Assignment-II (B. Tech. 2nd Sem. 2013)
Mathematics-II (AM 111), Topic: Differential Equations[1]

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1. Discuss linear independence and linear dependence of the following functions:
   (*a*) $e^{-x},\ e^{x}$
   (*b*) $1,\ x,\ x^2$
   (*c*) $\sin x,\ \cos x,\ x$
   (*d*) $e^{x},\ \sin x,\ x^2$

2. Define superposition principle, and discuss homogeneous and non-homogeneous differential equations with examples.

3. Define fundamental set of solutions of a differential equation and show that e^{x}, e^{2x} and e^{3x} constitute the fundamental solution set of $y''' - 6y'' + 11y' - 6y = 0$ on any interval I.

4. Solve the following differential equations:
   (*a*) $y'' - 4y' - 5y = 0$
   (*b*) $y'' + 6y' + 9y = 0, \quad y(0) = 2,\ y'(0) = 3$
   (*c*) $y'' + 4y' + 13y = 0, \quad y(0) = 0,\ y'(0) = 1$
   (*d*) $(4D^2 - 4D + 17)y = 0$
   (*e*) $[D^2 - 2aD + (a^2 + b^2)]y = 0$
   (*f*) $y'' + 25y = 0, \quad y(0) = 1,\ y(\pi) = -1$
   (*g*) $y^{(4)} + 2y''' + 11y'' + 18y' + 18y = 0, \quad y(0) = 0,\ y'(0) = 3,\ y''(0) = -1,\ y'''(0) = -23$
   (*h*) $y^{(4)} + 32y'' + 256y = 0$
   (*g*) $y''' + \pi^2 y' = 0, \quad y(0) = 0, y(1) = 0,\ y'(0) + y'(1) = 0$

5. Find the conditions under which the following equations hold:
   (*a*) $(D + a)[D + b(x)]f(x) = [D + b(x)](D + a)f(x)$, where a is a constant.
   (*b*) $[D + a(x)][D + b(x)]f(x) = [D + b(x)][D + a(x)]f(x)$

6. Solve the following using method of variation of parameters:
   (*a*) $y'' + 3y' + 2y = 2e^{x}$
   (*b*) $y'' + 4y = \cos x$
   (*c*) $y'' + 4y' + 4y = e^{-2x}\sin x$
   (*d*) $y'' + 6y' + 9y = e^{-3x}/x$

7. Find the general solution for the following differential equations:
   (*a*) $y'' - 4y' + 13y = 12e^{2x}\sin x$
   (*b*) $y'' + 25y = 50\cos(5x) + 30\sin(5x)$
   (*c*) $y''' + 3y'' - 4y = 12e^{-2x} + 9e^{x}$
   (*d*) $y''' - 2y'' + 4y' - 8y = 8(x^2 + \cos(2x))$
   (*e*) $y'' - 4y' + 3y = 4\cosh(3x)$
   (*f*) $(2D^2 + 7D - 4)y = xe^{-4x}$
   (*g*) $(D^3 - 4D^2 + 9D - 10)y = 24e^{x}\sin(2x)$
   (*h*) $(4D^2 + 8D + 3)y = xe^{-x/2}\cos x$

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[1] **Typeset by Dr. Suresh Kumar**

8. Solve the following differential equations:
   (*a*) $2x^2y'' + xy' - 6y = 0$
   (*b*) $4x^2y'' + 8xy' + 17y = 0$
   (*c*) $x^2y'' - 5xy' + 13y = 30x^2$
   (*d*) $x^3y''' + 3x^2y'' + 14xy' + 34y = 0$
   (*e*) $4x^2y'' + xy' - 6y = 25\sin(\ln x)$

9. Solve the following systems of differential equations:

   (*a*) $y_1' + 2y_2' - 2y_1 - y_2 = e^{2t}, \quad y_2' + y_1 - 2y_2 = 0$

   (*b*) $(3D + 1)y_1 + 3Dy_2 = 3t + 1, \quad (D - 3)y_1 + Dy_2 = 2t$

   (*c*) $y_1' = -2y_1 + y_2, \quad y_2' = y_1 - 2y_2$

   (*d*) $(D - 1)y_1 - (D + 1)y_2 = t, \quad (D + 1)y_1 + (2D + 1)y_2 = e^t$

   (*e*) $y_1' + y_1 + 3y_2 = 4e^{-t}, \quad y_2' + 4y_1 + 3y_2 = 8t$

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