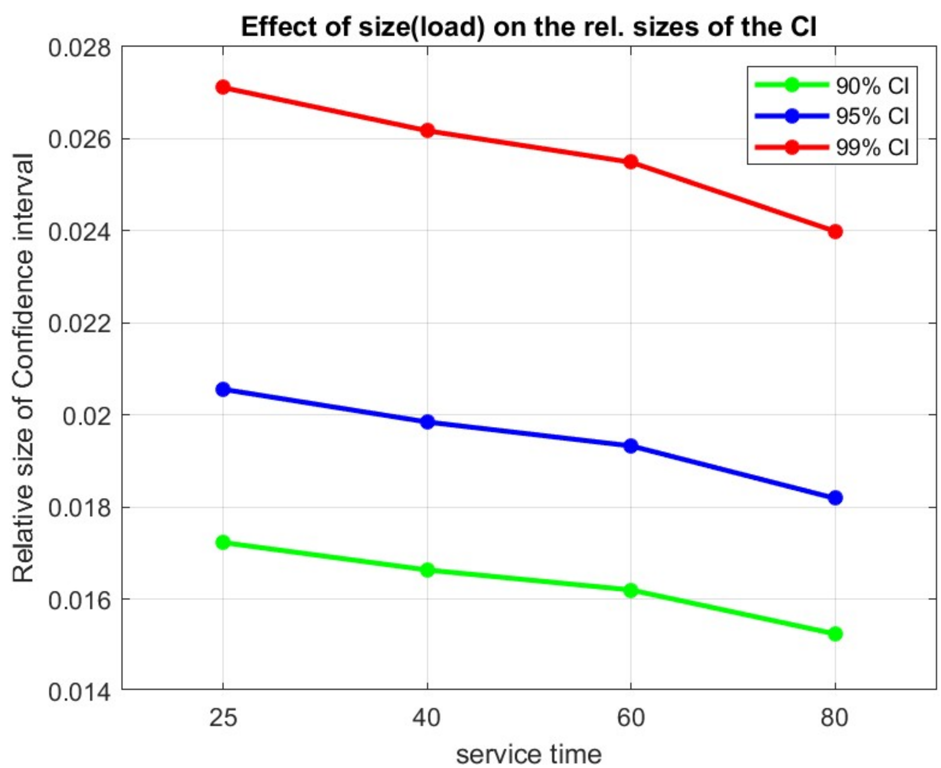


Figure 10.1

The results obtained are exactly opposite to the one obtained in Q8 and we can say that as the service time increases the relative half-size decreases.

2. Impact of the absolute confidence interval on queueing ratio by varying service time:

Service-time	90% CI	95% Ci	99%CI
25	[-0.026 0.026]	[-0.031 0.031]	[-0.0413 0.0413]
40	[-0.0207 0.0207]	[-0.024 0.024]	[-0.032 0.032]
60	[-0.0173 0.0173]	[-0.0207 0.0207]	[-0.027 0.027]
80	[-0.01 0.0115]	[-0.0181 0.0181]	[-0.023 0.023]

Table 10.2

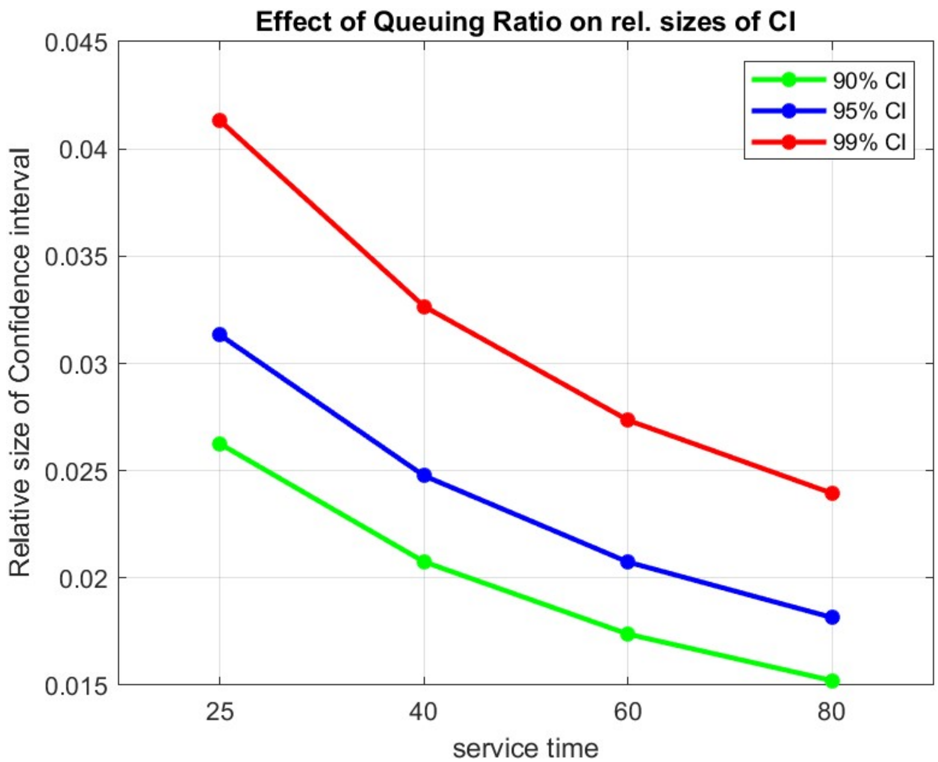


Figure 10.2

The results obtained are exactly opposite to the one obtained in Q8 and we can say that as the service time increases the relative half-size of the Confidence interval decreases.

