PH 354: hw 2, problem 7

Alankar Dutta

January 30, 2019

For a finite square well, the energy eigenvalues are roots of the following transcendental equations,
$\sqrt{u_0^2 - v^2} = v\tan(v)$ for symmetric wave functions
$\sqrt{u_0^2 - v^2} = -v\cot(v)$ for anti-symmetric wave functions
where
$k = \frac{\sqrt{2mE}}{\hbar}$
$v = \frac{kL}{2}$ and $u_0^2 = \frac{mL^2}{2\hbar^2}V$
However solving the transcendental equations given above is numerically awkward to carry on because $tan(x)$ and $cot(x)$ possess discontinuities in the range of interest. So its better to change the equation to sines and cosines which are continuous and well-behaved functions.
The roots corresponding to energies below V of the two transcendental equations are the energy eigenvalues and they are nearly equally spaced. This information is used as an initial guess to numerically evaluate the roots of the equation.