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De-Broglie λ of particle in a box is:

$\lambda = \frac{2l}{n}$. $l \rightarrow l$ is length.

& $E_n = \frac{n^2h^2}{8ml^2}$ where, m is mass.

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(*) How did the classical theory fail to explain the particle behaviour? How the failure of classical theory leads to the formation of Quantum theory.

(*) What do you mean by compton effect? Show that compton effect is given by $\Delta\lambda = \frac{h}{m_0 c}(1-\cos\phi)$.

(*) Show that pair production cannot occur in empty space.

Failure of Classical Theory :-

The overwhelming sucess of classical physics, classical Electromagnetism made us believe that the ultimate description of nature has been achieved and every physical phenomenon can be explained in general framework of radiation and matter. But in 20th century, classical physics has been challenged by 2 major fronts:

(i) Relativistic Domains :- Einstein's theory of relativity showed that the velocity

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(11) Microscopic Domain: Classical physics failed in providing proper explaination of newly discovered phenomena ~~sue~~ at microscopic level such as black body radiation, photoelectric effect, atomic stability, etc.
So, in 1900 max planck introduced the concept of quantum of energy to explain the phenomena

* Quantum ~~Effect~~ Mechanics :-

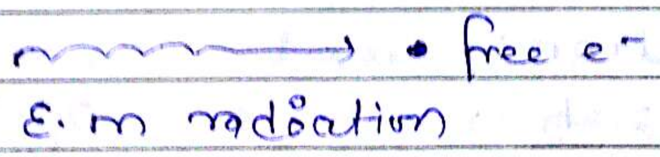
Compton Effect :-
→ Assumptions :-
Compton applied Einstins Quantum theory of light with the assumption

that the incident photon possed momentum in addition to energy.
(1) The beam of incident monochromatic of frequency ν consist of photon of each of energy $h\nu$

and momentum $p = \frac{h\nu}{c}$ along the direction of beam with velocity c.

(ii) The scattering is due to the elastic collision betⁿ photon and free electron at rest. Hence, the conservation laws of Energy & momentum holds good in Scattering Process.

Compton Effect: Reduction in the energy of Electromagnetic Radiations when they are scattered by free Electron is called as Compton Effect.



Energy of Scattered Photon < Energy of incident photon

or

freq. of Scattered Photon < freq. of incident photon

or

Wavelength of Scattered Photon > Wavelength of incidend photon

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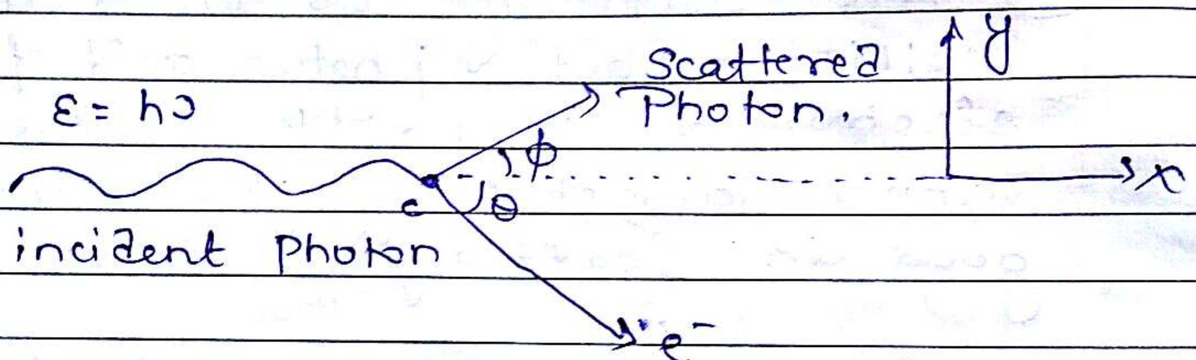
When Photon falls or strikes electron then photon transfer a part of Energy to the electron.



	Photon		Electron	
	Before Collision	A.C.	B.C.	A.C.
Energy	$h\nu$	$h\nu'$	m_0c^2	mc^2
Momentum	$\frac{h\nu}{c}$	$\frac{h\nu'}{c}$	0	mv

1) Derivation:-

$$p = \frac{h}{\lambda} = \frac{h\nu}{c} \quad (c = \lambda\nu)$$

As collision is relativistic, energy of e^- after collision is :-

$$\varepsilon^2 = (mc^2)^2$$

After collision: $\varepsilon^2 = m_0^2c^4 + p^2c^2$.

