NEWTON'S LAW OF VISCOSITY

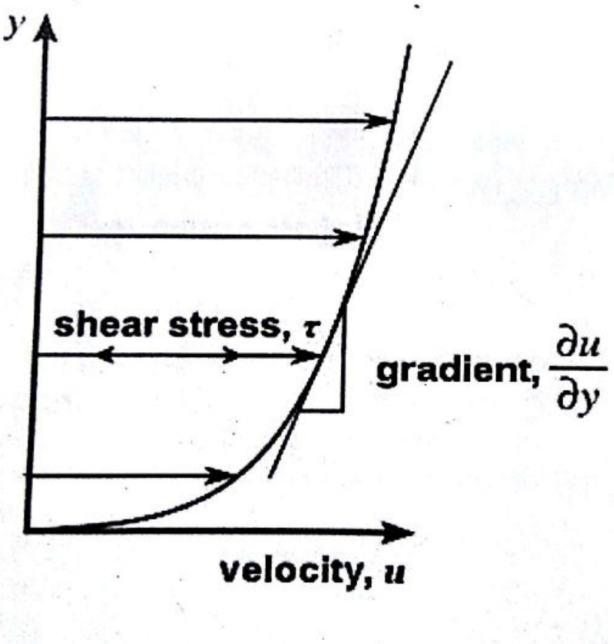
Viscosity is the physical property that characterizes the flow resistance of simple fluids. Newton's law of viscosity defines the relationship between the shear stress and shear rate of a fluid subjected to a mechanical stress. The ratio of shear stress to shear rate is a constant, for a given temperature and pressure, and is defined as the viscosity or coefficient of viscosity. Newtonian fluids obey Newton's law of viscosity. The viscosity is independent of the shear rate.

Non-Newtonian fluids do not follow Newton's law and, thus, their viscosity (ratio of shear stress to shear rate) is not constant and is dependent on the shear rate.

Dynamic viscosity is the coefficient of viscosity as defined in Newton's law of viscosity. Kinematic viscosity is the dynamic viscosity divided by the density.

Viscosity/Dynamic viscosity:

- Viscosity is the physical property that characterizes the flow resistance of simple fluids.
- Viscosity is the property of a fluid by virtue of its offers resistance to the movement of one layer of fluid over an adjacent layer.



Viscosity, $\tau = \mu \frac{du}{dy}$

Where,

μ = Dynamic viscosi

τ = Shear stress= F/A

$\frac{du}{dy}$ = Rate of shear de

Kinematic viscosity/ momentum diffusivity:

- It is defined as the ratio of dynamic viscosity to the density of the fluid.

Kinematic Viscosity, $\nu = \mu/\rho$

Where,.

ν = Kinematic viscosity

μ = Dynamic viscosity

ρ = Density of the fluid

Newton's Law of Viscosity:

Newton's viscosity law's states that, the shear stress between adjacent fluid layers is proportional to the velocity gradients between the two layers.

The ratio of shear stress to shear rate is a constant, for a given temperature and pressure, and is defined as the viscosity or coefficient of viscosity.

Newton's Law of viscosity, $\tau \propto \frac{du}{dy}$

$$\tau = \mu \frac{du}{dy}$$

Where,.

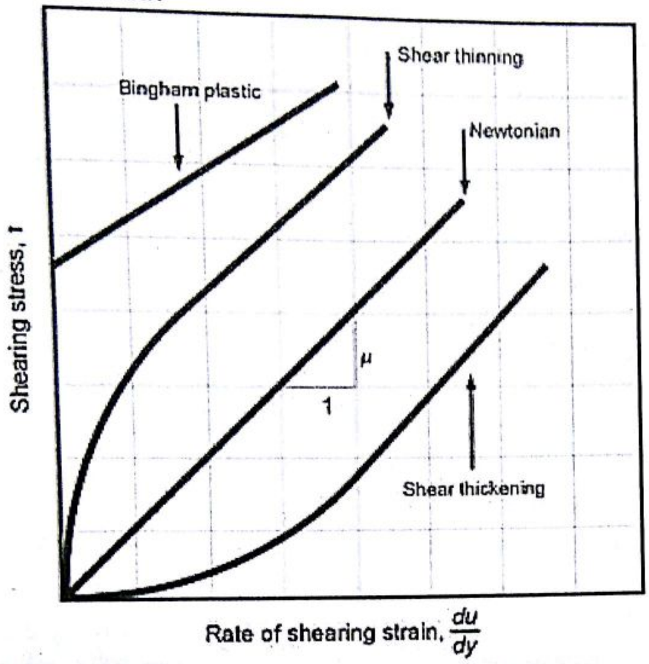
μ = Viscosity

τ = Shear stress= F/A

$\frac{du}{dy}$ = Rate of shear deformation

Newtonian Fluid:

