Homework 1

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Section 7

1.2 # 37

$$v = \begin{bmatrix} 3 \\ 8 \end{bmatrix}, \quad S = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$$

$$3 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 8 \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix} + 0 \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$$

$\begin{bmatrix} 0 \\ \end{bmatrix} \quad \begin{bmatrix} -1 \\ \end{bmatrix} \quad \begin{bmatrix} 0 \\ \end{bmatrix} \quad \begin{bmatrix} 8 \\ \end{bmatrix}$

1.3 #51

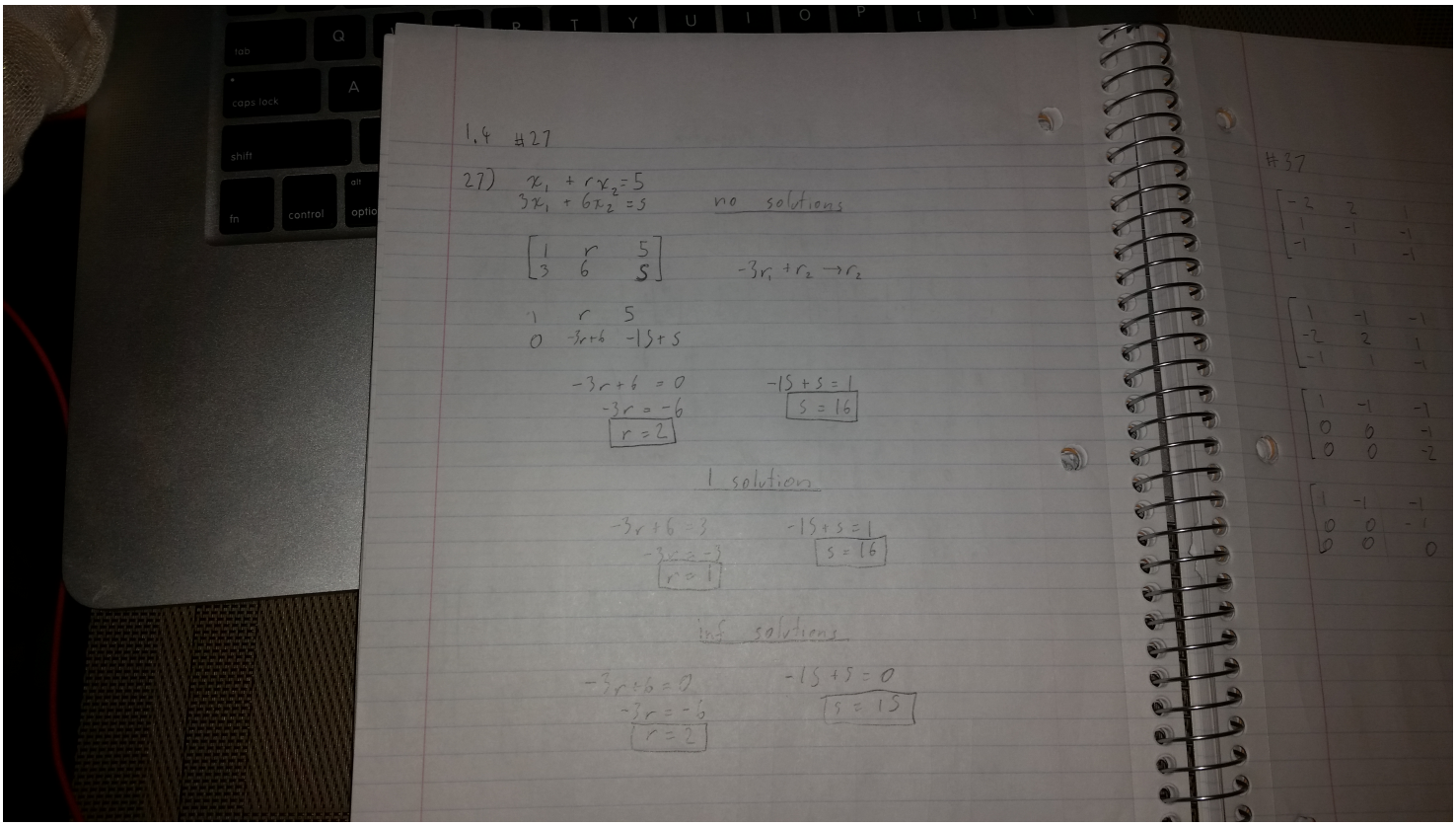
$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & -2 & 6 \\ 0 & 0 & 1 & 4 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

