NODAL ANALYSIS

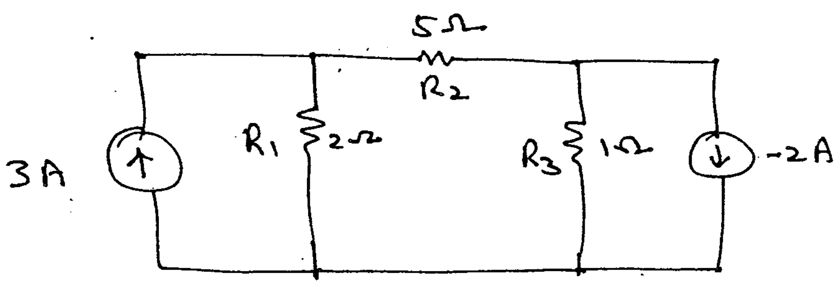
1. A reference node is selected and a set of node voltages is assigned as unknowns.
   - These node voltages are wrt reference node
   - Node connected to maximum no. of branches is preferably to be selected as reference node.

2. KCL equations are written for each node except the reference node.

3. These equations are solved to give node voltages

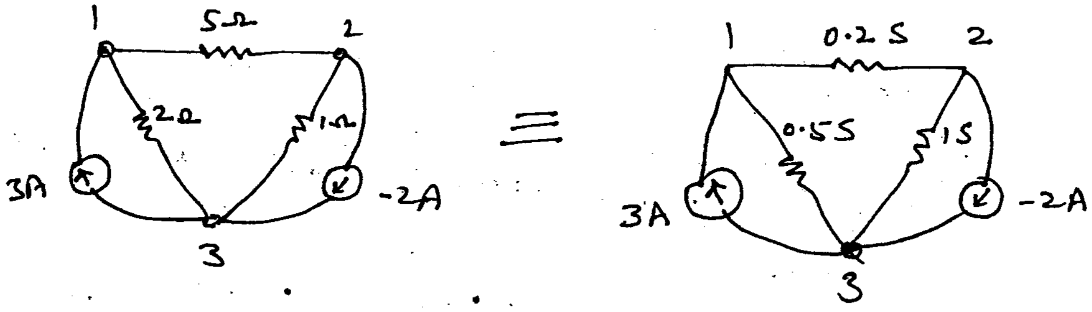
4. Next, the current through different branches can be calculated.

Example: Using nodal analysis, determine the current through the 2Ω resistor in the network shown below



Soln: For nodal analysis it is more convenient to work with conductance (G) values rather than resistance (R) values

The given network can be redrawn as



Write KCL for node 1

$$0.5\,V_1 + 0.2(V_1 - V_2) = 3$$

$$0.7\,V_1 - 0.2\,V_2 = 3 \qquad \text{--- (1)}$$

At node 2

$$1V_2 + 0.2(V_2 - V_1) = 2$$

$$-0.2\,V_1 + 1.2\,V_2 = 2 \qquad \text{--- (2)}$$

We can write (1) and (2) as

$$\begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 1.2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \qquad \text{--- (3)}$$

$$V_1 = \frac{\begin{vmatrix} 3 & -0.2 \\ 2 & 1.2 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.2 \\ -0.2 & 1.2 \end{vmatrix}} = \frac{(3\times1.2)+(2)(0.2)}{(0.7\times1.2)-(-0.2)(-0.2)} = \frac{3.6+0.4}{0.84-0.04} = \frac{4.0}{0.8}$$

$$= \underline{5V}$$

$$V_2 = \underline{2.5V}$$

So Current through R_1 of $2\Omega = \frac{5}{2} = \underline{2.5A}$

Generalised Form of Nodal Equations

It is possible to write the nodal equations too by inspection.

Eq. ③ of the above example can be rewritten as

$$\begin{bmatrix} (0.5+0.2) & -0.2 \\ -0.2 & 0.2+1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\equiv \begin{bmatrix} G_{11} & -G_{12} \\ -G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{s_1} \\ I_{s_2} \end{bmatrix}$$

This is ~~con~~ known as conductance matrix

G_{11} = Sum of all conductances connected to node 1. It called self-conductance of node 1

$G_{12} = G_{21} = 0.2$ = Sum of all conductances connected between nodes 1 and 2.

I_{s_1} = algebraic sum of source currents entering node 1

I_{s_2} = " " " " " 2

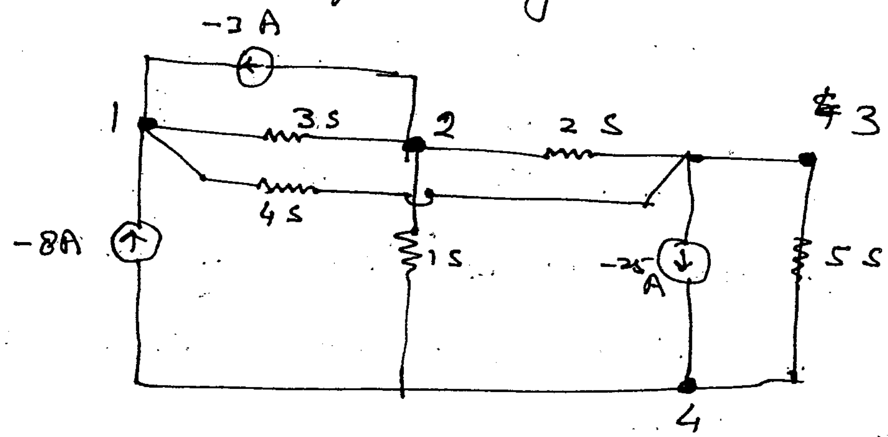
If we have a network of $n+1$ nodes out of which one is reference node. Then for n nodes, the nodal equations can be written in general form as

$$\begin{bmatrix} G_{11} & -G_{12} & \cdots & -G_{1n} \\ -G_{21} & G_{22} & \cdots & -G_{2n} \\ \vdots & \vdots & & \vdots \\ -G_{n1} & -G_{n2} & \cdots & G_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} I_{s1} \\ I_{s2} \\ \vdots \\ I_{sn} \end{bmatrix}$$

