Roll No. 244

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B.Tech (All groups)

2nd SEMESTER

MAY 2014

END SEMESTER EXAMINATION

AM – 111 Mathematics-II

Time : 3hr

Max. Marks: 70

Note: Attempt all questions selecting ***two parts*** from each question. All question carry equal marks. Assume missing data, if any.

1 (a) Find the inverse of the matrix using Cayley Hamilton theorem

$$A = \begin{pmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

(b) Test for the consistency, and if consistent then solve the system of equation

$$2x - 3y + 7z = 5$$
$$3x + y - 3z = 13$$
$$2x + 19y - 47z = 32$$

(c) Find the eigen values and eigen vectors of the matrix and if possible construct diagonalising matrix.

$$A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

2 (a) Solve the differential equation $\frac{d^4y}{dx^4} + n^4y = 0.$

(b) Solve the simultaneous differential equations

$$y_1' + y_1 + 3y_2 = 4e^{-t} \text{ and } y_2' + 4y_1 + 3y_2 = 8t$$

(c) Solve the differential equation $\frac{d^2y}{dx^2} + 4y = sin^2x + x^2e^{2x}$

3 (a) Obtain the series solution of the differential equation

$$x(1-x)y'' + (1-x)y' + y = 0$$

(b) State and prove the orthogonality of Bessel's function of first kind of order 'n'.

(c) State and prove the Rodrigue's formula for $P_n(x)$. Express x^3 in terms of Legendre's Polynomial.

4 (a) Find Laplace transform of the function

$$f(t) = \sin(at) - (at)\cos(at)$$

(b) If $f(t)$ is a periodic function of period 'a', then find it's Laplace transform.

(c) Solve the differential equation using Laplace transform

$$y'' + 9y = cos2t\,,\quad y(0) = 1\,,\quad y\left(\frac{\pi}{2}\right) = -1$$

5 (a) Find the Fourier series for the function

$$f(x) = x + x^2, -\pi < x < \pi$$

and deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

(b) Find a series of cosine of multiples of x which will represent $xsinx$ in the interval $(0\,, \pi)$.

(c) Find Fourier transform of $f(x) = \begin{cases} 1 - x^2, & -1 < x < 1 \\ 0\,, & elsewhere \end{cases}$

and hence evaluate the integral

$$\int_0^\infty \frac{xcosx - sinx}{x^3} cos\frac{x}{2}\,dx\,.$$