CIS 3223 Homework 1

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1 (12 pts) Complete the following table by writing "T"for true or "F "for false in each box. No justification required.

f	g	$f = O(g)$	$f = \Omega(g)$	$f = \Theta(g)$
$n \log n$	$5n \log 10n$	T	T	T
$n^{4/5}$	$n^{2/3}$	F	T	F
$\log 5n$	$\log 2n$	T	T	T
$n^{1.03}$	$n \log^3 n$	F	T	F
$(\log n)^2$	$\sqrt{n}$	T	F	F
$\Sigma_{i=1}^{n} i^k$	n^k	F	T	F
$n2^n$	3^n	T	F	F
$(\log n)^{\log n}$	$2^{\log n^2}$	F	T	F

2 (2 pts) **Give** as good big$-\Theta$ estimate for each of the following functions.

(a) $f(n) = (n! + n5^n)(n + \log(n^7 + 1))$

$\Theta(n!)$ $\Theta(n)$

$\Theta(n \cdot n!)$

(b) $f(n) = (n^3 + 4n^2)(n\sqrt{n} + 1000)(n + (\log n)^3$

Θ(n^3) Θ(n√n) Θ(n)

$\Theta(n^5 \cdot \sqrt{n})$

3 (2 pts) Evaluate $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{14}$

$\begin{pmatrix} 610 & 377 \\ 377 & 233 \end{pmatrix}$

I have the work for this attached

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{k} = \begin{pmatrix} F_{k+1} & F_k \\ F_k & F_{k-1} \end{pmatrix} \text{ for } n \geq 1 \text{ where } F_0 = 0$$

Let F_n be the n-th Fibonacci number.

4 (4 pts) Use strong induction to prove the following:

$$F_n \leq 1.7F_{n-1}, \quad n \geq 4$$

Base cases: Show true for $n = 4$ and 5:

$n = 4:$

lhs $= F_4 =$ 3

rhs $= 1.7F_3 =$ 3.4

$n = 5:$

lhs $=$ 5

rhs $=$ 5.1

Inductive case: Assume true for $n = s, \ s \in \{4, 5, \ldots k\}, \ k \geq 5,$

Show true for $n = k + 1$:

lhs $= F_{k+1} = F_k + F_{k-1}$

rhs $= 1.7F_{k+1-1} = 1.7F_k$

$$\text{lhs} = F_k + F_{k-1} \overset{\text{IH}}{\leq} 1.7F_{k-1} + 1.7_{k-2} = 1.7(F_{k-1} + F_{k-2}) = \boxed{1.7F_k} = \text{rhs}$$

$$\implies \text{lhs} \leq \text{rhs}$$

Therefore $\forall n \geq 5, \ F_n \leq 1.7F_{n-1}$

Why do we need to start with $n = 4$ (table useful)?

We must start with $n = 4$ since the formula ($F_n \leq 1.7F_{n-1}$) **does not** hold for any $n < 4$.

(Bonus. 1 point) Give a pair better than (1.7, 4). (1.667 , 4)

NOTE: This is based on zero based indexing. This means that $F_0 = 0, F_1 = 1$