Parth Johri

2K20/B17/33

Maths Assignment

No :- 5

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1) $f(x) = \begin{cases} 1-|x| & |x|<1 \\ 0 & |x|>1 \end{cases}$

$= \begin{cases} 1+x & -1<x<0 \\ 1-x & 0<x<1 \\ 0 & |x|>0 \end{cases}$

$$F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ipx} f(x)\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \left\{ \int_{-\infty}^{-1} e^{ipx} (0)\, dx + \int_{-1}^{0} e^{ipx}(1+x)\, dx + \int_{0}^{1} e^{ipx}(1-x)\, dx + \int^{\infty} e^{ipx} \cdot 0\, dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-1}^{\infty} e^{ipx}(1+x)\, dx + \int_{0}^{1} e^{ipx}(1-x)\, dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left\{ (1+x)\frac{e^{ipx}}{ip} \Big|_{-1}^{0} - \frac{e^{ipx}}{i^2p^2} \Big|_{-1}^{0} \right] + \left[\frac{(1-x)e^{ipx}}{ip} \Big|_{0}^{1} + \frac{e^{ipx}}{i^2p^2} \right\}$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{1}{ip} + \frac{1}{p^2} - \frac{e^{-ip}}{p^2} - \frac{1}{ip} - \frac{e^{ip}}{p^2} + \frac{1}{p^2} \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{2}{p^2} - \frac{(e^{-ip}+e^{ip})}{p^2} \right] = \frac{1}{\sqrt{2\pi}} \left[\frac{2-2\cos p}{p^2} \right]$$

$$F(f(x)) = \sqrt{\frac{2}{\pi}} \frac{(1-\cos p)}{p^2}$$

2.) $$f(x) = \begin{cases} 0 & x < -1 \\ 1 & -1 < x < 1 \\ 0 & x > 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 0 & |x| > 1 \\ 1 & |x| < 1 \end{cases}$$

$$\Rightarrow F(f(x)) = \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{-1} e^{ipx}(0)\,dx + \int_{-1}^{1} e^{ipx} 1 \cdot dx + \int_{1}^{\infty} e^{ipx}(0)\,dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-1}^{1} e^{ipx}\,dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-1}^{1} \cos px + i \sin px \,dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[2 \int_{0}^{1} \cos px \,dx \right]$$

$$= \frac{2}{\sqrt{2\pi}} \left[\frac{\sin px}{p} \right]_0^1$$

$$= \sqrt{2/\pi} \left(\frac{\sin p}{p} \right)$$

3.) $f(x) = e^{-ax^2}, a > 0$

$$f(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ax^2} e^{-ikx} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a\left(x^2 + \frac{ikx}{a}\right)}$$

$$f(k) = \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} e^{\left[-a\left(x + \frac{ik}{2a}\right)^2 - \frac{k^2}{4a}\right]} \right]$$

$$f(k) = \frac{1}{\sqrt{2\pi}} e^{-k^2/4a} \int_{-\infty}^{\infty} e^{\left[-a\left(x + \frac{ik}{2a}\right)^2\right]} dx$$

Put $y = x + \frac{ik}{2a} \longrightarrow dy = dx$

$$F(k) = \frac{1}{\sqrt{2\pi}} e^{-k^2/4a} \int_{-\infty}^{\infty} e^{-ay^2} dy$$

$$I = 2\int_{0}^{\infty} e^{-ay^2} dy \quad \text{Put } ay^2 = t \rightarrow 2ay\,dy = dt$$

$$I = 2\int_{0}^{\infty} e^{-t} \frac{dt}{2\sqrt{at}} = \frac{1}{\sqrt{a}} \int_{0}^{\infty} e^{-t} t^{1/2} dt = \frac{1}{\sqrt{a}} \Gamma\tfrac{1}{2}$$

$$I = \sqrt{\frac{\pi}{a}}$$

$$f(k) = \frac{1}{\sqrt{2\pi}} e^{-\frac{k^2}{4a}} \cdot \sqrt{\frac{\pi}{a}} = \frac{1}{\sqrt{2a}} e^{-k^2/4a}$$

