PH 354: hw 5, problem 10

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$$P(\theta) = \frac{1}{2}\sin(\theta)$$

$$P(\phi) = \frac{1}{2\pi}$$

Polar angle θ is from 0 to π while azimuthal angle ϕ is from 0 to 2π
Integrating, $\int_0^{\pi} P(\theta)d\theta = 1$ and $\int_0^{2\pi} P(\phi)d\phi = 1$
$P(\phi)$ can be sampled from a uniform distribution with support from 0 to 2π
However, $P(\theta)$ needs an inverse sampling from a uniform distribution from 0 to 1
The Cumulative Distribution for $P(\theta)$ is

$$C(\theta) = \int_0^{\theta} P(\theta')d\theta' = \int_0^{\theta} \frac{1}{2}\sin(\theta')d\theta' = \frac{1}{2}(1 - \cos(\theta))$$

Let u be sampled from a uniform distribution with support from 0 to 1
Obviously, $C(C^{-1}u)) = u$
Therefore substituting gives the following $C^{-1}(u)$ transformation that samples from $P(\theta)$

$$C^{-1}(u) = \begin{cases} \cos^{-1}(1-2u) & u \leq \frac{1}{2} \\ \pi - \cos^{-1}(2u-1) & u > \frac{1}{2} \end{cases}$$