- A fluid, whose viscosity does not change with the rate of deformation or shear stain (V/Y), is called Newtonian fluid.
- A fluid which obeys Newton's law of viscosity is termed as Newtonian fluid.



Non-Newtonian Fluid:

- A fluid, whose viscosity changes with the rate of deformation or shear stain (V/Y), is called Non-Newtonian fluid.
- A fluid which does not obey Newton's law of viscosity is termed as Non-Newtonian fluid.

STEADY AND UNSTEADY FLUID FLOW

We have noted previously (see Velocity Field) that velocity, pressure and other properties of fluid flow can be functions of time (apart from being functions of space). If a flow is such that the properties at every point in the flow do not depend upon time, it is called a steady flow. Mathematically speaking for steady flows,

$$\frac{\partial P}{\partial t} = 0 \qquad (3.4)$$

where P is any property like pressure, velocity or density. Thus,

$$P = P(x, y, z) \qquad (3.5)$$

Unsteady or **non-steady** flow is one where the properties do depend on time.

It is needless to say that any start up process is unsteady. Many examples can be given from everyday life- water flow out of a tap which has just been opened. This flow is unsteady to start with, but with time does become steady.

Some flows, though unsteady, become steady under certain frames of reference. These are called **pseudosteady** flows. On the other hand a flow such as the wake behind a bluff body is always unsteady.

Unsteady flows are undoubtedly difficult to calculate while with steady flows, we have one degree less complexity.

FLUID FLOW AND CONTINUITY EQUATION

Fluids, by definition can flow, but are essentially incompressible. This provides some very useful information about how fluids behave when they flow through a pipe, or a hose. Consider a hose whose diameter decreases along its length, as shown in the Figure below. The ``continuity equation" is a direct consequence of the rather trivial fact that what goes into the hose must come out. The volume of water flowing through the hose per unit time (i.e. the **flow rate** at the left must be equal to the flow rate at the right or in fact anywhere along the hose. Moreover, the flow rate at and point in the hose is equal to the area of the hose at that point times the speed with which the fluid is moving:

flow rate = (area) x (velocity)

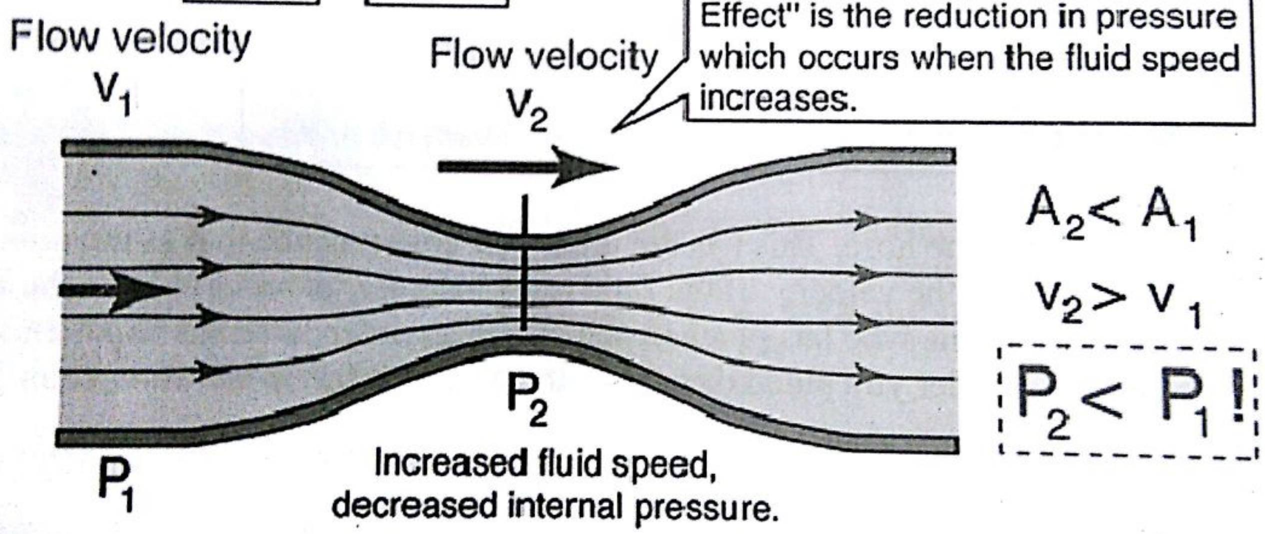
You can easily verify that (area)x(velocity) has units m^3/t which is correct for volume per unit time.

