INTERFERENCE

Condition for sustained interference

→ two sources should continously emitt waves of same wavelength or frequency.

→ the two sets of wave from the two source should either ~~at~~ have same phase or constant phase diff.

→ Distance b/w the ~~coherent~~ coherent ~~are~~ should be small

→ Distance b/w the source and screen should be reasonable

→ The coherent source should be narrow.

→ The source should be mono-chromatic ~~at~~ to get constant fringe width

→ The interfering wave if polarised should be in same state of polarization.

YOUNG'S DOUBLE SLIT EXP.

$$\boxed{I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta}$$

$$\boxed{\tan\phi = \frac{a_2\sin\delta}{a_1 + a_2\cos\delta}}$$

Condition for max. intensity :

$$\cos\delta = +1$$

$$\delta = 2n\pi$$

Path diff $= \frac{\lambda}{2\pi}$ (phase diff.)

$$= \frac{\lambda}{2\pi} \times 2n\pi$$

$\boxed{\text{Path diff} = n\lambda}$ → even multiple of $\lambda/2$

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2$$

$$\boxed{I_{max} = (a_1 + a_2)^2}$$

If $a_1 = a_2 \Rightarrow \boxed{I_{max} = 4a^2}$

Condition of minimum intensity

$$\cos\delta = -1$$

$$\delta = (2n-1)\pi$$

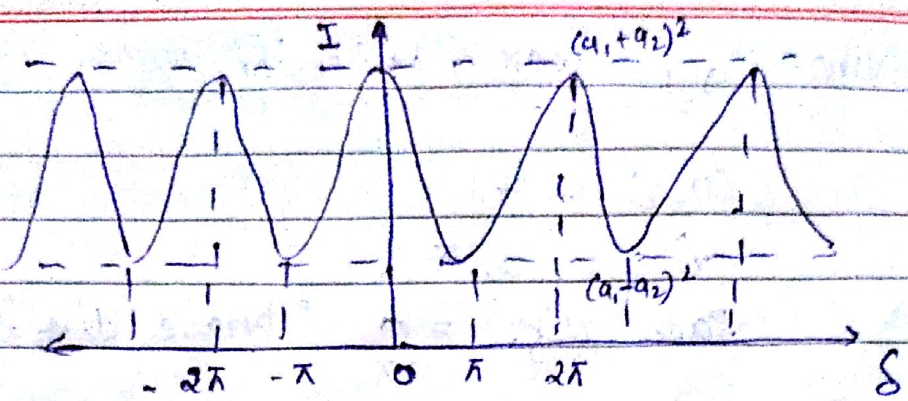
Path diff $= \frac{\lambda}{2\cancel{\pi}} \times (2n-1)\cancel{\pi}$

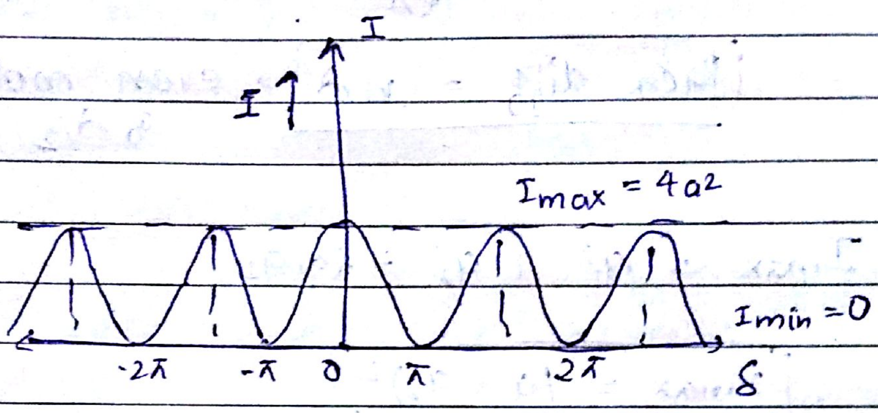
$\boxed{\text{Path diff} = \frac{\lambda}{2}(2n-1)}$ → odd multiple of $\lambda/2$

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2$$

$$\boxed{I_{min} = (a_1 - a_2)^2}$$

If $a_1 = a_2 = \boxed{I_{min} = 0}$





Average Intensity

$$I_{avg} = \frac{\int_0^{2\pi} I\, d\phi}{\int_0^{2\pi} d\phi}$$

$$= \frac{\int_0^{2\pi} (a_1^2 + a_2^2 + 2a_1a_2 \cos\phi)\, d\phi}{2\pi}$$

$$= \frac{1}{2\pi}\left[a_1^2\phi + a_2^2\phi + 2a_1a_2 \sin\phi\right]_0^{2\pi}$$

$$= \frac{\cancel{2\pi}(a_1^2 + a_2^2)}{\cancel{2\pi}} = a_1^2 + a_2^2$$

$$\boxed{I_{avg} = a_1^2 + a_2^2}$$

If $a_1 = a_2 = I_{avg} = 2a_1^2$

Interference produced by

- Division of wavefront — CASE I
  - Young's double slit exp; Fresnel biprism
- Division of amplitude — CASE - II
  - Thin film interference, Newton's Ring, Wedge shaped film, Michelson interferometer

CASE - I

In this case the wavefront is divided into two part by making use of reflection, refraction, deflection and defraction. ~~Sa~~ The two parts of wavefront travels ~~unequal~~, unequal distance and reunite to produce inteference fringes.

