SECOND SEMESTER

STANLOUX Roll No.

B. TECH. (ALL)

END SEMESTER EXAMINATION

MAY-2011

AM-111 MATHEMATICS-II

Time: 3 Hours

Max. Marks: 70

Note:

Answer ALL questions selecting any TWO from each question.

Assume suitable missing data, if any.

1[a] Discuss the consistency of the system of linear equations. Test the consistency of the following system of linear equations and solve, if consistent, using rank method.

$$x + 2y + z = 3;$$

 $2x + 3y + 2z = 5;$
 $3x - 5y + 5z = 2;$
 $3x + 9y - z = 4.$

3.5, 3.5

[b] Write any two properties of eigen values and eigen vectors and hence find the eigen vectors for the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

[c] Define diagonalization of a matrix. Show that the matrix

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

is diagonalizable and obtain the digaonalizing matrix.

1,6

2, 5

2[a] Solve the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \tan x$$

ob

A mechanical system with two degrees of freedom satisfies the equations

$$2\frac{d^2x}{dt^2} + 3\frac{dy}{dt} = 4; \qquad 2\frac{d^2y}{dt^2} - 3\frac{dx}{dt} = 0$$

Solve for x and y when x, y, $\frac{dx}{dt}$ and $\frac{dy}{dt}$ all vanish at t = 0.

A body executes damped forced oscillations given by $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + b^2x = e^{-kt}\sin\omega t.$

Solve the above equation for both cases when $\omega^2 \neq b^2 - k^2$ and $\omega^2 = b^2 - k^2.$

[b] Show that

State and prove Rodrigue's formula and hence find the Legendre polynomial $P_4(x)$.

Find the Laplace transform of the following functions

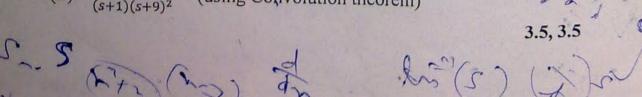
(i)
$$\int_0^t \frac{e^{-2t}sint}{t} dt$$
 (ii) $t^2 e^{-3t}sin2t$

(ii)
$$t^2e^{-3t}\sin 2t$$

Find the Inverse Laplace transform of the functions.

(i)
$$\frac{(s+2)}{(s^2+4s+5)^2}$$

(ii)
$$\frac{1}{(s+1)(s+9)^2}$$
 (using Convolution theorem)



Solve in series the differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$ (i) $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{3-x^2}{x^2} sinx - \frac{3}{x} cosx \right)$ (ii) $J_4(x) = \left(\frac{48}{r^3} - \frac{8}{r}\right)J_1(x) + \left(1 - \frac{24}{r^2}\right)J_0(x)$

[6

[c]

3.5, 3.5

3.5, 3.5

[c] Using Laplace transform to solve

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \mu(t - \pi)\cos(t - \pi)$$

where y(0) = 1, y'(0) = 0. and $\mu(t - \pi)$ is the unit step function.

[a] Find the Fourier series of the function defined in (-2, 2) as

$$f(x) = 2,$$
 $-2 \le x \le 0$
= x, $0 < x < 2.$

[b] Find the Fourier Transform of f(x), where

$$f(x) = 1 - x^2,$$
 $|x| < 1$
= 0 , $|x| > 1.$

and use it to evaluate

$$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) \cos \frac{x}{2} dx.$$

ry for

3.5, 3.5

c] The following values of y give the displacement in inches of a certain machine part for the rotation x of the flywheel. Expand y in the form of a Fourier series upto second harmonic.

			$2\pi/6$				
y:	0	9.2	14.4	17.8	17.3	11.7	0