FACULTY OF ENGINEERING & TECHNOLOGY

SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

B.Tech	**Mechanical Engineering**
Subject Code	**18MEO113T**
Subject Title	**Design of Experiments**
Course coordinator / Key prepared by	Dr. S. Murali / Dr. S. Murali
Key Approved by	Dr. S.Muralidharan
Exam Date and session	05-01-2024 & AN
Regulation	R2018
Regular / Arrear	Regular & Arrear

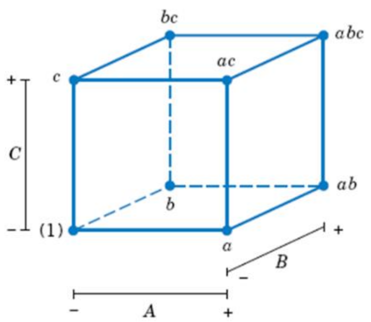
Part B

<table>
<tr><th>Q.No</th><th>Answer</th><th>Marks</th><th>BL</th><th>CO</th></tr>
<tr><td>21</td><td>The conventional strategy has various types including One factor at a time; several factors, one at a time; and several factors, all at the same time. Disadvantages: OFAT method fails to consider any possible interaction between the factors. Interactions between factors are very common, and if they occur, the OFAT strategy will usually produce poor results. Not possible to attribute the change in result to any of the factor/s. Effect of interaction between factors cannot be studied. These are poor experimental strategies and there is no scientific basis. The results cannot be validated.</td><td>4</td><td>2</td><td>1</td></tr>
<tr><td>22</td><td>Factor: A variable or attribute which influences or is suspected of influencing the characteristic being investigated. All input variables which affect the output of a system are factors. It is also known as an independent variable.
Level: Levels are the specific values or settings of each factor that you want to compare, such as high, low, or medium.</td><td>4</td><td>2</td><td>2</td></tr>
<tr><td>23</td><td>Interaction effect and Sum of square:

$$AB = \frac{ab + (1)}{2n} - \frac{a + b}{2n} = \frac{1}{2n}[ab + (1) - a - b]$$

$$SS_A = \frac{[a + ab - b - (1)]^2}{4n}$$

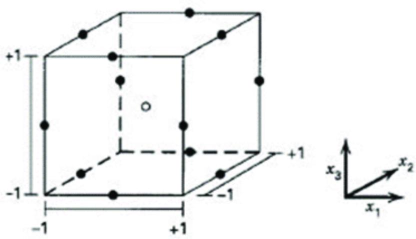
$$SS_B = \frac{[b + ab - a - (1)]^2}{4n}$$

$$SS_{AB} = \frac{[ab + (1) - a - b]^2}{4n}$$
</td><td>4</td><td>2</td><td>3</td></tr>
<tr><td>24</td><td>


(a) Geometric view

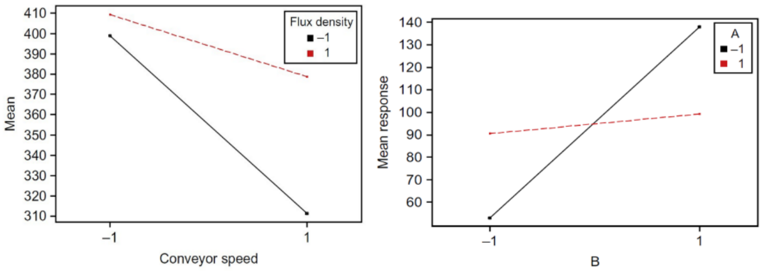
<table>
<tr><th>Run</th><th>A</th><th>B</th><th>C</th></tr>
<tr><td>1</td><td>−</td><td>−</td><td>−</td></tr>
<tr><td>2</td><td>+</td><td>−</td><td>−</td></tr>
<tr><td>3</td><td>−</td><td>+</td><td>−</td></tr>
<tr><td>4</td><td>+</td><td>+</td><td>−</td></tr>
<tr><td>5</td><td>−</td><td>−</td><td>+</td></tr>
<tr><td>6</td><td>+</td><td>−</td><td>+</td></tr>
<tr><td>7</td><td>−</td><td>+</td><td>+</td></tr>
<tr><td>8</td><td>+</td><td>+</td><td>+</td></tr>
</table>

