ASSIGNMENT NO:-1

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2K20/B17/33
MATHS

Ques 1.

i) $A = \begin{bmatrix} 0 & -1 & 5 \\ 2 & 4 & -6 \\ 1 & 1 & 5 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$\begin{bmatrix} 2 & 4 & -6 \\ 0 & -1 & 5 \\ 1 & 1 & 5 \end{bmatrix}$

$R_1 \to R_1 \div 2$

$\begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 5 \\ 1 & 1 & 5 \end{bmatrix}$

$R_3 \to R_3 - R_1$

$\begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 5 \\ 0 & -1 & 8 \end{bmatrix}$

$R_3 \to R_3 - R_2$

$\begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 5 \\ 0 & 0 & 3 \end{bmatrix}$

Matrix rank is 3 Ans.

(ii) $B = \begin{bmatrix} 5 & 3 & 0 \\ 1 & 2 & -4 \\ -2 & -4 & 8 \end{bmatrix}$ $R_1 \to \frac{R_1}{5}$ $\begin{bmatrix} 1 & \frac{3}{5} & 0 \\ 1 & 2 & -4 \\ -2 & -4 & 8 \end{bmatrix}$

$R_2 \to R_2 - R_1$ $\begin{bmatrix} 1 & \frac{3}{5} & 0 \\ 0 & 7/5 & -4 \\ -2 & -4 & 8 \end{bmatrix}$

$R_1 \to R_1 \times (-2)$

$\begin{bmatrix} -2 & -\frac{6}{5} & 0 \\ 0 & 7/5 & -4 \\ -2 & -4 & 8 \end{bmatrix}$

$R_3 \to R_3 - R_1$
$R_1 \to R_1 \div (-2)$

$\begin{bmatrix} 1 & \frac{3}{5} & 0 \\ 0 & \frac{7}{5} & -4 \\ 0 & \frac{-14}{5} & 8 \end{bmatrix}$

$R_1 \to R_1 \times 5$
$R_2 \to R_2 \div (7/5)$

$\begin{bmatrix} 5 & 3 & 0 \\ 0 & 1 & -\frac{20}{7} \\ 0 & \frac{-14}{5} & 8 \end{bmatrix}$

$R_2 \to R_2 \times \left(\frac{-14}{5}\right)$

$\begin{bmatrix} 5 & 3 & 0 \\ 0 & \frac{-14}{5} & 8 \\ 0 & -\frac{14}{5} & 8 \end{bmatrix}$

$R_3 \to R_3 - R_2$
$R_2 \to R_2 \div \left(\frac{-14}{5}\right)$

$\begin{bmatrix} 5 & 3 & 0 \\ 0 & 1 & -20/7 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore$ Matrix Rank is 2 Ans

PJohri

iii) $C = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{bmatrix}$

$R_1 \to R_1 \times 2$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 2 & 4 & -2 & 6 \\ 0 & 0 & 3 & -8 \\ 3 & 6 & 3 & -7 \end{bmatrix} R_1 \to R_1 \times \frac{3}{2}$$

$$\begin{bmatrix} 3 & 6 & -3 & 9 \\ 0 & 0 & 3 & -8 \\ 3 & 6 & 3 & -7 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 3 & 6 & -3 & 9 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 6 & -16 \end{bmatrix}$$

$R_1 \to R_1 \div 3$

$R_2 \to R_2 \div 3$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 1 & 8/3 \\ 0 & 0 & 6 & -16 \end{bmatrix}$$

$R_2 \to R_2 \times 6$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 6 & -16 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_2 \to R \div 2 \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Matrix rank will be 2

Ans

Srijohri

Ques 2.)

(i.) $X = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ $\begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_3 \end{matrix}$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -1 & -2 \end{bmatrix}$ $R_3 \to R_3 - R_2$.

