Delhi Technological University
Department of Applied Maths
B.Tech Sem-I 2017
Assignment No. 5

Que 1. Find $d\vec{w}/dt$
(a)$\vec{w}(t) = (3t\hat{i} + 5t^2\hat{j} + 6\hat{k}).(t^2\hat{i} - 2t\hat{j} + t\hat{k})$
(b)$\vec{w}(t) = (t\hat{i} + e^t\hat{j} - t^2\hat{k})X(t^2\hat{i} + \hat{j} + t^3\hat{k})$

Que 2. Show that
(a)$\vec{\nabla}^2(logr) = 1/r^2$ (b)$\vec{\nabla}^2 r^n = n(n+1)r^{n-2}$.

Que 3. If $\vec{r} = xi + yj + zk$, Show that
(a) $gradr = \vec{r}/r$ (b)$grad(1/r) = -\vec{r}/r^3$
(c)$\nabla r^n = nr^{n-2}\vec{r}$.

Que 4. If u and $\vec{r}$ have continuous second order partial derivative , then show that
(a) $curl(gradu) = 0$
(b) $div(curl\vec{r}) = 0$.

Que 5. If $\vec{r} = xi + yj + zk$ and $r = |\mathbf{r}|$. Show that
(a) $div(\mathbf{r}/r^3) = 0$ (b) $grad(1/r) = -\hat{r}/r^2$.

Que 6. Find a unit normal vector to the surface $xy^2 + 2yz = 8$ at the point (3,-2,1). (Ans-$(2i - 5j - 2k)/\sqrt{33}$)

Que 7. Find the normal vector and the equation of the tangent plane to the surface $z = \sqrt{x^2 + y^2}$ at point (3,4,5).

Que 8. Find the directional derivative of $F(x, y, z) = x^2y^3 + xy$ at (2,1), in the direction of a unit vector which makes an angle of $\pi/3$ with x-axis. (Ans-$(5 + 14\sqrt{3})/2$)

Que 9. Find the directional derivative of $F(x, y, z) = xy^2 + 4xyz + z^2$ at the point (1,2,3) in the direction of $3i + 4j - 5k$. (Ans-$78/5\sqrt{2}$)

DELHI TECHNOLOGICAL UNIVERSITY

Assignment 6
B.tech.1st Semester-2017

Vector Integral Calculus

1. If $\vec{r} = t\ \hat{i} - t^2\ \hat{j} + (t-1)\ \hat{k}$ and $\vec{s} = 2t^2\ \hat{i} + 6t\ \hat{k}$, evaluate $\int_0^2 \vec{r} \times \vec{s}\ dt$. (ans: $-24\hat{i} - \frac{40}{3}\hat{j} + \frac{64}{5}\hat{k}$)
2. Evaluate $\int\limits_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = xy\ \hat{i} + (x^2+y^2)\ \hat{j}$ and C is the arc of the parabola $y = x^2 - 4$ from $A(2,0)$ to $B(4,12)$ in the $xy - plane$. (ans: 732)
3. Evaluate $\iint\limits_S (yz\ \hat{i} + zx\ \hat{j} + xy\ \hat{k}) \cdot dS$ where S is the surface of the surface $x^2+y^2+z^2 = a^2$ in the first octant. (ans: $\frac{3a^4}{8}$)
4. Evaluate $\iint\limits_S \vec{F} \cdot \hat{n}\ dS$ where $\vec{F} = 2\ \hat{i} + x\ \hat{j} - 3y^2z\ \hat{k}$ and S is the surface of the cylinder $x^2+y^2 = 16$ included in the first octant between $z=0$ and $z=5$. (ans: 90)
5. Evaluate $\int\limits_C (y - sinx)\ dx + cosx\ dy$ by using *Green's theorem*, where C is the triangle whose vertices are $(0,0), (\frac{\pi}{2}, 0), (\frac{\pi}{2}, 1)$. (ans: $\frac{-2}{\pi} - \frac{\pi}{4}$)
6. Evaluate by *Green's theorem*, $\int\limits_C (x^2 - cosh\ y)\ dx + (y + sinx)\ dy$, where C is the rectangle with vertices $(0,0), (\pi,0), (\pi,1), (0,1)$. (ans: $\pi\ cosh\ 1 - \pi$)
7. Using *Stokes' theorem* prove that $curl\ grad\ \phi = 0$.
8. Verify *Stokes' theorem* for $\vec{F} = xy^2\ \hat{i} + y\ \hat{j} + z^2x\ \hat{k}$ for the surface of a rectanglar lamina bounded by $x=0, y=0, x=1, y=2, z=0$.
9. Verify *Stokes' theorem* for $\vec{F} = (x^2-y^2)\ \hat{i} + 2xy\ \hat{j}$ in the rectangular region in the $xy - plane$ given by $(0,0), (a,0), (0,b), (a,b)$.
10. Using *divergence theorem*, evaluate $\iint\limits_S \vec{r} \cdot dS$, where S is the surface of the sphere $x^2+y^2+z^2 = 9$. (ans: 108π)
11. Verify *divergence theorem* for $\vec{F} = (4x^3\ \hat{i} - x^2y\ \hat{j} + x^2z\ \hat{k})$ taken over the region bounded by the cylinder $x^2+y^2 = a^2, z=0, z=b$. (ans: $3\pi a^4 b$)
12. Evaluate $\iint (xdydz + ydzdx + zdxdy)$ over the surface of a sphere of radius 2. (ans: 32π)