Delhi Technological University

Department of Applied Mathematics

Assignment III(B.Tech.I Semester-2017)

Calculus of Several Variables

Q1. If $Z = f(x, y)$,$x = r\cosh\theta$,$y = r\sinh\theta$ then

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 - \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

Q2. Verify Euler's theorem for the functions

i) $u = \left(x^{1/2} + y^{1/2}\right)(x^n + y^n)$

ii) $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{x}{y}\right)$

Q3. If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$,prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = \sin 2u$$

Q4. If $w = \sin^{-1}u$,$u = \frac{(x^2+y^2+z^2)}{(x+y+z)}$,then

$$x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z} = \tan w$$

Q5. If $u = \tan^{-1}\frac{y^2}{x}$ prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -2\sin 2u\sin^2 u$$

Q6. If $u = \sin\frac{y}{x} + x\sin^{-1}\frac{y}{x}$,prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$$

Q7. If$z = f(x, y)$,$x = e^{2u} + e^{-2v}$,$y = e^{-2u} + e^{2v}$,then show that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = 2\left[x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}\right]$$

Q8. If $u = f\left(x^2 + 2yz, y^2 + 2zx\right)$ then prove that

$$\left(y^2 - zx\right)\frac{\partial u}{\partial x} + \left(x^2 - yz\right)\frac{\partial u}{\partial y} + \left(z^2 - xz\right)\frac{\partial u}{\partial z} = 0$$

Q9. Find the quadratic Taylor's series expansion of $e^x\cos y$ about the point$(1, \frac{\pi}{4})$

Ans: $\frac{e}{\sqrt{2}} + \left[(x-1)\frac{e}{\sqrt{2}} + (y - \frac{\pi}{4})(-\frac{e}{\sqrt{2}})\right] + \frac{1}{2!}\left[(x-1)^2\frac{e}{\sqrt{2}} + 2(x-1)(y + \frac{\pi}{4})(-\frac{e}{\sqrt{2}}) + (y - \frac{\pi}{4})^2(-\frac{e}{\sqrt{2}})\right]$.

Q10. Using Taylor's series find a quadratic approximation of $\cos x \cos y$ at the origin.

Ans. $1 - \frac{x^2}{2} - \frac{y^2}{2}$.

Q11. Expand $f(x,y) = \tan^{-1} xy$ in powers of $(x-1)$ and $(y-1)$ upto second degree terms using Taylor's series. Hence compute $f(1.1, 0.8)$.

Ans. $\tan^{-1} xy = 0.7854 + \frac{1}{2}(x-1) + \frac{1}{2}(y-1) - \frac{1}{4}(x-1)^2 - \frac{1}{4}(y-1)^2$; and ; $f(1.1, 0.8) = 0.7229$.

Q12. Find the shortest distance between the line $y = 10 - 2x$ and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

Ans. shortest distance is $\sqrt{5}$.

Q13. Find the points on the curve $x^2 + xy + y^2 = 16$, which are the nearest and farthest from the origin.

Ans. The points $(4,-4)$, $(-4,4)$ are farthest, $d^2 = 32$. The points $(\frac{4}{\sqrt{3}}, \frac{4}{\sqrt{3}})$, $(-\frac{4}{\sqrt{3}}, -\frac{4}{\sqrt{3}})$ are nearest $d^2 = \frac{32}{3}$.

Q14. Find the smallest and the largest distance between the points P and Q such that P lies on the plane $x + y + z = 2a$ and Q lies on the sphere $x^2 + y^2 + z^2 = a^2$, where a is any constant.

Ans. $P(\frac{2a}{3}, \frac{2a}{3}, \frac{2a}{3})$, $Q(\pm\frac{a}{\sqrt{3}}, \pm\frac{a}{\sqrt{3}}, \pm\frac{a}{\sqrt{3}})$; $\frac{a}{\sqrt{3}}\sqrt{(7-4\sqrt{3})}$; $\frac{a}{\sqrt{3}}\sqrt{(7+4\sqrt{3})}$

Q15. Show that $f(x,y) = y^2 + x^2 y + x^4$, has a minimum at $(0,0)$.