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ Number of non zero rows = 2

Rank = 2

Echelon form

ii.) $Y = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 3 & 1 & 1 & 3 \end{bmatrix}$ $\begin{matrix} R_1 \leftrightarrow R_2 \\ R_3 \to R_3 - 3R_1 \end{matrix}$ $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & -5 & -8 & -3 \end{bmatrix}$

$R_3 \to R_3 + 5R_2$

$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 2 \end{bmatrix}$ $\therefore$ Rank = 3

Echelon form

iii.) $Z = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{bmatrix}$ $\begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$ $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 1 & 2 & -2 \end{bmatrix}$

$R_3 \to R_3 - R_2$

$R_2 \to R_2 \times (-1)$ $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

Echelon form

$\therefore$ Rank = 3

Bjohri

Ques 3)

i) $2x+y+z=5$

$x+y+z=4$

$x-y+2z=1$

$$\boxed{AX=B}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix} R_1 \to R_1 \div 2 \quad \begin{bmatrix} 1 & 0.5 & 0.5 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & -1.5 & 1.5 \end{bmatrix} \begin{matrix} R_2 \to R_2 \times 2 \\ R_3 \to R_3 + 1.5R_2 \end{matrix} \quad \begin{bmatrix} 1 & 0.5 & 0.5 \\ 1 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

$R_3 \to R_3 \div 3$

$$\begin{bmatrix} 1 & 0.5 & 0.5 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \therefore \rho(A) = 3 \text{ Ans.}$$

Augmented matrix $C = [A, B] \sim \begin{bmatrix} 2 & 1 & 1 & 5 \\ 1 & 1 & 1 & 4 \\ 1 & -1 & 2 & 1 \end{bmatrix}$

$$\begin{matrix} R_1 \to R_1 \div 2 \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix} \begin{bmatrix} 1 & 0.5 & 0.5 & 2.5 \\ 0 & 0.5 & 0.5 & 1.5 \\ 0 & -1.5 & 1.5 & -1.5 \end{bmatrix} \begin{matrix} R_2 \to R_2 \times 2 \\ R_3 \to R_3 + 1.5R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0.5 & 0.5 & 2.5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 3 & 3 \end{bmatrix} R_3 \to R_3/3 \quad \begin{bmatrix} 1 & 0.5 & 0.5 & 2.5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$\therefore \rho(C) = 3$

$\because \rho(A) = \rho(C) =$ No. of variable $\therefore$ It will have unique

Now, $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -\frac{1}{3} & 1 & -1/3 \\ -\frac{2}{3} & 1 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \text{Ans}$$

ii) $x + y + z = 6$
$x + 2y + 3z = 14$
$x + 4y + 7z = 30$

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \therefore \rho(A) = 2$$

Augmented Matrix $C = [A, B] \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{bmatrix} R_3 \to R_3 - 3R_2 \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \therefore \rho(C) = 2$$

Jyoti

$\because \rho(A)=\rho(C) <$ No of variables $\therefore$ It has infinite solutions.

Let $z=k$

$x=6-y-k$

$(6-y-k)+2y+3k=14 \Rightarrow y=8-2k$

$x=6-8+2k-k = k-2$

$z=k$; $k \in R$ Ans

4.) $x-4y+7z=14$

$3x+8y-2z=13$ $\qquad AX=B$

$7x-8y+26z=5$

$$\begin{bmatrix} 1 & -4 & 7 \\ 3 & 8 & -2 \\ 7 & -8 & 26 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4 & 7 \\ 3 & 8 & -2 \\ 7 & -8 & 26 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 7R_1 \end{matrix} \begin{bmatrix} 1 & -4 & 7 \\ 0 & 20 & -23 \\ 0 & 20 & -23 \end{bmatrix} \begin{matrix} R_2 \to R_2 \div 2 \\ R_3 \to R_3 - R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & -4 & 7 \\ 0 & 1 & -1.15 \\ 0 & 0 & 0 \end{bmatrix} \quad \therefore \rho(A)=2$$

Augmented matrix $C=[A, B] \sim \begin{bmatrix} 1 & -4 & 7 & 14 \\ 3 & 8 & -2 & 13 \\ 7 & -8 & 26 & 5 \end{bmatrix}$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 7R_1$

Jyothi

$$\begin{bmatrix} 1 & -4 & 7 & 14 \\ 0 & 20 & -23 & -29 \\ 0 & 20 & -23 & -9 \end{bmatrix} \begin{matrix} R_2 \to R_2 \div 20 \\ R_3 \to R_3 - 20R_2 \end{matrix} \begin{bmatrix} 1 & -4 & 7 & 14 \\ 0 & 1 & -1.15 & -1.45 \\ 0 & 0 & 0 & -64 \end{bmatrix}$$

$$R_3 \to R_3 \div (-64) \begin{bmatrix} 1 & -4 & 7 & 14 \\ 0 & 1 & -1.15 & -1.45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \therefore \rho(C) = 3$$

