Robust Principle Component Analysis

EMMANUEL J. CANDES
and XIAODONG LI
Stanford University

YI MA
University of Illinois at Urbana-Champaign
Microsoft Research Asia

JOHN WRIGHT
Microsoft Research Asia

Report By : Parth Satodiya (1401056)

***Abstract*—This paper is mainly focused on separation problem and its application. We have a matrix combining of low-rank component and sparse component. It is possible to recover both law-rank and sparse component using *Principle Component Analysis*. This decomposition is possible by minimize a weighted combination of a nuclear norm and *l1*-norm. In this paper, writer also successful to recover principle components even though a positive fraction of its entries are corrupted.**

***Index Terms*—Singular Values, convex optimization, nuclear norm, *l1*-norm.**

I. Introduction

Suppose we have some large dimension matrix M and we know that it can be decompose in two matrix

$$M = L_0 + S_0$$

Where L_0 is low rank matrix and S_0 is sparse matrix. We don't know the dimensions of these matrix and we get various data from various fields like science, engineering, social issues which are very higher dimensional data. We can represent it on some lower dimensional subspace. We can do this using principle component analysis.

$$\begin{array}{rl} minimize & ||M - L|| \\ subject\ to & rank(L) \leq k \end{array}$$

$||M||$ denotes the 2-norms; that is,the largest singular value of M. Principle Component Analysis is the most widely used tool but if there is a single grossly corrupted entry in M then may be the estimated L̂ is arbitrarily far from the true L_0.

This problem can also be solved by tractable convex optimization which is also discussed in this article.$||M||_*$ denote the nuclear norm of the matrix, that is, the sum of the singular values of M and $||M||_1$ denote the $l1$-norm, that is, the sum of absolute norm of M. Then the *Principle Component Pursuit (PCP)* estimate solving

$$\begin{array}{rl} minimize & ||L||_* \ + \ \lambda||S||_1 \\ subject\ to & L + S = M \end{array}$$

exactly recovers the low-rank L_0 and sparse S_0 and the cost of this algorithm is not so higher than classical PCA. Matrix with singular vectors are spread enough can be recovered perfectly. The only thing that affect the algorithm is the position of nonzero entries in S_0.

II. Matrix completion from Grossly Corrupted Data

So, we can recover low-rank matrix even though a significant entries are corrupted. In some application, some entries may be missing as well. Let P_ω be the orthogonal projection onto the linear space of matrices supported on $\omega \subset [n_1] \times [n_2]$,

$$P_\Omega X = \begin{cases} X_{ij} & (i,j) \in \Omega \\ 0 & (i,j) \notin \Omega \end{cases}$$

We only have available a few entries of $L_0 + S_0$ which we can write as,

$$Y = P_{\Omega_{obs}}(L_0 + S_0)$$

so, we can only see those entries $(i,j) \in \Omega_{obs} \subset [n_1] \times [n_2]$. Now the problem is we only see a few entries of L_0, and among those a fraction happens to be corrupted, we of course do not know which one. They propose recovering L_0 by solving the following problem:

$$\begin{array}{rl} minimize & ||L_*|| + \lambda||S||_1 \\ subject\ to & P_{\Omega_{obs}}(L + S) = Y \end{array}$$

So, perfectly recovery from incomplete and corrupted entries is possible by convex optimization.

III. Experiments

In this paper, they investigate Principle Component Pursuit's ability to correctly recover matrices of various rank and errors of various density. They have also show applications like background modeling form video and removing shadows from face images.

They verify the correct recovery phenomenon of theorem $minimize \quad ||L||_* \ + \ \lambda||S||_1$ on randomly generated problems. They consider square matrices of varying dimension n = 500,...,3000 and generate rank-r matrix L_0 as a product $L_0 = X \times Y$ where X and Y are n$n\times$r matrices with entries independently sampled from a $N(0, 1/n)$ distribution.

From **Fig.1** we can say that Principal Component Pursuit still gives visually appealing solutions to this practical low-rank and sparse separation problem, without using any additional information about the spatial structure of the error. We emphasize that our goal in this example is not to engineer a complete system for visual surveillance. Such a system would have to cope with real-time processing requirements, and also perform nontrivial post-processing for object detection, tracking, and so on. Our goal here is simply to demonstrate the potential real-world applicability of the theory and approaches of this article.

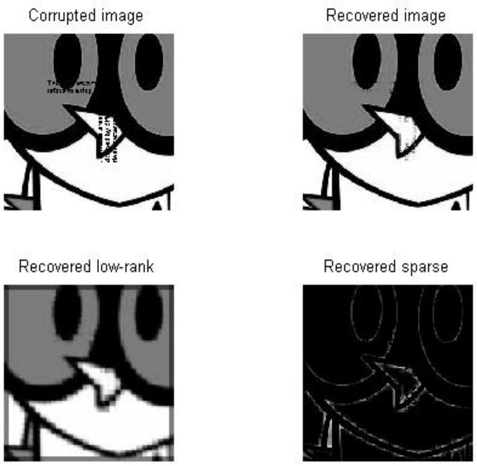

Fig.1 Simulation Result

From the above figure, corrupted image contains some unnecessary text. After applying algorithm, we can separate corrupted image into low-rank matrix and sparse matrix.

IV. Applications

Many important data applications can naturally be modeled as a low-rank and sparse components. Following are some real life applications.

From the surveillance video frames, we always need to differentiate foreground from background. L_0 is represent background and sparse is represent moving objects in foreground. However, each image frame has thousands or tens of thousands of pixels, and each video fragment contains hundreds or thousands of frames.

This algorithm also use from Face Recognition in particular, images of a humans face can be well-approximated by a low-dimensional subspace.Being able to correctly retrieve this subspace is crucial in many applications such as face recognition and alignment. However, realistic face images often suffer from self-shadowing, specularity, or saturations in brightness, which make this a difficult task and subsequently compromise the recognition performance.

V. Further Discussion

This article delivers some encouraging news: one can disentangle the low-rank and sparse components exactly by convex programming, and this provably works under quite broad conditions. Further, our analysis has revealed rather close relationships between matrix completion and matrix recovery (from sparse errors) and our results even generalize to the case when there are both incomplete and corrupted entries. In addition, Principal Component Pursuit does not have any free parameter and can be solved by simple optimization algorithms with remarkable efficiency and accuracy. More importantly, our results may point to a very wide spectrum of new theoretical and algorithmic issues together with new practical applications that can now be studied systematically.

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