FIRST SEMESTER

B.Tech.(Group A & B)

END SEMESTER EXAMINATION

NOVEMBER-2010

AM- 101 MATHEMATICS-I

Time: 3 Hours

Note:

Max. Marks: 70

-23 (x37452) 3x2

Answer ALL questions taking TWO parts out of the three set in each question.

Assume suitable missing data, if any.

[a] Discuss the convergence of the series

$$1 + \frac{x}{2} + \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \cdots + (x > 0).$$

[b] State a test to check for the convergence of an alternating series. Examine the convergence/absolute convergence of the series 5 - S

$$\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \cdots$$

(ii) Show that the radius of curvature at any point of the cardiod

$$r = a(1 - \cos\theta)$$
 varies as \sqrt{r} .

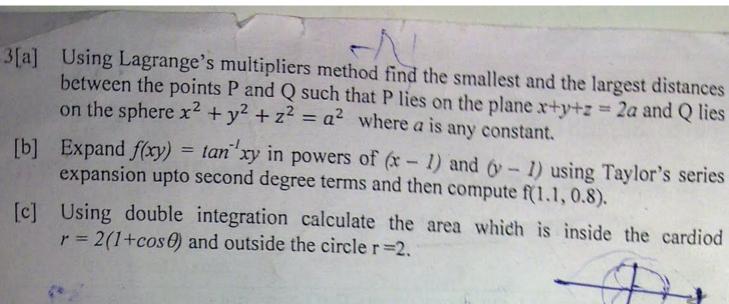
(iii) Expand $e^{asin^{-1x}}$ upto the term containing x^4 .

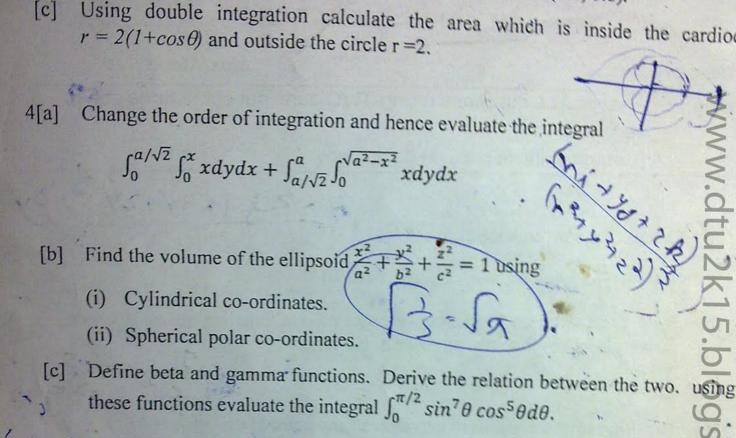
The area lying inside the cardiod $r = 2a(1 - \cos\theta)$ and outside the parabola 2[a] $r(1 + \cos \theta) = 2a$ is revolved about the initial line $\theta = 0$. Find the volume of The solid generated.

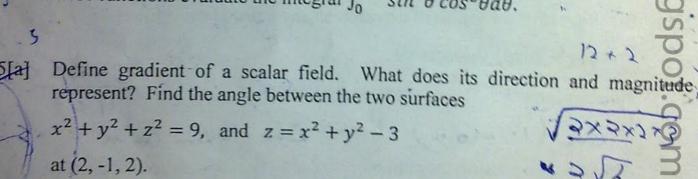
If $u = \sin^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$, then prove that $\frac{\partial^{2} u}{\partial x^{2}} + 2xy \frac{\partial^{2} u}{\partial x \partial y} + y^{2} \frac{\partial^{2} u}{\partial y^{2}} = -\frac{\sin u \cos 2u}{4\cos^{3} u}$

$$x^{2} \frac{\partial^{2} u}{\partial x^{2}} + 2xy \frac{\partial^{2} u}{\partial x \partial y} + y^{2} \frac{\partial^{2} u}{\partial y^{2}} = -\frac{\sin u \cos 2u}{4\cos^{3} u}$$

A balloon is in the form of a right circular cylinder of radius 1.5 meter and length 4 meter and is surmounted by hemispherical ends. If the radius is increased by 0.01 meter and length by 0.05 meter, find the percentage change in the volume of balloon.







at (2, -1, 2).

Prove that $div(r^n\vec{r}) = (n+3)r^n$. Hence show that \vec{r}/r^3 is solenoidal. Is. \vec{r}/r^3 irrotatinal also? Verify your answer. Here \vec{r} is the position vector of a point P(x, y, z) and $r = |\vec{r}|$.

If $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$, evaluate $\iint_S \vec{F} \cdot \hat{N} dS$ where S is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.