PH 354: hw 2, problem 10

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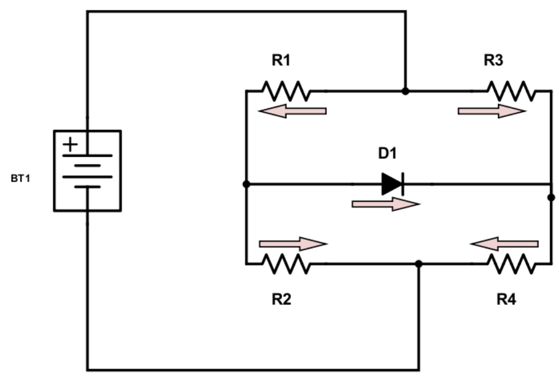

The equations for the circuit are as follows,

$$F = \begin{bmatrix} V_+ - I_1R_1 - I_2R_2 \\ V_+ - I_3R_3 - I_4R_4 \\ I_1 - I2 - I_0\left[\exp\left(\frac{I_3R_3 - I_1R_1}{V_T}\right) - 1\right] \\ I_3 - I4 - I_0\left[\exp\left(\frac{I_3R_3 - I_1R_1}{V_T}\right) - 1\right] \end{bmatrix}$$

where diode current $I_D = I_0\left[\exp\left(\frac{I_3R_3 - I_1R_1}{V_T}\right) - 1\right]$ and $V_1 - V_2 = I_3R_3 - I_1R_1$ is the forward bias voltage across the diode
This is an equation of four unknown variables (current). For solving this system by Newton's method requires setting up the Jacobian to find the solution matrix:

$$I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

The Jacobian is as follows

$$J = \begin{bmatrix} -R_1 & -R_2 & 0 & 0 \\ 0 & 0 & -R_3 & -R_4 \\ g & -1 & h & 0 \\ m & 0 & d & -1 \end{bmatrix}$$

where

$$g = 1 + \frac{I_0 R_1}{V_T} \exp\left(\frac{I_3 R_3 - I_1 R_1}{V_T}\right)$$

$$h = -\frac{I_0 R_3}{V_T} \exp\left(\frac{I_3 R_3 - I_1 R_1}{V_T}\right)$$

$$m = 1 - g$$

$$d = 1 - h$$

So for each iteration (starting from an initial guess matrix I) the following linear equation is solved, $J(I^{(n)})\Delta I = -F(I^{(n)})$

The solution for the currents is updated to $I^{(n+1)} = I^{(n)} + \Delta I$