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Assignment : Parameter estimation.

1. Let us consider a random Sample $(X_1, X_2, X_3 \ldots X_n)$. where

$\mu = \theta_1$ (mean) and $\sigma^2 = \theta_2$ (variance)

So likelihood f^n $$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}.$$

to max. (logarithm)

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[-\frac{1}{2} \ln(2\pi\theta_2) - \frac{(x_i-\theta_1)^2}{2\theta_2} \right].$$

(i) $$\frac{d \ln L(\theta_1, \theta_2)}{d\theta_1} = \sum_{i=1}^{n} \left[\frac{(x_i-\theta_1)^2}{\theta_2} \right] = 0.$$

$$\Rightarrow x_i - n\theta_1 = 0$$

$$\boxed{\theta_1 = \frac{\sum_{i=1}^{n} x_i}{n}}$$ (mean).

(ii) $$\frac{d \ln L(\theta_1, \theta_2)}{d\theta_2} = \sum_{i=1}^{n} \left[-\frac{1}{2\theta_2} + \frac{(x_i-\theta_1)^2}{2\theta_2^2} \right] = 0.$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i-\theta_1)^2.$$

$$\boxed{\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\theta_1)^2}$$

(variance).

Parameter estimation

2: Binomial distribution $B(n, \theta)$.

$p = \theta \qquad q = 1-\theta$

Probability mass function.

$$f(x; n, \theta) = {}^{n}C_{x}\, \theta^{x} (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

taking log.

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left[\ln {}^{n}C_{x_i} + x_i \ln\theta + (n-x_i) \ln(1-\theta) \right]$$

diff. w.r.t θ.

$$\frac{d}{d\theta}(\ln(L(\theta))) = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{(n-x_i)}{1-\theta} \right] = 0.$$

To find θ. $\sum_{i=1}^{n} \left[\frac{(1-\theta)x_i - \theta(n-x_i)}{\theta(1-\theta)} \right] = 0.$

$$\sum_{i=1}^{n} ((1-\theta)x_i - \theta(n-x_i)) = 0.$$

$$\theta \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i . m$$

$$\boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{n.m}}$$