Important Points :

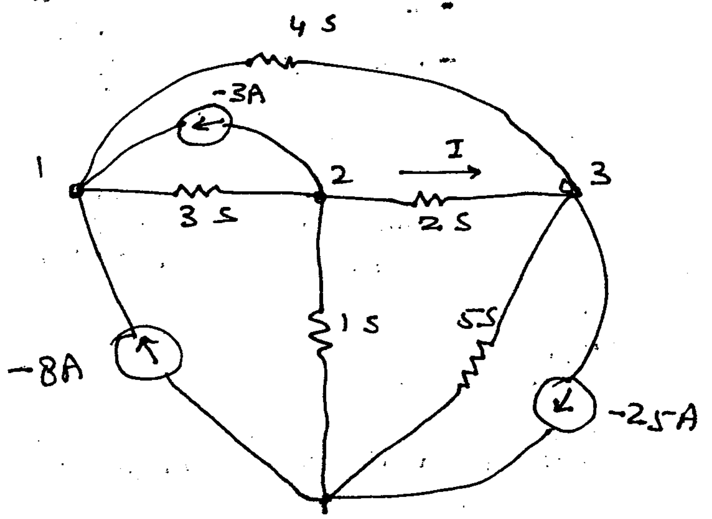
1. Nodal equations are for current sources. If there are some voltage sources present, these must be converted into the ~~volta~~ current source first.
2. Diagonal elements are +ve and cannot be zero
3. Off-diagonal elements are -ve and can be zero
4. Conductance matrix can be written by inspection.

Example: Determine the current through 2 S is resistor in the network shown below using node-voltage analysis.



There are 4 nodes in all, one of these is taken as reference node (4). Remaining are 3 nodes only

~~Sol~~ Redrawing the network as follows



Next node voltage equations can be written by inspection as follows

$$\begin{bmatrix} 4+3 & -3 & -4 \\ -3 & 3+2+1 & -2 \\ -4 & -2 & 4+2+5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -(8+3) \\ +3 \\ 25 \end{bmatrix}$$

$$\equiv \begin{bmatrix} 7 & -3 & -4 \\ -3 & 6 & -2 \\ -4 & -2 & 11 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 3 \\ 25 \end{bmatrix}.$$

We have to find current through 2 S resistor i.e. I i.e. Current between node 2 and 3 $\therefore$ we need to find out V_2 and V_3

So

$$V_2 = \frac{\begin{vmatrix} 7 & -11 & -4 \\ -3 & 3 & -2 \\ -4 & 25 & 11 \end{vmatrix}}{\begin{vmatrix} 7 & -3 & -4 \\ -3 & 6 & -2 \\ -4 & -2 & 11 \end{vmatrix}} = \underline{2V}$$

$$V_3 = \frac{\begin{vmatrix} 7 & -3 & 11 \\ -3 & 6 & 3 \\ -4 & -2 & 25 \end{vmatrix}}{G} = 3V$$

$$I = \frac{V_2 - V_3}{R} = \frac{2-3}{1/2} = \underline{-2\ A}$$

Example: Find the current through 5-Ω resistor
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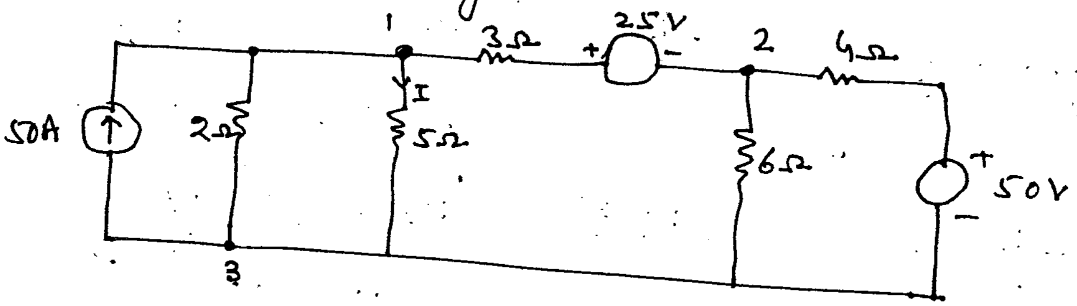
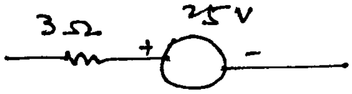
in the circuit of network shown by node analysis.



Soln.
- There are three nodes
- Node 3 can be taken as reference node
- There are only 2 nodes for nodal analysis
- There is one current source
- There are 2 voltage sources. These should be converted into equivalent current sources.
- All resistances should be converted into



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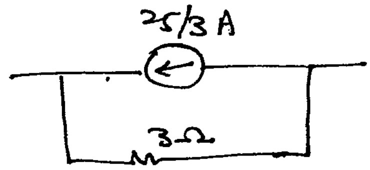

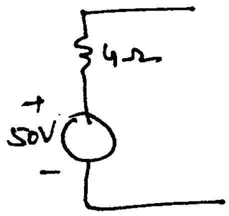

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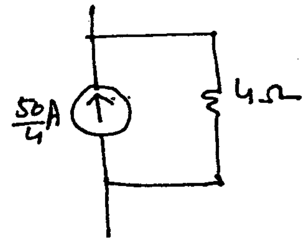
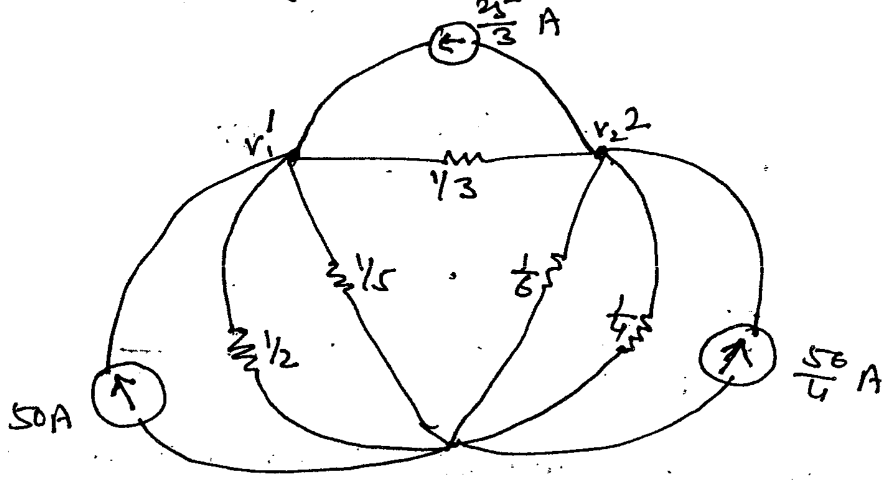


The circuit can be redrawn with conductances shown on each branch.



