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Unit 2: Nuclear Physics

→ Neutrino, which has no mass plays ~~a~~ or imp. role in disease carrier of energy.

Proton - Neutron Theory:-
Total no. of protons & neutrons = A and, protons by Z, and (A-Z) neutrons.

$n \rightarrow p + e^-$ (emission of e^-)
$p \rightarrow n + e^+$ (emission of positron)

→ Together, they're known as ~~pa~~ nucleon.

① **Isotopes:-**
Species having diff. mass number A but same atomic number Z, are called isotopes

Eg. $^{3}_{1}H$, $^{2}_{1}H$, $^{1}_{1}H$,
$^{12}_{6}C$, $^{13}_{6}C$, $^{14}_{6}C$, $^{15}_{6}C$, $^{16}_{6}C$, ...

② **Isobars:-**
Species having same mass number A but diff. atomic number Z are isobars.

Eg: $^{22}_{11}Na$, $^{22}_{10}Ne$, $^{40}_{20}Ca$, $^{40}_{18}Ar$, etc.

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③ Isotones:-

Same ~~no~~ number of neutrons.

Eg: $^{13}_{6}C$, $^{14}_{7}N$, $A - Z = 7$.

④ Isomers:- Species having same number of A, Z, N however different intend structure due to diff. intend energies due to diff. intend ~~energies~~ decay periods, are called Isomers.

Eg: a nucleus in the excited state.

X^* & X are isomers

↳ longer life-time.

GENERAL PROPERTIES OF NUCLEUS

① Nuclear Mass:- Expressed in amu (u).

$$1u = \frac{\text{mass of } ^{12}_{6}C \text{ atom}}{12} = 1.66 \times 10^{-27} kg$$

② Nuclear Charge:- Charge on a nucleus = Z_e as neutrons ~~are~~ are ~~p~~ neutral.

Eg: $^{12}_{6}C$ = 6p + 6e, etc.

③ Nuclear Size:-

$R = R_0 A^{1/3}$, $R_0 = 1.07 fm$

$1 fm = 10^{-15} m$

A = mass number.

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Hence, $V \propto R^3 \propto A$

Hence, density $= \frac{mass}{Vol} \propto \frac{A}{A}$

Same for all nuclei.

$\approx 3 \times 10^{17}\ kg/m^3$

(4) Nuclear Quantum States:-

Spacing of the levels in nuclei is many times layer than that in atoms. Hence, the photon that is released due to transitions in nuclear level is much higher than that in atomic levels. Inside the extra-nuclear region, electrostatic Columbic force dominates while in the nuclear region, short range nuclear force dominates.

(5) Angular Momentum

$$J = |J| = \sqrt{j(j+1)}\,\hbar$$

(1) If A is odd, Z is even or odd, j is half integer.

(2) If A is even and Z is odd, j has integral value.

(3) If A & Z are even, j is zero

(4) Magnetic Moment: The total (or spin) angular momentum of a nucleus gives rise to a

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magnetic moment to it, and is given by

$$\boxed{\mu = g_N \mu_N J}$$

g_N = nuclear g-factor
μ_N = nuclear magneton

$$\boxed{\mu_N = \frac{e\hbar}{2m_p}} = 5.051 \times 10^{-27} \, J T^{-1}$$

→ Positive sign of moment means that the spin angular momentum and the magnetic moment are in the same direction while negative sign indicates that they are in the opposite directions.

$$\boxed{\frac{\mu_B}{\mu_N} = \frac{m_p}{m_e} = \frac{1}{1836}}$$

NUCLEAR FORCE

→ The force responsible to hold the nucleons together is called the nuclear ~~for~~ force. It's a very strong force.

→ Nuclear force rapidly falls as the seperation increases. It's active only upto 1-2 fm and decreases at 2-3 fm. apart.