Question-11: Validate the $f2$ rule-of-thumb.

"f^2 rule-of-thumb" states that reduction of the Confidence interval by factor 'f' means f^2 the effort.

To validate the above statement we have calculated the Confidence interval size for reference simulation parameters. We initially find out the confidence interval size for the number of batches = 200 and found it to be 0.0287 (95% CI) and wished to decrease it by a factor of 3 (f=3) and hence to see the effect on the confidence interval we have multiplied the number of batches by 9 (f^2), so the new number of batches is 200*9 = 1800.

The results obtained are:

Number of batches	90% CI	95% CI	99% CI
200	[0.0240]	[0.0286]	[0.0377]
1800	[0.0072]	[0.0086]	[0.0114]

Table 11.1

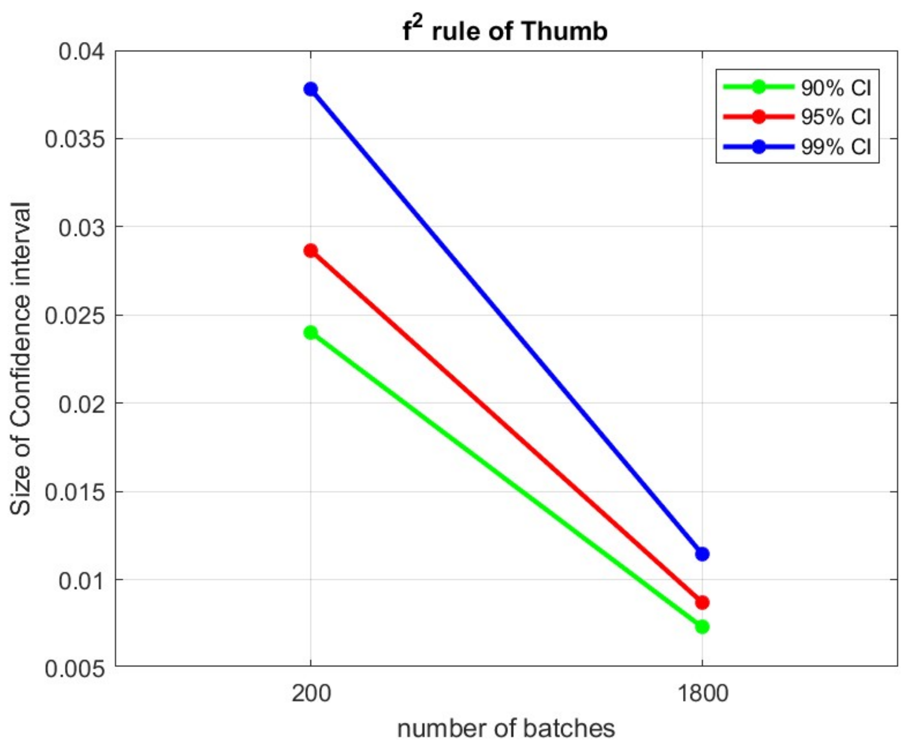


Figure 11

From the above table and Figure 11, we can validate the use of "f^2 rule-of-thumb".

Question-12: **Compare loss ratio, queuing ratio, and load in the following cases:**

a)- K = 0; b)- K = 2; c)- K = 10.

In this, we will change the queue length and observe their effect on the Confidence interval. K=0 indicates that there is no queue and hence the person who comes will be directly rejected if there is any ongoing service we assume that the loss ratio will be relatively high for the K=0 case compared to rest two cases.

As for the load and queuing ratio we assume will increase as the K increases as there will be the effect of customers waiting in the queue on the delay of customers in queue and load on the system.

