A8/1176

Total No. of Pages 2 Roll No. ......

FIRST SEMESTER **B.Tech. [Common for all branches]**

END SEMESTER EXAMINATION ***November-2014***

AM 101 Mathematics I

Time: 3:00 Hours *Max. Marks : 70*

Note : Answer all questions by selecting any two parts out of the three set in a question. Assume suitable missing data, if any. All questions carry equal marks.

1. (a) What is absolute convergence of an infinite series. Show that absolute convergence implies convergence. What about the converse? Justify.

(b) Test the convergence of the series

$$1+\frac{m}{n}x+\frac{m(m+1)}{n(n+1)}x^2+\frac{m(m+1)(m+2)}{n(n+1)(n+2)}x^3+\cdots, m,n>0, x>0.$$

(c) Expand $\tan^{-1}x$ up to the fifth power of x and using this evaluate the value of π.

2. (a) I) Find the length of the curve $y=\left(\frac{x}{2}\right)^{\frac{2}{3}}$ from $x=0$ to $x=2$.

II) Show that the radius of curvature at any point of the cardiod $r=a(1-\cos\theta)$ varies as $\sqrt{r}$.

(b) For the surface $z=f(x,y)$, what is the geometrical interpretation for the partial derivative of z with respect to x. Explain this by considering a surface of your choice.

(c) The diameter and altitude of a can in the shape of a right circular cylinder, with top and bottom both closed, are measured as 4 cm and 6 cm respectively. The possible error in each measurement is at the most 0.1 cm. Find approximately the maximum possible error in the values computed for volume and surface.

3.(a) I) Give an example of a homogeneous function of order n in x and y. Verify that its partial derivative with respect to x is homogeneous function of order $n-1$.

II) If $u = \sin^{-1}\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$, then find the value of $xu_x + yu_y$.

(b) Plot the area lying inside the circle $x^2 + y^2 - 4x = 0$ and outside the circle $x^2 + y^2 = 4$. Using double integration evaluate it. Can you evaluate it using single integration? Just explain how?

(c) Define gamma and beta functions. What is the relation between these two? Using the evaluate the integral

$$\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^5\theta \, d\theta$$

4. (a) Find the volume of the sphere with centre at $(2, 2, 2)$ and diameter equal to 4 *units* using

(I) Cylindrical coordinates , (II) Spherical polar coordinates.

(b) Define gradient of a scalar field. What is its geometrical interpretation? Find the directional derivative of $\phi = 5x^2y - 5y^2z + 2.5\,z^2x$ at the point $(1, 1, 1)$ in the direction of the line

$$\frac{x-1}{2} = \frac{y-3}{-2} = z.$$

(c) (I) If $\vec{a}$ and $\vec{b}$ are constant vectors, then find $\nabla \times \{\vec{a} \times (\vec{b} \times \vec{r})\}$, where $\vec{r}$ is a position vector of the point $P(x, y, z)$.

(II) Show that the vector $\nabla\phi \times \nabla\psi$ is solenoidal.

5. (a) Evaluate $\int_c [(x^2 + xy)dx + (x^2 + y^2)dy]$, where c is the square formed by the line $y = \pm 1, x = \pm 1$.

(b) Evaluate $\int\int_S \vec{F}.\hat{N}\, ds$, where $\vec{F} = z^2\,\hat{\imath} + xy\,\hat{\jmath} - y^2\hat{k}$ and S is the portion of the surface of the cylinder $x^2 + y^2 = 36, 0 \le z \le 4$ included in the first octant.

(c) Evaluate $\int\int_S \vec{F}.\hat{N}\, ds$, where $\vec{F} = ax\hat{\imath} + by\,\hat{\jmath} + cz\,\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.