CASE - II

In this case amplitude of incident light is divided into two parts by partial reflection or refraction. These light waves with divided amplitude reinforce after travelling diff distances and produce interference fringes.

MICHELSON MORLEY EXPERIMENT [Ether hypothesis]

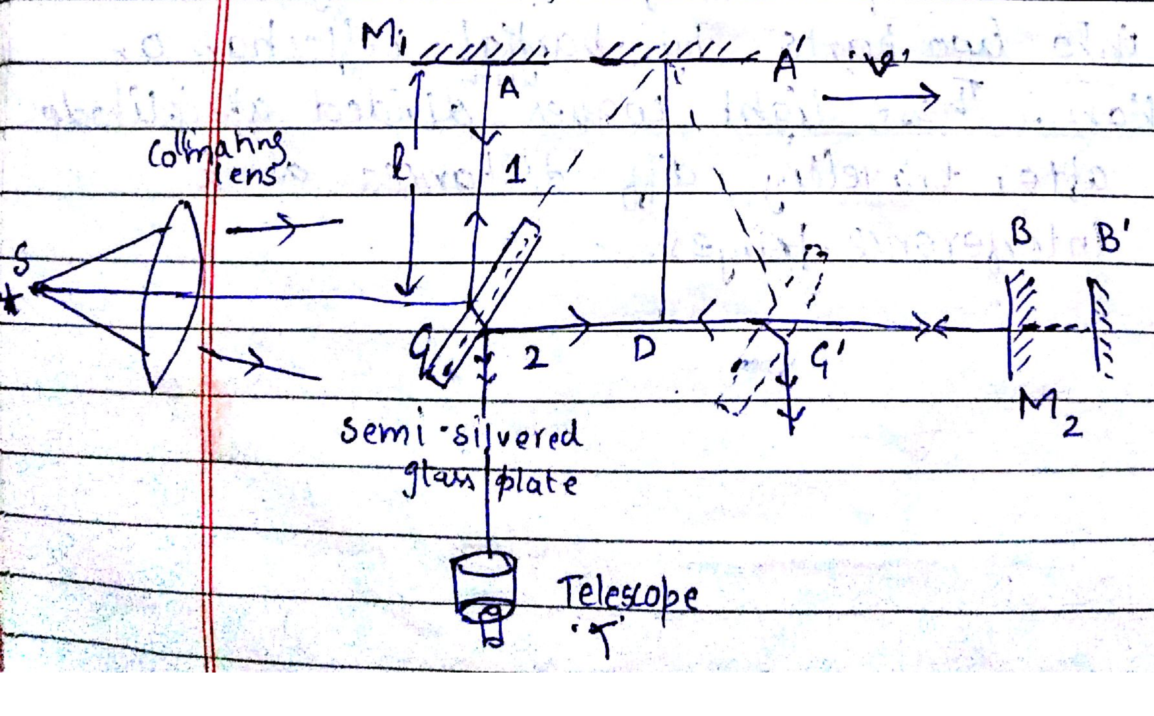
This exp

OBJECTIVE (1) To confirm the exisistence of stationary frame of reference that is ether frame.
(2) To detect the change in velocity of light due to relative motion between earth and hypothetical medium ether.

EXPERIMENTAL ARRANGEMENT

(1) Light from monochromatic axtended source 'S' after being rendered parallel by a collimating lens ~~fallen~~ falls on semi-silvered glass plate 'g' inclined at an 45° to the beam. It is ~~divi~~

(2) It is divided into two part one being reflected from semi-silvered surface 'g' giving rise to ray 1 which travelled towards M1 and other being transmitted which giving rise to ray 2 which travelled towards M2



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(3) The two rays falls normally on M_1 & M_2 resp. and are reflected back along their original path.

(4) The reflected rays again meat at semi-silvered glass plate g and enter the telescope 'T' where interference pattern is obtained.

(5) Optical distances of mirrors M_1 & M_2 from g are made equal.

(6) If apparatus is at rest in ether the two reflected rays would take equal time to return to glass plate 'g'.

(7) But actually whole apparatus is moving along with the earth lets us suppose that ~~there~~ direction of motion of earth is in direction of beam.
∴ whole apparatus is moving with earth through ether with 'v' velocity.

(8) due to the motion of the earth optical path traversed by both the rays are not same, thus time taken by two rays to travell the mirror and come back to 'g' will be diff in this case.