For load:

Queue-length	95% CI	Lag-1 autocorrelation
0	0.0133	-0.0868
2	0.019	-0.027
10	0.028	0.165

Table 12.1

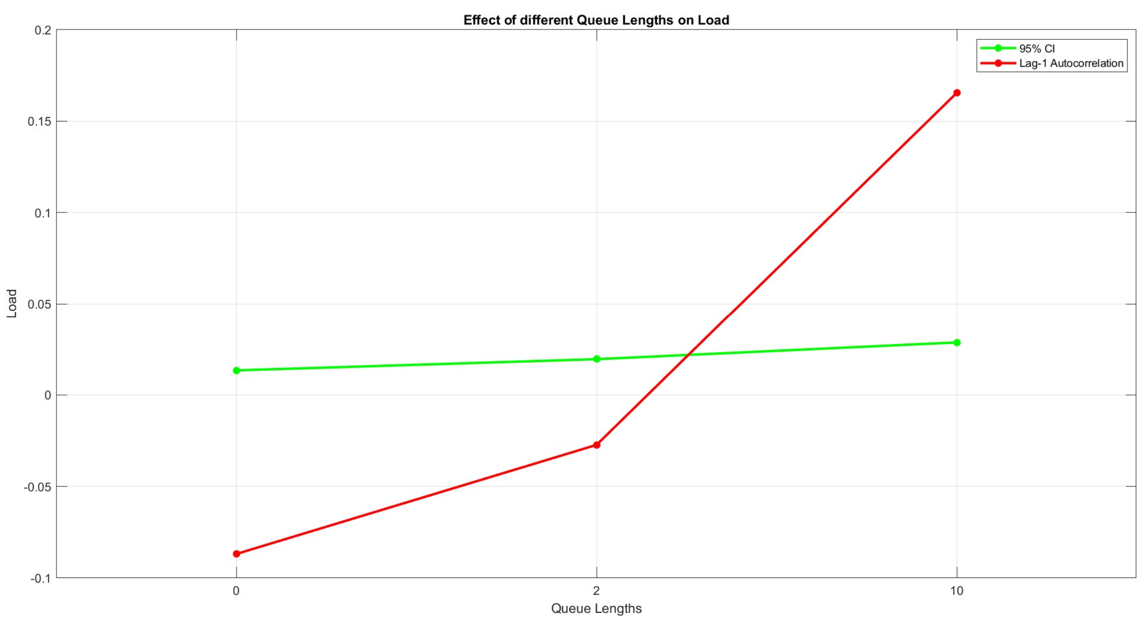


Figure 12.1

For loss-ratio:

Queue-length	95% CI	Lag-1 autocorrelation
0	0.0134	0.0088
2	0.0163	0.1176
10	0.0093	0.1679

Table 12.2

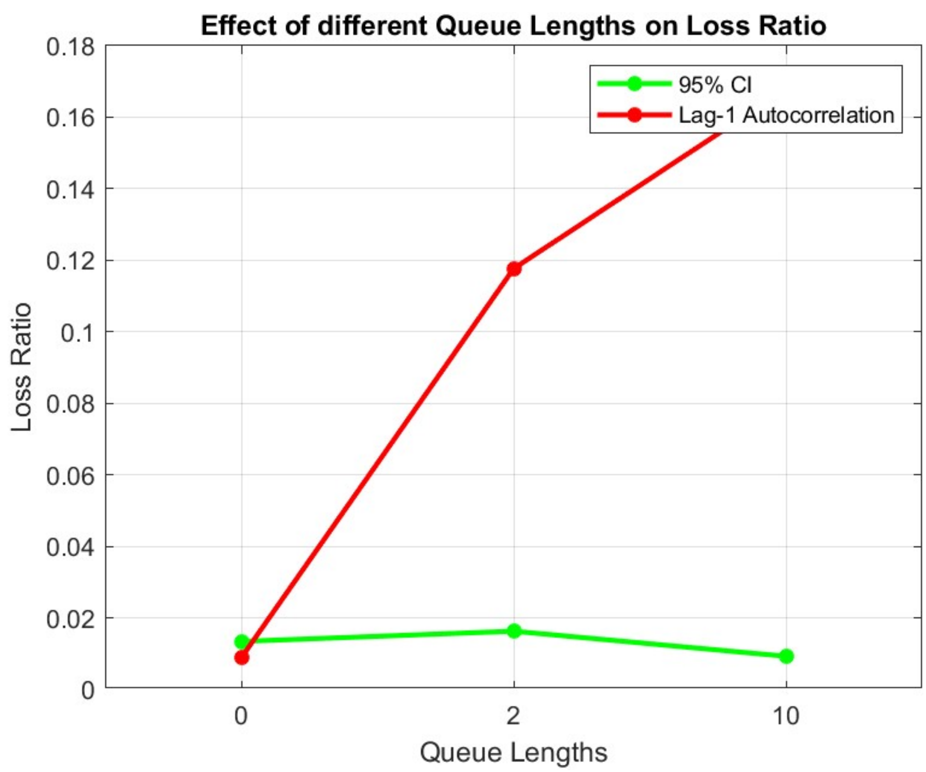


Figure 12.2

For queuing-ratio:

Queue-length	95% CI	Lag-1 autocorrelation
0	0.0	0
2	0.0139	-0.0744
10	0.0276	0.0901

