

(1) (a) by the boy

$$x=0$$

$$y = 20t - \frac{g}{2}t^2$$

(b) by observer on the ground

$$x = 100t$$

$$y = 20t - \frac{g}{2}t^2$$

(2)  $\Delta n = \frac{2x}{\lambda} \frac{v^2}{c^2}$

$$0.005 = \frac{2(11)}{5900 \times 10^{-10}} \times \frac{v^2}{(3 \times 10^8)^2}$$

$$v = \sqrt{\frac{5 \times 10^{-3} \times 59 \times 10^{-8} \times 9 \times 10^{14}}{22}}$$

$$= \sqrt{12.068 \times 10^4}$$

$$= 3.47 \times 10^3 \text{ m/s}$$

Ans.

(3)

$$x = 100 \text{ km}, y = 10 \text{ km}, z = 1 \text{ km}, t = 5 \times 10^{-4} \text{ sec.}$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{100 \times 10^3 - 0.8(3 \times 10^8)(5 \times 10^{-4})}{\sqrt{1 - (0.8)^2}}$$

$$= \frac{10^5 - 12 \times 10^{12}}{0.6} \text{ m}$$

$$y = y' = 10 \text{ km}$$

$$z = z' = 1 \text{ km}$$

$$t' = t - \frac{vx}{c^2}$$

$$\frac{\sqrt{1 - \frac{v^2}{c^2}}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t' = \frac{5 \times 10^{-4} - \frac{0.8 \times 10^5}{3 \times 10^8}}{0.6}$$

$$t' = \frac{5 \times 10^{-4} - 2.66 \times 10^{-4}}{0.6} \text{ sec. Ans.}$$

(5)

$$l_0^3 = 1000 \text{ cm}^3$$

$$l_0 = 10 \text{ cm}$$

So, Vol. =  $l_0 \times l_0 \times l_0 \sqrt{1 - \frac{v^2}{c^2}}$

$$= l_0^3 \sqrt{1 - \frac{v^2}{c^2}}$$

$$= 1000 \sqrt{1 - \frac{(0.8c)^2}{c^2}}$$

$$= 600 \text{ cm}^3$$

(4) Given:

$$\Delta x_0 = 600 \text{ m}$$

$$\Delta t_0 = 8 \times 10^{-9} \text{ sec}$$

$$\Delta t' = 0$$

$$\Delta t' = \frac{\Delta t_0 - \frac{\Delta x_0 v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow 0 = \frac{8 \times 10^{-9} - \frac{600 v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$8 \times 10^{-9} = \frac{600 v}{\frac{c^2}{8 \times 10^{-9} \times 8 \times 10^{16}}}$$

$$= 12 \times 10^3$$

$$v = 1.2 \times 10^8 \text{ m/s Ans.}$$

$$(2) \Delta t_0 = 0 \quad \Delta x_0 = 600 \text{ m}$$

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$600 = 1200 \sqrt{1 - \frac{v^2}{c^2}}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{4}$$

$$v = -\frac{\sqrt{3}}{2}c$$

$$\Delta t' = \Delta t_0 - \frac{\Delta x_0 v}{c^2} = 0 + \frac{600 \sqrt{3} c}{2c^2} \sqrt{1 - \frac{3}{4}}$$

$$= \frac{600 \sqrt{3}}{c} = 200 \sqrt{3} \times 10^{-8} \text{ s}$$

$$= 3.46 \times 10^{-6} \text{ s} \quad \underline{\text{Ans.}}$$

$$(8) \text{ given equation: } x^2 + y^2 + z^2 + c^2 t^2 = 0$$

also  $t = t' \Rightarrow$   
at origin

$$\Rightarrow \left( \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \right)^2 + (y')^2 + (z')^2 + c^2 \left( t' + \frac{vx'}{c^2} \right)^2 = 0$$

$$\frac{1}{1 - \frac{v^2}{c^2}} \left[ x'^2 + v^2 t'^2 + 2x'vt' + c^2 \left( t'^2 + \frac{v^2 x'^2}{c^4} + \frac{2t'vx'}{c^2} \right) \right] + y'^2 + z'^2$$

$$\frac{1}{1 - \frac{v^2}{c^2}} \left[ \frac{c^2 x'^2 + c^2 v^2 t'^2 - c^4 t'^2 - v^2 x'^2}{c^2} \right] + y'^2 + z'^2 = 0$$

$$\frac{1}{1 - \frac{v^2}{c^2}} \left[ (x'^2 - c^2 t'^2) \left( \frac{c^2 - v^2}{c^2} \right) \right] + y'^2 + z'^2 = 0$$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = 0 \quad \underline{\text{Proved.}}$$

9

for observed.