Node voltage equations can be written by inspection

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{5}+\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3}+\frac{1}{6}+\frac{1}{4} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 50+\frac{25}{3} \\ \frac{50}{4} \end{bmatrix}$$

$$\begin{bmatrix} \frac{31}{30} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{3}{4} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{175}{3} \\ \frac{25}{6} \end{bmatrix}$$

$$V_1 = \frac{\begin{bmatrix} \frac{175}{3} & -\frac{1}{3} \\ \frac{25}{6} & \frac{3}{4} \end{bmatrix}}{\begin{bmatrix} \frac{31}{30} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{3}{4} \end{bmatrix}} = 68V$$

$$I = \frac{68}{5} = \underline{13.6\,A}$$

The choice between Mesh Analysis and Nodal Analysis

Let.

n = number of ~~branches~~ nodes

b = " " branches

No. of equations needed for nodal analysis = $n-1$

" " " " mesh " = $b-(n-1)$ [can be proved]

$= b-n+1$

We should use the method for which no. of eq=ns to be solved is lesser.

Consideration of Controlled Sources

- For nodal and mesh analysis controlled sources are treated in same manner as ~~of~~ independant sources
- Regarding value of source, it is treated as a ~~contained~~ constraint

Details we don't cover in class. Do yourself.

Reference: Basic electrical Engg. by

A.E. Fitzgerald

David E. Higginbotham

Arvin ...

~~PTO~~

Network Theorems

Need for network Theorems

- Kirchhoff's Laws are sufficient to solve the network FULLY.
- But Fully only not Partially
- For solving a network Partly we need network theorems
- Advantage is the reduced computational effort
- Some theorems ~~are~~ have universal applications
- Some can be applied to ~~lim~~ linear elements only.
- R, L and C are linear, Diodes are not.

PTO.

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Superposition Theorem :

This theorem states that in a linear network containing several sources (including dependent sources) the overall response (loop current or node voltage) at any point in the network equals the sum of the responses of each individual source considered seperately with all other sources made inoperative, i.e. replaced by impedances equal to their internal impedances.

A few terms need further explanation:

1. **Linear network** : It is a combination of linear elements, independent sources and linear dependent sources

2. **Linear Elements** : It is defined as a passive element that has a linear voltage current relationship. R, L and C are linear elements. Diodes are non-linear.

3. **Linear Dependent Source** : It is a source whose output current/voltage is proportional to the first power of some current and/or voltage variable in the network or to the sum of such quantities.

- A dependent voltage source

$v_s = 0.6 i_1 - 16 v_2$ is linear

but
$$\left.\begin{array}{l} v_s = 0.6 i_1^2 \\ v_s = 0.6 i_1 v_2 \end{array}\right\} \text{ are not linear}$$

- The dependent sources are generally considered alongwith with independent sources.

- It is quite difficult to analyse the non-linear networks simply because the principle of superposition is not applicable to non-linear networks.

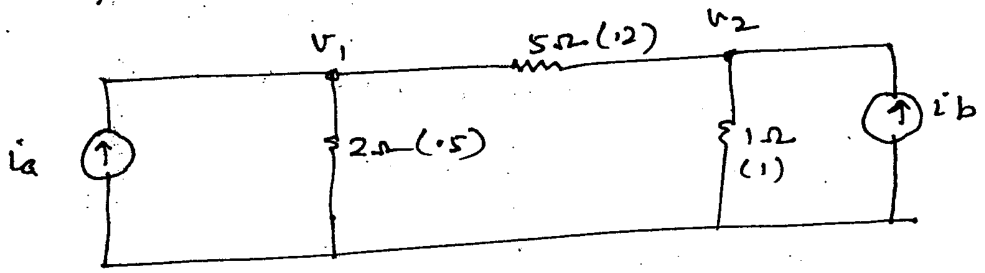
How to make a source inoperative

Voltage source : It is short-circuited leaving behind its internal resistance.

Current source : It is ~~short~~ open-circuited leaving behind its internal resistance.

Proof of Superposition Theorem

- Actually we are developing principle of superposition for electrical networks here.



i_a, i_b : forcing functions, input, excitation

v_1, v_2 = Response

Two nodal equations are (by inspection)

$$.7 v_1 - .2 v_2 = i_a \quad \text{——} (1)$$

$$-.2 v_1 + 1.2 v_2 = i_b \quad \text{——} (2)$$

Let us perform ~~and~~ an experiment X

excitations = i_{ax} , i_{bx}

Responses : v_{1x} , v_{2x}

Nodal eq=ns become

$$0.7\, v_{1x} - 0.2\, v_{2x} = i_{ax} \quad \text{—} (3)$$

$$-0.2\, v_{1x} + 1.2\, v_{2x} = i_{bx} \quad \text{—} (4)$$

Let us perform another experiment Y

excitations = i_{ay} , i_{by}

Responses = v_{1y} , v_{2y}

Then nonal eq=ns become

$$0.7\, v_{1y} - 0.2\, v_{2y} = i_{ay} \quad \text{—} (5)$$

$$-0.2\, v_{1y} + 1.2\, v_{2y} = i_{by} \quad \text{—} (6)$$

Superposing equs (3) + (5) , ~~(4) + (6) we have~~

$$(0.7\, v_{1x} + 0.7\, v_{1y}) - (0.2\, v_{2x} + 0.2\, v_{2y}) = i_{ax} + i_{ay} \quad \text{—} (7)$$

$$0.7\, v_1 - 0.2\, v_2 = i_a \quad \text{—} (1)$$

Superposing (4) and (6)

$$-(0.2\, v_{1x} + 0.2\, v_{1y}) + (1.2\, v_{2x} + 1.2\, v_{2y}) = i_{bx} + i_{by} \quad \text{—} (8)$$

$$-0.2\, v_1 + 1.2\, v_2 = i_b \quad \text{—} (2)$$

Discuss eq=ns (7) and (8)

if ~~$a =$~~ $i_a = i_{ax} + i_{ay}$ excitation

Then $v_1 = v_{1x} + v_{1y}$ Response

Condition of Linearity for the Superposition principle.