(b) The 2^3 design matrix
</td><td>4</td><td>2</td><td>4</td></tr>
</table>

<table>
<tr><td>25</td><td>The Taguchi Loss Function (or Quality Loss Function) is a mathematical representation that quantifies the relationship between product quality and the financial loss (loss to society) associated with deviations from a target value. The Taguchi Loss Function is an equation used to quantify the financial loss incurred due to deviations in product quality from the target value. It is based on the principle that any deviation from the ideal or target value increases costs associated with poor quality, customer dissatisfaction, or product failures.
The Taguchi Loss Function equation is:
L(x) = K * (x - T)^2
Where:
L(x) represents the financial loss associated with a specific deviation from the target value.
K is the loss coefficient, a constant that determines the rate at which financial loss increases with deviations from the target value.
x is the actual value of the quality characteristic being evaluated.
T is the target value, the ideal or optimal value of the quality characteristic.</td><td>4</td><td>2</td><td>5</td></tr>
<tr><td>26</td><td>Box-Behnken designs have treatment combinations that are at the midpoints of the edges of the experimental space and require at least three continuous factors. A Box-Behnken design is a type of response surface design that does not contain an embedded factorial or fractional factorial design. The Box–Behnken design for 3 factors involves three blocks, in each of which 2 factors are varied through the 4 possible combinations of high and low. The following figure shows a three-factor Box-Behnken design. Points on the diagram represent the experimental runs that are done.
<table>
<tr><th>Run</th><th>x_1</th><th>x_2</th><th>x_3</th></tr>
<tr><td>1</td><td>−1</td><td>−1</td><td>0</td></tr>
<tr><td>2</td><td>−1</td><td>1</td><td>0</td></tr>
<tr><td>3</td><td>1</td><td>−1</td><td>0</td></tr>
<tr><td>4</td><td>1</td><td>1</td><td>0</td></tr>
<tr><td>5</td><td>−1</td><td>0</td><td>−1</td></tr>
<tr><td>6</td><td>−1</td><td>0</td><td>1</td></tr>
<tr><td>7</td><td>1</td><td>0</td><td>−1</td></tr>
<tr><td>8</td><td>1</td><td>0</td><td>1</td></tr>
<tr><td>9</td><td>0</td><td>−1</td><td>−1</td></tr>
<tr><td>10</td><td>0</td><td>−1</td><td>1</td></tr>
<tr><td>11</td><td>0</td><td>1</td><td>−1</td></tr>
<tr><td>12</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>13</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>14</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>15</td><td>0</td><td>0</td><td>0</td></tr>
</table>

</td><td>4</td><td>2</td><td>4</td></tr>
<tr><td>27</td><td>Analysis of Variance (abbreviated as ANOVA). The ANOVA technique enables us to perform to examine the significance of the difference amongst more than two sample means at the same time. ANOVA is a procedure for testing the difference among different data groups for homogeneity. The responses for each factor level have a normal population distribution. These distributions have the same variance. The data are independent.</td><td>4</td><td>2</td><td>5</td></tr>
</table>