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$E^2 = m_0^2c^4 \rightarrow$ ($p = 0$ before collision)

$h\nu + m_0c^2 = h\nu' + mc^2$ — (i)

↳ Conservation of Energy.

Momentum is a vector quantity & in collision b/w 2 bodies, it is conserved in each ~~each~~ of 2 mutually ⊥r direction :-

In 'x' dirⁿ :- $\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + mv\cos\theta$ ↳ (2)

In 'y' dirⁿ :- $0 + 0 = \frac{h\nu'\sin\phi}{c} - mv\sin\theta$ ↳ (3)

from eqⁿ (i) :-

$$mc^2 = h(\nu - \nu') + m_0c^2$$

Squaring :-

$$m^2c^4 = h^2(\nu - \nu')^2 + m_0^2c^4 + 2h(\nu - \nu')m_0c^2$$

$$E^2 = m^2c^4 = h^2(\nu^2 + \nu'^2 - 2\nu\nu') + m_0^2c^4 + 2h(\nu - \nu')m_0c^2$$ ↳ (4)

From eqⁿ (2), multiply by 'c'.

$$h\nu = h\nu'\cos\phi + mvc\cos\theta$$

$$mvc\cos\theta = h\nu - h\nu'\cos\phi \quad \text{— (5)}$$

From eqⁿ (3), multiply by 'c'

$$h\nu'\sin\phi = mvc\sin\theta \quad \text{— (6)}$$

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Square and add; 5 & 6;

$$m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2\cos\phi - 2h^2\nu\nu'\cos\phi + h^2\nu'^2\sin^2\phi$$

$$m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\phi \quad - (7)$$

$$E^2 = p^2c^2 + m_0^2c^4$$

$$\therefore m_0^2c^4 = E^2 - p^2c^2 \quad - (8)$$

now: (4) - (7); we get:

$$m^2c^4 - p^2c^2 = h^2(\nu^2 + \nu'^2 - 2\nu\nu') + m_0^2c^4 + 2h(\nu - \nu')m_0c^2 - h^2\nu^2 - h^2\nu'^2 + 2h^2\nu\nu'\cos\phi$$

from (8):

$$m_0^2c^4 = 2h^2\nu\nu'(\cos\phi - 1) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

$$-2h^2\nu\nu'(\cos\phi - 1) = 2h(\nu - \nu')m_0c^2$$

$$\left(\frac{\nu - \nu'}{\nu\nu'}\right) = \frac{h}{m_0c^2}(1 - \cos\phi)$$

$$\left(\frac{1}{\nu'} - \frac{1}{\nu}\right) = \frac{h}{m_0c^2}(1 - \cos\phi)$$

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$$\left(\nu = \frac{c}{\lambda} ; \nu' = \frac{c}{\lambda'}\right) \Rightarrow \left(\frac{\lambda'}{c} - \frac{\lambda}{c}\right) = \frac{h}{m_0 c^2}(1-\cos\phi)$$

This gives:-

$$(\lambda' - \lambda) = \Delta\lambda = \frac{h}{m_0 c}(1-\cos\phi)$$

Compton's Wavelength :-

$$\lambda_c = \frac{h}{m_0 c} = 0.02426\ \text{Å}$$

(1) $(\Delta\lambda)_{max} = \phi = 180^\circ = (\Delta\lambda = 2\lambda_c)$

(2) $(\Delta\lambda)_{min} = \phi = 0^\circ = (\Delta\lambda = 0)$.

Conclusions:-

(i) Compton shift is independent of the wavelength of incident radiation and scattering nature of material.

(ii) Compton shift depends upon scattering angle ϕ.

(iii) Larger is the rest mass of the scatterer smaller the Compton shift.

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(*) Find the wavelength of waves associated with an electron having energy 1 MeV.