$\therefore \rho(A) \neq \rho(C)$,

$\therefore$ Equations are in consistent

Ques 5)

$x + 2y - 3z = -2$

$3x - y - 2z = 1$

$2x + 3y - 5z = K$

$$\boxed{AX = B}$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & -2 \\ 2 & 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ K \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & -2 \\ 2 & 3 & -5 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix} \begin{bmatrix} 1 & 2 & -3 \\ 0 & -7 & 7 \\ 0 & -1 & 1 \end{bmatrix} \begin{matrix} R_2 \to R_2 \div (-7) \\ R_3 \to R_3 + R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \therefore \rho(A) = 2$$

Augmented Matrix $C = [A, B] \sim \begin{bmatrix} 1 & 2 & -3 & -2 \\ 3 & -1 & -2 & 1 \\ 2 & 3 & -5 & K \end{bmatrix}$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$ $\begin{bmatrix} 1 & 2 & -3 & -2 \\ 0 & -7 & 7 & 7 \\ 0 & -1 & 1 & K+4 \end{bmatrix} \begin{matrix} R_2 \to R_2 \div 7 \\ R_3 \to R_3 - R_2 \end{matrix}$

$$\begin{bmatrix} 1 & 2 & -3 & -2 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & k+3 \end{bmatrix}$$

pjohri

To be consistent, $\rho(C) = 2$

$\therefore k+3=0$

$\underline{k=-3}$ Ans

Ques 6.)

$x+y+z=7$

$x+2y+3z=18$

$y+kz=6$

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 18 \\ 6 \end{bmatrix}$$

A

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & k \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & k \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & k-2 \end{bmatrix}$$

To be inconsistent

$\rho(A) \neq \rho(C)$

$\therefore k-2=0$; $k=2$

for $k=2$,

$\rho(A) = 2$

$\rho(C) = 3$

Augmented Matrix $= [A, B]$

$$\begin{bmatrix} 1 & 1 & 1 & 7 \\ 1 & 2 & 3 & 18 \\ 0 & 1 & k & 6 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & k-2 & -5 \end{bmatrix}$$

Ques 7)

$x+ay+z=3$
$x+2y+2z=b$
$x+5y+3z=9$

$$\left[\begin{array}{ccc|c} 1 & a & 1 & 3 \\ 1 & 5 & 3 & 9 \\ 1 & 2 & 2 & b \end{array}\right]$$

(Augmented Matrix)

$R_2 \to R_2 - R_3$
$$\left[\begin{array}{ccc|c} 1 & a & 1 & 3 \\ 0 & 3 & 1 & 9-b \\ 1 & 2 & 2 & b \end{array}\right]$$
$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & a & 1 & 3 \\ 0 & 3 & 1 & 9-b \\ 0 & 2-a & 1 & b-3 \end{array}\right]$$
For consistent $\rho(A)=\rho(C)$

i) For $a=2$ and $b \in R$, the system will be consistent and have unique solution because

$\rho(A)=\rho(C)=3=$ No of variables

ii) For $a=-1$ and $b=6$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 3 & 1 & 0 \\ 0 & 3 & 1 & 0 \end{array}\right]$$
$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Pishori

The system will be consistent but have infinite solution because

$\rho(A)=\rho(C)=2<$ No of variables.

Ques 8

i) $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$

$A - \lambda I = 0$

$-\lambda^3 + 6\lambda^2 - 10\lambda + 4 = 0$

$(2-\lambda)(x^2 - 4x + 2) = 0$

$\therefore$ Eigen values will be $2-\sqrt{2},\ 2+\sqrt{2},\ 2.$

For eigen vectors, $\begin{bmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

when $\lambda = 2-\sqrt{2}$, $\sqrt{2}x_1 - x_2 = 0$

$-x_1 + \sqrt{2}x_2 - x_3 = 0$

$-x_2 + \sqrt{2}x_3 = 0$

$\frac{x_1}{1-0} = \frac{x_2}{0+\sqrt{2}} = \frac{x_3}{2-1}$

$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix}$

When $\lambda = 2+\sqrt{2}$, $-\sqrt{2}x_1 - x_2 = 0$

$-x_1 - \sqrt{2}x_2 - x_3 = 0$

$-x_2 - \sqrt{2}x_3 = 0$

$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix}$

pjohri

When $\lambda = 2$

$$-x_2 = 0$$
$$-x_2 - x_3 = 0$$
$$-x_2 = 0$$

$$\frac{x_1}{1-0} = \frac{x_2}{0-0} = \frac{x_3}{0-1}$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