Put $a = 1/2$ $\therefore$ $\underline{\underline{f(k) = e^{-k^2/2}}}$

4.) $F(x) = \frac{e^{-ax}}{x}$

$$F_s(s) = \int_0^\infty \frac{e^{-ax}}{x} \sin sx \, dx$$

Differentiate wrt 'x'

$$\frac{d F_s(s)}{ds} = \int_0^\infty \frac{e^{-ax}}{x} (x \cos sx) \, dx$$

$$\frac{d F_s(s)}{ds} = \int_0^\infty e^{-ax} (\cos sx) \, dx$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2} [\cos bx + \sin bx]$$

$$\frac{d F_s(s)}{ds} = \frac{1}{a^2+s^2} [0 - 1(-a+0)] = \frac{a}{a^2+s^2}$$

Integrating both sides

$$\int \frac{d f_s(s)}{ds} = \int \frac{a \, ds}{a^2+s^2}$$

$$= a \cdot \frac{1}{a} \tan^{-1}(s/a) + c$$

$$f_s(s) = \tan^{-1}(s/a) + c$$

Put $s=0$

$F_s(0) = c$

$\because F_s(0) = 0$

$\therefore c = 0$

$\therefore F_s(s) = \tan^{-1}(s/a)$

Note :-> $F_s\{f(x)\} = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin px \, dx$

$\therefore F_s(s) = \sqrt{\frac{2}{\pi}} \tan^{-1}\left(\frac{s}{a}\right)$

5.) $f(x) = \begin{cases} 1 & 0 < x < a \\ 0 & x > a \end{cases}$

$$F(f(x)) = \sqrt{\frac{2}{\pi}} \left[\int_0^a \sin px \, dx + \int_a^\infty 0 \cdot dx \right]$$

$$= \sqrt{\frac{2}{\pi}} \left[-\frac{\cos px}{p} \right]_0^a$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{-\cos px}{p} \right]_0^a$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{1}{p} - \frac{\cos ap}{p} \right]$$

$$F_c(f(x)) = \sqrt{2/\pi} \left[\int_0^a \cos px \, dx + \int_a^\infty 0 \cdot \cos px \, dx \right.$$

$$= \sqrt{2/\pi} \left[\frac{\sin px}{p} \right]_0^a$$

$F_e(f(x)) = \sqrt{\frac{2}{\pi}}\left[\frac{\sin ap}{p}\right]$

6.) $f(x) = \begin{cases} x & 0 < x < 1/2 \\ 1-x & 1/2 < x < 1 \\ 0 & x > 1 \end{cases}$

$f_c(s) = \int_0^\infty f(x)\cos(sx)\,dx$

$f_c(s) = \int_0^{1/2} x\cos(sx)\,dx + \int_{1/2}^{1}(1-x)\cos sx\,dx + \int_1^\infty 0 \cdot \cos sx\,dx$

$f_c(s) = \left|\frac{x\sin sx}{s} + \frac{\cos sx}{s^2}\right|_0^{\frac{1}{2}} + \left|\frac{(1-x)\sin(sx)}{s} - \frac{\cos sx}{s^2}\right|_{1/2}^{1}$

$f_c = \left|\frac{\sin(s/2)}{2s} + \frac{\cos(s/2)}{s^2} - 0 - \frac{1}{s^2}\right] + \left|\frac{-\cos x}{s^2}\right.$

$\left.-\frac{\sin(s/2)}{2s} + \frac{\cos(s/2)}{s^2}\right]$

$= \frac{2\cos(s/2)}{s^2} - \frac{1}{s^2}(1+\cos s)$

Note:- $F_c(y(x)) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sx\,dx$

$\therefore F_c(s) = \sqrt{\frac{2}{\pi}}\left[\frac{2\cos(s/2)}{s^2} - \frac{1}{s^2}(\cos x + 1)\right]$

Ans

7) $f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases}$

i) $I = \int_{-\infty}^{\infty} \frac{\sin(ap)\cos(px)}{p}\, dp$ ii) $\int_{0}^{\infty} \frac{\sin p}{p}\, dp$

$$f(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ipx} f(x)\, dx = \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} e^{ipx}\, dx$$

$$f(p) = \frac{1}{\sqrt{2\pi}} \left[\frac{e^{ipx}}{ip}\right]_{-a}^{a} = \frac{2\sin ap}{p\sqrt{2\pi}} \quad p \neq 0 \rightarrow (i)$$

