PH 354: hw 3, problem 17

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For a point charge q at (x_0, y_0), the electric potential Φ at any place (x, y) is given by,

$$\Phi(x,y) = \frac{q}{4\pi\epsilon_0}\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2}}$$

Electric field is given by
$E_x = -\frac{\partial\Phi}{\partial x}$ and $E_y = -\frac{\partial\Phi}{\partial y}$
$E_x = \frac{q}{4\pi\epsilon_0}\frac{x-x_0}{[(x-x_0)^2+(y-y_0)^2]^{\frac{3}{2}}}$ and $E_y = \frac{q}{4\pi\epsilon_0}\frac{y-y_0}{[(x-x_0)^2+(y-y_0)^2]^{\frac{3}{2}}}$
Φ, E_x or E_y can be superposed for any arbitrary configuration of point charges. Similarly, for any arbitrary two dimensional surface of charge density $\sigma(x, y)$, the potential and the electric field goes as,

$$\Phi(x,y) = \frac{1}{4\pi\epsilon_0}\int dx'\int dy'\frac{\sigma(x',y')}{\sqrt{(x-x')^2+(y-y')^2}}$$

$$E_x = -\frac{\partial\Phi}{\partial x} = \frac{1}{4\pi\epsilon_0}\int dx'\int dy'\frac{\sigma(x',y')}{[(x-x')^2+(y-y')^2]^{\frac{3}{2}}}(x-x')$$

$$E_y = -\frac{\partial\Phi}{\partial y} = \frac{1}{4\pi\epsilon_0}\int dx'\int dy'\frac{\sigma(x',y')}{[(x-x')^2+(y-y')^2]^{\frac{3}{2}}}(y-y')$$