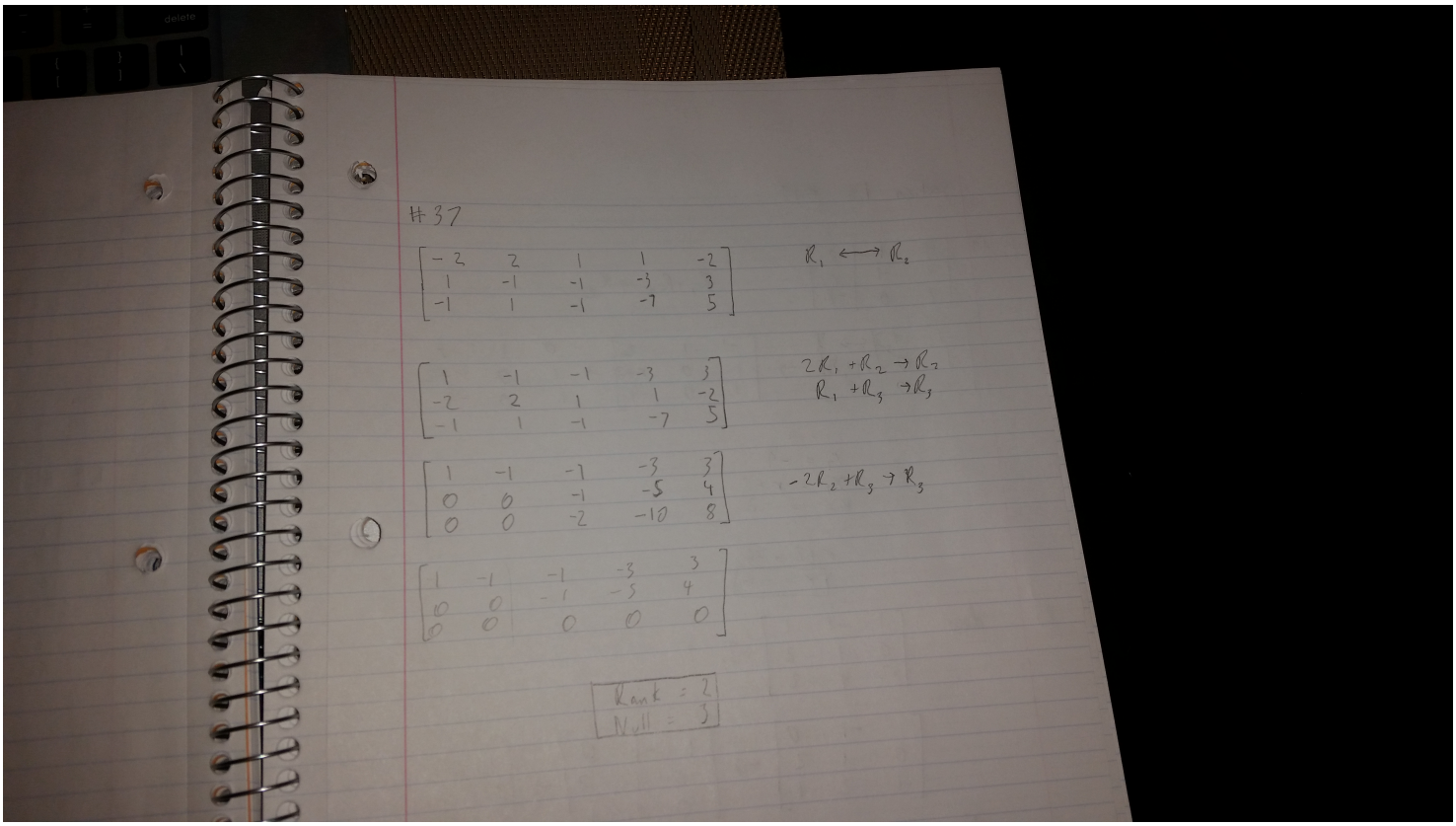
it is consistent

$x_1 + 3x_2 - 2x_4 = 6$
$x_3 + 4x_4 = 7$

so, $x_4 = x_4$
$x_3 = 7 - 4x_4$
$x_2 = x_2$
$x_1 = 6 - 3x_2 + 2x_4$

$$x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ 0 \\ -4 \\ 1 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ 7 \\ 0 \end{bmatrix}$$


1.4 #27
27) $x_1 + rx_2 = 5$
$3x_1 + 6x_2 = s$
no solutions
$\begin{bmatrix} 1 & r & 5 \\ 3 & 6 & s \end{bmatrix}$
$-3r_1 + r_2 \to r_2$
$\begin{bmatrix} 1 & r & 5 \\ 0 & -3r+6 & -15+s \end{bmatrix}$
$-3r + 6 = 0$
$-3r = -6$
$r = 2$
$-15 + s = 1$
$s = 16$
1 solution
$-3r + 6 = 3$
$-3r = -3$
$r = 1$
$-15 + s = 1$
$s = 16$
inf solutions
$-3r + 6 = 0$
$-3r = -6$
$r = 2$
$-15 + s = 0$
$s = 15$
#37



#37

$$\begin{bmatrix} -2 & 2 & 1 & 1 & -2 \\ 1 & -1 & -1 & -3 & 3 \\ -1 & 1 & -1 & -7 & 5 \end{bmatrix} \quad R_1 \longleftrightarrow R_2$$