Bernoulli Equation

The Bernoulli Equation can be considered to be a statement of the conservation of energy principle appropriate for flowing fluids. The qualitative behavior that is usually labeled with the term "Bernoulli effect" is the lowering of fluid pressure in regions where the flow velocity is increased. This lowering of pressure in a constriction of a flow path may seem counterintuitive, but seems less so when you consider pressure to be energy density. In the high velocity flow through the constriction, kinetic energy must increase at the expense of pressure energy.

Energy per unit volume before = Energy per unit volume after

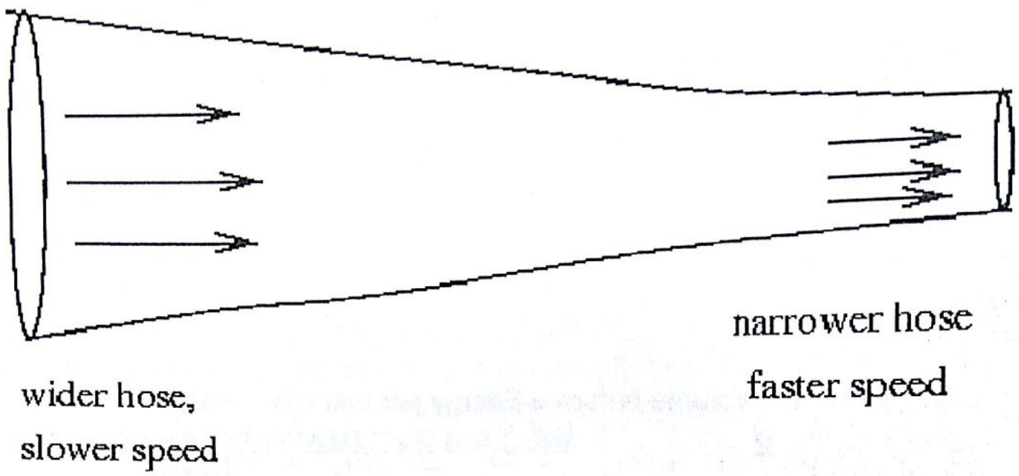
$$\underbrace{P_1}_{\text{Pressure Energy}} + \underbrace{\tfrac{1}{2}\rho v_1^2}_{\text{Kinetic Energy per unit volume}} + \underbrace{\rho g h_1}_{\text{Potential Energy per unit volume}} = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$$



Pressure at a point within a fluid

Consider a fluid at rest as shown in Fig. 2.2. From around the point of interest, P in the fluid let us pull out a small wedge of dimensions **dx x dz** x **ds** . Let the depth normal to the plane of paper be **b.** In some of the derivations we chose z to be the vertical coordinate. This is consistent with the use of z as the elevation or height in many applications involving atmosphere or an ocean. Let us now mark the surface and body forces acting upon the wedge.

Figure 7.1: Fluid flow in a hose of variable size



These considerations lead us directly to the continuity equation, which states that

Area x velocity = constant

everywhere along the hose. This has the important consequence that as the area of the hose decreases, the velocity of the fluid must increase, in order to keep the flow rate constant. Anyone who has pinched one end of a garden hose has experienced this effect: the smaller you pinch the end of the hose, the faster the water comes out.

