COMPUTATIONAL PHYSICS – PH 354

HOMEWORK DUE ON 15TH APR 2019

Exercise 1: Gaussian elimination and Conjugate gradient

Consider the matrix

$$B = 2 * \texttt{np.diag(np.full(200,1.0))} + \texttt{np.random.rand(200, 200)}/\sqrt{200}$$

This matrix (of size 200×200) has $2 + \frac{1}{\sqrt{200}} * random$ on the diagonal and offdiagonal entries are $\frac{1}{\sqrt{200}} * random$ where $random$ is a random number between $(0,1)$. Construct a symmetric matrix $A = \frac{1}{2}(B + B^T)$. For

$$b = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

Solve for x in $Ax = b$ using

  a) LU decomposition (Gaussian elimination).

  b) Conjugate gradient iteration.

Exercise 2: Eigenvalues using in-built eig and Lanczos

Consider the matrix

$$B = 2 * \texttt{np.diag(np.full(200,1.0))} + \texttt{np.random.rand(200, 200)}/\sqrt{200}$$

This matrix (of size 200×200) has $2 + \frac{1}{\sqrt{200}} * random$ on the diagonal and offdiagonal entries are $\frac{1}{\sqrt{200}} * random$ where $random$ is a random number between $(0,1)$. Construct a symmetric matrix $A = \frac{1}{2}(B + B^T)$.

Find the eigenvalues of A using:

  a) `numpy.linalg.eig`

  b) Lanczos iteration.