Part C

<table>
<tr><th>Q.No</th><th>Answer</th><th>Marks</th><th>BL</th><th>CO</th></tr>
<tr><td>28
(a)
(i)</td><td>
Types are:
- Best guess approach First, suppose the initial best-guess does not produce the desired results. Now the experimenter has to take another guess at the correct combination of factor levels. This could continue for a long time, without any guarantee of success. Second, suppose the initial best-guess produces an acceptable result. Now the experimenter is tempted to stop testing, although there is no guarantee that the best solution has been found.
- Conventional strategy (One factor at a time; Several factors, one at a time; Several factors, all at the same time). The OFAT method consists of selecting a starting point, or baseline set of levels, for each factor, and then successively varying each factor over its range with the other factors held constant at the baseline level. After all tests are performed, a series of graphs are usually constructed showing how the response variable is affected by varying each factor with all other factors held constant. Not possible to attribute the change in result to any of the factor/s. Effect of interaction between factors cannot be studied. These are poor experimental strategies and there is no scientific basis. The results cannot be validated.
<table>
<tr><th>Trial</th><th>Factor</th><th>Test Result</th><th></th><th>Test Average</th></tr>
<tr><td>1</td><td>A1</td><td>-</td><td>-</td><td>Y1</td></tr>
<tr><td>2</td><td>A2</td><td>-</td><td>-</td><td>Y2</td></tr>
</table>
<table>
<tr><th></th><th colspan="4">Factor</th><th colspan="2">Test Result</th><th>Test</th></tr>
<tr><th>Trial</th><th>A</th><th>B</th><th>C</th><th>D</th><th></th><th></th><th>Average</th></tr>
<tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>-</td><td>-</td><td>Y1</td></tr>
<tr><td>2</td><td>2</td><td>1</td><td>1</td><td>1</td><td>-</td><td>-</td><td>Y2</td></tr>
<tr><td>3</td><td>1</td><td>2</td><td>1</td><td>1</td><td>-</td><td>-</td><td>Y3</td></tr>
<tr><td>4</td><td>1</td><td>1</td><td>2</td><td>1</td><td>-</td><td>-</td><td>Y4</td></tr>
<tr><td>5</td><td>1</td><td>1</td><td>1</td><td>2</td><td>-</td><td>-</td><td>Y5</td></tr>
</table>
<table>
<tr><th></th><th colspan="4">Factor</th><th colspan="2">Test Result</th><th>Test</th></tr>
<tr><th>Trial</th><th>A</th><th>B</th><th>C</th><th>D</th><th></th><th></th><th>Average</th></tr>
<tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>-</td><td>-</td><td>Y1</td></tr>
<tr><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td><td>-</td><td>-</td><td>Y2</td></tr>
</table>
- Strategic design: Example, full factorial design, it is balanced and also orthogonal, both factor and interaction effects can be estimated.
<table>
<tr><th></th><th colspan="2">Factor</th><th colspan="2"></th></tr>
<tr><th>Trial</th><th>A</th><th>B</th><th colspan="2">Response</th></tr>
<tr><td>1</td><td>1</td><td>1</td><td>-</td><td>-</td></tr>
<tr><td>2</td><td>1</td><td>2</td><td>-</td><td>-</td></tr>
<tr><td>3</td><td>2</td><td>1</td><td>-</td><td>-</td></tr>
<tr><td>4</td><td>2</td><td>2</td><td>-</td><td>-</td></tr>
</table>
</td><td>6</td><td>2</td><td>1</td></tr>
<tr><td>28
(a)
(ii)</td><td>
For modern industrial processes, the interactions between the factors or process parameters are a major concern to many engineers and managers, and therefore should be studied, analyzed and understood properly for problem solving and process optimisation problems. For many process optimisation problems in industries, the root cause of the problem is sometimes due to the interaction between the factors rather than the individual effect of each factor on the output performance characteristic (or response).
Interactions occur when the effect of one process parameter depends on the level of the other process parameter. In other words, the effect of one process
</td><td>6</td><td>2</td><td>1</td></tr>
</table>

<table>
<tr><td></td><td>parameter on the response is different at different levels of the other process parameter. In order to study interaction effects among the process parameters, we need to vary all the factors simultaneously.
To determine whether two process parameters are interacting or not, one can use a simple but powerful graphical tool called **interaction graphs**. If the lines in the interaction plot are **parallel**, there is no interaction between the process parameters. This implies that the change in the mean response from low to high level of a factor does not depend on the level of the other factor.
On the other hand, if the lines are **non-parallel**, an interaction exists between the factors. The greater the degree of departure from being parallel, the stronger the interaction effect.
Synergistic interaction: Combined effect of two factors is greater than the individual factors on the response.
Antagonistic Interaction: Combined effect of two factors is lesser than the individual factors on the response.