THEORY

let the two mirrors M_1 & M_2 be at same distance 'l' from 'g'

$c \rightarrow$ velocity of light

$v \rightarrow$ velocity of earth or Apparatus.

Reflected ray I from 'g' strikes mirror M_1 at A' & not A

Total path of Ray from G to A' & back will be GA'G'.

From $\Delta G'AD$:- $(GA')^2 = (AA')^2 + (A'D)^2$

If 't' is time taken by ray to move from G to A' then

$$(ct)^2 = (vt)^2 + l^2$$

$$t^2 (c^2 - v^2) = l^2$$

$$\boxed{t = \frac{l}{\sqrt{c^2 - v^2}}}$$

If 't_1' be time taken by ray to travell whole path GA'G', then

$$t_1 = 2t = \frac{2l}{\sqrt{c^2 - v^2}} = \frac{2l}{c(1 - v^2/c^2)^{1/2}}$$

$$t_1 = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

(Binomial expansion)

$$\boxed{t_1 = \frac{2l}{c}\left[1 + \frac{v^2}{2c^2}\right]} \quad - \text{①}$$

Now, ray 2 which is moving longitudinally towards mirror M_2. It has velocity $(c - v)$ relative to the apparatus when it is moving from G to B. During its return journey, its velocity relative to apparatus is $(c + v)$. If 't_2' be total time taken by longitudinal ray to reach G' then

$$t_2 = \frac{l}{(c-v)} + \frac{l}{(c+v)}$$

$$= \frac{l(c+v) + l(c-v)}{(c-v)(c+v)}$$

$$= \frac{2lc}{(c^2-v^2)} = \frac{2lc}{c^2(1-v^2/c^2)}$$

$$t_2 = \frac{2l}{c}\left(1-\frac{v^2}{c^2}\right)^{-1}$$

$$\Rightarrow \boxed{t_2 = \frac{2l}{c}\left(1+\frac{v^2}{c^2}\right)} \quad -(2)$$

Thus, the difference in times of travel of longitudinal & transverse journey is -

$$\Delta t = t_2 - t_1$$

$$= \frac{2l}{c}\left(1+\frac{v^2}{c^2}\right) - \frac{2l}{c}\left(1+\frac{v^2}{2c^2}\right)$$

$$\boxed{\Delta t = \frac{lv^2}{c^3}}$$

∴ Optical path difference b/w 2 rays :-
path diff = velocity × Δt = $c\Delta t$

$$\Delta t = \frac{clv^2}{c^3} \Rightarrow \boxed{\Delta = \frac{lv^2}{c^2}}$$

Michelson and morley perform the exp in two steps firstly by setting the diagram and secondly turning the apparatus through 90° in this case positions of the two mirror are changed now the part diff is in opposite direction

$$\Delta = \frac{l v^2}{c^2}$$

If 'λ' is wavelength of light, then path diff

$$\Delta = \frac{l v^2}{c^2 \lambda}.$$

$$\Delta = -\frac{l v^2}{c^2 \lambda}$$

Resultant path diff becomes $= \frac{l v^2}{\lambda c^2} - \left(-\frac{l v^2}{\lambda c^2}\right)$

$$\boxed{= \frac{2 l v^2}{c^2 \lambda}}$$

APPLICATIONS :

Optical path diff corresponds to shift of one fringe & hence path diff corresp. to fringe shift $\Rightarrow$ $\boxed{\Delta n = \frac{2 l v^2}{\lambda c^2}}$

EXPERIMENTAL DATA :-

$l = 1.0 \times 10^3$ cm

$\lambda = 5 \times 10^{-5}$ cm

$v = 3 \times 10^6$ cm/s

$c = 3 \times 10^8$ m/s

$$\Delta n = \frac{2 l v^2}{\lambda c^2}$$

$$\boxed{\Delta n = 0.4 \text{ fringe}}$$

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Thus a shift of less than half of a fringe was only observe. michelson & morley repeated the exp. at diff points on earth and they could not detect any measurable shift so it was a negative result. This -ve results suggest that it is impossible to measure the speed of earth relative to ether or concept of fixed frame of ref. cannot be checked by exp.

This suggested that speed of light in vaccum is same in all frame of ref. which are in uniform relative motion.

SIGNIFICANCE OF -VE RESULTS

⇒ Velocity of light is invarient

⇒ The effect of presence of ether in entire space of universe are undetectable ∴ all attempts to make ether as universal frame of refrence is meaningless

⇒ A new ~~of~~ theory having diff concept of time, mass, space is needed, apart from gallilean theory.