Let ~~f(x) =~~

$$y = f(x)$$

$$y_1 = f(x_1)$$

$$y_2 = f(x_2)$$

Then $y_1 + y_2 = f(x_1 + x_2)$ not merely $f(x_1) + f(x_2)$

This is true for

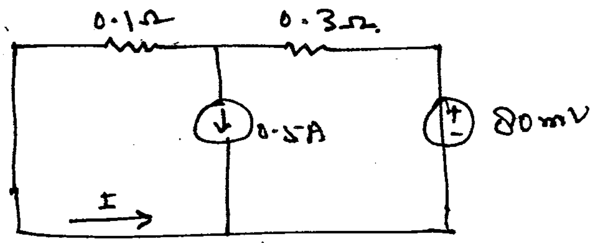
$$y = kx$$

but not for $y = kx^2$

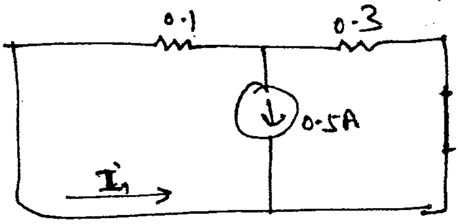
and also

$y = x + 7$ - - - - Demonstrate.

Example: Find the current I in the network shown given using superposition Theorem.

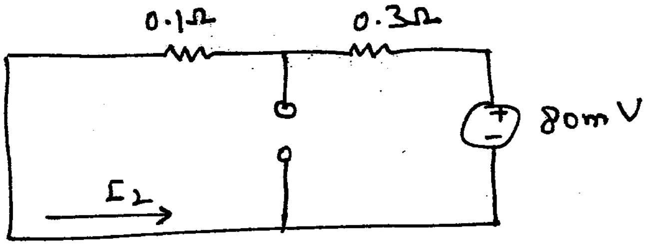


Soln. Making the voltage source of 80 mV is operative



$$I_1 = -\frac{0.5 \times 0.3}{0.1 + 0.3} = -\frac{0.15}{0.4} = -0.375\,A \quad (1)$$

Making current source of 0.5A inoperative we have

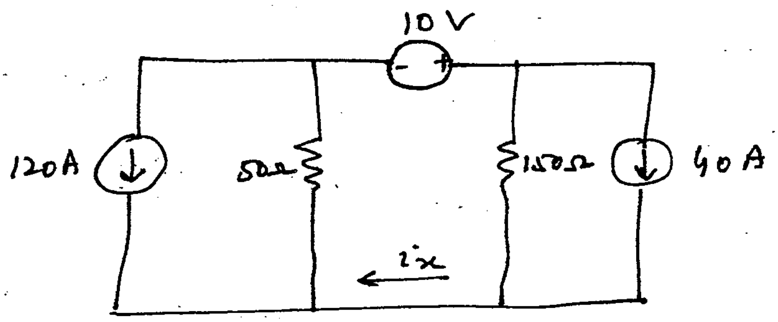


$$I_2 = \frac{80 \times 10^{-3}}{0.1 + 0.3} = 0.2\,A \quad \text{———} \quad (2)$$

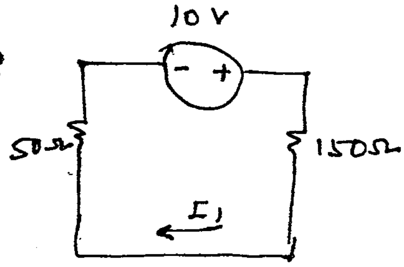
from (1) and (2)

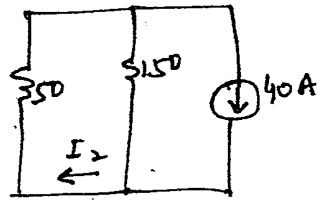
$$I = I_1 + I_2 = -0.375 + 0.2 = \underline{-0.175\,A}$$

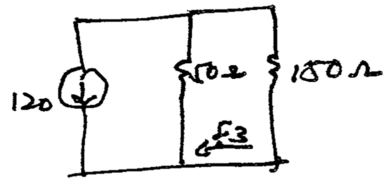
__Example:__ Find i_x in the network shown using superposition theorem.



__Soln__







$$I_1 = \frac{10}{50+150} = 0.05\,A \qquad I_2 = \frac{40 \times 150}{50+150} = 30\,A \qquad I_3 = -\frac{120 \times 50}{50+150} = -30\,A$$

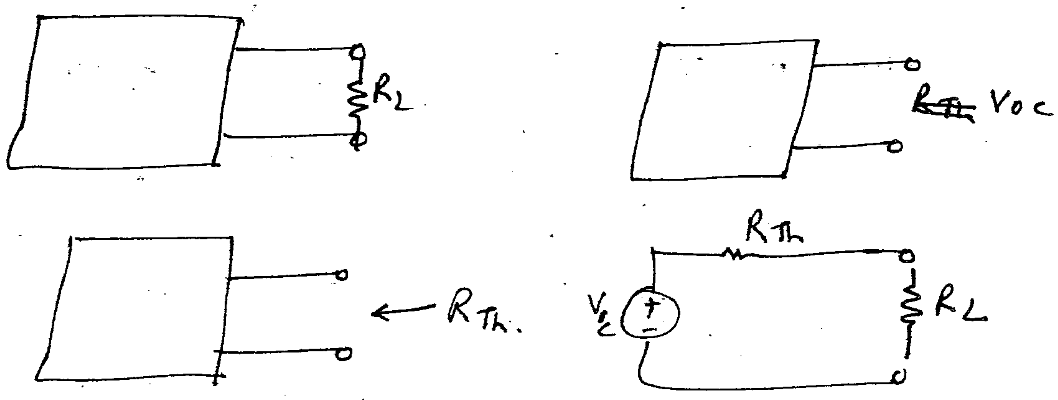
$$I = I_1 + I_2 + I_3 = .05\,A + 30 - 30 = 0.05\,A$$

Thevenin's Theorem

It is also known as Helmholtz - Thevenin theorem.