</td><td></td><td></td><td></td></tr>
<tr><td>28
(b)
(i)</td><td>**Randomization:** A statistical tool used to minimize potential uncontrollable biases in the experiment by randomly assigning material, people, order in the experimental trails to be conducted. The purpose of randomization is to remove bias and variation. Failure to randomize the trial conditions mitigates/reduces the statistical validity of an experiment. In other words, randomisation can ensure that all levels of a factor have an equal chance of being affected by noise factors.
Blocking: A technique used to increase the precision of an experiment by breaking the experiment into homogeneous segments (blocks) in order to control block variability. It basically deals with noise factors. The main purpose of the principle of blocking is to increase the efficiency of an experimental design by decreasing the experimental error.</td><td>6</td><td>2</td><td>1</td></tr>
<tr><td>28
(b)
(ii)</td><td>The conventional strategy has various types including One factor at a time; several factors, one at a time; and several factors, all at the same time. Disadvantages: OFAT method fails to consider any possible interaction between the factors. Interactions between factors are very common, and if they occur, the OFAT strategy will usually produce poor results. Not possible to attribute the change in result to any of the factor/s. Effect of interaction between factors cannot be studied. These are poor experimental strategies and there is no scientific basis. The results cannot be validated.</td><td>6</td><td>2</td><td>1</td></tr>
<tr><td>29
(a)</td><td>**A:** time on the stove → 160 or 200 sec (numerical factor)
B: type of popcorn → white or yellow corn (categorical factor)
It is 2^2 factorial design.<table><tr><th>Factor</th><th>Low Level (-1)</th><th>High Level (+1)</th></tr><tr><td>**Types of corn**</td><td>White</td><td>Yellow</td></tr><tr><td>**Heating Time**</td><td>160 sec</td><td>200 sec</td></tr></table></td><td>12</td><td>3</td><td>2</td></tr>
</table>

		Heating Time		
		160 sec (-1)	200 sec (+1)	
Corn	White (-1)	50	70	120
	Yellow (+1)	60	80	140
		110	150	

Effect of time on the amount of popcorn = (150/2) – (110/2) = 20.
When the heating time increased from 160 sec to 200 sec, the increase of total amount of popcorns measured **20 pieces.**

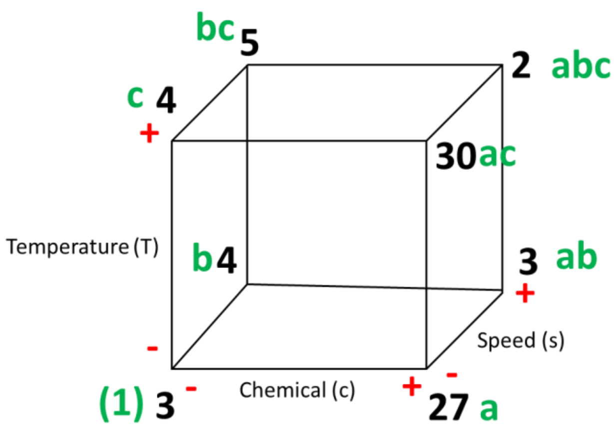
In the same way effect of corn type on the amount of popcorn measured = (140/2) – (120/2) = 10.

In here when the popcorn type switched white to yellow, the increase of the total number of popcorns measured **10 pieces.**

As a result, the heating time effect is larger than corn type.

29 (b)

Std. Order	Run Order	C (Chemicals)	T (Temp)	S (Speed)	Outcomes	
1	6	-1	-1	-1	3	(1)'
2	2	1	-1	-1	27	a
3	5	1	1	1	4	b
4	3	1	1	-1	30	ab
5	7	-1	-1	1	4	c
6	1	1	-1	1	3	ac
7	8	-1	1	1	5	bc
8	4	1	1	1	2	abc



For Chemical (C):

Temp (T)	Speed (S)	
+	+	2 – 5 = -3
+	-	30 – 4 = 26
-	+	3 -4 = -1
-	-	27 – 3 = 24
Average		= 11.5