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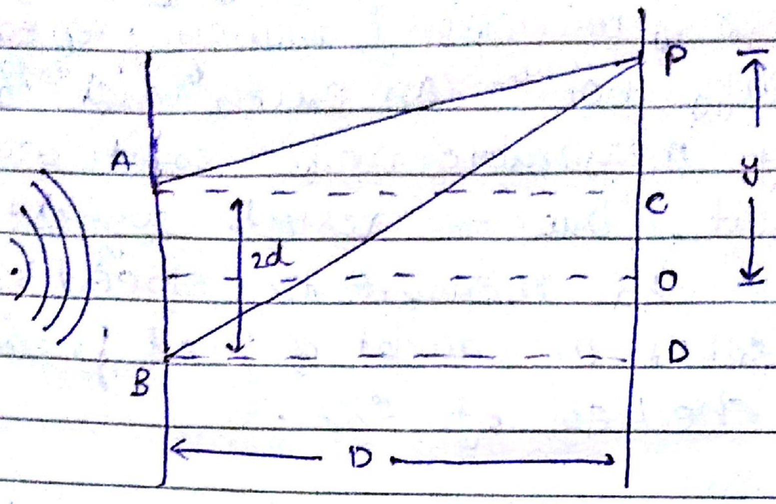
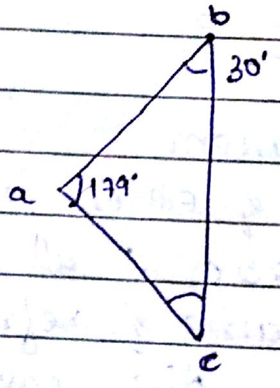
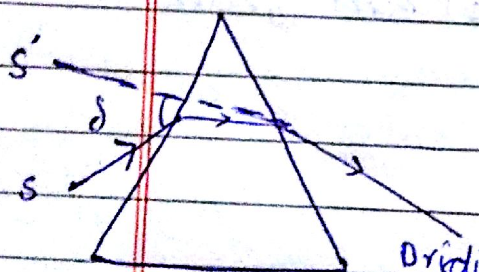
YOUNG's DOUBLE SLIT EXP. (Div. of wavefront)

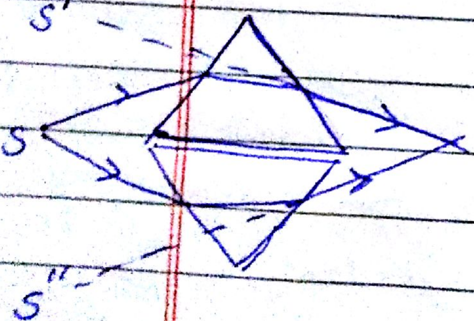


FRESNEL BIPRISM (Div. of wavefront)





Ordinary prism



→ Biprism

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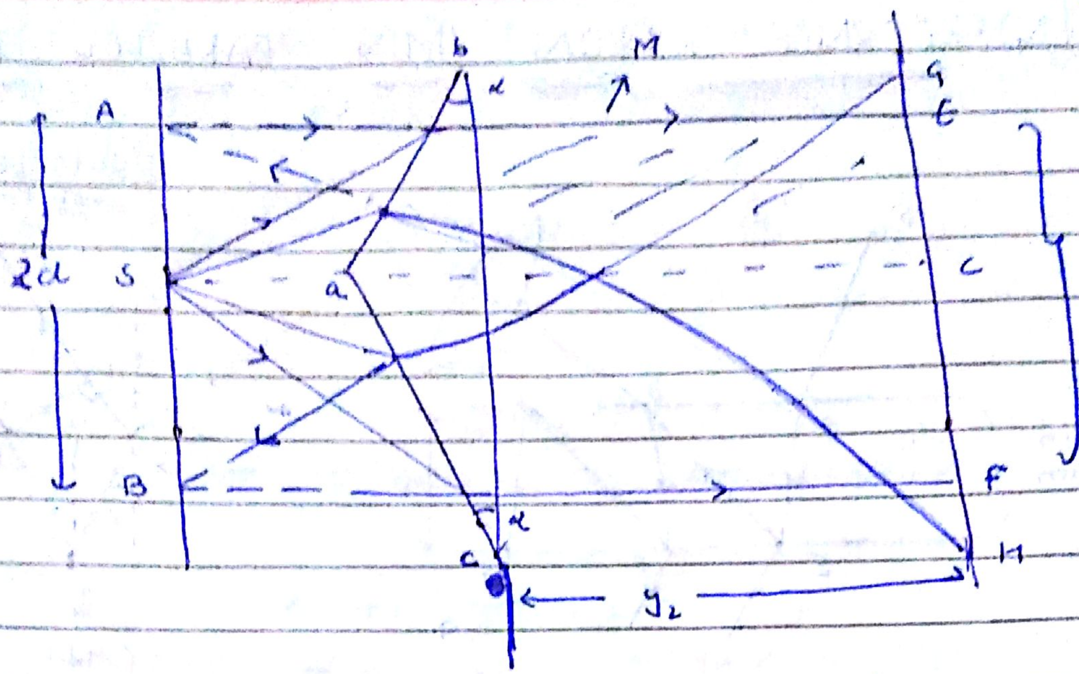
FRESNEL BIPRISM