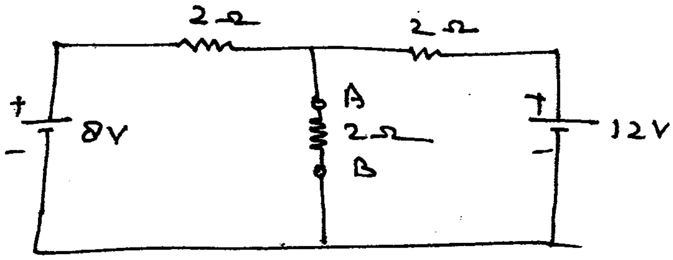
Any two terminals AB ~~is~~ of a network composed of linear passive ~~and~~ active ~~elements~~ may be replaced by a simple equivalent circuit consisting of an equivalent voltage source V_{oc} in series with an equivalent resistance R_{Th}. The voltage source V_{oc} is equal to the potential difference between the two terminals AB caused by the active network with no external resistance connected to these terminals. The series resistance R_{Th} is the equivalent resistance looking back into the network at terminals AB with all the sources within the network made inoperative.

This term has been ~~made~~ explained earlier.



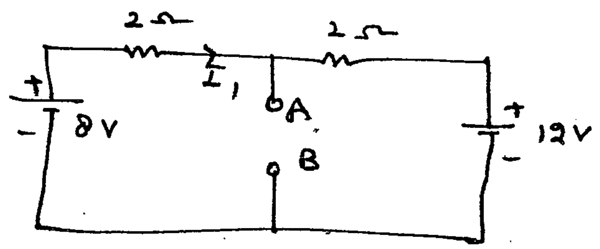
Example: Using Thevenin's theorem find the current and voltage in the 2-Ω resistors connected across the points A and B in the network shown.

p 15·2



Voc

- Remove 2-Ω resistor across AB



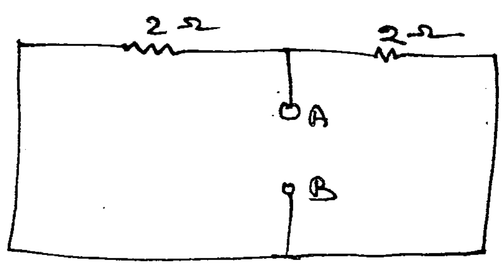
$$8 - (2+2)\,I_1 - 12 = 0$$

$$4I_1 = -4 \quad \therefore \quad I_1 = -1\,A$$

$$V_{oc} = 8 - (-1.2) = 8 + 2 = \underline{10\,V}$$

R_{Th}

- make the sources inoperative



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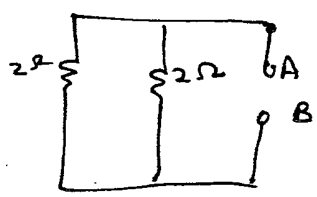
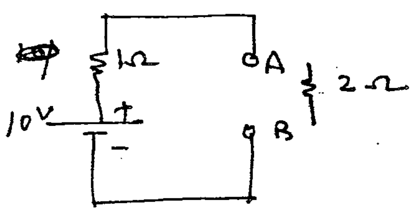
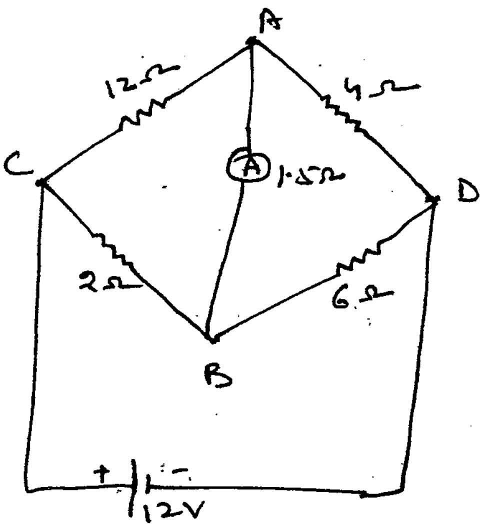


$$R_{Th} = \underline{1\,\Omega}$$

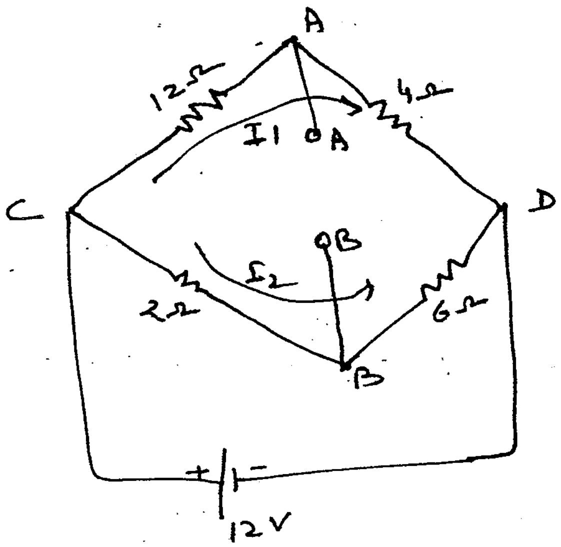


$$I_L = \frac{10}{1+2} = \underline{3.33\,A}$$

Example 153. Using Thevenin's ~~theorem~~ equivalent circuit, find the current in the ammeter A of resistance 1·5 Ω connected in an unbalanced Wheatstone bridge shown below.