Hence, Y = (11.5/2)Xc = 5.75 Xc, where, **5.75 is the coefficient** for Chemical (Xc).
Intercept = (3+27+3+30+2+4+4+5)/8 = **9.75**

12 3 2

<table>
<tr>
<td>30
(a)
(i)</td>
<td>Step 1: Choose control factors and their levels
Step 2: Identify uncontrollable (noise) factors and decide on how they will be simulated.
Step 3: Select the response variable(s) and determine the performance measures (mean, standard deviation, SNR, etc.)
Step 4: Setup the experimental layout (choose appropriate design array (s))
Step 5: Conduct the experiments and collect data
Step 6: Analyze the data (effects, ANOVA, regression)
Step 7: Choose optimal control factor levels and predict the performance measure at these levels
Step 8: Confirm the optimal levels by experimentation.</td>
<td>8</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>30
(a)
(ii)</td>
<td>The Taguchi Loss Function (or Quality Loss Function) is a mathematical representation that quantifies the relationship between product quality and the financial loss (loss to society) associated with deviations from a target value. The Taguchi Loss Function is an equation used to quantify the financial loss incurred due to deviations in product quality from the target value. It is based on the principle that any deviation from the ideal or target value increases costs associated with poor quality, customer dissatisfaction, or product failures.
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Where:
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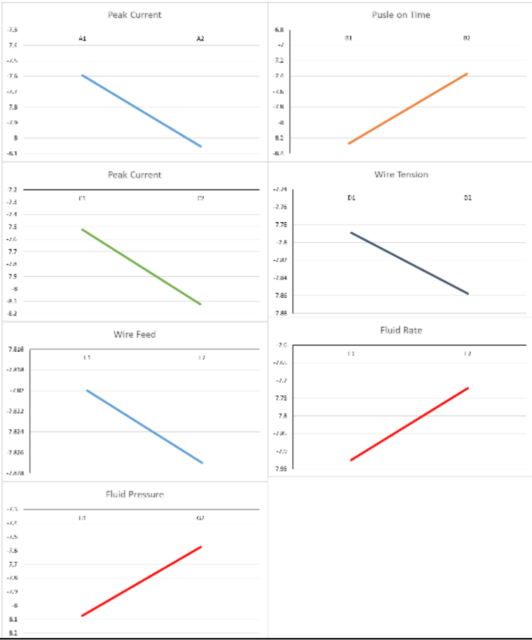
x is the actual value of the quality characteristic being evaluated.
T is the target value, the ideal or optimal value of the quality characteristic.</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>30
(b)</td>
<td>Since, the number of factors given are 7 with 2 levels. Hence, we can choose L8 orthogonal array. Smaller-the-Better: $\eta = -10 \log10(y^2)$
<table>
<tr><th rowspan="2">Expt. No</th><th colspan="7">Columns</th><th rowspan="2">Surface Roughness (y)</th><th rowspan="2">η</th><th rowspan="2"></th></tr>
<tr><th>Peak current (A)</th><th>Pulse on time (B)</th><th>Peak Current(C)</th><th>Wire Tension (D)</th><th>Wire Feed (E)</th><th>Fluid Rate (F)</th><th>Fluid Pressure (G)</th></tr>
<tr><td>1</td><td>125 (1)</td><td>35 (1)</td><td>11 (1)</td><td>1000 (1)</td><td>7 (1)</td><td>8 (1)</td><td>13 (1)</td><td>2.5275</td><td>-8.053823285</td><td>η1</td></tr>
<tr><td>2</td><td>125 (1)</td><td>35 (1)</td><td>11 (1)</td><td>1200 (2)</td><td>8 (2)</td><td>9 (2)</td><td>15 (2)</td><td>2.3520</td><td>-7.428746348</td><td>η2</td></tr>
<tr><td>3</td><td>125 (1)</td><td>40 (2)</td><td>12 (2)</td><td>1000 (1)</td><td>7 (1)</td><td>9 (2)</td><td>15 (2)</td><td>2.2540</td><td>-7.059078234</td><td>η3</td></tr>
<tr><td>4</td><td>125 (1)</td><td>40 (2)</td><td>12 (2)</td><td>1200 (2)</td><td>8 (2)</td><td>8 (1)</td><td>13 (1)</td><td>2.4650</td><td>-7.836338472</td><td>η4</td></tr>
<tr><td>5</td><td>130 (2)</td><td>35 (1)</td><td>12 (2)</td><td>1000 (1)</td><td>8 (2)</td><td>8 (1)</td><td>15 (2)</td><td>2.7000</td><td>-8.627275283</td><td>η5</td></tr>
<tr><td>6</td><td>130 (2)</td><td>35 (1)</td><td>12 (2)</td><td>1200 (2)</td><td>7 (1)</td><td>9 (2)</td><td>13 (1)</td><td>2.8125</td><td>-8.981850622</td><td>η6</td></tr>
<tr><td>7</td><td>130 (2)</td><td>40 (2)</td><td>11 (1)</td><td>1000 (1)</td><td>8 (2)</td><td>9 (2)</td><td>13 (1)</td><td>2.350</td><td>-7.421357245</td><td>η7</td></tr>
<tr><td>8</td><td>130 (2)</td><td>40 (2)</td><td>11 (1)</td><td>1200 (2)</td><td>7 (1)</td><td>8 (1)</td><td>15 (2)</td><td>2.2875</td><td>-7.187222055</td><td>η8</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>μ</td><td>-7.82446144</td><td></td></tr>
</table>
$mA_1 = 1/4 (\eta_1 + \eta_2 + \eta_3 + \eta_4) = 0.25 * (-8.053-7.428-7.059-7.836) = -7.5945$
$mA_2 = -8.053$;
$mB_1 = -8.272$;
$mB_2 = -7.37$;
$mC_1 = -7.522$;
$mC_2 = -8.125$;
$mD_1 = -7.789$;
$mD_2 = -7.858$;
$mE_1 = -7.82$;
$mE_2 = -7.827$;
$mF_1 = -7.925$;
$mF_2 = -7.722$;
$mG_1 = -8.072$;
$mG_2 = -7.575$;</td>
<td>12</td>
<td>3</td>
<td>3</td>
</tr>
</table>