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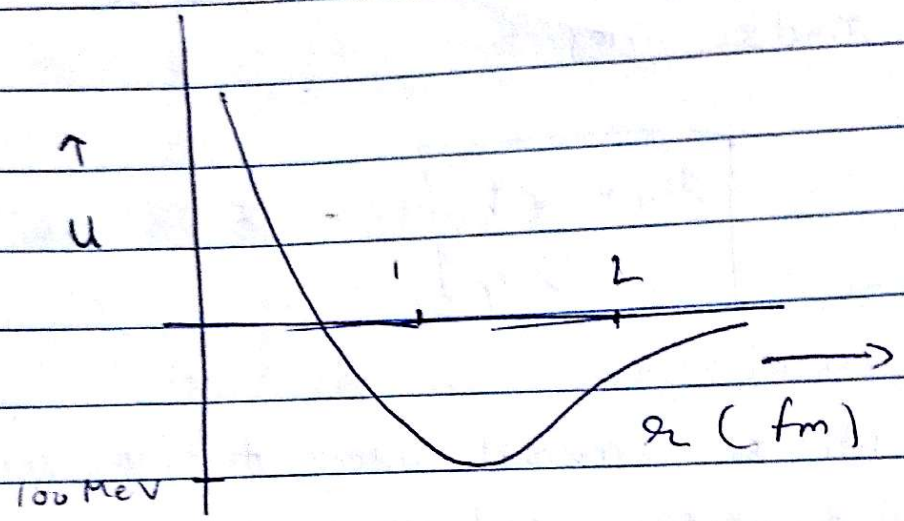
→ Nuclear force isn't purely attractive. If the separation reduces below 1fm, then it is highly repulsive.



Charge Independence :-
Nuclear force b/w two nucleons is the same whether they are protons or neutrons or p-n. Hence, there are equal number of neutrons & protons.

Nuclear force and Electrons :-
Nuclear ~~force~~ ~~does~~ force doesn't act on electrons at all.

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Nuclear Binding Energy - Stability of Nuclei

→ The energy reqd. to break the nucleus into its constituents is called Binding Energy. (energy reqd. to break the nucleus into its constituents so that it doesn't interact w each other)

→ Higher the B.E, higher the stability.

→ B.E is due to the 'mass defect' i.e due to the difference in mass between the mass of nucleus of sum of masses of nucleons.

$$E_b = [Zm_p + Nm_p - m({}^A_Z X)]c^2$$

$N = (A - Z)$

$m_{atom} = m_{nucl} + Zm_e$ (mass of electrons)

Now,

$$E_b = [Zm_H + Nm_n - m_{atom}]c^2$$

$m_H = m_p + m_e$ (m_H)

