PH 354: hw 3, problem 10

Alankar Dutta

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$$E = \frac{1}{2}\left(\frac{dx}{dt}\right)^2 + V(x)$$

For mass $m = 1$
At maximum displacement $x = a$, $\frac{dx}{dt} = 0$ and total Energy $E = V(a)$ and the oscillator travels for a quarter of its time period T

$$\therefore T = 4\left|\int_0^a \frac{dx}{\sqrt{\frac{2}{m}[E - V(x)]}}\right| = 4\left|\int_0^a \frac{dx}{\sqrt{\frac{2}{m}[V(a) - V(x)]}}\right|$$

Oscillator getting faster meaning time period T decreases. For potential $V(x) = x^4$, as a increases, $E = V(a)$ increases. So for all x, oscillator becomes fast (large $\frac{dx}{dt}$) to keep the total energy constant and hence its time period decreases. Going in the opposite direction, as $a \to 0$, the time period $T \to \infty$ which implies the particle doesn't swing at all.