ii.) $B = \begin{pmatrix} 10 & -2 & 4 \\ -20 & 4 & -10 \\ -30 & 6 & -13 \end{pmatrix}$

$$B - \lambda I = 0$$
$$-\lambda^3 + \lambda^2 + 2\lambda = 0$$
$$(0-\lambda)(2-\lambda)(-1-\lambda) = 0$$

$\therefore$ Eigen values will be $0, -1, 2$.

For Eigen vectors $\begin{bmatrix} 10-\lambda & -2 & 4 \\ -20 & 4-\lambda & -10 \\ -30 & 6 & -13-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

When $\lambda = 0$,

$$10x_1 - 2x_2 + 4x_3 = 0$$
$$-20x_1 + 4x_2 - 10x_3 = 0$$
$$-30x_1 + 6x_2 - 13x_3 = 0$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}$$

When $\lambda = -1$

$$\begin{bmatrix} 11 & -2 & 4 \\ -20 & 5 & -10 \\ -30 & 6 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$11x_1 - 2x_2 + 4x_3 = 0$$
$$-20x_1 + 5x_2 - 10x_3 = 0$$
$$-30x_1 + 6x_2 - 12x_3 = 0$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

Jyotshree

When $\lambda = 2$,

$$\begin{bmatrix} 8 & -2 & 4 \\ -20 & 2 & -10 \\ -30 & 6 & -15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$8x_1 - 2x_2 + 4x_3 = 0$$
$$-20x_1 + 2x_2 - 10x_3 = 0$$
$$-30x_1 + 6x_2 - 15x_3 = 0$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$\Rightarrow 4x_1 - x_2 + 2x_3 = 0$$
$$-10x_1 + x_2 - 5x_3 = 0$$
$$-10x_1 + 2x_2 - 5x_3 = 0$$

$$x_1 = \begin{bmatrix} 3k \\ 0k \\ -6k \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$\frac{x_1}{5-2} = \frac{x_2}{-20+20} = \frac{x_3}{4-10} = k.$$

<u>Ques 9></u>

i) We have $(A - \lambda I)' = A' - \lambda I' = A' - \lambda I$

$= |(A - \lambda I)'| = |A' - \lambda I|$

$= |A - \lambda I| = |A' - \lambda I|$

$\therefore |A - \lambda I| = 0$ if and only if $|A' - \lambda I| = 0$

i.e., λ is an eigen value of A if and only if it is an eigen value of A'

Jjohri

ii) If X be the eigen vector corresponding to λ, $AX = \lambda X$, Premultiplying both sides by A^{-1}, we get
$A^{-1}AX = A^{-1}\lambda X$
i.e. $IX = \lambda A^{-1}X$ or $X = \lambda(A^{-1}X)$, i.e., $A^{-1}X = \left(\frac{1}{\lambda}\right)X$
This being of the same form as (i), shows that $1/\lambda$ is an eigen value of the inverse matrix A^{-1}

iii) Let A_i be the eigen value of A and X_i the corresponding eigen vectors, then,

$$AX_i = \lambda_i X_i$$

we have $A^2 X_i = A(AX_i) = A(\lambda_i X_i) = \lambda_i(AX_i) = \lambda(\lambda_i X_i)$
$= \lambda_i^2 X_i$
Similarly $A^3 X_i = \lambda_i^3 X_i$. In general, $A^m X_i = \lambda_i^m X_i$ which is of the same form as (i)
Hence, λ_i^m is an eigen value for A^m
The corresponding eigen vector is the same X_i.