Voc



$$I_1 = \frac{12}{12+4} = \frac{12}{16} = 0.75\,A \qquad I_2 = \frac{12}{2+6} = 1.5\,A$$

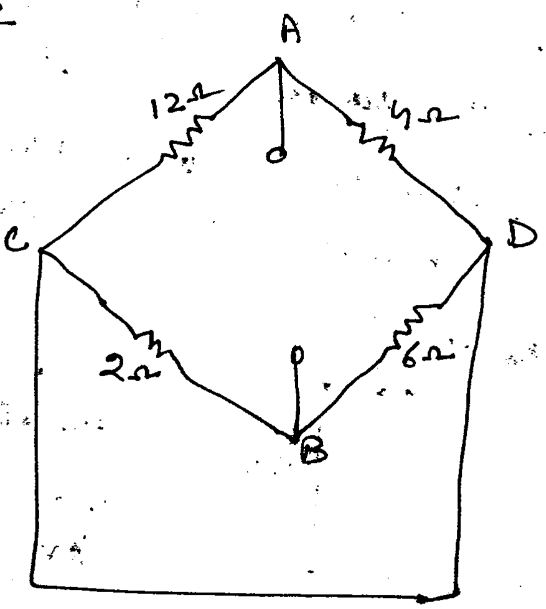
$$V_{oc} = V_{BC} - \cancel{V_{CA}} = \cancel{1.5\times 2 - 12\times .75}$$

$$\cancel{= 3 - 9 = -6V = V_{BA}}$$

$$V_{oc} = V_{AB} = V_{AD} - V_{DB} = 4\times 0.75 - 1.5\times 6 = 3 - 9 = \underline{-6V}$$

−ve sign means A is −ve w.r.t B.

R_{Th}



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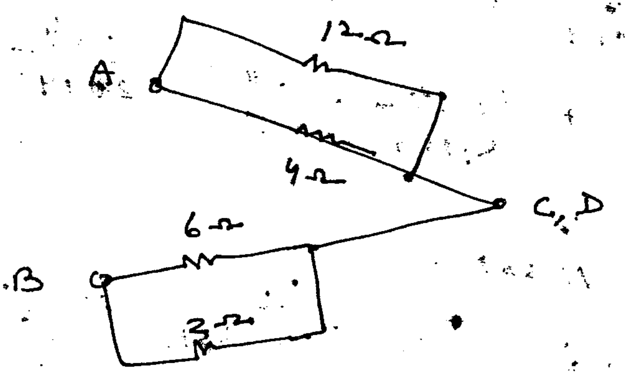

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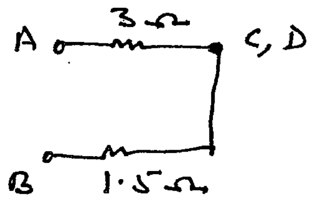

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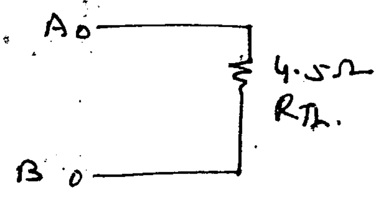
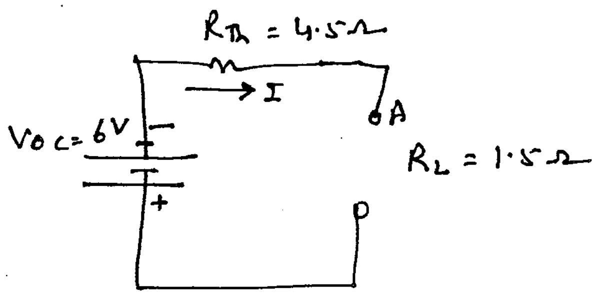
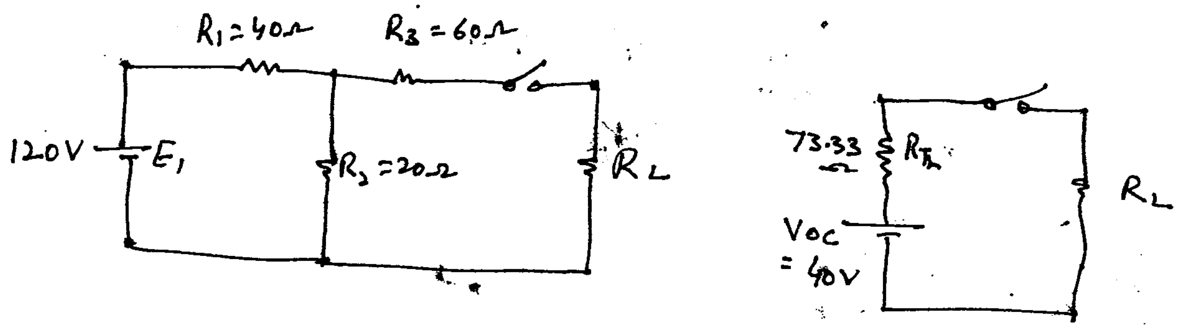




$$I = \frac{-6}{6} = \underline{-1A}$$

Example: In the network shown, the load
p-154
resistance R_L is changed so that its value becomes 10Ω, 50Ω and 200Ω. Find the current through R_L for each of these values.