m_H = mass of hydrogen atom

$N = A - Z$

Average binding energy per nucleon $= \left(\frac{E_b}{A}\right)$

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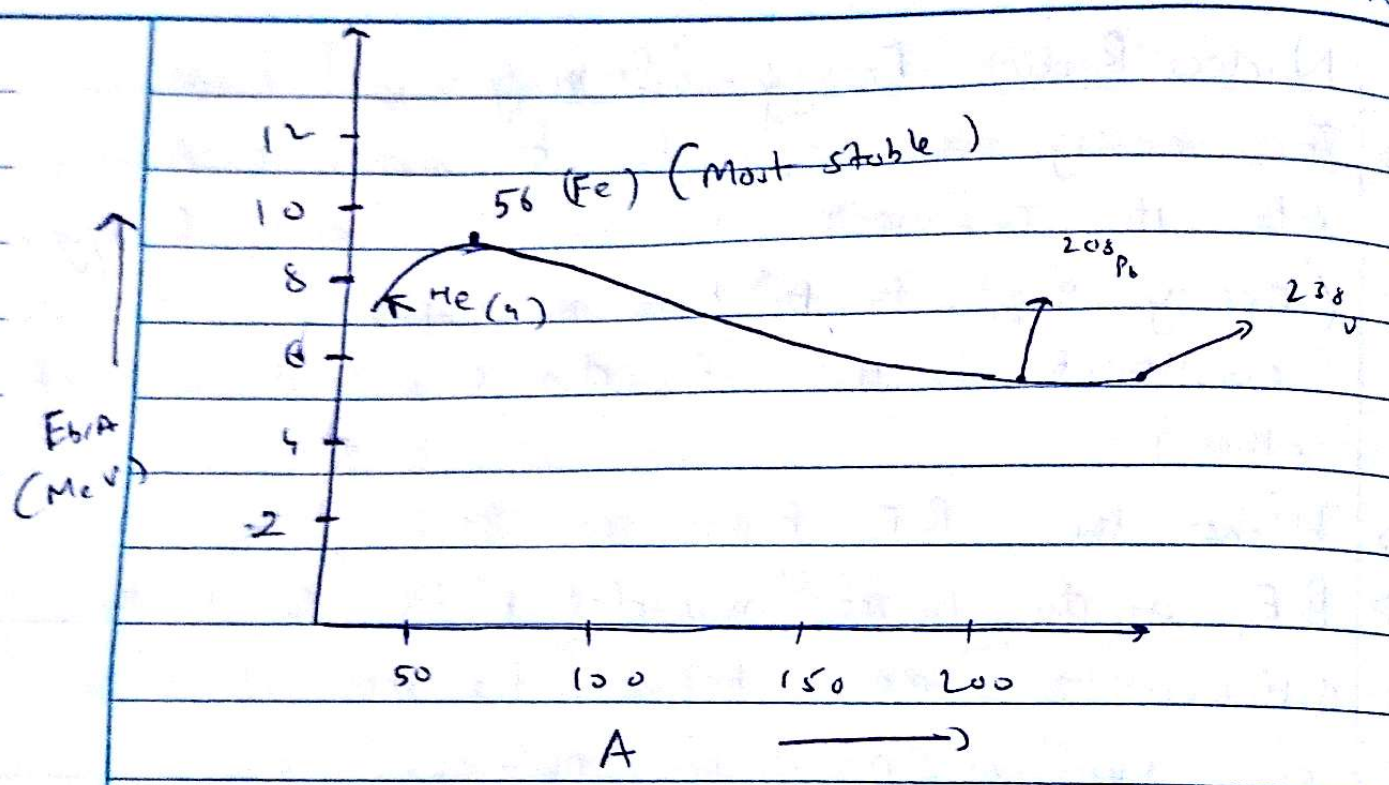

① For low atomic mass number ($A < 20$), the Binding energy per nucleon is very low. As A falls below 20, (E_b/A) falls rapidly to zero for hydrogen, which has no nuclear binding energy. This is due to large surface energy.

② For most nuclei, the binding energy per nucleon is 8MeV, except Fe (8.8 MeV)

③ Beyond iron, E_b/A decreases slowly with A becoming 7.6MeV for ^{238}U. This is due to increase in columbic repulsion b/w the protons in the nuclei.

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→ Lower (E_b/A) when A is large implies that energy is released when heavy nucleus splits.

→ Lower (E_b/A) when A is small implies that energy is released when small nucleus combines with another.

→ Most nuclei that are stable have <u>even Z</u> and <u>even N</u>.

Decay of Unstable Nuclei

→ $Z \geq 83$ are naturally occuring radioactive nuclei

① Radioactive Decay is a spontaneous process independent of ~~external~~ ~~condition~~ conditions like temperature, pressure, etc.

② Nuclei decay one by one over a period of time.

③ The rate of decay at any instant is directly proportional to the number of radioactive atoms present at that instant. This is <u>radioactive decay law</u>.

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N$$

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λ = disintegration constant

$$\int \frac{dN}{N} = \int -\lambda dt$$

$$\log N \quad \ln N = -\lambda t + C$$

$$\ln N = -\lambda t + \ln N_0$$

$$\ln \frac{N}{N_0} = -\lambda t$$

$$N = N_0 e^{-\lambda t}$$

Decay Constant:

Reciprocal of the time during which the number of nuclei (N_0) in a radioactive sample reduces to $\left(\frac{N_0}{e}\right)$. $\left[t = \frac{1}{\lambda}\right]$.

Half Life:

the time in which half of the nuclei ~~deca~~ initi present decays.

$$t = T_{1/2} = , \quad N = \frac{N_0}{2}$$

$$\Rightarrow \quad \frac{N_0}{2} = N_0 e^{-\lambda t} \qquad e^{\lambda t} = 2$$

$$\lambda t = \ln 2$$

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$$

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Mean Life :-

The mean life of a radioactive nucleus is the avg. time the nucleus survives before it decays.