For $p = 0$; it becomes $0/0$ form

$$= \frac{2\cos 0 \cdot a}{\sqrt{2\pi}(\pi)} = \frac{2a}{\sqrt{2}(\pi)}$$

$$\because F(p) = \frac{1}{\sqrt{2}(\pi)} \int_{-\infty}^{\infty} f(x)\, e^{ipx}\, dx$$

$$f(x) = \frac{1}{\sqrt{2}(\pi)} \int_{-\infty}^{\infty} f(p)\, e^{ipx}\, dp$$

$$\begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases} = \frac{1}{\sqrt{2}(\pi)} \int_{-\infty}^{\infty} \frac{2\sin ap\, e^{ipx}}{p\sqrt{2\pi}}\, dp$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin(pa)}{p} e^{-ipx}\, dp$$

$\left(\because e^{-ipx} = \cos px - i\sin px\right)$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin pa}{p}\cos px\, dp - \frac{i}{\pi}\underbrace{\int_{-\infty}^{\infty} \frac{\sin pa \sin px}{p}\, dp}_{0}$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin pa \cdot \cos px}{p}\, dp$$

$$\therefore \int_{-\infty}^{\infty} \frac{\sin pa \cos px}{p}\, dp = \begin{cases} \pi & |x| < a \\ 0 & |x| > a \end{cases}$$

If $x = 0$ & $a = 1$

$$\int_{-\infty}^{\infty} \frac{\sin p}{p}\, dp = \pi \Rightarrow 2\int_{0}^{\infty} \frac{\sin p}{p}\, dp = \pi$$

$$\int_{0}^{\infty} \frac{\sin p}{p}\, dp = \frac{\pi}{2} \quad \text{Ans}$$

8.) $f(x) = \frac{1}{1+x^2}$, $F(x) = \frac{x}{1+x^2}$

$$F_c(s) = \int_0^\infty f(x)\cos(sx)\,dx = \int_0^\infty \frac{1}{1+x^2}\cos(sx)\,dx = I \quad (1)$$

Differentiate wrt s,

$$\frac{dI}{ds} = \int_0^\infty \frac{1}{1+x^2}(x)(-\sin sx)\,dx = -\int_0^\infty \frac{x\sin sx}{1+x^2}\,dx$$

$$\frac{dI}{ds} = -\int_0^\infty \frac{x^2 \sin sx\,dx}{x(1+x^2)}$$

$$\frac{dI}{ds} = -\int_0^\infty \left[\frac{(x^2+1)\sin(sx)}{x(1+x^2)} - \frac{\sin(sx)}{x(1+x^2)}\right]dx$$

$$\frac{dI}{ds} = -\int_0^\infty \frac{\sin(sx)}{x}\,dx + \int_0^\infty \frac{\sin(sx)}{x(1+x^2)}\,dx$$

$$\because \int_0^\infty \frac{\sin sx}{x}\,dx = \frac{\pi}{2}$$

$$= -\frac{\pi}{2} + \int_0^\infty \frac{\sin sx}{x(1+x^2)}\,dx \longrightarrow (2)$$

Differentially wrt "s",

$$\frac{d^2I}{ds^2} = 0 + \int_0^\infty \frac{\cos sx}{1+x^2}\,dx = I = \frac{d^2 s}{ds^2} - I = 0$$

$$(D^2 - 1)I = 0$$

Auxillary eqn is $m^2-1=0 \Rightarrow m=\pm 1$

$\therefore C.F. = c_1 e^{s} + c_2 e^{-s} = I \longrightarrow (3)$

$\frac{dI}{ds} = c_1 e^{s} - c_2 e^{-s} \longrightarrow (4)$

put $s=0$

$$(I)_{s=0} = c_1 + c_2 \longrightarrow (5)$$

$$\left(\frac{dI}{ds}\right)_{s=0} = c_1 - c_2 \rightarrow (6)$$

$$(I)_{s=0} = \int_0^{\infty} \frac{dx}{1+x^2} = \left|\tan^{-1}x\right|_0^{\infty} = \pi/2$$

$$\left(\frac{dI}{ds}\right)_{s=0} = -\frac{\pi}{2}.$$

$\therefore$ eqn (5); $c_1 + c_2 = \pi/2 \Rightarrow c_1 = 0$

eqn (6); $c_1 - c_2 = \pi/2$ $\quad c_2 = \pi/2$

$\therefore I = f_c(s) = F_c(f(x)) = \left(\frac{\pi}{2}\right)e^{-s}$

$\sqrt{\frac{2}{\pi}} \int^{\infty} \frac{1}{1+x^2} \cos(sx)\, dx = \frac{\pi}{2} e^{-s} \sqrt{2/\pi}$ $\left\{ \text{From (3)} \right\}$