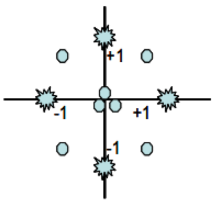
A1, B2, C1, D1, E2, F2, G2 are giving the minimum effect in reducing the surface roughness and these are the optimum process parameters.



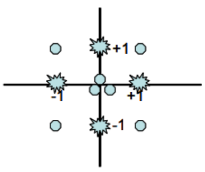
31 (a)

Central Composite designs can fit a full quadratic model. They are often used when the design plan calls for sequential experimentation because these designs can include information from a correctly planned factorial experiment. A central composite design is the most commonly used response surface designed experiment. Central composite designs are a factorial or fractional factorial design with center points, augmented with a group of axial points (also called star points) that let you estimate curvature. Central composite designs are especially useful in sequential experiments because you can often build on previous factorial experiments by adding axial and center points.

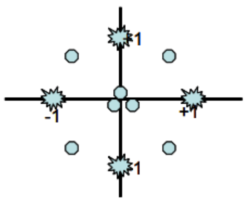
Central Composite Design (CCD) has three different design points: edge points as in two- level designs (±1), star points at ±α; |α1≥| | that take care of quadratic effects and centre points. Three variants exist: Circumscribed (CCC); Face centered (CCF); Inscribed (CCI).



. CCC design for two factors.



. CCF design for two factors.



. CCI design for two factors

CCD (Contd.)

		K=2	K=3	K=4	K=5
CCD	Factorial points	4	8	16	32
	Axial points	4	6	8	10
	Centre points	5	5	6	6
	Total	13	19	30	48
3^k Designs		9	27	81	243
Choice of α	Spherical	1.4	1.73	2	2.24
	Rotatable	1.4	1.68	2	2.38

12 | 3 | 4

31 (b)

Box-Behnken designs have treatment combinations that are at the midpoints of the edges of the experimental space and require at least three continuous factors.