10)

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Bijohri

$$\begin{pmatrix} -\lambda & 1 & 0 \\ -1 & -\lambda & 0 \\ 0 & 0 & 2-\lambda \end{pmatrix} = (2-\lambda)(1+\lambda^2)$$

$\therefore$ Eigen value $= 2, i, -i$
$\hookrightarrow$ Real value

Eigen vector

$$\begin{bmatrix} -\lambda & 1 & 0 \\ -1 & -\lambda & 0 \\ 0 & 0 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda = 2$,

$$\frac{x_1}{0} = \frac{x_2}{0} = \frac{x_3}{5} = K$$

$$x = \begin{bmatrix} 0K \\ 0K \\ 5K \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Ques 11)

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 3-\lambda & -1 & 0 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & -5-\lambda \end{bmatrix}$$

$$-\lambda^3 + \lambda^2 + 21\lambda - 45 = 0$$

$$(-5-\lambda)(3-\lambda)^2$$

Eigen values $\longrightarrow$ $-5, \underbrace{3, 3}_{\text{Multiplicity } 2}$

$$\begin{bmatrix} 3-\lambda & -1 & 0 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & -5-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda = -5$

Eigen vector $x_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

For $\lambda = 3$

Eigen value for

$$x_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Bjohri

$$B = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 3-\lambda & 0 & 0 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & -5-\lambda \end{pmatrix}$$

$$= -\lambda^3 + \lambda^2 + 21\lambda - 45 = 0$$

Eigen values $\longrightarrow -5, \underbrace{3, 3}_{\text{Multiplicity } 2}$

a.) $\lambda = -5$

$$\begin{bmatrix} 3-\lambda & 0 & 0 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & -\lambda-5 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The null space (eigen vector) of this matrix is

$$X_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

b.) $\lambda = 3$

$$\begin{bmatrix} 3-\lambda & 0 & 0 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & -\lambda-5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -8 \end{bmatrix}$$

The null space (eigen vectors) of this matrix is

$$x_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad x_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

pjohri

12.)

$$A = \begin{bmatrix} -4 & -4 & -8 \\ 4 & 6 & 4 \\ 6 & 4 & 10 \end{bmatrix}$$

The characteristic equation $\Rightarrow -\lambda^3 + 12\lambda^2 - 44\lambda + 48 = 0$

$$(6-\lambda)(4-\lambda)(2-\lambda) = 0$$

Eigen values are 6, 4, 2.

Eigen vector is given by $\begin{bmatrix} -4-\lambda & -4 & -8 \\ 4 & 6-\lambda & 4 \\ 6 & 4 & 10-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

For $\lambda = 2$ For $\lambda = 4$ For $\lambda = 6$

$X_1 = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$ $X_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $x_3 = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$

Modal Matrix $P = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 0 & -1 \\ 1 & 1 & -2 \end{bmatrix}$

$$P^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & -2 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

$$D = P^{-1}AP = \begin{bmatrix} 1 & 0 & 1 \\ -1 & -2 & 0 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} -4 & -4 & -8 \\ 4 & 6 & 4 \\ 6 & 4 & 10 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 0 & -1 \\ -1 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 2 \\ -4 & -8 & 0 \\ -6 & -6 & -6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 0 & -1 \\ -1 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix} \text{Ans}$$

Jyoti

Ques 13.)

$A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 0 & -3 & -3 \\ -3 & 0 & -3 \\ -3 & -3 & 0 \end{bmatrix}$

Characteristic Equation

$(3-\lambda)^2(-5-\lambda)$

$\underbrace{3, 3}, -5$

[Multiplicity 2]

Eigen vector is given by

$\begin{bmatrix} 3-\lambda & -1 & 0 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & -5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

for $\lambda = -5, \lambda = 3$

$X_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\therefore$ It has only two eigen vector

$\therefore$ A is non-diagonalizable

$(3-\lambda)^2(-6-\lambda)$

$\underbrace{3, 3}, -6$

(Multiplicity 2)

$\begin{bmatrix} -\lambda & -3 & -3 \\ -3 & -\lambda & -3 \\ -3 & -3 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

for $\lambda = -6$

$X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

for $\lambda = 3$

$X_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

$X_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

Modal Matrix $P = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$P^{-1} \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix}$

$D = P^{-1}BP$

$= \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ Ans

Sjohri

Ques 14.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

The characteristic equation of the matrix A is

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$$

According to Cayley Hamilton theorem, we have

$$A^3 - 5A^2 + 7A - 3I = 0$$

Multiplying it by A^{-1}, we get

$$A^2 - 5A + 7I - 3A^{-1} = 0$$

$$A^{-1} = \frac{1}{3}\left[A^2 - 5A + 7I\right]$$

But,

$$A^2 = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix}$$

$$\therefore A^2 - 5A + 7I = \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} - 5\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & -1 \\ 0 & 3 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

Anjohri

$$\therefore A^{-1} = \frac{1}{3}\left[A^2 - 5A + 7I\right] = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ 0 & 3 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

Now, $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$

$= A^5(A^3-5A^2+7A-3I)+A(A^3-5A^2+7A-3I)$.