De-Broglie Relation

1. It connects the momentum (characteristics of a particle) with the wavelength (ch. of a wave).

2. Consider an e-m wave of energy 'ε' and freq. 'ν', energy of photon acc. to quantum theory of radiation is

$$\varepsilon = h\nu \quad -(i)$$

Energy of relativistic particle, $\varepsilon = mc^2$ —(ii)

m → mass of photon.

$$h\nu = mc^2$$

$$\frac{h\nu}{c} = mc$$

$$\frac{h}{\lambda} = mc$$

$$p = \frac{h}{\lambda} \quad \text{or} \quad \lambda = \frac{h}{p} \quad -(3)$$

If 'v' is velocity of particle of mass 'm';

$$mv = \frac{h}{\lambda} \quad -(4)$$

now, if an electron having charge 'e' & mass 'm_0' is accelerated through a p.d. of 'V' volts, then;

energy = eV

As e's are free, this energy will appear in the form of K.E.

$$E_K = \frac{1}{2} m_0 v^2$$

$$\therefore v = \sqrt{\frac{2eV}{m_0}} \quad - (5)$$

$$\lambda = \frac{h}{m_0 v} \quad (\text{from } - ③)$$

$$\lambda = \frac{h}{m_0 \sqrt{\frac{2eV}{m_0}}}$$

$$\lambda = \frac{h}{\sqrt{2m_0 eV}} = \left(\frac{h}{\sqrt{2m_0 e}}\right)\left(\frac{1}{\sqrt{V}}\right) = \frac{12.28 \mathring{A}}{\sqrt{V}}$$

(i) lighter the particle, more will be de-broglie wavelength.

(ii) faster the particle moves, lower will be its de-Broglie wavelength.

(iii) de-Broglie wavelength is independent of charge or nature of particle.

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Q. Find the λ of waves having energy 1 MeV. 0.0087 Å

Q. Cal. the λ of an e^- moving with vel. 3.5 c. 0.0323 Å.

Case 1:

When K.E. of particle is small as compared to m_0c^2;

$$\lambda = \frac{h}{\sqrt{2m_0K}}$$

Case 2 :- $\lambda = \frac{h}{p} = \frac{hc}{\sqrt{K(K+2m_0c^2)}} = \frac{h}{\sqrt{2m_0K\left(1+\frac{K}{2m_0c^2}\right)}}$

If K.E. << $2m_0c^2$;

$\frac{E_K}{2m_0c^2} \rightarrow$ negligible

$$\therefore \lambda = \frac{h}{\sqrt{2m_0K}} = \frac{h}{\sqrt{2m_0Ve}}$$

$$\lambda = \frac{12.28}{\sqrt{V}} \text{ Å}$$

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$K.E + m_0c^2 = T.E.$

$\sqrt{p^2c^2 + m_0^2c^4} = K.E. + m_0c^2$

$p^2c^2 = K^2 + m_0^2c^4 + 2Km_0c^2$

$$p = \frac{\sqrt{K(K+2m_0c^2)}}{c}$$

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{(K+2m_0c^2)K}}$$

Case 3: When 'K' is comparable with m_0c^2;

relativistic form of λ is

$$\lambda = \frac{hc}{\sqrt{K(K+2m_0c^2)}}$$

* An e^- has a de-broglie λ of 2×10^{-12} m. Find its K.E. ~~and~~, the phase and group velocity of its de-broglie waves. The rest mass of the electron is $0.511\ MeV/c^2$.

* X-rays of λ 10 picometer are scattered from a target.

a. find λ of x-rays scattered through $45°$.

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b. Find the maximum λ present in the scattered x-rays.

c. Find the maximum K.E. of recoil electrons.

*. An electron and positron are moving side by side in +ve dir^n, with 0.5c, when they annihilate eachother. Two photons are produced that move along x-axis. What is energy of each photon.

Pair-Production:-

The theory of quantum mechanics that schrodinger - Heisenberg proposed works only with non-relativistic phenomenon. This theory is called non-relavistic Q. mechanics. Combining the theory special relative with quantum mechanics, Dirak suceeded in extending the quantum mechanics to the relativistic phenomena. This new theory is called as relativistic quantum mechanics & predicted the existance of particle 'Positron'.

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This particle is defined as the anti-particle of electron having same mass & opposite charge.