12 | 3 | 4

A Box-Behnken design is a type of response surface design that does not contain an embedded factorial or fractional factorial design.
For example, you would like to determine the best conditions for injection-molding a plastic part. The factors you can set are:

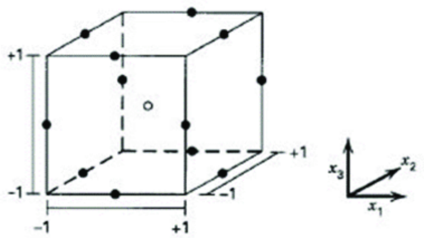
- Temperature: 190° and 210°
- Pressure: 50Mpa and 100Mpa
- Injection speed: 10 mm/s and 50 mm/s

For a Box-Behnken design, the design points fall at combinations of the high and low factor levels and their midpoints:

- Temperature: 190°, 200°, and 210°
- Pressure: 50Mpa, 75Mpa, and 100Mpa
- Injection speed: 10 mm/s, 30 mm/s, and 50 mm/s

The Box–Behnken design for 3 factors involves three blocks, in each of which 2 factors are varied through the 4 possible combinations of high and low. The following figure shows a three-factor Box-Behnken design. Points on the diagram represent the experimental runs that are done.

Run	x_1	x_2	x_3
1	−1	−1	0
2	−1	1	0
3	1	−1	0
4	1	1	0
5	−1	0	−1
6	−1	0	1
7	1	0	−1
8	1	0	1
9	0	−1	−1
10	0	−1	1
11	0	1	−1
12	0	1	1
13	0	0	0
14	0	0	0
15	0	0	0



32 (a) (i)	Analysis of Variance (abbreviated as ANOVA). The ANOVA technique enables us to perform to examine the significance of the difference amongst more than two sample means at the same time. ANOVA is a procedure for testing the difference among different data groups for homogeneity. The responses for each factor level have a normal population distribution. These distributions have the same variance. The data are independent.	4	4	5
32 (a) (ii)	One Way ANOVA: Only one factor is investigated. One independent variable (With 2 levels). Analysis of Variance could have one independent variable. • Evaluate the difference among the means of three or more groups. • A one way ANOVA is used to compare two means from two independent (unrelated) groups using the F-distribution. • The null hypothesis for the test is that the two means are equal. • Therefore, a significant result means that the two means are unequal. • Alternate hypothesis …? Two means are not equal.. • The independent variable is the categorical variable that defines the compared groups. E.g., instructional methods, grade level, or marital status. • The dependent variable is the measured variable whose means are being compared e.g., level of job satisfaction or text anxiety. **Assumptions:** • Your dependent variable should be measured at the interval or ratio scales (i.e., they are continuous) • Your independent variable should consist of two or more categorical, independent groups.	8	4	5

- You should have independence of observations, meaning there is no relationship between the observations in each group or between the groups themselves.
- There should be no significant outliers. Outliers are simple single data points within your data that do not follow the usual pattern.
- Your dependent variable should be approximately normally distributed for each category of the independent variable.
- There needs to be homogeneity of variances.

Source of Variation	Degrees of Freedom	Sum Of Squares	Mean Square (Variance)	F
Among Groups	c - 1	SSA	MSA = $\frac{SSA}{c - 1}$	$F_{STAT} = \frac{MSA}{MSW}$
Within Groups	n - c	SSW	MSW = $\frac{SSW}{n - c}$	c = number of groups n = sum of the sample sizes from all groups
Total	n − 1	SST		

32 (b)

SUMMARY				
Groups	*Count*	*Sum*	*Average*	*Variance*
Compact Cars	3	18	6	1
Mid Size Cars	3	18	6	4
Full Size Cars	3	17	5.666667	2.333333

ANOVA						
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.222222	2	0.111111	0.045455	0.955889	5.143253
Within Groups	14.66667	6	2.444444			
Total	14.88889	8				

12 | 4 | 5