CIS 3223 Homework 4

Dr Anthony Hughes

Name: Parth Patel
Temple ID (last 4 digits): 5761

Simple non-graphing calculator | **Make: TI-30Xa**

1 6 pts) Give a big-θ bound for the solutions of the following relations (show work).

(a) $T(n) = 2T(n/3) + 1$

$\boxed{\Theta(n^{log_3 2})}$

$$a = 2$$
$$b = 3$$
$$d = 0$$

Since $d < \log_b a = \log_3 2 \approx 0.6309, T \in \Theta(n^{log_3 2})$

(b) $T(n) = 5T(n/4) + n$

$T \in \Theta(n^{\log_4 5})$

$$a = 5$$
$$b = 4$$
$$d = 1$$

Since $d < \log_4 5 \approx 1.16096$,

(c) $T(n) = 9T(n/3) + n^2$

$\boxed{\Theta(n^2 \log_3 n)}$

$$a = 9$$
$$b = 3$$
$$d = 2$$

Since $d = \log_b a = \log_3 9 = 2, T \in \Theta(n^2 \log_3 n)$

2 (4.5 pts) P71 Q4

Choice: Algorithm

Algorithm A: $\Theta(n^{\log_2 5})$

$$a = 5$$
$$b = 2$$
$$d = 1$$

Since $d < \log_b a = \log_2 5 \approx 2.3219, T \in \Theta(n^{\log_2 5})$

Algorithm B: $\Theta(2^n)$

Subtract and conquer

$$a = 2$$
$$b = 1$$
$$d = 0$$

Since $a > 1,\ T \in \Theta(n^d a^{n/b}) = \Theta(2^n)$

Algorithm C: $\Theta(n^2 \log_3 n)$

divide and conquer

$$a = 9$$
$$b = 3$$
$$d = 2$$

Since $d = \log_b a = \log_3 9 = 2, T \in \Theta(n^2 \log_3 n)$

I would pick algorithm C to solve the problem since for larger n it is more runtime efficient.

3 (4 pts) Give a big-θ bound for the solutions of the following relations (show work).

(a) $T(n) = T(n-1) + 1$

$\boxed{\Theta(n)}$

$$a = 1$$
$$b = 1$$
$$d = 0$$

Since $a = 1$, $T \in \Theta(n^{d+1}) = \Theta(n)$

(b) $T(n) = 4T(n-2) + n$

$\boxed{\Theta(n \cdot 2^n)}$

$$a = 4$$
$$b = 2$$
$$d = 1$$

Since $a > 1$, $T \in \Theta(n^d a^{n/b}) = \Theta(n \cdot 4^{\frac{n}{2}}) = \Theta(n \cdot 2^n)$

4 (2.5 pts) Show that for any positive integer n, there must be some power of 3 lying in the range $[n, 3n]$.

Consider the following equality: $\log_3 n \leq x \leq 1 + \log_3 n$ for $n \in \mathbb{Z}^+$

We can note that $x \in \mathbb{Z}^+$ since there must exists an integer between two numbers that differ by one. Either the end points are integers *or* they are decimals where since they differ by 1 there is an integer between them.

Using properties of logs we can note that:

$$\log_3 n \leq x \leq \log_3 3 + \log_3 n$$

Combine logs:

$$\log_3 n \leq x \leq + \log_3 3n$$

Raise to third power:

$$n \leq 3^x \leq 3n$$

Since $x \in \mathbb{Z}^+$ we can say that an power of 3 exists in the interval $[n, 3n]$

5 (3 pts) P42 Q38(a) p=13 $\boxed{6}$ p=17 $\boxed{16}$

$p = 13$

Since $p = 13$, $p - 1 = 12$
Therefore $s \in \{2, 3, 4, 6, 12\}$

$$10^2 = 100 = 13(7) + 9 \equiv_{13} 9$$
$$10^3 = 10 \cdot 10^2 \equiv_{13} 9 * 10 = 13(6) + 12 \equiv_{13} 12$$
$$10^6 = 10^{3^2} \equiv_{13} 12^2 = 144 = 13(11) + 1 \equiv_{13} \boxed{1}$$

Suppose $a = 10$, since 13 is prime $a \in \mathbb{Z}_{13}^*$

The smallest $r \in s$ s.t 10^r is 6.

Therefore the groupings must be size 6

$p = 17$

Since $p = 17$, $p - 1 = 16$
Therefore $s \in \{4, 16\}$

Suppose $a = 10$, since 17 is prime $a \in \mathbb{Z}_{17}^*$

$$10^4 = 10000 = 588(17) + 4 \equiv_{17} 4$$
$$10^{16} = 10^{4^4} \equiv_{17} (4)^4 = 256 = 15(17) + 1 \equiv_{17} 1$$

The smallest $r \in s$ s.t 10^r is 16.

Therefore the groupings must be size 16