$= \sqrt{\pi/2}\, e^{-s}$

Differentiating w.r.t. s.

$$\int_0^\infty \frac{1}{1+x^2}\, x\,(-\sin sx)\,dx = \frac{\pi}{2} e^{-s}(-1)$$

$$-\int_0^\infty \frac{x}{1+x^2}\sin(sx)\,dx = \frac{-\pi}{2} e^{-s}$$

$$\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{x}{1+x^2}\sin sx\,dx = \frac{\pi}{2} e^{-s}\sqrt{2/\pi}$$

$$= \sqrt{\frac{\pi}{2}}\, e^{-s}$$

9.) $f(x) = |\cos x|$, $-\pi < x < \pi$

$\because f(x) = f(-x) \Rightarrow f(x)$ is even function

$\therefore b_n = 0$

1 $\rightarrow f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi x}{b-a}\right)$ Here $b-a = 2\pi$

$|\cos x| = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \rightarrow$ (1)

$a_0 = \frac{2}{b-a}\int_{-\pi}^{\pi} |\cos x|\,dx = \frac{1(2)}{\pi}\left[\int_0^{\pi/2}\cos x\,dx + \int_{\pi/2}^{\pi} -\cos x\,dx\right.$

$$a_0 = 4/\pi$$

$$a_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} |\cos x| \cos nx \, dx = \frac{1}{\pi} \times 2 \int_0^{\pi} |\cos x| (\cos nx) \, dx$$

$$= 2/\pi \left[\frac{1}{2}\int_0^{\pi/2} \cos nx . 2\cos x \, dx + \frac{1}{2}\int_{\pi/2}^{\pi} 2\cos nx (-\cos x) \, dx \right]$$

$$= \frac{1}{\pi}\left[\left| \frac{\sin (n+1)x}{n+1} - \frac{\sin 0}{n+1} + \frac{\sin (n-1)\pi/2}{n-1} - \frac{\sin 0}{n-1} - \frac{\sin (n+1)\pi}{n+1} + \frac{\sin (n+1)\frac{\pi}{2}}{(n+1)} - \frac{\sin (n-1)\pi}{(n-1)} - 1\frac{\sin (n-1)}{(n-1)}\pi/2 \right]$$

$$= 2/\pi \left[\frac{\sin (n\pi/2 + \pi/2)}{(n+1)} + \frac{\sin (n\pi/2 - \pi/2)}{(n-1)} \right]$$

$$= 2/\pi \left[\frac{\cos n\pi/2}{n+1} - \frac{\cos n\pi/2}{n-1} \right]$$

$$= -\frac{4\cos n\pi/2}{\pi} \quad (\text{when } n \neq 1)$$

$a_1 = \frac{2}{\pi}\left[\int_0^{\pi/2} \cos^2 x \, dx - \int_{\pi/2}^{\pi} \cos^2 x \, dx\right] = \frac{2}{\pi}\left[\int_0^{\pi/2} \frac{(\cos 2x + 1)}{2}\right.$

$\left. - \int_{\pi/2}^{\pi} \frac{\cos 2x + 1}{2} \, dx \right]$

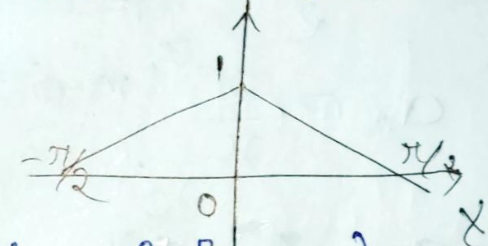
$= \frac{1}{\pi}\left[\frac{\pi}{2} - \pi + \frac{\pi}{2}\right] = 0$

from ①

$$|\cos x| = \frac{2}{\pi} - \frac{4}{\pi} \sum_{n=2}^{\infty} \frac{\cos n\pi/2 \cdot \cos nx}{n^2 - 1}$$

which is the required fourier series

10) $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi < x < 0 \\ 1 - \frac{2x}{\pi} & 0 < x < \pi \end{cases}$



