The wave equation :- Maxwell's eqs. are

(i) $\nabla \cdot D = \rho$ (ii) $\nabla \cdot B = 0$

(iii) $\nabla \times E = -\frac{\partial B}{\partial t}$ (iv) $\nabla \times H = J + \frac{\partial D}{\partial t}$

In a uniform linear media, having permittivity ε and permeability μ then $D = \varepsilon E$, $B = \mu H$, $J = \sigma E$. no charge ($\rho = 0$) $\rho = 0$
Maxwell's eqs. for this medium can be written as

(i) $\nabla \cdot E = 0$ (ii) $\nabla \cdot H = 0$

(iii) $\nabla \times E = -\mu \frac{\partial H}{\partial t}$ (iv) $\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$

$$\text{Curl}(\nabla \times E) = -\mu \frac{\partial}{\partial t}(\nabla \times H)$$ σ = conductivity of medium

Substituting the value of $\nabla \times H$

$$\text{curl curl } E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right)$$

$$= -\mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \quad \text{----} \quad (v)$$

Similarly, taking curl of maxwell's eq. (iv)

$$\text{curl}(\nabla \times H) = -\mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} \quad \text{----} \quad (vi)$$

Using vector identity curl curl A = grad div A $- \nabla^2 A$ eqs. (v) and (vi) becomes

$$\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \quad \text{----} \quad (vii)$$

and

$$\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0 \quad \text{----} \quad (viii)$$

Eqs. (vii) and (viii) are the wave eqs. of e.m. fields in a linear homogeneous medium where charge density is zero.
Its plane e.m. wave solution is of the form

$$E(r,t) = E_0 \cdot e^{ik \cdot r - i\omega t}$$, r is position vector

E_0 is the complex amplitude which is constant in space & time

k is the wave propagation vector defined as

$k = \frac{2\pi}{\lambda} \cdot n = \frac{\omega}{v} n$, n being unit vector along k.
$v \longrightarrow$ phase velocity of wave.

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One of the simplest and familiar wave form is sine wave

$$f(z,t) = A\cos[k(z-vt)+\delta]$$

A is the amplitude, δ is the phase constant.

The time period of wave $T = \frac{2\pi}{kv}$

$\therefore$ The frequency $\nu = \frac{1}{T} = \frac{kv}{2\pi} = \frac{v}{\lambda}$

angular frequency $\omega = 2\pi\nu = kv$

Interms of ω, the wave equation can be written as

$$f(z,t) = A\cos(kz-\omega t+\delta)$$

If the wave is travelling towards left, then

$$f(z,t) = A\cos(kz+\omega t-\delta)$$

The wave eq. can also be represented in complex notation using Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$

$$\therefore f(z,t) = Re\left[A e^{i(kz-\omega t+\delta)}\right]$$

where Re represents the real part of complex no.

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