Biprism consist of two prism. Each half of the biprism consist of acute angle prism placed base to base to form a single prism of obtuse angle 179°, the acute angle of 'α' are about 30'/ 30 minutes

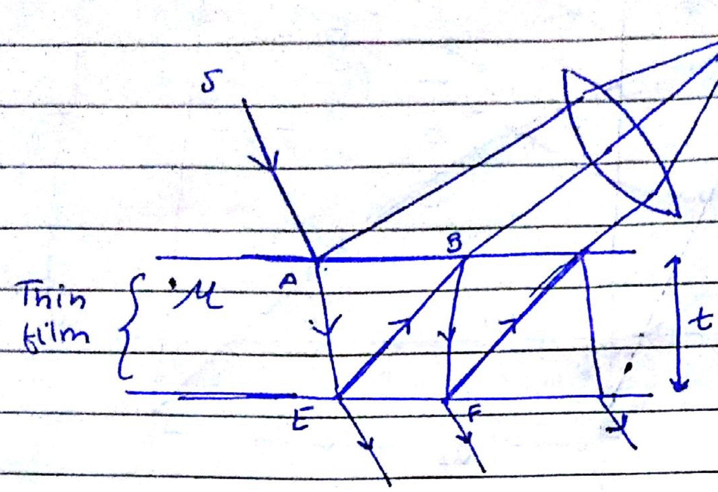
Each half of biprism produces a virtual image of 'S' by refraction. The distance b/w 'S' and prism are so adjusted that two virtual image S_1 & S_2 are quiet close together.

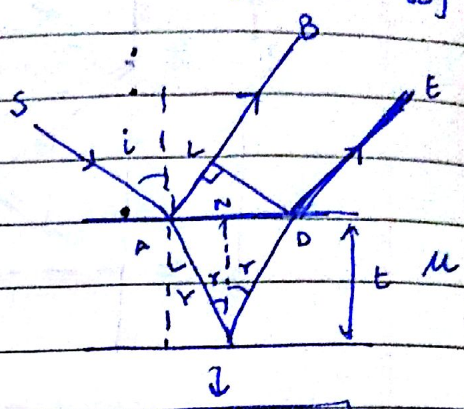
A closely spaced interference fringe system is produce only in region EF while the ~~whites~~ wide set of fringes are obtained at the edges of pattern due to diffraction.

If 'n' represent the refractive index of biprism & 'α' is base angle then $\boxed{(n-1)\alpha \text{ is approx angular deviation}}$

INTERFERNCE FROM THIN PARALLEL FILM

[Interference from reflected rays]





[CASE - 1]

(*) When its thickness is about the order of 1 wavelength of light (invisible light) is called thin film

Path diff b/w AB & DE

$$\boxed{\Delta = \mu(AM + MD) - AL} \quad — \text{(1)}$$

In ΔCAN :-

$$\cos r = \frac{CN}{AC}$$

$$AC = \frac{CN}{\cos r} = \frac{t}{\cos r} \quad — \text{(2)}$$

similarly In ΔCND :-

$$CD = \frac{CN}{\cos r} = \frac{t}{\cos r} \quad — \text{(3)}$$

In right ΔADL :-

$$\sin i = \frac{AL}{AD}$$

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$AL = AD \sin i$

$AL = AD \sin i$

$AL = (AN + ND) \sin i$ — (4)

From ΔACN & ΔCND :-

$\frac{AN}{NC} = \tan r$ & $\frac{ND}{NC} = \tan r$

$AN = NC \tan r = t \tan r$ — (5)

$ND = NC \tan r = t \tan r$ — (6)

Put (5) & (6) in (4)

$AL = [t \tan r + t \tan r] \sin i$

$= 2t \tan r \sin i$ $\leftarrow \mu = \frac{\sin i}{\sin r}$

$AL = \frac{2t \sin r \sin i}{\cos r} = 2\mu t \frac{\sin^2 r}{\cos r}$ — (7)

Put (2), (3), (7) in (1)

$\Delta = \mu \left[\frac{t}{\cos r} + \frac{t}{\cos r} \right] - 2\mu t \frac{\sin^2 r}{\cos r}$

$= \frac{2\mu t}{\cos r} (1 - \sin^2 r)$

$= \frac{2\mu t \cos^2 r}{\cos r}$

$\boxed{\Delta = 2\mu t \cos r}$

$$\boxed{\Delta = 2\mu t \cos r}$$ - path diff

Acc to stoke's law when the light ray get reflected from optically ~~to~~ densor medium then there occurs an additional path diff. of $\lambda/2$ or phase change

$\therefore$ Total path diff

$$\Delta = 2\mu t \cos r + \frac{\lambda}{2}$$

For maxima

$$2\mu t \cos r \pm \frac{\lambda}{2} = 2n\left(\frac{\lambda}{2}\right)$$

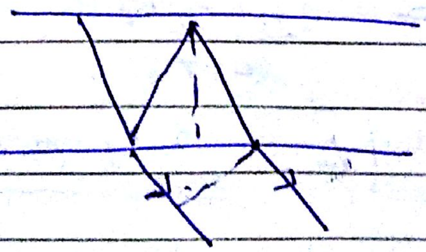
$$2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2} \rightarrow \text{Bright}$$

For minima

$$2\mu t \cos r \pm \frac{\lambda}{2} = (2n \pm 1)\lambda/2$$

$$\boxed{2\mu t \cos r = n\lambda} \rightarrow \text{Dark}$$

Conditions in case of ~~in~~ reflected and transmitted rays are reversed hence the film which appears bright in reflected light appears dark in transmitted light.