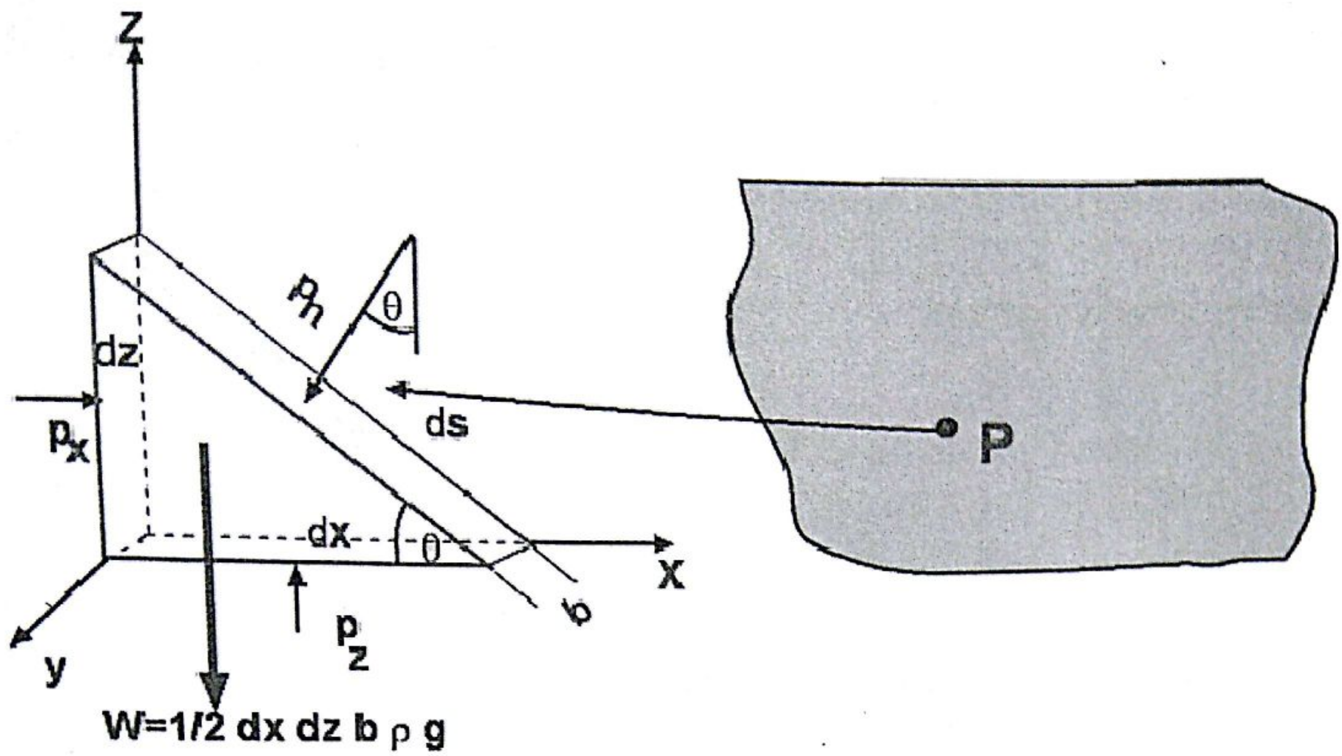


Figure 2.2 : Pressure at a point

The surface forces acting on the three faces of the wedge are due to the pressures, p_x , p_z and p_n as shown. These forces are normal to the surface upon which they act. We follow the usual convention that compression pressure is positive in sign. We again remind ourselves that since the fluid is at rest there is no shear force acting. In addition we have a body force, the weight, W of the fluid within the wedge acting vertically downwards.

Summing the horizontal and the vertical forces we have,

$$\Sigma F_x = 0; \text{ ie.}, p_x \, dz \, b - p_n \, b \, ds \, sin\theta = 0$$

$$\Sigma F_z = 0; \text{ ie.}, p_z \, dx \, b - p_n \, b \, ds \, cos\theta - W = 0$$

$$\text{i.e.}, = p_z \, dx \, b - p_n \, b \, ds \, cos\theta - \frac{1}{2}\rho \, g \, b \, dx \, dz = 0 \qquad (2.1)$$

$$ds\ sin\theta = dz, \quad \text{and} \quad ds\ cos\theta = dx \tag{2.2}$$

we have after simplification,

$$p_x = p_n, \quad \text{and} \quad p_z = p_n + \frac{1}{2}\rho\ g\ dz \tag{2.3}$$

We note that the pressure in the horizontal direction does not change, which is a consequence of the fact that there is shear in a fluid at rest. In the vertical direction there is a change in pressure proportional to density of the fluid, acceleration due to gravity and difference in elevation.