The positron was discovered experimentally by Carl. D. Anderson in 1932 by studying the trails left by cosmic rays in a cloud chamber. When high freq. e-m radiation passes through a foil, individual photons of this radiation disappear by producing a pair of particles consisting of an electron & a positron. This process is called as Pair Production.

$$\gamma = e^- + e^+ .$$

Due to charge, momentum & energy conservation, pair production cannot occur in empty space. For this process to occur, photon must interact with external field such as Coloumb field of atomic nucleus, to absorb some of its momentum. This is consequence of mass-energy equation of Einstein.
$E = mc^2$.

$$\varphi^* \varphi = |\varphi|^2 = a^2 + b^2$$

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v imp. Wave-Nature.

Variable Quantity characterizing the matter wave is called wavefunction, denoted by ~~ϕ~~ φ. It cannot be determined experimentally. φ has got ~~more~~ no physical significance.

We assumed wave associated with the particle in motion is represented by a complex variable quantity $\varphi(x,y,z,t)$ called as wave-function.

$|\varphi|^2 \rightarrow$ Probability ~~identity~~ density.

$\varphi(x,y,z,t) \rightarrow a+ib$

$\varphi^* \rightarrow a-ib$

$$\varphi^* \varphi = |\varphi|^2 = a^2 + b^2$$

$$|\varphi(x,y,z,t)|^2 = a^2 + b^2$$

(*) limitations on φ:

$|\varphi|^2 \propto$ Probability density in a given region in space.

Probability per unit ~~time~~ volume.

Probability of finding particle in volume element dv

Total prob. of finding particle in entire region of space is

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$$\propto \int_{-\infty}^{\infty} |\psi|^2 \, dV$$

(1) ψ must be positive, finite & real quantity.

i.e. $\int_{-\infty}^{\infty} |\psi|^2 dv \neq 0$

$$\int_{-\infty}^{\infty} |\psi|^2 dv \neq \infty$$

$$\int_{-\infty}^{\infty} |\psi|^2 dv \neq -ve \neq complex$$

→ real.

(2) ψ must be single valued; i.e. for each set of values of x, y, z ; ψ must have one value.

(3) ψ must be continuous in all regions except in those regions where the potential energy $V(x, y, z) = \infty$.

(4) Partial Derivatives $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$ must be continuous everywhere.

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Normalization condition;

$$\int_{-\infty}^{\infty} |\psi|^2 dV = 1$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \psi^*(x,y,z,t)\,\psi(x,y,z,t)\,dV = 1$$

In 1-D;

$$\int_{-\infty}^{\infty} \psi(x,t)\,dx = 1.$$

(*) Expectation Value :-

It is the most expected value of the expected value of a quantity.

$$\langle x \rangle = \frac{\int_{-\infty}^{\infty} \psi^* x \psi \, dx}{\int_{-\infty}^{\infty} |\psi|^2 dx}$$

$$\Downarrow$$

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx$$

Normalizing the ψ means finding the form of wave fn that makes the statement $\int_{-\infty}^{\infty} |\psi|^2 dv = 1$ true. It means finding exact value of ψ that ensures particle is found somewhere.

Q. A particle is in motion along a line between $x=0$ & $x=a$ with zero P.E., and at point for which $x<0$ & $x>a$ the P.E. is ∞. The wave function of the particle in n^{th} state is given by $\varphi_n = A\sin \frac{n\pi x}{a}$. Find the expression for normalized wave function.

Sol^n;

$$\int_{-\infty}^{\infty} \varphi^* \varphi \, dx = 1$$

$$\int_0^a A^2 \sin^2 \frac{n\pi x}{a} \, dx = 1$$

$$A^2 \int_0^a \frac{1-\cos \frac{2n\pi x}{a}}{2} \, dx = 1$$

$$\frac{A^2}{2} \int_0^a \left[1 - \cos \frac{2n\pi x}{a}\right] dx = 1$$

$$\left[\lim_{x\to\infty} \frac{\sin nx}{x} = 0\right] \quad \frac{A^2}{2}\left[x \Big|_0^a - \frac{\sin \frac{2n\pi x}{a}}{\frac{2n\pi}{a}} \Bigg|_0^a\right] = 1$$

$$\frac{A^2}{2}(a) = 1$$

$$A^2 = \frac{2}{a} \quad \therefore A = \sqrt{2/a}.$$

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OPERATORS:- It is a rule which changes a function into another function.

for eg. $\frac{d}{dx}$

$$f(x) = x^n$$

$$\frac{d}{dx} f(x) = nx^{n-1}$$

$$\underbrace{\hat{A}}_{\text{operator}} \underbrace{f(x)}_{\text{eigen function}} = \underbrace{a}_{\text{eigen value}} f(x)$$

Eigen value eq^n.