$$\begin{bmatrix} 1 & -1 & -1 & -3 & 3 \\ -2 & 2 & 1 & 1 & -2 \\ -1 & 1 & -1 & -7 & 5 \end{bmatrix} \quad \begin{matrix} 2R_1 + R_2 \to R_2 \\ R_1 + R_3 \to R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & -1 & -1 & -3 & 3 \\ 0 & 0 & -1 & -5 & 4 \\ 0 & 0 & -2 & -10 & 8 \end{bmatrix} \quad -2R_2 + R_3 \to R_3$$

$$\begin{bmatrix} 1 & -1 & -1 & -3 & 3 \\ 0 & 0 & -1 & -5 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = 2
Null = 3

Section 1.6 #19

19 $\left[\begin{array}{cc|c} -1 & 1 & 2 \\ 2 & -1 & r \\ 2 & 0 & -8 \end{array}\right]$ $R_1 + R_2 \to R_2$

$\left[\begin{array}{cc|c} -1 & 1 & 2 \\ 1 & 0 & r+2 \\ 2 & 0 & -8 \end{array}\right] \to \left[\begin{array}{ccc} -1 & 1 & 2 \\ 1 & 0 & r+2 \\ 1 & 0 & -4 \end{array}\right] \to \begin{array}{ccc} 0 & 1 & -2 \\ 1 & 0 & r+2 \\ 1 & 0 & -4 \end{array}$

$x_1 = -4$

$x_1 = r+2$

$x_2 = -2$

$r+2 = -4$

$\boxed{r = -6}$

$r + 12 = -4$

$\boxed{r = -6}$

25 does $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ -2 & 4 & 2 \end{bmatrix} = I_3$?

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$ yes