Table 12.3

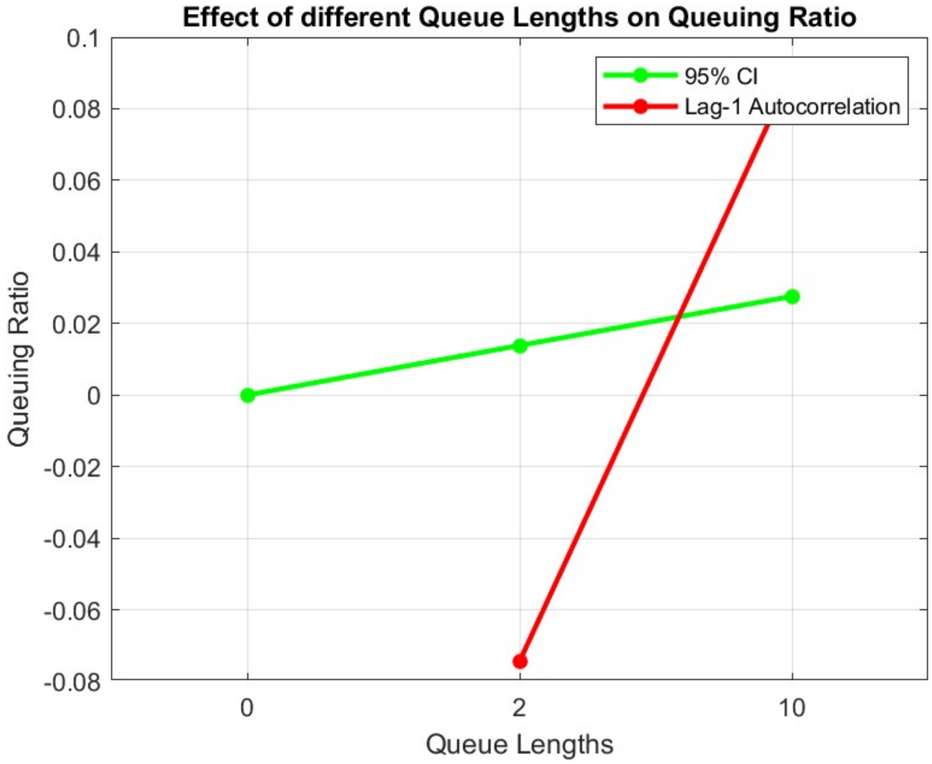


Figure 12.3

From the given diagrams, for load, as the queue length increases confidence interval tends to be smaller, showing higher accuracy in finding the dataset. In the case of the loss ratio, it shows similar results with lower rejection from queues with an increase in queue lengths. Autocorrelation Lag 1 values for queuing ratio tends to be very small though showing gradual increase showing increasing similarity in the data sets with increase of queue lengths.

<u>Question-13:</u> Discuss the impact of the type of the source process (D, M, 1) and (U, M,1) on loss.

A deterministic system is a system where an initial state completely determines the system's future states. Thus, there is no randomness in producing the future states.

We compare D/M/1, U/M/1 based on their 95% Confidence interval and autocorrelation.

System	95% CI	Lag-1 autocorrelation
D/M/1	0.0019	0.225
U/M/1	0.0046	-0.0731

Table 13.1

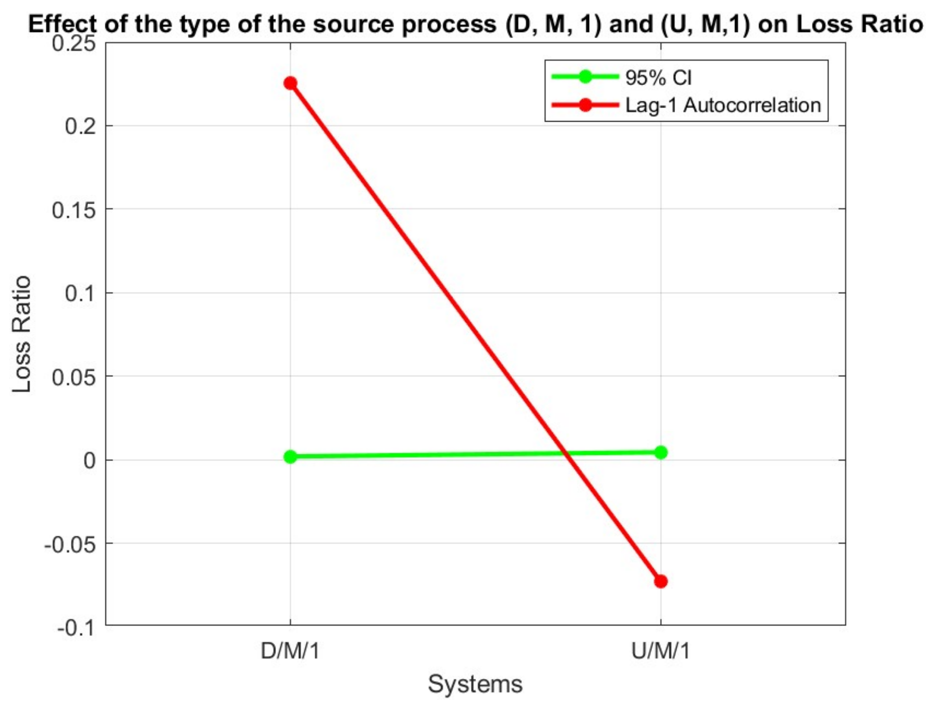


Figure 13

From the above figure we can conclude that the loss ratio in D/M/1 system has significantly high value of correlation compared to other systems while the others M/M/1 have less and for U/M/1 value is negative. When considering the confidence interval we can say that for D/M/1 system there is very little almost nil change and hence its CI is very low compared to rest two cases.