Eigen value EQ.: It states that an operator acting on a f^n. reproduces the same f^n multiplied by constant factor. All operator of quantum mechanics have eigen function & eigen value.

Operators:-

Momentum   K.E.   Total Energy

(I) Momentum :-

$$\psi(x,t) = Ae^{i/\hbar(p_x x - Et)} \rightarrow \text{wave Eq}^n,$$

$\hbar = \frac{h}{2\pi}$

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$$\frac{\partial\psi}{\partial x} = A\left(\frac{i}{\hbar}\right) p_x \, \underbrace{e^{\frac{i}{\hbar}(p_x x - Et)}}$$

$$\frac{\partial\psi}{\partial x} = \left(\frac{i}{\hbar}\right) p_x \psi$$

$$\frac{\hbar}{i} \cdot \frac{\partial\psi}{\partial x} = p_x \psi$$

$$\hat{p}_x(\psi) = \frac{\hbar}{i} \frac{\partial}{\partial x}(\psi)$$ → Eigen value eqn. for x-component ~~for~~ of momentum.

$$\hat{p} = -i\hbar \vec{\nabla}$$

$$\hat{p}_y = -i\hbar \frac{\partial}{\partial y} \; ; \quad \hat{p}_z = i\hbar \frac{\partial}{\partial z}$$

2. K.E. :-

$$\frac{\hbar}{i} \cdot \frac{\partial\psi}{\partial x} = p_x \psi$$

Differentiate $\frac{\hbar}{i} \cdot \frac{\partial^2\psi}{\partial x^2} = p_x \frac{\partial\psi}{\partial x}$

$$\frac{\hbar}{i} \frac{\partial^2\psi}{\partial x^2} = p_x \left(\frac{i}{\hbar} p_x \psi\right)$$

$$-\hbar^2 \cdot \frac{\partial^2\psi}{\partial x^2} = p_x^2 \psi$$, ÷ by 2m on both sides.

$$\Rightarrow -\frac{\hbar^2}{2m} \cdot \frac{\partial^2\psi}{\partial x^2} = \frac{p_x^2}{2m} \cdot \psi$$

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$$\therefore \frac{P_x^2}{2m} = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2}$$

$$k\psi = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \psi}{\partial x^2}$$

$$\hat{k}_x = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2}$$

$$\vec{\hat{k}} = -\frac{\hbar^2}{2m} \cdot \vec{\nabla}^2 .$$

(iii) Total Energy (TE) :-

$$T.E = \frac{P_x^2}{2m} + V(x)$$

$$-\frac{\hbar^2}{2m} \cdot \frac{\partial^2 \psi}{\partial x^2} = (\underbrace{T.E - V}_{K.E})\psi$$

$$-\frac{\hbar^2}{2m} \cdot \frac{\partial^2 \psi}{\partial x^2} = (E - V)\psi$$

$$\left(-\frac{\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} + V\right)\psi = E\psi .$$

$$\hat{T.E} = \left(-\frac{\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} + V\right)$$

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In 3-D ;

$$\hat{H} \cdot \varepsilon = \frac{-\hbar^2}{2m} \cdot \nabla^2 + V$$

Schrodinger Eq^n :-

Time Independent:-

1D. : $$\frac{d^2\varphi}{dx^2} + \frac{2m}{\hbar^2}(\varepsilon - V)\varphi = 0$$

3D. $$\nabla^2\varphi + \frac{2m}{\hbar^2}(\varepsilon - V)\varphi = 0$$

Time Dependent :-

$$-\frac{\hbar^2}{2m}\nabla^2\varphi + V\varphi = \underbrace{i\hbar \cdot \frac{\partial\varphi}{\partial t}}_{\varepsilon\varphi}$$

