domized online algorithm that achieves a competitive ratio

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(It is required that $t_k \geq a_n$ and $k \geq 1$.) The parameter k is called the *acknowledgement cost* and $\sum_j$ latency(j) is called the *latency cost* of the algorithm on that input. Of course in practice the acknowledgment times must be chosen online without knowledge of when future arrivals will occur.

Dooly *et al.* showed that the natural algorithm which waits until the latency since the previous acknowledgement equals the cost of the acknowledgement has a competitive ratio of 2. Subsequently, Seiden [13], and independently Noga [11], obtained a lower bound of $e/(e-1)$ on the competitive ratio of randomized online algorithms for this problem. A matching upper bound remained elusive, and in fact no randomized algorithm was known to beat the 2-competitive ratio achieved by the deterministic algorithm.

The variant of the problem where packet j has weight w_j and one wishes to minimize $k + \sum_j w_j$latency(j) was also studied, but it is easy to see that for our purposes it is equivalent to the original problem.

The Bahncard problem

The Bahncard problem models online ticket purchasing in the German Deutsche Bundesbahn, where one can opt to buy a Bahncard that entitles the traveler to a 50% discount