<u>Question-14:</u> Discuss the impact of the type of the source process (D, M, 1) and (U, M,1) on queuing ratio.

As we know A deterministic system is a system where an initial state completely determines the system's future states. Thus, there is no randomness in producing the future states.

We have compare D/M/1, U/M/1 based on their 95% Confidence interval and autocorrelation.

System	95% CI	Lag-1 autocorrelation
D/M/1	0.0358	0.1650
U/M/1	0.0312	0.0684

Table 14.1

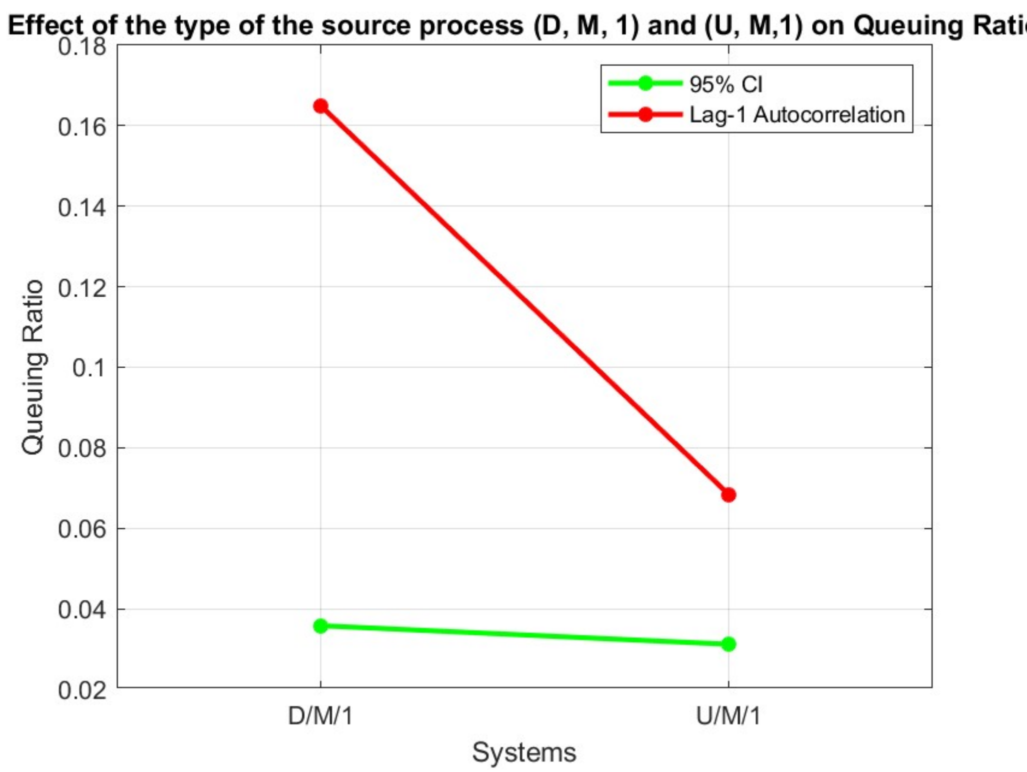


Figure 14

From the above figure we can get to know that there is slight delay in queuing of D/M/1 system and hence its confidence interval is slightly higher than the U/M/1 system while lag-1 autocorrelation is higher in D/M/1 as there is no much randomness in the data generated by particular system.

<u>Question-15:</u> Discuss the impact of the type of the server process (M, D, 1) and (M, U,1) on loss.

In this question we are asked to change server process. For both the systems Confidence interval and lag-1 autocorrelation are calculated in order to give conclusion.

