**CIS 3223 Homework 2**

Dr Anthony Hughes

| **Name: Parth Patel** |
| --- |
| **Temple ID (last 4 digits) : 5761** |

1 (3 pts) Show that $\sum_{i=1}^{n} \frac{1}{i} = \Theta(\log n)$

$$\sum_{i=1}^{n} \frac{1}{i} = \Theta\left(\int_1^n \frac{1}{x}\,dx\right)$$

$$= \Theta\left(\ln x \Big|_{n=1}^{n=x}\right)$$

$$= \Theta(\ln n) \overset{*}{=} \Theta(\log n)$$

* - logs only differ by a constant $\forall (b_0, b_1) > 1,\ \log_{b_0} n \in \Theta(log_{b_1}(n))$

2 (3 pts) Determine the following:

(a) 4231 (mod 17)

| 15 |
| --- |

(b) -75 (mod 17)

| 10 |
| --- |

(c) $15^7$ (mod 13)

**IMPORTANT:** $A^B \bmod C = ((A \bmod C)^B) \bmod C$

$$15^7 \bmod 13 = 2^7 \bmod 13$$

$$= \left[\left(2^4 \bmod 13\right)\cdot\left(2^3 \bmod 13\right)\right] \bmod 13$$

$$= \left[\left(16 \bmod 13\right)\cdot\left(8 \bmod 13\right)\right] \bmod 13$$

$$= \left(3 \cdot 8\right) \bmod 13$$

$$= \boxed{11}$$

3 (6 pts) Apply the non-recursive **division algorithm** to find the quotient and remainder when 83 is divided by 7. **Show all steps** (diagram carefully).

x = 83 y = 7 $83 = 1010011_2$

| digit | q | r | $r \geq y$ |
|---|---|---|---|
| 0 | 0 | 0 | FALSE |
| 1 | 0 | 1 | FALSE |
| 0 | 0 | 2 | FALSE |
| 1 | 0 | 5 | FALSE |
| 0 | 0<br>1 | 10<br>3 | TRUE<br>FALSE |
| 0 | 2 | 6 | FALSE |
| 1 | 4<br>5 | 13<br>6 | TRUE<br>FALSE |
| 1 | 10<br>11 | 13<br>6 | TRUE<br>FALSE |
| | | | |

quotient **11** remainder **6**

4 (6 pts) Use the **extended Euclidean algorithm** (using matrices) to find integers $s$ and $t$ such that $81s + 11t = \gcd(81, 11)$ (show all steps).

$a = 81, \qquad b = 11$

| $a$ | $b$ | $q$ | $r$ | $Q$ |
|---|---|---|---|---|
| 81 | 11 | 7 | 4 | $\begin{pmatrix} 0 & 1 \\ 1 & -7 \end{pmatrix}$ |
| 11 | 4 | 2 | 3 | $\begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & -7 \end{pmatrix} = \begin{pmatrix} 1 & -7 \\ -2 & 15 \end{pmatrix}$ |
| 4 | 3 | 1 | $\boxed{1}$ | $\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & -7 \\ -2 & 15 \end{pmatrix} = \begin{pmatrix} -2 & 15 \\ 3 & -22 \end{pmatrix}$ |
| 1 | 1 | 1 | 0 | $\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -2 & 15 \\ 3 & -22 \end{pmatrix} = \begin{pmatrix} 3 & -22 \\ -5 & 37 \end{pmatrix}$ |

$81\,\boxed{3}\,+11\,\boxed{-22} = \boxed{1}$

If $a$ and $b$ are two $n$-bit numbers, what is a good bound for the number of iterations of the loop in the algorithm? $O(\log n)$ $\underline{O(n)}$ $O(n \log n)$ $O(n^2)$

5 (2 pts). Exercise 1.2

Let $n$ be a number.

The length of $n$ is in **binary** $\lceil \log_2(n+1) \rceil$

Then,

$$\lceil \log_2(n+1) \rceil \overset{*}{=} \left\lceil \frac{\log_{10}(n+1)}{\log_{10}(2)} \right\rceil \leq \left\lceil \frac{1}{\log_{10}(2)} \right\rceil \lceil \log_{10}(n+1) \rceil \overset{**}{=} 4 \lceil \log_{10}(n+1) \rceil$$

Since $\lceil \log_{10}(n+1) \rceil$ represents the length of $n$ in decimal we can conclude that the length of a binary number is at most 4 times as long as its decimal format.

* - log Change of base
** - See that $\frac{1}{\log_{10}(2)} = \log_2(10) \approx 3.3219\ldots$

**Large $n$**

For $n$ the ratio of the $\log_{10} n$ and $\log_2 n$ is represented by:

$$\lim_{n\to\infty} \frac{\log_2 n}{\log_{10} n} = \lim_{n\to\infty} \frac{\log_2 n}{\frac{\log_2 n}{\log_2 10}} = \log_2 10 \approx 3.3219\ldots$$