University College London

COMP0043 Numerical Methods for Finance

Dr Guido Germano

Coursework 2, due 11 January 2020

1. Modify the script bs.m splitting it into a main and three functions for each of its three sections: analytical Black-Scholes-Merton, Fourier transform and Monte Carlo. To speed up the tests, comment out the code that produces the figures 1–4 in bs.m and the code that produces the figure at the end of payoff.m (which is called twice). (10 marks)

   If you prefer, you may rewrite everything in another programming language you like better, for instance Go, https://golang.org, developped since 2007 by Google and increasingly deployed there, or Julia, https://julialang.org, developped since 2011 at MIT and used since 2014 by Blackrock, the world's largest asset manager, for its next-generation analytics platform: https://juliacomputing.com/case-studies/blackrock.

2. Add to geometric Brownian motion, as a jump component, a normal compound Poisson process; the result is a Merton jump-diffusion process, whose Monte Carlo simulation we have seen in the script mjd.m. For the diffusion part use the same parameters as in bs.m, i.e. $\sigma = 0.4$; for the jump part use the same parameters as in mjd.m, i.e. $\lambda = 0.5, \mu_J = 0.1, \sigma_J = 0.15$.

   (a) Pricing with the Fourier transform: add a term $\lambda(e^{i\mu_J\xi - \frac{1}{2}(\sigma_J\xi)^2} - 1)$ (Ballotta and Fusai 2018, Table 6.4, page 182) to the characteristic exponent $\psi(\xi)$ and a corresponding constant to the drift parameter μ_{RN} in order to obtain the risk-neutral measure. For a call, use the damping parameter $\alpha = -1$. (20 marks)

   (b) Pricing with Monte Carlo: add a few lines of code inside the for loop over blocks adapting it from mjd.m. Like for arithmetic Brownian motion, you can do a single step from 0 to T. In order to perform the simulation with the risk-neutral measure, use the same corrected drift parameter μ_{RN} as for the Fourier transform method. (20 marks)

   Alternatively, instead than to MJD you may extend geometric Brownian motion to the Kou process. The extra term for the characteristic exponent is $\lambda\big(p\frac{\eta_1}{\eta_1 - i\xi} + (1-p)\frac{\eta_2}{\eta_2 + i\xi} - 1\big)$ (Ballotta and Fusai 2018, Table 6.5, page 185). Use the random number generator for the jumps in kou.m, the same parameters as in kou.m, i.e. $\lambda = 0.5, p = 0.4, \eta_1 = 4, \eta_2 = 3$, and the damping parameter $\alpha = -1$ for a call.

   The first three to five digits of the FT and MC prices should agree and be slightly higher than the Black-Scholes price because the jump process increases the volatility.

3. The price of an option is the discounted expected value of its payoff, and thanks to the Plancherel theorem it can be computed also in Fourier space,

$$
\begin{aligned}
V(X,0) &= e^{-rT} E\big[g(X(T))e^{-\alpha X(T)}\big] \\
&= e^{-rT} \int_{-\infty}^{+\infty} g(x)e^{-\alpha x} f_X(x,T)dx \\
&= \frac{e^{-rT}}{2\pi} \int_{-\infty}^{+\infty} \widehat{g}(\xi)\varphi_X^*(\xi + i\alpha, T)d\xi,
\end{aligned}
$$

where $X(t) = \log(S(t)/S_0)$ is the log-price, T is maturity, $g(x)$ is the payoff damped by the factor $e^{\alpha x}$, and $\varphi_X(\xi,T) = \widehat{f}_X(\xi,T)$ is the characteristic function (see page 14 of the lecture notes on pricing with Fourier transform methods).

Extend bs.m as modified in the previous two questions to show the following.

(a) As the price V is a real number, it does not change taking its complex conjugate and thus if the integrand in Fourier space is $\widehat{g}^*(\xi)\varphi_X(\xi + i\alpha, T)$. (10 marks)

(b) Currently the integration in Fourier space is overengineered by taking the zero-frequency value of an FFT. Using the trapezoidal integration rule (see Assignment 1) yields the same result. (10 marks)

(c) Even a simple sum (see again Assignment 1) yields the same result. (10 marks)

(d) Add a figure that plots the one-time PDF $f_X(x,T)$ at maturity computed (i) with Monte Carlo and (ii) with an inverse FFT of the characteristic function. To keep things simple, you may use just the data from the last Monte Carlo block. (10 marks)

(e) Do the integral in log-price space using the payoff $g(x)e^{-\theta\alpha x} = \max(\theta(S_0e^x - K), 0)$ and the one-time PDF $f_X(x,T)$ obtained in part (ii) of the previous subquestion from an inverse FFT of the characteristic function (the accuracy may be lower). (10 marks)