System	95% CI	Lag-1 autocorrelation
M/D/1	0.0030	0.0135
M/D/1	0.0050	0.1133

Table 15.1

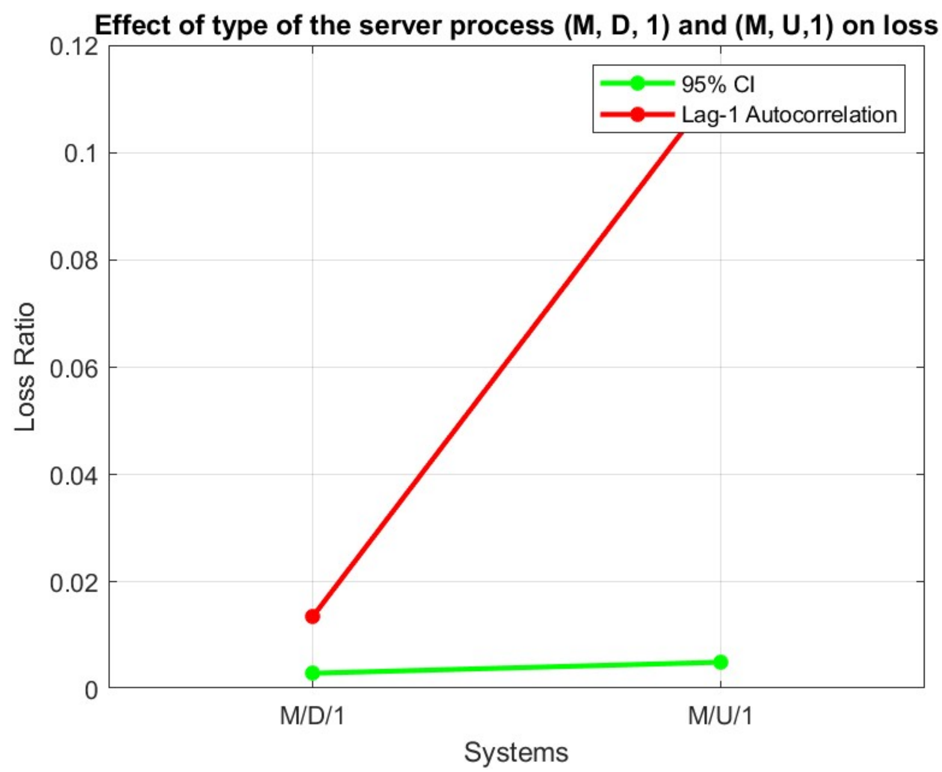


Figure 15

We can see that the results look similar to that of Q13 i.e., there is no much confidence interval while autocorrelation values are reversed in a manner. Indicating that there is more correlation when the server process is uniform than deterministic.

<u>Question 16</u>: Discuss the impact of the type of the server process (M, D, 1) and (M, U,1) on queuing ratio.

For both the systems Confidence interval and lag-1 autocorrelation are calculated for generated values of load.