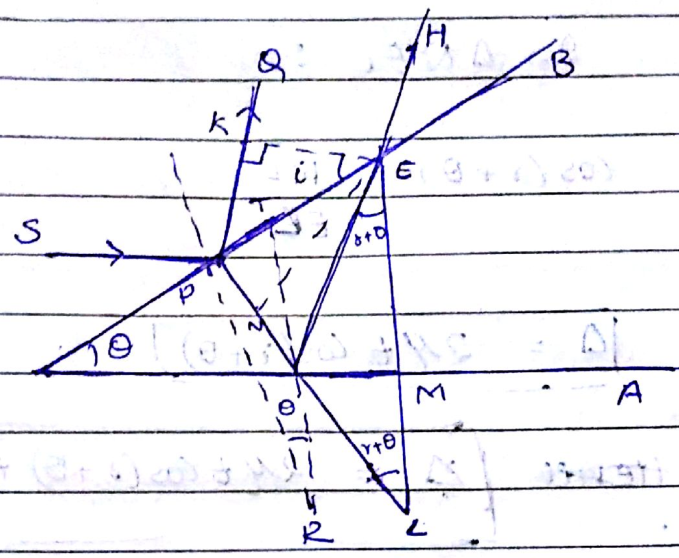
CASE - II

$$2\mu t \cos r = n\lambda \rightarrow \text{Brightness}$$

$$2\mu t \cos r = (2n \pm 1)\lambda/2 \rightarrow \text{darkness}$$

HAIDINGER'S FRINGES are obtain in case of thin films. These fringes are fringes of equal inclination

WEDGE SHAPED THIN FILM



↪ Thickness of the film increases from O to A

↪ Optical path diff $\Delta = \mu(PF + FE) - (PK)$ — ①

$$\Delta = \mu(PN + NF + FE) - PK$$

In ΔPKE :- $\sin i = \frac{PK}{PE}$

$$\sin r = \frac{PN}{PE}$$

$$\mu = \frac{\sin i}{\sin r}$$

$$\mu = \frac{PK}{PN}$$

$$\boxed{\mu PN = PK}$$

$$\therefore \Delta = \mu(NF + FE)$$

$$\boxed{\Delta = \mu\,(NF + FL)} \quad (\because FL = FE)$$

$$\boxed{\Delta = \mu(NL)} \quad — \quad (1)$$

In ΔNEL :-

$$\cos(r+\theta) = \frac{NL}{EL}$$

$$\boxed{\Delta = 2\mu t \cos(r+\theta)}$$

Total path diff. ~~Hence~~ $\boxed{\Delta = 2\mu t \cos(r+\theta) + \frac{\lambda}{2}}$

(★★) For Constructive interference :-

$$2\mu t \cos(r+\theta) + \frac{\lambda}{2} = n\lambda$$

$$\boxed{2\mu \cos(r+\theta) = (2n-1)\frac{\lambda}{2}}$$

$n = 1, 2, 3, \ldots$

(★★) For destructive interference

$$2\mu t \cos(r+\theta) + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\boxed{2\mu t \cos(r+\theta) = n\lambda}$$

$n = 0, 1, 2 \ldots$

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Case I

For normal incidence, $r \simeq 0$ & in practice $\theta \simeq$ small $\therefore \cos(r+\theta) \simeq 1$

$\mu = 1$ (air)

(*) Maxima condⁿ $\rightarrow$ $2\mu t = (2n-1)\frac{\lambda}{2}$

(*) Minima $\rightarrow$ $2\mu t = n\lambda$

Case II

For small 'θ' thickness of film at location 'x' can be takes as :- $\simeq x\theta$

$$\therefore \boxed{2\mu x\theta = (2n-1)\lambda/2} \rightarrow \text{Bright}$$

$$\therefore \boxed{2\mu x\theta = n\lambda} \rightarrow \text{Dark}$$

Case III

If 'x_n' is distance of n^{th} dark ~~fringe~~ fringe from the edge & x_{n+m} that of $(n+m)^{th}$ fringe then :

$$x_n = \frac{n\lambda}{2\mu\theta} \quad \& \quad x_{n+m} = \frac{(n+m)\lambda}{2\mu\theta}$$

$\therefore$ Fringe width .