Now if we take the limit as the wedge volume decreases to zero, i.e., the wedge collapses to the point P, we have,

$$p_x = p_z = p_n = p \tag{2.4}$$

This equation is known as Pascal's Law. It is important to note that it is valid only for a fluid at rest. In the case of a moving fluid, pressures in different directions could be different depending upon fluid accelerations in different directions. Hence, for a moving fluid pressure is defined as an average of the three normal stresses acting upon the fluid element.

PRESSURE VARIATION IN A STATIC FLUID

Consider a hypothetical differential cylindrical element of fluid of cross sectional area A and height $(z_2 - z_1)$.

Upward force due to pressure P_1 on the element = P_1A

Downward force due to pressure P_2 on the element = P_2A

Force due to weight of the element = mg = $\rho A(z_2 - z_1)g$

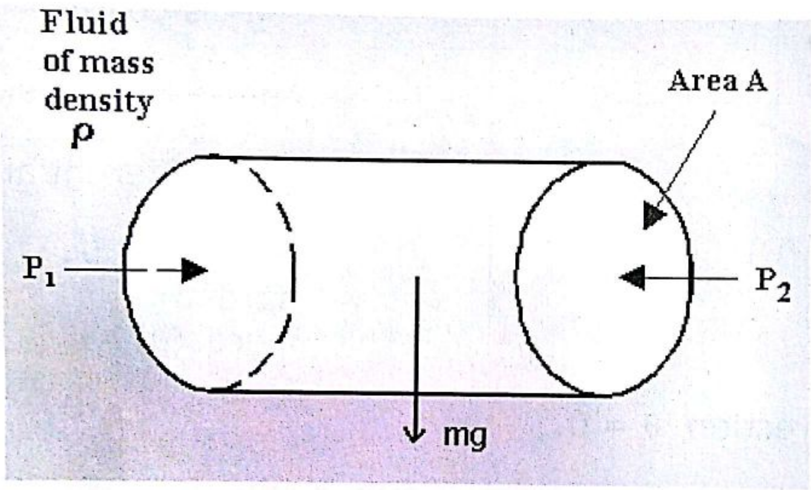
Equating the upward and downward forces,

$P_1A = P_2A + \rho A(z_2 - z_1)g$

$P_2 - P_1 = -\rho g(z_2 - z_1)$

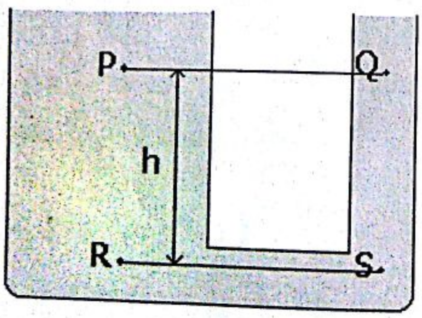
Thus in any fluid under gravitational acceleration, pressure decreases, with increasing height z in the upward direction.

Equality of pressure at the same level in a static fluid:



Equating the horizontal forces, $P_1A = P_2A$ (i.e. some of the horizontal forces must be zero)

Equality of pressure at the same level in a continuous body of fluid:



Pressures at the same level will be equal in a continuous body of fluid, even though there is no direct horizontal path between P and Q provided that P and Q are in the same continuous body of fluid.