System	95% CI	Lag-1 autocorrelation
M/D/1	0.0225	0.0259
M/U/1	0.0257	0.1012

Table 16.1

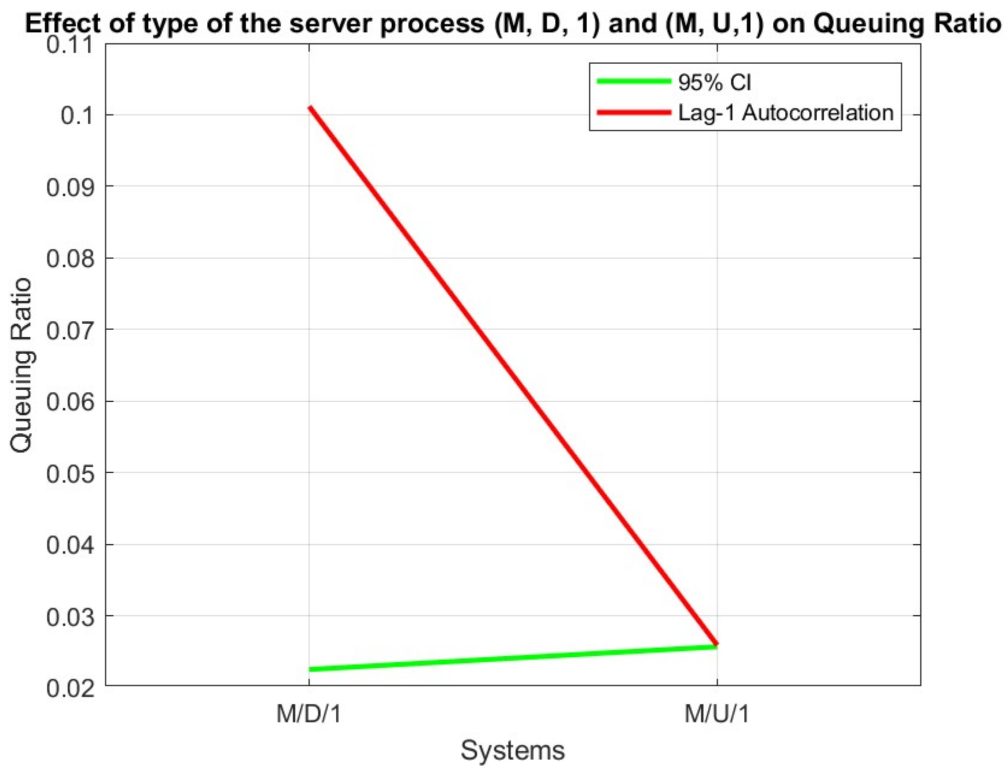


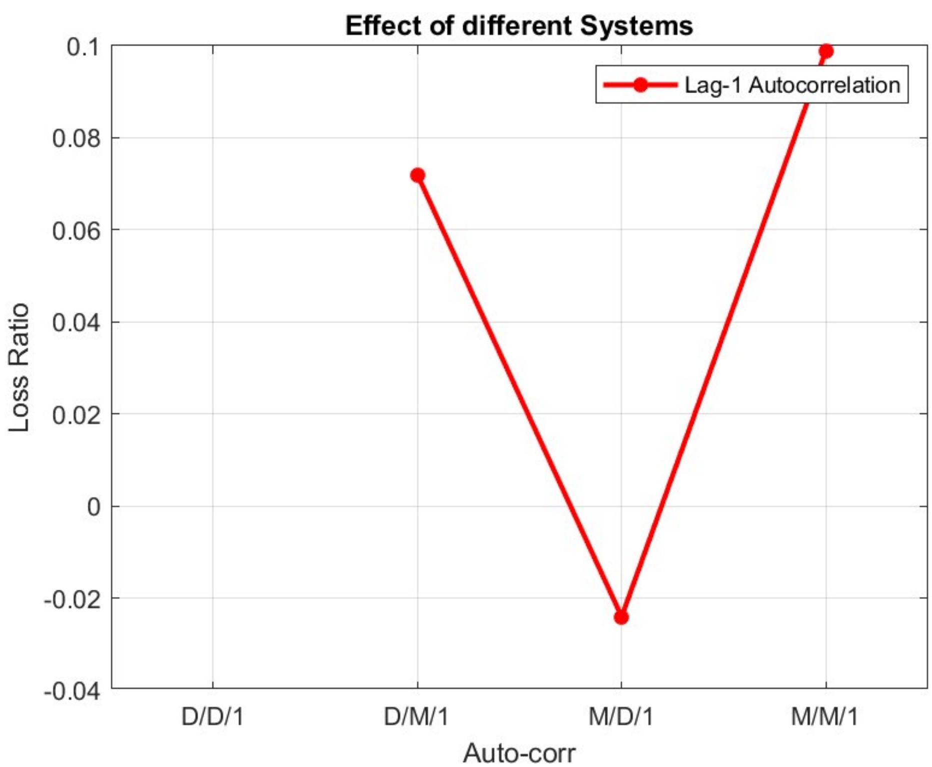
Figure 16

The above results are similar to that of one in Q14, but the values are a bit lower. So, we can say that not much of a difference is caused when either the server or system is changed.

<u>Question-17:</u> Compare the loss ratios for the M/M/K''=10 system with those obtained for the M/D/K'' and D/M/K'' systems at 80 % load, and motivate your observations. (Why is the D/D/K'' case trivial?)

The simulation is first done for single-server models and then results are used to estimate for multi-server systems.
The length of queue has chosen to be 5, to find the variation in loss ratio. We use Lag-1 Autocorrelation values to show the differences among different systems.