$$\beta = \frac{x_{n+m} - x_n}{m}$$

$$\boxed{\beta = \frac{\lambda}{2\mu\theta}}$$

V. IMP

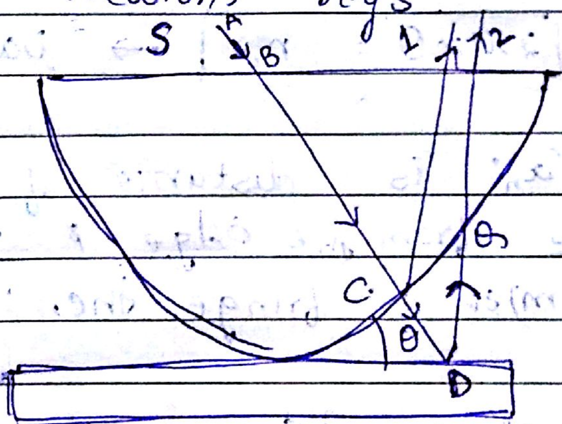
(*) If two surfaces are optically plan then fringes are of equal thickness and are straight

Each fringe is locus of points where the thickness of film has constant value.

NEWTON'S RINGS

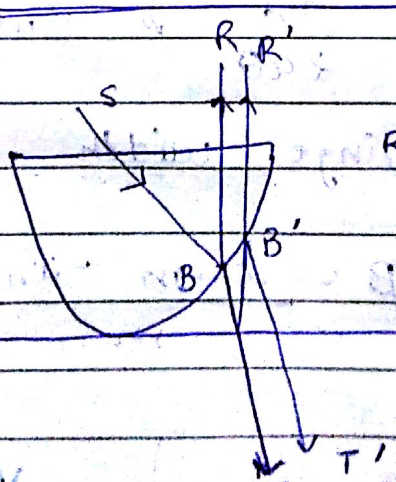
Air film is wedge shaped and the locus of points of equal thickness were circles concentric with the pt of contact at the centre

Interference fringes of equal thickness so formed were first investigate by newton so called as Newton's rings



1 Reflected

Generally



Reflected

2 Transmitted

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1. AB $\longrightarrow$ mono chromatic light

Part of ray $\longrightarrow$ reflected at C (glass - air interference)

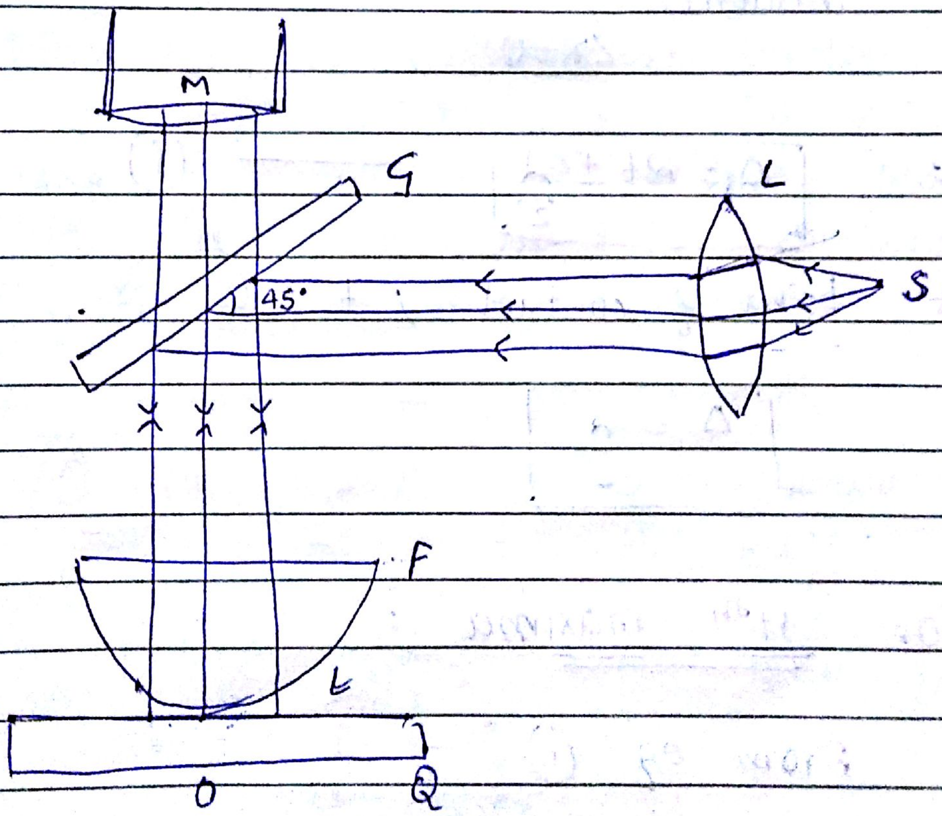
ray 1

Refracted along CD

At D, reflected to form (ray 2) with phase reversal of π

Ray 1 and 2 $\longrightarrow$ interference.

3.