Soln.

$$V_{oc} = \frac{120 \times 20}{20 + 40} = \underline{40V}$$

$$R_{Th} = R_3 + \frac{R_1 R_2}{R_1 + R_2} = 60 + \frac{20 \times 40}{20 + 40} = \underline{73.33\Omega}$$

$$I_L = \frac{V_{oc}}{73.33 + R_L}$$

For $R_L = 10\Omega$; $I_L = \dfrac{40}{73.33 + 10} = \underline{\underline{0.481\ A}}$

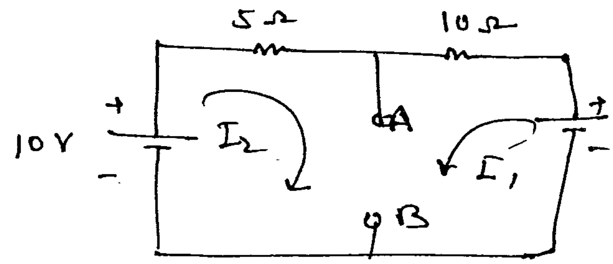
" " 50Ω, $I_L = \dfrac{40}{73.33 + 50} = \underline{\underline{0.324\ A}}$

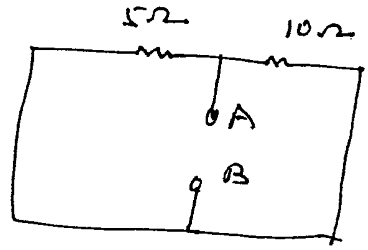
" " 200Ω, $I_L = \dfrac{40}{73.33 + 200} = \underline{\underline{0.146\ A}}$

Norton's Theorem

It states that a two terminal netw can be replaced by an equivalent circuit of a constant-current source in parallel with a resistance. The value of the constant-current source is the short-circuit current developed when the terminals of the original network are short-circuited. The parallel resistance is the resistance looking back into the original network, with all the sources with in the network made inactive (as in Thevenin's theorem).

Example: Obtain the Norton's equivalent circuit with respect to the terminals AB for the network shown and hence determine the value of current that would flow through a load resistance of $5\,\Omega$ if it were connected across the terminals AB.



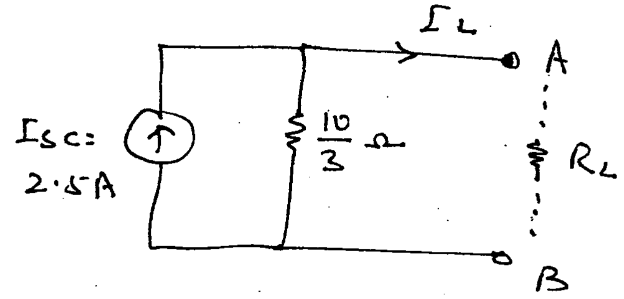


Soln:

$$I_{sc} = I_1 + I_2 = \frac{10}{5} + \frac{5}{10} = 2.5\,A$$

$$R_{Th} = \frac{5 \times 10}{5+10} = \frac{10}{3}\,\Omega$$

Norton's Equivalent is

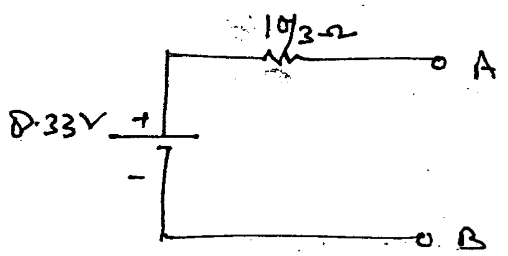


$$I_L = \frac{2.5 \times 10/3}{(10/3 + 5)} = \underline{\underline{1\,A}}$$

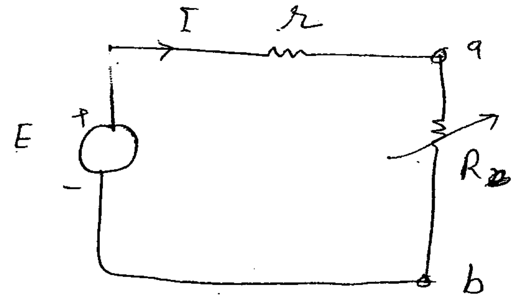
<u>Example</u>: Draw Thevenin's equivalent for the above example.

<u>Soln.</u> $V_{oc} = I_{sc} \times R_{Th} = 2.5 \times \frac{10}{3} = \underline{\underline{8.33V}}$

Thevenin's equivalent is



Maximum Power Transfer Theorem (CLW p-106)



Problem is to find R_L for which power transferred is maximum.

$$I = \frac{E}{r + R_L}$$

$$P = I^2 R = \frac{E^2}{(r+R)^2} R$$

For P to be maximum

$$\frac{dP}{dR} = 0$$

$$= \frac{E^2\left[(r+R)^2 - R\,2(r+R)\right]}{(r+R)^4} = 0$$

$$(r+R)^2 - 2R(r+R) = 0$$

$$r + R - 2R = 0$$

$$r - R = 0$$

$$\boxed{R = r}$$

$$P_{max} = \frac{E^2 r}{(2r)^2} = \frac{E^2}{4r} \quad \text{(Good)}$$

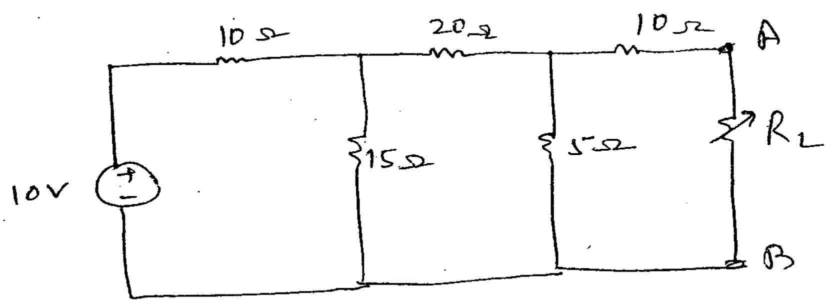
$$P_r = \frac{E^2}{4r}\,; \quad E_{ab} = E/2\,;$$

~~$E_{ab} = E/2$~~ $\eta = 50\%$ (poor)

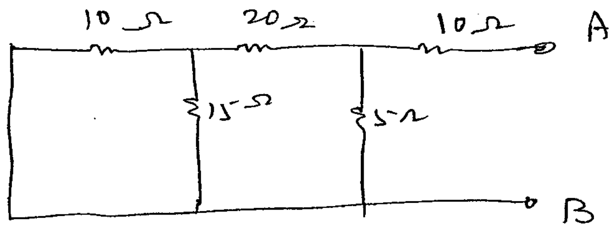
Voltage Regulation = 50% (poor)

It is used for impedance matching in electronic circuits for loud speakers.