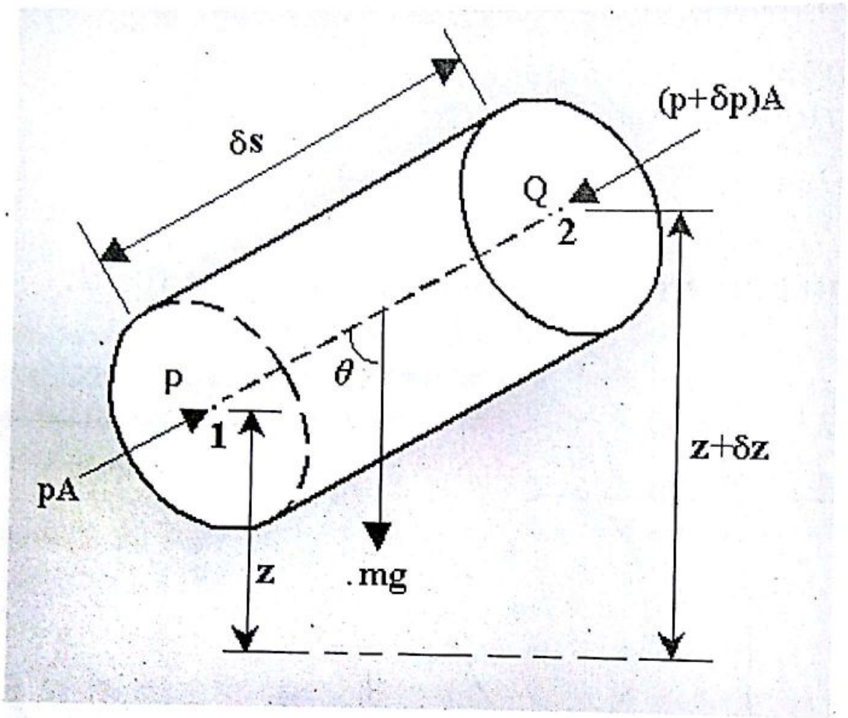
We know that, $P_R = P_S$

$P_R = P_P + \rho gh \rightarrow 1$

$P_S = P_Q + \rho g h \rightarrow 2$

From equn.1 and 2, $P_P = P_Q$

General equation for the variation of pressure due to gravity from point to point in a static fluid:



Resolving the forces along the axis PQ,

$pA - (p + \delta p)A - \rho g A \delta s \cos(\theta) = 0$

$\delta p = - \rho g \delta s \cos(\theta)$

or in differential form,

$dp/ds = - \rho g \cos(\theta)$

In the vertical z direction, $\theta = 0$.

Therefore,

$dp/dz = -\rho g$

This equation predicts a pressure decrease in the vertically upwards direction at a rate proportional to the local density.

Pascal's Law

Proof of Pascal's law

$90-\theta$

θ P_z

Overview

Pascal's Law states,

" The intensity of pressure at any point in a fluid at rest, is the same in all direction."

Contents

Blaise Pascal (1623-1662) wa French mathematician, physic inventor, writer and Catholic philosopher.

Theorem Proof

Consider a very small right angled triangular element **ABC** of a liquid as shown in figure.

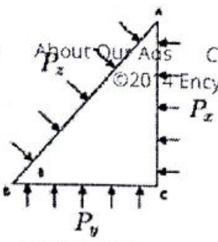


Fig: Element of liquid

Let:

- p_x = Intensity of horizontal pressure on the element of the liquid
- p_y = Intensity of vertical pressure on the element of the liquid
- p_z = Intensity of pressure on the diagonal of the triangular element of the liquid
- θ = Angle of the triangular element of the liquid

Now total pressure on the vertical side **AC** of the liquid,

$$P_x = p_x \times AC \qquad (1)$$

Similarly,total pressure on the horizontal side **BC** of the liquid,

$$P_y = p_y \times BC \qquad (2)$$

and total pressure on the diagonal side **AB** of the liquid,

$$P_z = p_z \times AB \qquad (3)$$

Since the element of the liquid is at rest, therefore sum of the horizontal and vertical components of the liquid pressure must be equal to zero.

Now using eqilibrium condition for horizontal pressure,

$$P_z \sin\theta = P_x$$

$$\Rightarrow p_z.AB.\sin\theta = p_x.AC$$

From the geometry of the figure, we find that,

$$AB\sin\theta = AC$$

$$\therefore p_z.AC = p_x.AC$$

$$\Rightarrow p_z = p_x \qquad (4)$$

Now using equilibrium condition for vertical pressure, I.e.,

$$p_z \cos\theta = p_y - W$$

(where W = Weight of the liquid)

As the triangular element is very small, the weight of the liquid *W* is neglected, so,

$$p_z \cos\theta = p_y$$

$$\therefore p_z.AB.\cos\theta = p_y.BC$$

From the geometry of the figure, we find that

$$AB\cos\theta = BC$$

$$\therefore p_z.BC = p_y.BC$$

$$\Rightarrow p_z = p_y$$

Now from equation *(4)* and *(5)*, we find that

$$p_x = p_y = p_z$$

Thus the intensity of pressure at any point in a fluid, at rest, is the same in all direction.