$f(x)$ is an even function of x in $[-\pi, \pi]$ which is clear from graph, it is symmetrical about y-axis,

$\therefore b_n = 0$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} \left(1 - \frac{2x}{\pi}\right) dx = 0$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} \left(1 - \frac{2x}{\pi}\right) \cos nx \, dx = \frac{2}{\pi}\left[\frac{2}{\pi n^2} - \frac{2\cos n\pi}{\pi n^2}\right.$$

$$= \frac{4}{\pi^2 n^2}[1-(-1)^n]$$

$$f(x) = \frac{4}{\pi^2}\sum_{n=1}^{\infty}[1-(-1)^n]\cos\left(\frac{nx}{n^2}\right)$$

$$f(x) = \frac{4}{\pi^2}\left[\frac{2\cos x}{1^2}+\frac{2\cos 3x}{3^2}+\frac{2\cos 5x}{5^2}-\cdots\right]$$

$$f(x) = \frac{8}{\pi^2}\left(\frac{\cos x}{1^2}+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}-\cdots\right)$$

Ans

11.) $f(x) = x;\ 0 \le x \le \pi$

$$x = \frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx \rightarrow (1)$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x\,dx = \pi$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx = \left[\frac{x\sin nx}{n}+\frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$a_n = \frac{2}{n^2\pi}[\cos n\pi - 1]$$

from (1)

$$x = \frac{\pi}{2}-\frac{4\cos x}{\pi}-\frac{4}{3^2\pi}\cos 3x-\frac{4}{5^2\pi}\cos 5x-\cdots$$

$$x = \frac{\pi}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2};\quad 0\le x\le\pi$$

$$f(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} \left[\int_0^{\pi} f(x) \sin(nx) dx \right] \sin nx$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx = \frac{2}{\pi} \int_0^{\pi} x \sin nx \, dx$$

$$b_n = \frac{2}{\pi} \left[-\frac{\pi}{n} \cos n\pi \right] = -\frac{2 \cos n\pi}{n}$$

$$f(x) = \sum_{n=1}^{\infty} -\frac{2 \cos n\pi}{x} (\sin nx)$$

$$f(x) = -2 \left[\frac{-\sin x}{1} + \frac{\sin 2x}{2} - \frac{\sin 3x}{3} - - \right]$$

$$x = 2 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin(nx)}{n}, \quad 0 \le x \le \pi$$

(2) $f(x) = x^2; \; -\pi < x < \pi$

$\because$ $f(x)$ is an even function

$\therefore$ $b_n = 0$

Let $f(x) = x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} x^2 dx = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} x^2 \cos nx \, dx = -\frac{4}{\pi n} \left[\left\{ x \left(-\frac{\cos nx}{n} \right) \right\}_0^{\pi} + \int_0^{\pi} \frac{\cos nx}{n} dx \right]$$

$$a_n = \frac{4}{n^2}(-1)^n$$

$$\therefore x^2 = \frac{\pi^2}{3} - 4\left[\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \frac{\cos 4x}{4^2} - - - - - -\right] \quad (1)$$

Put $x = \pi$ in (1)

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} - - - - - = \frac{\pi^2}{6} \rightarrow (2)$$

Ans Hence Proved

Put $x = 0$ in (1)

$$1/1^2 - 1/2^2 + 1/3^2 - 1/4^2 - - - - - - = \frac{\pi^2}{12} \rightarrow (3)$$

Ans

Hence Proved

Adding (2) & (3)

$$\frac{1}{1^2} + 1/3^2 + 1/5^2 - - - - - - - = \frac{\pi^2}{8}$$ Hence Proved

13.) $f(x) = 2x - x^2 \quad 0 < x < 3$

$b - a = 3, \; f(x) = 2x - x^2$

$\therefore$ The fourier series is given by

$$f(x) = 2x - x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n x}{3}\right)$$

$$+ \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n x}{3}\right) \rightarrow (1)$$

$$a_0 = \frac{2}{3}\int_0^3 (2x - x^2)\,dx = \frac{2}{3}\left[x^2 - \frac{x^3}{3}\right]_0^3 = 0 = a_0$$