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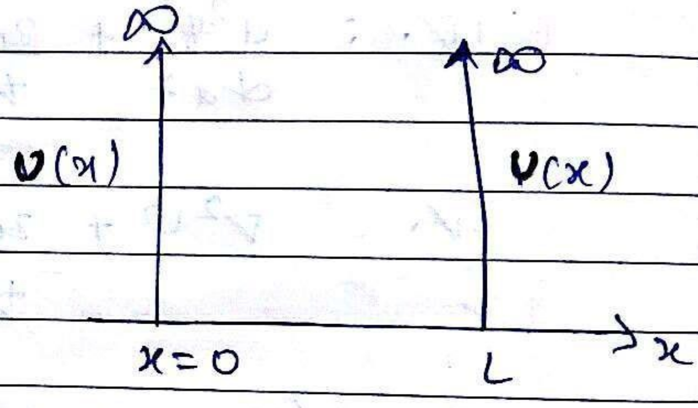
* Particle in 1-D Box :-

// A free electron trapped in a metal or charge carriers by a potential barrier of a double heterojunction can be approximated by an electron in an infinitely deep 1-D potential ~~barrier~~ well.



Boundary Conditions $V(x)=0, \quad 0<x<x$,

$V(x)=\infty; \quad L\leq x\leq 0$,

for simplicity,

we take $V(x)=0$ within the potential well & $V(x)=\infty$ outside the potential well.

since, $V(x)=0$ inside potential well.

$\hbar = \frac{h}{2\pi}$

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Inside potential well :-

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2} E\psi(x) = 0 \quad \text{— (i)}$$

wave eq^n of a free particle

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad \text{— (ii)}$$

where; $k^2 = \frac{2mE}{\hbar^2}$ — (iii)

So, possible solution to eq^n (2) is :-

$\psi(x) = A\sin kx + B\cos kx$

(A & B are constants)

now, since the particle cannot penetrate an infinitely high potential barrier so we have these boundary conditions.

for $\psi(x) = 0$ at $x = 0$; B must be 0.

for $\psi(x) = 0$ at $x = L$; $k.L$ must be integral multiple of π.

$\therefore K = \frac{n\pi}{L}$ — (iv)

$\therefore \psi(x) = A\sin\frac{n\pi x}{L}$ — (5)

Substitute; $\frac{n^2\pi^2}{L^2} = \frac{2mE}{\hbar^2}$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \quad \text{— (6)}$$

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$$\int_0^L \psi^* \psi \, dx = 1$$

$$A = \sqrt{\frac{2}{L}}$$

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \quad — (7)$$

In 3-D ;

$$\psi_n = \sqrt{\frac{2}{L}} \cdot \sqrt{\frac{2}{L}} \cdot \sqrt{\frac{2}{L}}$$

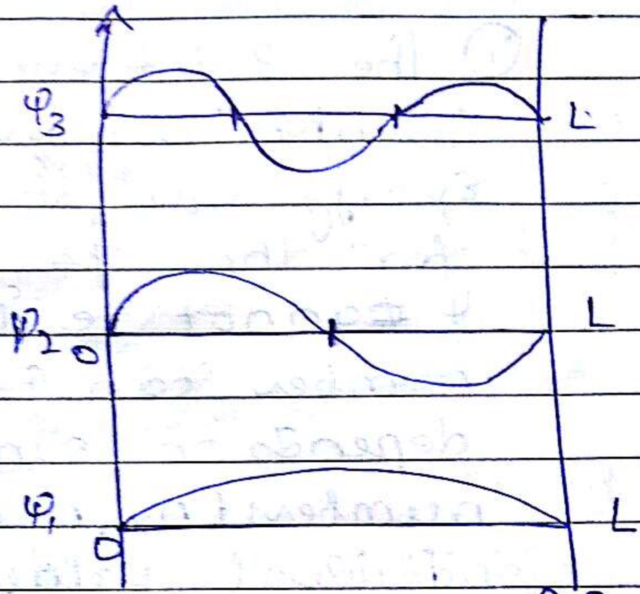
$$\sin \frac{n_x \pi x}{L} \cdot \sin \frac{n_y \pi y}{L} \cdot \sin \frac{n_z \pi z}{L}$$

$$= \sqrt{\frac{8}{L^3}} \sin \frac{n_x \pi x}{L} \cdot \sin \frac{n_y \pi y}{L} \cdot \sin \frac{n_z \pi z}{L}$$

