Assignment 4 B.tech.1 Semester-2017

Multiple Integral

1. Evalute the integral $\iint\limits_R (x+y)dxdy$ over the region R in the positive quadrant bounded by ellipse $x^2/a^2 + y^2/b^2 = 1$. (ans: $ab(a+b)\backslash 3$)
2. Evaluate$\iint\limits_R (x^2+y^2)dxdy$ in the positive quadrant for which $x+y \leqslant 1$. (ans:$\frac{1}{6}$)
3. Evaluate $\int_0^1 \int_{\sqrt{y}}^{2-y} x^2 dxdy$. (ans:$\frac{67}{60}$)
4. Evaluate $\int_0^{\frac{\pi}{4}} \int_0^{\sqrt{\cos 2\theta}} \frac{r}{(1+r^2)^2} drd\theta$.(ans:$\frac{\pi-2}{8}$)
5. Use polar double integral to find the area enclosed by the three petaled rose r=sin 3θ.(ans:$\frac{\pi}{4}$)
6. Change the order of integration I=$\int_0^1 \int_{x^2}^{2-x} f(x,y)dydx$.(ans:$\int_1^2 \int_0^{2-y} f(x,y)dxdy + \int_0^1 \int_0^{\sqrt{y}} f(x,y)dxdy$)
7. Changing the order of integration of $\int_0^\infty \int_0^\infty e^{-xy} dxdy$.Show that $\int_0^\infty \frac{\sin(nx)}{x} dx = \frac{\pi}{2}$
8. Find the area bounded by the parabola $y^2 = 4ax$ and the line $x+y=3a$ in the positive quadrant.(ans:$\frac{10a^2}{3}$sq. units)
9. Find the area of the cardiod $r = a(1+cos\theta)$ by double integration.(ans:$\frac{3a^2\pi}{2}$sq. units)
10. A circular hole of radius b is made centrally through a sphere of radius a.Find the volume of the remaning sphere.(ans:$\frac{4(a^2-b^2)^{3/2}\pi}{3}$sq. units)
11. Find the volume bounded by cylinder $x^2+y^2=4$ and the plane $y+z=4$ and $z=0$.ans:16π
12. Evaluate$\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2-z^2}} \frac{1}{\sqrt{a^2-x^2-y^2-z^2}} dzdydx$. (ans:$\frac{\pi^2 a^2}{8}$)
13. By changing into polar coordinates, evaluate$\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$. (ans:$\frac{a\pi}{4}$)
14. Evaluate the following by changing into polar coordinates $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)dxdy$.(ans:$\frac{a^4\pi}{8}$)
15. Find the volume of the portion of the sphere $x^2+y^2=a^2$ lying inside the cylinder $x^2+y^2=ay$.(ans:$\frac{2a^3(3\pi-4)}{9}$)

16. Use triple integration in cylindrical coordinates to find the volume of the solid R that is bounded above by the hemisphere $z = \sqrt{25 - x^2 - y^2}$, below by the $xy-$plane, and laterally by the cylinder $x^2 + y^2 = 9$. (ans:$\frac{122\pi}{3}$)

17. By using gamma function evaluate the integral $\int_0^1 x^5 \log(\frac{1}{x})^3 dx$. (ans:$\frac{6}{625}$)

18. Given that $\int_0^\infty \frac{x^{n-1}}{1+x} = \frac{\pi}{sin(n\pi)}$. Show that $\Gamma(n)\Gamma(1-n) = \frac{\pi}{sin(n\pi)}$.

19. Show that(i) β function is a symmeteric function $i.e \beta(m,n) = \beta(n,m)$.
(ii) Calculate $\Gamma(7/2)$.

20. Prove that $\int_0^\infty e^{-x^2} = \frac{\sqrt{\pi}}{2}$.

21. Show that $\int_0^{\pi/2} \sqrt{tanx} = \frac{\pi}{\sqrt{2}}$.

22. Prove that $\frac{\beta(m+1,n)}{\beta(m,n)} = \frac{m}{m+n}$ where $m > 0, n > 0$.