$$a_n = \frac{2}{3}\int_0^3 (2x - x^2)\cos\left(\frac{2n\pi x}{3}\right)dx$$

$$a_n = \frac{2}{3}\left| \frac{(2x - x^2)\sin\left(\frac{2\pi n x}{3}\right)}{\left(\frac{2n\pi}{3}\right)} - (2 - 2x)\left\{\frac{-\cos 2n\pi x/3}{4n^2\pi^2/9}\right\} + \frac{\sin 2n\pi x/3}{4n^3\pi^3/27}\right|_0^3$$

$$= \frac{2}{3}\left[\frac{3(-3)}{2n\pi}\sin(2n\pi) - 0 + \frac{(-36)}{4n^2\pi^2}\cos(2n\pi) - \frac{18}{4n^2\pi^2}\cos 0 + \frac{54\sin 2n\pi}{8n^3\pi^3}\right]$$

$$\Rightarrow -\frac{2}{3}\times\frac{9}{n^2\pi^2}\left(\frac{3}{2}\right) \Rightarrow -\frac{9}{\pi^2 n^2}$$

$$b_n = \frac{2}{3}\int_0^3 (2x - x^2)\sin\left(\frac{2n\pi x}{3}\right)dx$$

$$\frac{2}{3}\left| (2x - x^2)\left\{\frac{-\cos(2n\pi x/3)}{2n\pi/3}\right\} - (2 - 2x)\left\{\frac{-\sin \frac{2n\pi}{3}}{4n^2\pi^2/9}\right\} - \frac{2\cos\left(\frac{2n\pi x}{3}\right)}{8n^3\pi^3/27}\right|_0^3$$

$$b_n = \frac{2}{3}\left[\frac{9}{2n\pi}\cos 2n\pi - \frac{36}{4n^2\pi^2}\sin 2n\pi - \frac{18}{4n^2\pi^2}\sin 0 - \frac{27}{4n^3\pi^3}\cos 2n\pi + \frac{27}{4n^3\pi^3}\cos 0\right]$$

$$b_n = \frac{\cancel{2}}{\cancel{3}}\frac{3}{n\pi} \quad \therefore \text{from } (1)$$

$$2x - x^2 = \frac{-9}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\left(\frac{2\pi n x}{3}\right) + \frac{3}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\sin\left(\frac{2n\pi x}{3}\right)\right)$$

Put $x = 3/2$

$$\therefore \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} - \cdots = \frac{\pi^2}{12}$$ Ans

14) $f(x) = \begin{cases} \pi x & 0 < x < 1 \\ (2-x)\pi & 1 < x < 2 \end{cases}$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x + \sum_{n=1}^{\infty} b_n \sin n\pi x$$

Then, $a_0 = \int_0^1 \pi x\, dx + \int_1^2 \pi(2-x)\, dx = \pi$

$$a_n = \int_0^1 \pi x \cos(n\pi x)\, dx + \int_1^2 \pi(2-x)\cos(n\pi x)\, dx$$

$$a_n = \left[\pi x \cdot \frac{\sin \pi n x}{n\pi} - \pi\left(\frac{-\cos x n\pi}{n^2\pi^2}\right)\right] + \left[\pi(2-x)\frac{\sin(n\pi)}{n\pi} + \pi - \cancel{\cos n\pi} - \frac{\cos n\pi x}{n^2\pi^2}\right]$$

$$= \frac{2}{n^2\pi}(\cos n\pi - 1) = \frac{2}{n^2\pi}[(-1)^n - 1] = 0 \text{ or } -\frac{4}{n^2\pi}$$

$$b_n = \int_0^1 \pi x \sin n\pi x\,dx + \int_1^2 \pi(2-x)\sin n\pi x\,dx$$

$$= \left[\pi x\left(-\frac{\cos n\pi x}{n\pi}\right) - \pi\left(-\frac{\sin n\pi x}{n^2\pi^2}\right)\right]_0^1 +$$

$$= \left[\pi(2-x)\left(-\frac{\cos n\pi x}{n\pi}\right) + \pi\left(-\frac{\sin n\pi x}{n^2\pi^2}\right)\right]_1^2$$

$$b_n = 0$$

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left[\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + \cdots\right]$$

Put $x = 0$

$$\Rightarrow f(x) = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} - - - = \frac{\pi}{2} \times \frac{\pi}{4} =$$

$$f(x) = \frac{\pi^2}{8} \text{ Ans}$$

Hence Proved