$\because [A^3-5A^2+7A-3I=0]$

$= A^2+A+I$

$$= \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

Ques 15.)

$$A = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}$$

The characteristic equation is $\begin{bmatrix} 1-\lambda & 2 & 6 \\ 2 & 5-\lambda & 15 \\ 6 & 15 & 46-\lambda \end{bmatrix} = 0$

i.e., $-\lambda^3+52\lambda^2-16\lambda+1=0$

By cayley Hamilton theorem, $-A^3+52A^2-16A+I=0$

$A^{-1} = A^2-52A+16I$

$$A^2 = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}\begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix} = \begin{bmatrix} 41 & 102 & 312 \\ 102 & 254 & 777 \\ 312 & 777 & 2377 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 41 & 102 & 312 \\ 102 & 254 & 777 \\ 312 & 777 & 2377 \end{bmatrix} - \begin{bmatrix} 52 & 104 & 312 \\ 104 & 260 & 780 \\ 312 & 780 & 2392 \end{bmatrix} + \begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix}$$

$\therefore A^{-1} = \begin{pmatrix} 5 & -2 & 0 \\ -2 & 10 & -3 \\ 0 & -3 & 1 \end{pmatrix}$

16.)

(i) $\{(3,4,5), (-3,0,5), (4,4,4), (3,4,0)\}$

$$\begin{bmatrix} 3 & 4 & 5 \\ -3 & 0 & 5 \\ 4 & 4 & 4 \\ 3 & 4 & 0 \end{bmatrix} R_1 \to \frac{R_1}{3} \begin{bmatrix} 1 & \frac{4}{3} & \frac{5}{3} \\ -3 & 0 & 5 \\ 4 & 4 & 4 \\ 3 & 4 & 0 \end{bmatrix}$$

$R_2 \to R_2 + 3R_1$
$R_3 \to R_3 - 4R_1$
$R_4 \to R_4 - 3R_1$

$$\begin{bmatrix} 1 & \frac{4}{3} & \frac{5}{3} \\ 0 & 4 & 10 \\ 0 & -\frac{4}{3} & -8/3 \\ 0 & 0 & -5 \end{bmatrix} R_2 \to R_2 \div 4$$

$$\begin{bmatrix} 1 & 4/3 & 5/3 \\ 0 & 1 & 2.5 \\ 0 & -4/3 & -8/3 \\ 0 & 0 & -5 \end{bmatrix} R_3 \to R_3 + \frac{4}{3}R_2 \begin{bmatrix} 1 & 4/3 & 5/3 \\ 0 & 1 & 2.5 \\ 0 & 0 & 2/3 \\ 0 & 0 & -5 \end{bmatrix}$$

$$R_3 \to R_3 \div \left(\frac{2}{3}\right) \begin{bmatrix} 1 & 4/3 & 5/3 \\ 0 & 1 & 2.5 \\ 0 & 0 & 1 \\ 0 & 0 & -5 \end{bmatrix} R_4 \to R_4 + 5R_3$$

$$\begin{bmatrix} 1 & 4/3 & 5/3 \\ 0 & 1 & 2.5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank $= 3 \neq$ No of vectors

Hence, vectors are linearly dependent

Spohie

ii) $\{(1,1,1), (1,2,0), (0,-1,1)\}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix} R_2 \to R_2 - R_1 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} R_3 \to R_3 + R_2 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank $= 2 \neq$ No of vectors

$\therefore$ The vectors are linearly dependent.

iii) $\{(1,1,1), (1,2,0), (0,-1,2)\}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & -1 & 2 \end{bmatrix} R_2 \to R_2 - R_1 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$R_3 \to R_3 + R_2 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Rank $= 3 =$ No of vectors

$\therefore$ The vectors are linearly dependent.

Jyothi