Total No. of Pages 2 | Roll No. ......

FIRST SEMESTER | **B.Tech. [ALL]**

END SEMESTER EXAMINATION | **NOV.-DEC.-2011**

AM-101 MATHEMATICS-I

Time: 3 Hours | ***Maximum Marks : 70***

Note : Answer *ALL* by selecting *TWO* parts from each question. Assume suitable missing data, if any.

1[a] State Cauchy's integral test. Hence or otherwise discuss the convergence of $\sum \frac{1}{n^p}$.

[b] Discuss the convergence and absolute convergence of the following series;

$$x - \frac{x^2}{2} + \frac{x^3}{3} \ldots\ldots\ldots\ldots + (-1)^{n+1}\frac{x^n}{n} + \cdots \ldots\ldots$$

[c] Calculate the approximate value of $\sqrt{17}$ by choosing a suitable function and writing its Taylor's series, correct upto 4th decimal place.

7+7

2[a] Show that the curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the folium

$x^3 + y^3 = 3xy$ is $-\frac{8\sqrt{2}}{3a}$.

[b] Find the surface area of the solid generated by revolving an arc of the cycloid $x = a(\theta - sin\theta)$, $y = a(1 - cos\theta)$ $0 \leq \theta \leq 2\pi$ about the x-axis.

[c] Find the area included between the cardiod $r = a(1 + cos\theta)$ and the circle $r = a$.

7+7

3[a] If $u = log(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 . u = \frac{-9}{(x+y+z)^2}$$

[b] Locate the stationary point of
$f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ and determine their nature.

[c] At a distance of 50 meter from the foot of a tower, the elevation of its top is 30°. The possible errors in the measuring of distance and the elevation are 2 cm and 0.05 degree respectively. Find the approximate error in the calculated height.

7+7

4[a] Evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x dy dx}{\sqrt{x^2+y^2}}$ by changing in the polar co-ordinates

(ii) Show that $\Gamma n = \int_0^1 \left(log \frac{1}{y}\right)^{n-1} .dy$

[b] Find the volume bounded by cylinder $x^2 + y^2 = 4$ and the hyperboloid $x^2 + y^2 - z^2 = 1$

[c] Find the volume bounded by *xy*-plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$

7+7

5[a] (i) If $u = x + y + z$, $v = x^2 + y^2 + z^2$ $w = yz + zx + xy$ prove that grad u, grad v and grad w are coplanar.

(ii) Find whether the vector field $\bar{F} = \cosh(x+y)\,(\hat{\imath} + \hat{\jmath})$ is conservative.
If it is so, find the potential function.

[b] State the Divergence theorem. Verify it for $\bar{F} = 4xz\hat{\imath} - y\hat{\jmath} + yz\hat{k}$ taken over the cube bounded by the planes $x = 0 = y = z$ and $x = 1 = y = z$.

[c] Verify Green's theorem in the plane for $\int_c [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region bounded by $x = 0$, $y = 0$ and $x + y = 1$.

7+7