$$U_x = 0.8c \cos 30^\circ$$

$$= 0.4\sqrt{3}c$$

$$U_y = 0.8c \sin 30^\circ$$

$$= 0.4c$$

$$v = 0.6c$$

$$U'_x = \frac{U_x - v}{1 - \frac{vU_x}{c^2}}$$

$$= \frac{0.4\sqrt{3}c - 0.6c}{1 - \frac{0.6(0.4\sqrt{3})}{1}}$$

$$= \frac{0.092823 \times 10^8 \text{ m/s}}{0.58432}$$

$$= 4.76 \times 10^7 \text{ m/s. Ans.}$$

$$U'_y = U_y \sqrt{\frac{1 - v^2}{c^2}} = \frac{0.4c \times 0.8}{0.58432}$$

$$= \frac{0.32 \times 3 \times 10^8}{0.58432}$$

$$= 1.64 \times 10^8 \text{ m/s Ans.}$$

10

$$V_{rel} = \frac{v_{en} + v_{nl}}{1 + \frac{v_{en}v_{nl}}{c^2}} = \frac{0.9c + 0.5c}{1 + \frac{0.9 \times 0.5}{1}} = \frac{1.4c}{1 + 0.45}$$

$$= 0.965c = 2.895 \times 10^8 \text{ m/s.}$$

(11)

$$\frac{A \rightarrow B}{0.9c} \quad \frac{B \rightarrow A}{0.9c}$$

$$V_{AO} = 0.95c \quad V_{BO} = -0.95c$$

$$V_{BA} = \frac{V_{BO} + V_{OA}}{1 + \frac{V_{BO} \cdot V_{OA}}{c^2}}$$

$$= \frac{-0.95c - 0.95c}{1 + (0.95)(0.95)}$$

$$= \frac{-1.9c}{1.9025} = -0.998c$$

$$= -2.996 \times 10^8 \text{ m/s}$$

(12)

$$m_1 = \frac{m_1}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{3}{\sqrt{1 - (0.8)^2}} = \frac{3}{0.6} = 5 \text{ kg}$$

$$m_2 = \frac{m_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} = \frac{5}{\sqrt{1 - (0.6)^2}} = \frac{5}{0.8} = 6.25 \text{ kg}$$

$$\therefore Mc^2 = m_1 v_1^2 + m_1 c^2 + m_2 v_2^2 + m_2 c^2$$

$$Mc^2 = 5(0.8)^2 c^2 + 3c^2 + 6.25(0.6)^2 c^2 + 5c^2$$

$$= 3.2 + 3 + 2.25 + 5$$

$$M = 13.45 \text{ kg}$$

$$M = \text{Rest mass of original body} = 13.45 \text{ kg}$$

$\therefore$  Rest mass of original body is not conserved.

$\therefore$  Rest mass is not conserved.

(13)

$$\frac{0.8c}{0.6c} \cdot \frac{0.6c}{0.6c}$$

$$V_{EL} = 0.6c$$

$$V_{OL} = 0.8c$$

$$V_{EO} = \frac{V_{EL} + V_{LO}}{1 + \frac{V_{EL} V_{LO}}{c^2}}$$

$$= \frac{(0.6 + 0.8)c}{1 + 0.6 \times 0.8}$$

$$= \frac{1.4c}{1.48} = 0.945c$$

$$V_{EO} = 0.945c$$

$$\therefore E = mc^2 = m_0 \frac{c^2}{\sqrt{1 - \frac{V_{EO}^2}{c^2}}}$$

$$= \frac{9.1 \times 10^{-31} \times 9 \times 10^{16}}{0.9249}$$

$$= 8.85 \times 10^{-14} \text{ J.}$$

(14)

$$m = m_0 \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$2m_0 = \frac{m_0}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$1 - \frac{V^2}{c^2} = \frac{1}{4}$$

$$\frac{V^2}{c^2} = \frac{3}{4}$$

$$\therefore V = \frac{\sqrt{3}}{2} c = 2.58 \times 10^8 \text{ m/s}$$

(15)

$V = c/2$  rest mass =  $m_0$

$$m = \frac{m_0}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \frac{1}{4}}} = \frac{2m_0}{\sqrt{3}}$$

$\therefore \text{momentum} = mV$   
 $= \frac{2m_0}{\sqrt{3}} \times \frac{c}{2}$   
 $= \frac{\sqrt{3}m_0c}{3} \text{ Ans.}$

Total Energy =  $mV^2 + m_0c^2 = mc^2$   
 $= \frac{2m_0c^2}{\sqrt{3}}$   
 $= \frac{2\sqrt{3}m_0c^2}{3}$

Kinetic Energy =  $(m - m_0)c^2$   
 $= \left(\frac{2}{\sqrt{3}} - 1\right)m_0c^2$