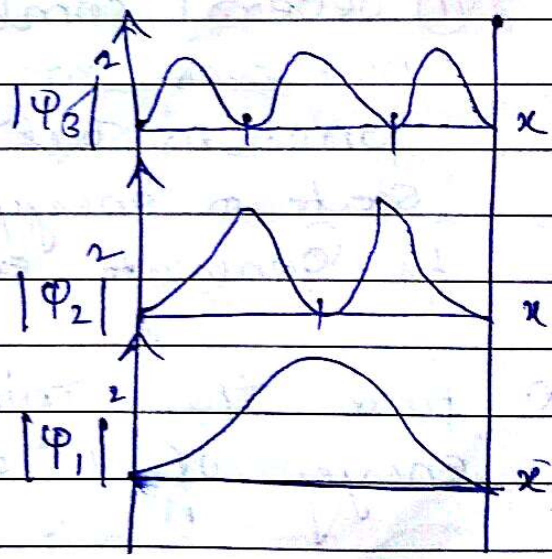
$$\therefore E_n = \frac{(n_x^2 + n_y^2 + n_z^2)}{2mL^2} \pi^2 \hbar^2$$

Energy level	Quantum Numbers $(n_x \; n_y \; n_z)$	Degree of Degeneracy
$\frac{3\pi^2\hbar^2}{2mL^2}$	1 1 1	non-degenerate
$\frac{6\pi^2\hbar^2}{2mL^2}$	(2 1 1) (1 2 1) (1 1 2)	3-fold degenerate.
$\frac{9\pi^2\hbar^2}{2mL^2}$	(2 2 1) (2 1 2) (1 2 2)	3-fold.
$\frac{11\pi^2\hbar^2}{2mL^2}$	(3 1 1) (1 3 1) (1 1 3)	3-fold.

Figure show the ground state and the first two excited ~~ex~~ states of an electron in a potential well.



Electron wave f?.



Only 1 combinate → non-degenerate.

Corresponding probability density function.

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(i) The 3-integers n_x, n_y & n_z are called Quantum numbers and are required to specify each energy state completely. Since, for the particle inside the box, ψ cannot be '0' so no quantum number can be '0'. Energy 'E' depends on sum of squares of quantum numbers (n_x, n_y & n_z) and not of their individual values.

(ii) Several combinations of 3 quantum numbers may gives different wave functions but of same value. Such state or energy levels are called as De-Generate energy levels.

*Q. Find the minimum energy or energy of 1st excited state.

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One Dimension Harmonic Oscillator.

The time independent schrodinger wave-equation for linear motion of a particle along x-axis is given by

$$-\frac{\hbar^2}{2m} \cdot \frac{d^2\psi}{dx^2} + V\psi = E\psi \rightarrow \text{dependent.}$$

Independent of

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{—(i)}$$

$$V = \frac{1}{2} m\omega^2 x^2 \quad \text{(ii)}$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - \frac{1}{2} m\omega^2 x^2\right)\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \left[\frac{2mE}{\hbar^2} - \frac{m^2\omega^2 x^2}{\hbar^2}\right]\psi = 0 \quad \text{— (iii)}$$

is

Schrodinger wave eqn. for Oscillator.

Soln, $\psi_n = N_n e^{-y^2/2} H(y)$

$$\psi_n = \left(\frac{\alpha}{\pi}\right)^{1/4} (2^n n!)^{-1/2} e^{-\alpha x^2/2} H_n(x\sqrt{\alpha})$$

$$\alpha = \frac{m\omega}{\hbar} \qquad \therefore y = x\sqrt{\alpha}$$

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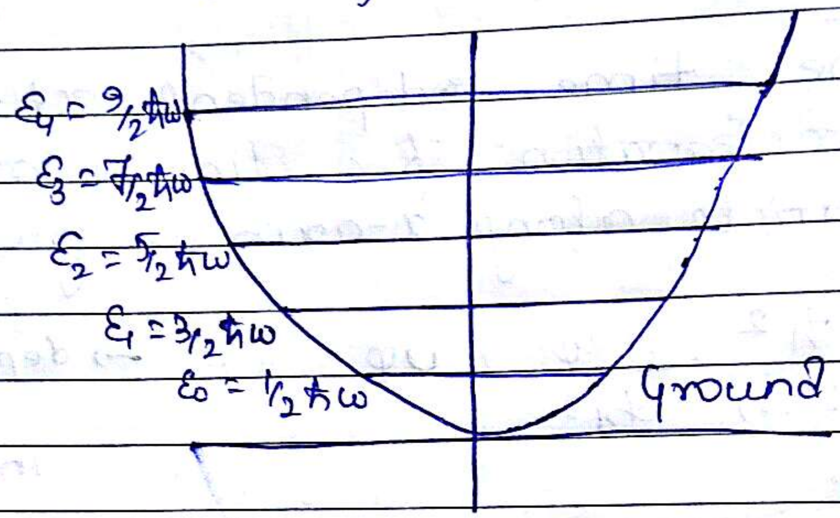
$$\varepsilon_n = \left(n+\tfrac{1}{2}\right)\hbar\omega,$$