System	Lag-1 Autocorrelation
D/D/1	0
D/M/1	0.0719
M/D/1	-0.240
M/M/1	0.989

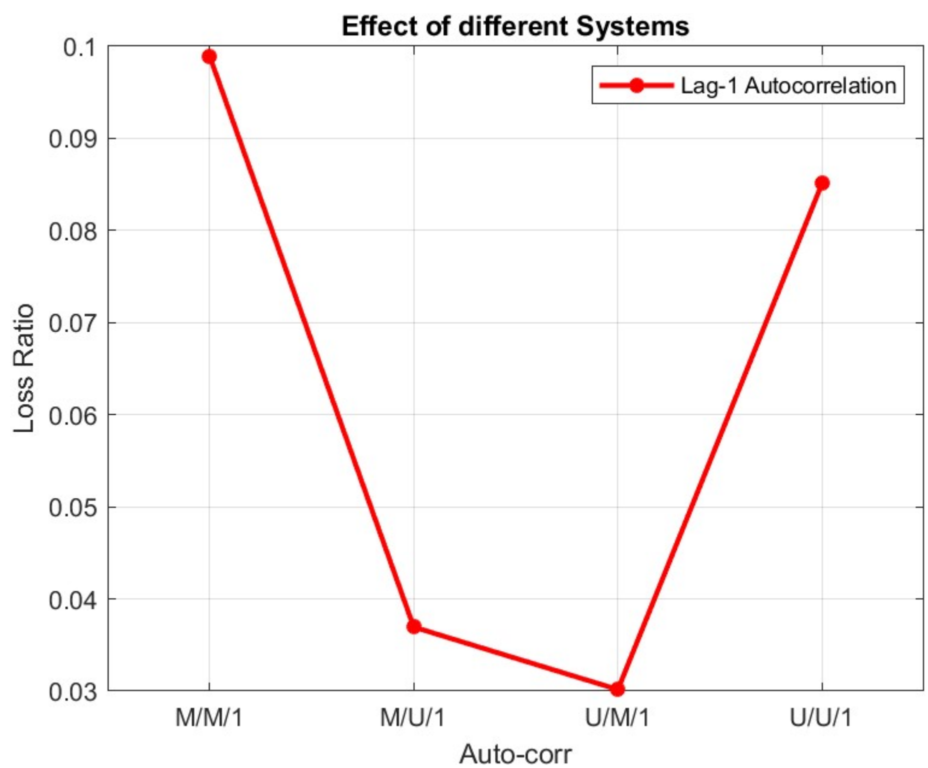


From the figure, D/D/K system doesn't tend to have any loss or rejection. Its values remain undefined as the arrival and service times are deterministic in nature. This case is trivial because the deterministic arrival times and departure times means that they are spread out entirely evenly, and while the load is less than 100%, that means each previous arrival has always had time to depart before the next arrival. This kills the randomness of the datasets. D/M/K and M/M/K tend to show similarities in values of autocorrelation as both are different only in arrival process. The negative values in M/D/K system shows opposite nature in the dataset and its Lag-1 values. M/D/K system shows more randomness.

<u>Question 18:</u> Compare the loss ratios for the M/M/K'' system with those obtained for the M/U/K'', U/M/K'' and U/U/K'' systems at 80 % load, and motivate your observations.

The simulation is first done for single-server models and then results are used to estimate for multi-server systems.
The length of queue has chosen to be 5, to find the variation in loss ratio. We use Lag-1 Autocorrelation values to show the differences among different systems.

System	Lag-1 Autocorrelation
M/M/1	0.0989
M/U/1	0.0369
U/M/1	0.0302
U/U/1	0.0851

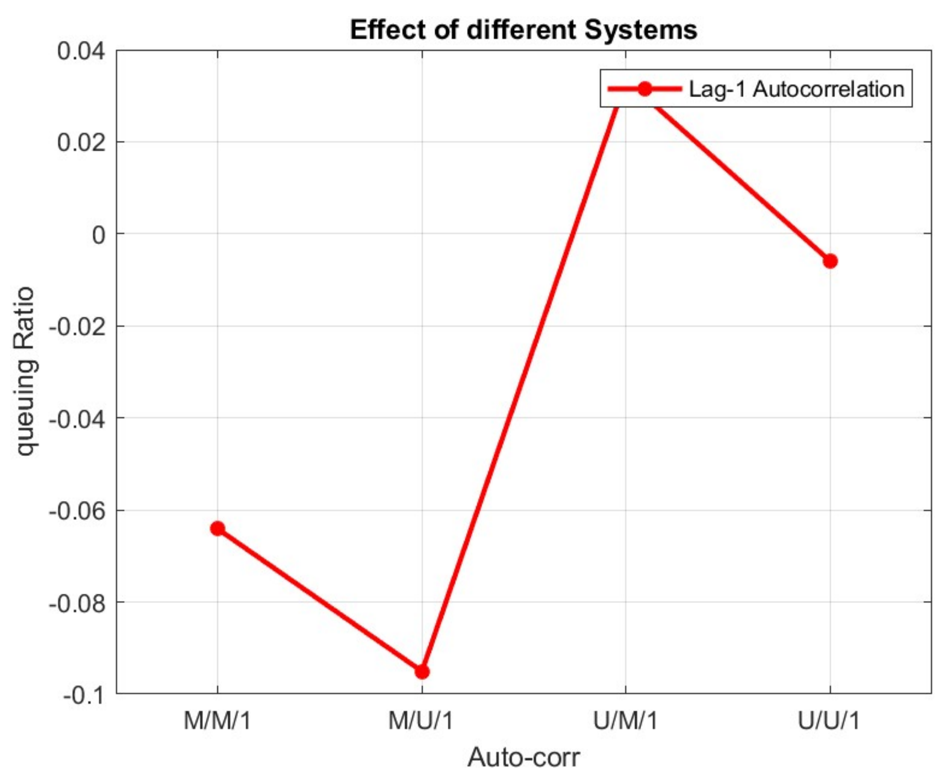


From the figure, M/U/k and U/M/k comparison is done on the basis of comparison of single server systems run for more batches to estimate for the required systems. U/U/k system show most similar values of autocorrelation in comparison to M/U/k and U/M/k sytems which are having most randomness.

<u>Question 19:</u> Compare the queuing ratios for the M/M/K''=10 system with those obtained for the M/U/K'', U/M/K'' and U/U/K'' systems at 80 % load, and motivate your observations.

The simulation is first done for single-server models and then results are used to estimate for multi-server systems.
The length of queue has chosen to be 5, to find the variation in queuing ratio.
We use Lag-1 Autocorrelation values to show the differences among different systems.

System	Lag-1 Autocorrelation
M/M/1	-0.0641
M/U/1	-0.0951
U/M/1	0.0335
U/U/1	-0.0059s



From the figure, M/U/k and U/M/k show most diverse values in autocorrelation showing dissimilarities in queuing ratio. U/U/1 tend to have the most randomness in queueing values at 80% load among the other 3 systems. We should note that with change in buffer size, the system model effects on queuing ratio may change, since each system behaves differently to the queuing ratios.