Assignment-I (B. Tech. 2nd Sem. 2013)
Mathematics-II (AM 111), Topic: Matrices[1]

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1. Find the rank of the following matrices:

$$(a)\ \begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix} \qquad (b)\ \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 13 & 0 & -7 \end{bmatrix}$$

$$(c)\ \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix} \qquad (d)\ \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$$

Ans. (a) 3 $\quad (b)$ 3 $\quad (c)$ 3 $\quad (d)$ 2

2. Using Gauss-Jordan method, find the inverse of the following matrices:

$$(a)\ \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \qquad \text{Ans.}\ \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$(b)\ \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \qquad \text{Ans.}\ \begin{bmatrix} -1 & 1 & 2 \\ 0 & 1/2 & -1/2 \\ 1 & -1/2 & -3/2 \end{bmatrix}$$

$$(c)\ \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} \qquad \text{Ans.}\ \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

$$(a)\ \begin{bmatrix} 14 & 3 & -2 \\ 6 & 8 & -1 \\ 0 & 2 & -7 \end{bmatrix} \qquad \text{Ans.}\ -\frac{1}{654}\begin{bmatrix} -54 & 17 & 13 \\ 42 & -98 & 2 \\ 12 & -28 & 94 \end{bmatrix}$$

3. Show that no skew symmetric matrix can be of rank 1.

4. Test the consistency and solve:

$$(a)\ \begin{aligned} 2x - 3y + 7z &= 5 \\ 3x + y - 3z &= 13 \\ 2x + 19y - 47z &= 32 \end{aligned} \qquad (b)\ \begin{aligned} x + 2y + z &= 3 \\ 2x + 3y + 2z &= 5 \\ 3x - 5y + 5z &= 2 \\ 3x + 9y - z &= 4 \end{aligned}$$

Ans. (a) Inconsistent $\quad (b)$ Consistent : $x = -1,\ y = 1,\ z = 2.$

5. Discuss the existence and nature of solutions for all values of λ for the following system of equations:

$x + y + 4z = 6\ ;\quad x + 2y - 2z = 6\ ;\quad \lambda x + y + z = 6\ .$

Ans. Inconsistent for $\lambda = 7/10$ and unique solution otherwise.

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[1] **Typeset by Dr. Suresh Kumar**

6. Determine the values of a and b for which the following system of equations:

   $x + 2y + 3z = 4\ ; \quad x + 3y + 4z = 5\ ; \quad \lambda x + 3y + az = b$

   have (a) no solution (b) unique solution (c) infinite number of solutions.

   Ans. (a) $a = 4,\ b \neq 5$ (b) $a \neq 4$ (c) $a = 4,\ b = 5$

7. Find the eigen values and eigen vectors of the following matrices:

   $$(a)\ \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \qquad \text{Ans. } 2, 2, 8\ ;\ \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}.$$

   $$(b)\ \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \qquad \text{Ans. } 0, 3, 15\ ;\ \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}.$$

8. If the characteristic roots of a matrix A are $\lambda_1,\ \lambda_2,\ ....,\ \lambda_n$, show that the characteristic roots of A^2 are $\lambda_1^2,\ \lambda_2^2,\ ....,\ \lambda_n^2$.

9. Show that the matrices A and $B^{-1}AB$ have the same characteristic roots.

10. State and prove Cayley-Hamilton theorem.

11. Verify Cayley-Hamilton theorem for the matrix

    $$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

    and hence find its inverse.

    Ans. $A^{-1} = \begin{bmatrix} -3 & 0 & 2 \\ -1 & 1/2 & 1/2 \\ 2 & 0 & -1 \end{bmatrix}.$

12. Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalizable. Obtain the diagonalizing matrix P.

    Ans. $P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$; diag.$(-1, -1, 3)$.

13. Diagonalize the matrix $\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}.$

    Ans. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$

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