(i) lowest energy of the oscillator is obtained by putting, $n=0$;
$\varepsilon_0 = \tfrac{1}{2}\hbar\omega$, is called as "0 point vibration energy of oscillator or ground state energy."

(ii) Energy Eigen values depend only on 1-quantum number 'n' ∴ All energy level of 1-D oscillator are non-degenerate.

(iii) The sucessive energy levels are equally space and sepperation bet^n 2 adjacent energy levels is equal to $\hbar\omega$.

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3-D harmonic Oscillator :-

$$E = (n_x + n_y + n_z + 3/2)\hbar\omega .$$

Energy	n	n_x n_y n_z	degeneracy
$3/2\,\hbar\omega$	0	(0) (0) (0)	non-degenerate
$5/2\,\hbar\omega$	1	(1) (0) (0)	3-fold degenerate
		(0) (1) (0)	
		(0) (0) (1)	
$7/2\,\hbar\omega$	2	(1) (1) (0)	6-fold degenerate
		(0) (1) (1)	
		(2) (0) (0)	
		(1) (0) (1)	
		(0) (2) (0)	
		(0) (0) (2)	

And so-on.

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*. Well-Behaved wave function:-

→ The ch. of a ψ.

Degenerate and Non-Degenerate Eigenfunction.

*. Group Velocity And Phase (or Wave) Velocity:

$$\lambda = \frac{h}{mv}$$

$$\& \quad E = mc^2 = h\nu$$

$$\therefore \nu = \frac{mc^2}{h}$$

$$\therefore V = \lambda\nu$$

$$= \frac{h}{mv} \times \frac{mc^2}{h}$$

$$V = c \times \frac{c}{v}$$

$$V > c$$

- Which is not possible & implies that a moving material particle cannot be single wave train.

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∴ Schrodinger stated that "a moving material particle is equivalent to ~~a~~ wave packet."

Consider a wave ; $A\cos(kx-\omega t)=0$

~~diff w.r.to time;~~

For the condition such that wave looks stationary, our 2 wave velocity must be equal i.e.

$$kx-\omega t=0$$

diff. w.r. to time;

$$k\frac{dx}{dt}+x\frac{dk}{dt}-\omega\frac{dt}{dt}-t\frac{d\omega}{dt}=0$$

now, k & ω are time independent.

$$\therefore k\frac{dx}{dt}-\omega=0$$

$$\text{or}\quad k\frac{dx}{dt}=\omega \qquad \therefore \frac{dx}{dt}=\frac{\omega}{k}=V_g$$

now, for Group velocity = Particle Velocity.

$$\cancel{\exists}\quad \frac{1}{2}mv^2=E-V$$

$$v=\sqrt{\frac{2(E-V)}{m}}$$

$$\lambda=\frac{h}{m}*\sqrt{\frac{m}{2(E-V)}}$$

now,

$$V_g=\frac{d\omega}{dk}=\frac{d(\nu)}{d(1/\lambda)}$$

i.e. $$\frac{1}{V_g}=\frac{d(1/\lambda)}{d(\nu)}=\frac{d\frac{m}{h}\sqrt{\frac{2(E-V)}{m}}}{d\nu}$$

$$\frac{1}{V_g} = \frac{1}{h}\frac{d}{d\vartheta}\sqrt{2m(E-V)}$$

$$= \frac{1}{h} \times \frac{1}{2} \times \left(2m(E-V)\right)^{-1/2} \cdot 2mh$$

$$= \frac{m}{\sqrt{2m(E-V)}} = \frac{1}{V_p}.$$

$\therefore$ Group Velocity = Particle Velocity.