CIS 3223 Homework 2

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1 (3 pts) Show that $\sum_{i=1}^{n} \frac{1}{i} = \Theta(\log n)$

$$\sum_{i=1}^{n} \frac{1}{i} = \Theta\left(\int_{1}^{n} \frac{1}{x}\, dx\right)$$

$$= \Theta\left(\ln x \Big|_{n=1}^{n=x}\right)$$

$$= \Theta(\ln n) \overset{*}{=} \Theta(\log n)$$

* - logs only differ by a constant $\forall(b_0, b_1) > 1,\ \log_{b_0} n \in \Theta(log_{b_1}(n))$

2 (3 pts) Determine the following:

(a) 4231 (mod 17)

15

(b) -75 (mod 17)

10

(c) 15^7 (mod 13)

IMPORTANT: $A^B \bmod C = ((A \bmod C)^B) \bmod C$

$$15^7 \bmod 13 = 2^7 \bmod 13$$

$$= \left[\left(2^4 \bmod 13\right) \cdot \left(2^3 \bmod 13\right)\right] \bmod 13$$

$$= \left[\left(16 \bmod 13\right) \cdot \left(8 \bmod 13\right)\right] \bmod 13$$

$$= \left(3 \cdot 8\right) \bmod 13$$

$$= \boxed{11}$$

3 (6 pts) Apply the non-recursive **division algorithm** to find the quotient and remainder when 83 is divided by 7. **Show all steps** (diagram carefully).

x = 83   y = 7 $83 = 1010011_2$

digit	q	r	r ≥ y
0	0	0	FALSE
1	0	1	FALSE
0	0	2	FALSE
1	0	5	FALSE
0	0 1	10 3	TRUE FALSE
0	2	6	FALSE
1	4 5	13 6	TRUE FALSE
1	10 11	13 6	TRUE FALSE

quotient **11**   remainder **6**

4 (6 pts) Use the **extended Euclidean algorithm** (using matrices) to find integers s and t such that $81s + 11t = \gcd(81, 11)$ (show all steps).

$a = 81, \qquad b = 11$

a	b	q	r	Q
81	11	7	4	$\begin{pmatrix} 0 & 1 \\ 1 & -7 \end{pmatrix}$
11	4	2	3	$\begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & -7 \end{pmatrix} = \begin{pmatrix} 1 & -7 \\ -2 & 15 \end{pmatrix}$
4	3	1	$\boxed{1}$	$\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & -7 \\ -2 & 15 \end{pmatrix} = \begin{pmatrix} -2 & 15 \\ 3 & -22 \end{pmatrix}$
1	1	1	0	$\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -2 & 15 \\ 3 & -22 \end{pmatrix} = \begin{pmatrix} 3 & -22 \\ -5 & 37 \end{pmatrix}$

$81 \boxed{3} + 11 \boxed{-22} = \boxed{1}$

If a and b are two n-bit numbers, what is a good bound for the number of iterations of the loop in the algorithm? $O(\log n)$ $\quad O(n)$ $\quad O(n \log n)$ $\quad O(n^2)$

5 (2 pts). Exercise 1.2

Let n be a number.

The length of n is in **binary** $\lceil \log_2(n+1) \rceil$

Then,

$$\lceil \log_2(n+1) \rceil \overset{*}{=} \left\lceil \frac{\log_{10}(n+1)}{\log_{10}(2)} \right\rceil \leq \left\lceil \frac{1}{\log_{10}(2)} \right\rceil \lceil \log_{10}(n+1) \rceil \overset{**}{=} 4 \lceil \log_{10}(n+1) \rceil$$

Since $\lceil \log_{10}(n+1) \rceil$ represents the length of n in decimal we can conclude that the length of a binary number is at most 4 times as long as its decimal format.

* - log Change of base

** - See that $\frac{1}{\log_{10}(2)} = \log_2(10) \approx 3.3219\ldots$

Large n

For n the ratio of the $\log_{10} n$ and $\log_2 n$ is represented by:

$$\lim_{n\to\infty} \frac{\log_2 n}{\log_{10} n} = \lim_{n\to\infty} \frac{\log_2 n}{\frac{\log_2 n}{\log_2 10}} = \log_2 10 \approx 3.3219\ldots$$