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Path diffrenence

$$\boxed{\Delta = 2\mu t \cos(r+\theta) \pm \frac{\lambda}{2}}$$

'θ' $\rightarrow$ angle of wedge

Due to large radius of curvature of plano - convex lense

'θ' is very small & neglected.

$$\therefore \Delta = 2\mu t \cos r \pm \frac{\lambda}{2}$$

$\hookrightarrow$ For air thin film $\mu = 1$ & for normal incidence

$$\angle r = 0$$

$$\therefore \boxed{\Delta = 2t \pm \frac{\lambda}{2}} \quad \text{——— (1)}$$

At point of contact ; $t \simeq 0$.

$$\boxed{\Delta = \frac{\lambda}{2}}$$

(I) For n^{th} maxima :

From eq (1).

$$\Delta = 2t \pm \frac{\lambda}{2}$$

$$\hookrightarrow 2t \pm \frac{\lambda}{2} = n\lambda.$$

$$\boxed{2t = (2n \pm 1)\frac{\lambda}{2}} \quad \text{— (2)}$$

For $\mu \neq 1$

$$\boxed{2\mu t = (2n \pm 1)\frac{\lambda}{2}}$$

(II) For n^{th} minima

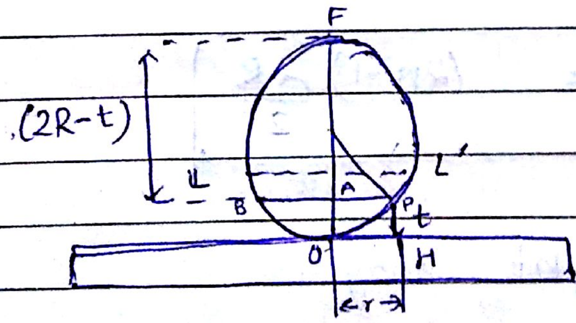
$$\Delta = 2t \pm \frac{\lambda}{2} = (2n-1)\lambda/2$$

$$\boxed{2t = n\lambda}$$

For $\mu \neq 1$, $\boxed{2\mu t = n\lambda}$

Maximum of a particular order 'n' will occur for a constant value of 't'. In this case of a system 't' remains constant along a circle. //ly for minima diff circular forms are obtained.

TO CALCULATE THE DIAMETER OF RINGS



R → radius of curved surface of lens

By Property of circle $(AP)^2 = OA \times AF$

Now $AP = AB = r$ → radius of Ring passing through P.

$OA = PN = t$

We have, $r^2 = t(2R - t)$

$r^2 = 2Rt - t^2$

In actual practice
'R' is Quite large & 't' is very small

$t <<< R$

So neglect 't^2' as compared to $2Rt$

$\therefore r^2 = 2Rt$

$$\boxed{t = \frac{r^2}{2R}} \quad \text{——— (3)}$$

(I) For Bright Rings :-

$2t = (2n-1)\lambda/2$

$\frac{\cancel{2} r^2}{\cancel{2} R} = (2n-1)\lambda/2$

$$\boxed{r^2 = \frac{(2n-1)\lambda R}{2}}$$

For n^{th} Bright ring :-

$r_n^2 = \frac{(2n-1)\lambda R}{2}$

If 'D_n' is diameter of ring

$$r_n = \frac{D_n}{2}$$

$$\left(\frac{D_n}{2}\right)^2 = \frac{(2n-1)\lambda R}{2}$$

$$D_n^2 = \frac{\cancel{4}^{2}(2n-1)\lambda R}{\cancel{2}}$$

$$\boxed{D_n^2 = 2(2n-1)\lambda R} \quad — \quad (4)$$

If $\mu \neq 1$

$$\boxed{D_n^2 = \frac{2(2n-1)\lambda R}{\mu}} \quad — \quad (5)$$

↳ Diameter of the successive bright rings are proportional to the odd natural number

(II) For Dark Rings :-

$$2t = n\lambda$$

$$\cancel{2} \times \frac{r^2}{\cancel{2}R} = n\lambda$$

$$r^2 = Rn\lambda$$

$$\left(\frac{D_n}{2}\right)^2 = Rn\lambda$$

$$D_n^2 = 4Rn\lambda$$

$$\boxed{D_n = \sqrt{4n\lambda R}}$$

∴ Diameter of dark rings are probational to square root of natural number

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↳ Fringe width decreases with increase in order of the fringe

Fringes get closer with ↑se in order of fringe

↳ Diameter of 16^{th} & 9^{th} dark

$D_{16} = \sqrt{4 \times 16 \lambda R} = 2 \times 4 \sqrt{\lambda R} = 8\sqrt{\lambda R}$

$D_9 = \sqrt{4 \times 9 \lambda R} = 6\sqrt{\lambda R}$

↳ $D_{16} - D_9 = (8-6)\sqrt{\lambda R} = 2\sqrt{\lambda R}$

↳ $D_4 - D_1 = 2\sqrt{\lambda R}$