(16)

Total power received by earth =  $1.4 \times 2\pi \times 10^{22} \text{ kW}$   
 $= 1.4 \times 2\pi (1.5 \times 10^{11})^2$   
 $= 6.3 \times 10^{22} \text{ kW}$

$\therefore \text{Energy loss per sec} = 6.3 \times 10^{25} \text{ J}$

$E = mc^2$

$m = \frac{6.3 \times 10^{25}}{9 \times 10^{16}} = 2.198 \times 10^9 \text{ kg}$

$\therefore \text{Mass lost per sec} = 2.198 \times 10^9 \text{ kg}$



(17) total time taken acc. to person on earth,  
 $= 2 \times \frac{40}{8} = 10 \text{ yrs.}$

$$t = t_0 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\rightarrow t_0 = t \sqrt{1 - \frac{v^2}{c^2}} = 10 \sqrt{1 - (0.8)^2} = 6 \text{ yrs.}$$

$\therefore$  The twin who went in the spaceship will be younger by 4 yrs.

(18)  $\lambda = \lambda_0 \sqrt{1 - \frac{v^2}{c^2}}$

$$\lambda = 20 \sqrt{1 - \frac{v^2}{c^2}}$$

$$\therefore 26 \times 10^{-9} = 20 \sqrt{1 - \frac{v^2}{c^2}}$$

$$2.6 \times 10^{-8} = 20 \sqrt{1 - \frac{v^2}{c^2}}$$

$$(1.3 \times 10^{-9})^2 = 1 - \frac{v^2}{c^2}$$

$$1.69 \times 10^{-18} v^2 = 1 - \frac{v^2}{c^2}$$

$$v^2 (1.69 \times 10^{-18} + 11.1 \times 10^{-18}) = 1$$

$$v^2 = \frac{1}{12.79 \times 10^{-18}}$$

$$= 7.8 \times 10^{16}$$

$$\therefore v = 2.79 \times 10^8 \text{ m/s.}$$



(19)

$$60 = \frac{5.9}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\left(\frac{5.9}{60}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{5.9}{60}\right)^2$$

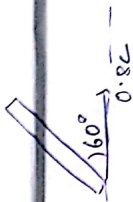
$$v = c \sqrt{1 - \left(\frac{5.9}{60}\right)^2}$$

$$= c \sqrt{0.9330}$$

$$= 0.18c$$

$$= 5.4 \times 10^9 \text{ m/s}$$

(20)



Let the actual length be  $l$ .

$$\therefore l \cos 60^\circ = \frac{l}{2}$$

$$ly = l \sin 60^\circ = \frac{l\sqrt{3}}{2}$$

$$l' = \frac{ly}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{l}{2 \sqrt{1 - \frac{v^2}{c^2}}} = \frac{5l}{6}$$

$$\begin{aligned} \therefore \text{New length} &= \sqrt{\frac{25l^2}{36} + \frac{3l^2}{4}} \\ &= \frac{\sqrt{82}}{6} l \\ &= 1.2l \end{aligned}$$

$$l' = l \sqrt{1 - \frac{v^2}{c^2}} = l \sqrt{1 - (0.8)^2} = 0.3l$$

$$\therefore \text{New length } l' = \sqrt{(0.09 + 0.75)} l = 0.916l$$

$$\begin{aligned}
 \% \text{ contraction} &= \frac{l^0 - l'}{l} \times 100 \\
 &= \frac{1 - 0.9161}{1} \times 100 \\
 &= 8.4\% \text{ Ans.}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Apparent orientation} &= \frac{l_y}{l_x} \\
 \tan \theta &= \frac{\sqrt{3} \times 10}{2 \times 3} \\
 &= 2.886
 \end{aligned}$$

$$\begin{aligned}
 \therefore \theta &= \tan^{-1}(2.886) \\
 &= 1.237 \text{ rad.}
 \end{aligned}$$

② Applying conservation of energy,  
 $E_i = E_f$

$$2 \left( \frac{m_0 c^2}{\sqrt{1 - \left(\frac{3}{5}\right)^2}} \right) = M c^2$$

$$\therefore M = \frac{5 m_0}{2} \text{ Ans.}$$

⑥ Relative vel =  $0.6c$   
 in frame of  $O$ ,  $O'$  appears to move at speed of  $0.6c$   
 & dis is  $20\text{m}$   
 $\therefore \text{time} = \frac{d}{s} = \frac{20}{0.6 \times 3 \times 10^8} \text{ sec} = \frac{1}{9} \times 10^{-6} \text{ sec}$   
 $\therefore \text{similarly for } O', t = \frac{1}{9} \times 10^{-6} \text{ sec.}$