Example Determine the value of R_L in the network shown for maximum power transfer. ~~and calculate the value of Power~~

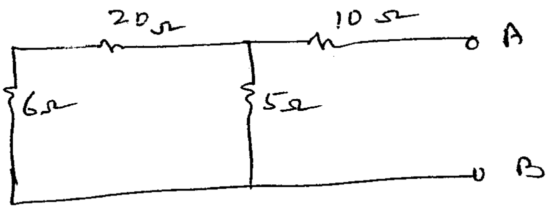


Soln:



$$\frac{10\times15}{25} = 6\,\Omega$$

=



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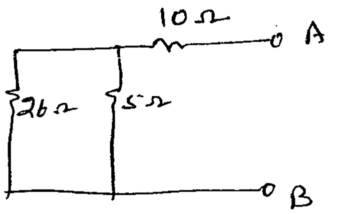

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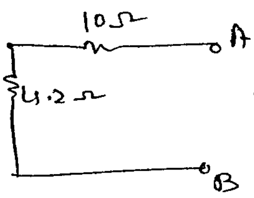

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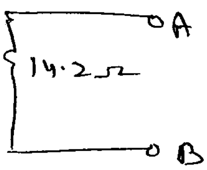
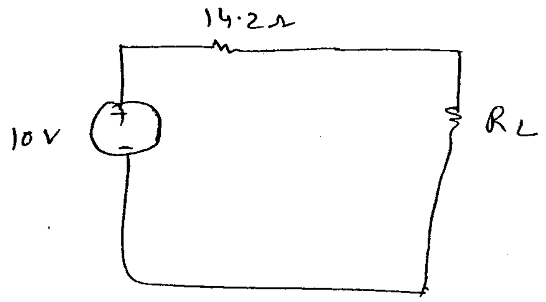


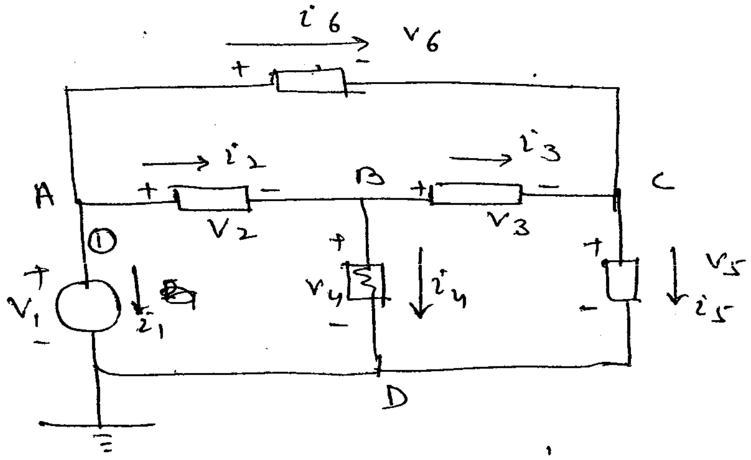
$$\frac{26\times5}{31} = 4.2\,\Omega$$



For P_{max} : $R_L = 14.2\,\Omega$

Telegon's Theorem

- This needs two Kirchhoff's laws to be applicable
- This is applicable to networks having elements which may be
  - linear / nonlinear
  - passive / active
  - time variant / invariant
- This is, therefore, the most powerful tool.



We need to choose i's to satisfy Kirchhoff's current law

and v's to satisfy Kirchhoff's voltage law.

KVL

for loop ABDA $\quad V_1 = 5, \quad V_2 = 3 \quad \therefore V_4 = 2$

" BCDB $\quad V_3 = 3V, \quad$ ~~$V_5 = -1V$~~ $\quad \therefore V_5 = -1V$

" ABCA $\quad V_2 = 3 \quad V_3 = 3V \quad \therefore V_6 = 6$

KCL

Node A $\quad i_2 = 4A, \quad i_6 = 2A \quad \therefore i_1 = -6A$

Node B $\quad i_4 = 1A, \quad$ ~~$i_3 = 3A$~~ $\quad \therefore i_3 = 3A$

Node C $\quad i_5 = i_3 + i_6 = 3+2 = 5A$

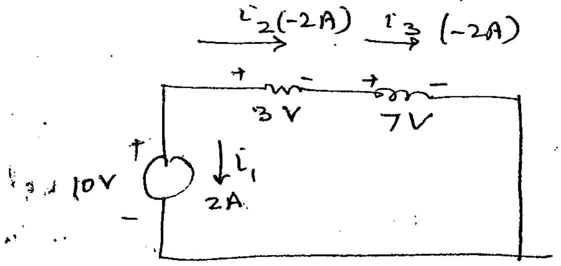
$$\sum_{k=1}^{b} v_k i_k = 0$$

verify

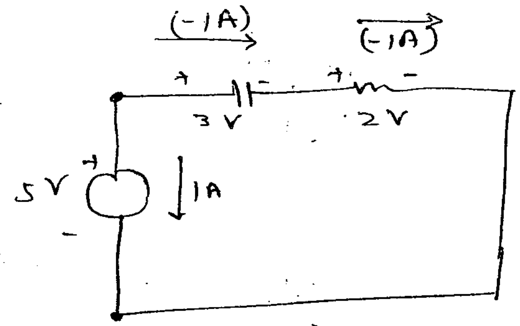
b. No.	v	i	vi
1	5	-6	-30
2	3	4	12
3	3	3	9
4	2	1	2
5	-1	5	-5
6	6	2	12

$\sum v_i = 0$

Example: verify Tellegen's theorem in respect of two networks shown



(i)



(ii)

For (i) $\sum_{k=1}^{3} v_k i_k$ = ~~10 × 2 + (3)×(-2) + (7)×2 = 0~~

$10 \times 2 + (3)(-2) + 7(-2) = 0$

(ii) $\sum_{k=1}^{3} v_k i_k = 5 \times 1 + (-1) \times 3 + (-1) \times 2 = 0$

$5 \times 1 + 3($

For v(I) and i(ii)

$\sum = 10(1) + 3(-1) - 7(-1) = 0$.

for v(ii) and i(i)

$5 \times 2 + 3(-2A) + (2) \times (-2) = 0$