Total lifetime of all the nuclei = $\int_0^{N_0} t \cdot dN$

$$\tau = \int_0^{N_0} t \cdot \frac{dN}{N_0}$$

$$\boxed{\tau = \frac{1}{\lambda}}$$ mean lifetime

$$\boxed{\tau = 1.44\, T_{1/2}}$$

Rate of Decay :- Also known as Activity of the nuclei.

$$R(t) = \left|\frac{dN}{dt}\right| = \lambda N(t)$$

$$\boxed{R(t) = R_0 e^{-\lambda t}}$$

R_0 = initial activity at $t = 0$.

Units = Curie (Ci)

$1\ Ci = 3.7 \times 10^{10}$ ~~dis~~ nuclei per second (dps)

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Artificial Radioactivity

The phenomenon of inducing radioactivity in otherwise stable nuclei by bombarding them with suitable particle is called radioactivity.

$$^{27}_{13}Al + ^{4}_{2}He \rightarrow ^{31}_{15}P_o \rightarrow ^{30}_{15}P_o + ^{1}_{0}n$$

↳ Polonium.

$$^{30}_{15}P \rightarrow ^{30}_{14}Si + ^{0}_{1}e + \nu$$

$$^{10}_{5}B + ^{4}_{2}He \rightarrow ^{14}_{7}N \rightarrow ^{13}_{7}N + ^{1}_{0}n$$

$$^{13}_{7}N \rightarrow ^{13}_{6}C + ^{0}_{1}e$$

→ Around 1300 artificial radionuclei exist.

Applications of Radioactivity

1) Dating :- To estimate the age of archaeological and geological specimens, the exponential decay law of radioactive nuclei is used. This is dating. Eg: $t_{1/2}$ of $^{14}_{6}C$ is 5730

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The age of an archaeological specimen can be determined by the amount of $^{14}_{6}C$ it contains and comparing it with the content of $^{14}_{6}C$ in a live similar organism.

② Tracer Technique:

~~Chemical properties of radioisotopes of an element~~

③ Therapeutic Use

④ Activation Analysis

⑤ Preservative

⑥ Agriculture

ALPHA - DECAY * (At the end)

Beta - Decay

→ Atomic number changes by one unit but mass number remains same.

$$n \rightarrow p + e^-$$

$$Q = (m_n - m_p - m_e).c^2$$

$$= (m_n - m_H).c^2$$

$$\approx 0.728 \text{ MeV}$$

→ However, ~~eae~~ K.E of e^- ranges from 0 to 0.728 M

→ ~~A~~ Another particle called neutrino (ν) is present.

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① Electron Emission:- (β^+) (β^-)

$n \rightarrow p + e^- + \bar{\nu}$

↳ anti neutrino

Eg: $^{A}_{Z}X \rightarrow ^{A}_{Z+1}X' + e^- + \bar{\nu}$

$$Q = \left[m\left(^{A}_{Z}X\right) - m\left(^{A}_{Z+1}X'\right) \right] c^2$$

$$\boxed{Q \approx KE_{e^-} + KE_{\bar{\nu}}}$$

② Positron Emission

In some β - decay process, positron (β^+ - particle) is ejected.

$p \rightarrow n + e^+ + \nu$

e^+ → positron

ν ↳ neutrino

$^{A}_{Z}X \rightarrow ^{A}_{Z-1}X + e^+ + \nu$

Eg: $^{22}_{11}Na \rightarrow ^{22}_{10}Ne + ^{0}_{1}e + \nu$

$$\boxed{Q = \left[m\left(^{A}_{Z}X\right) - m\left(^{A}_{Z-1}X\right) - 2m_e \right] c^2}$$

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* For positron emission, Q must be positive i.e.

③ Electron Capture

Sometimes, a proton in a nucleus captures one of its atomic electrons and converts itself into a neutron & a neutrino.

$$p + e^- \rightarrow n + \nu$$

⊛ Usually, the electron captured is from K-shell, hence it's called K-capture.

~~Gamma~~ Decay

NUCLEAR REACTIONS

→ Transformation of a nucleus into another is called transmutation, while the process is called nuclear reaction.

→ In a nuclear reaction, number of protons, number of neutrons, energy, linear momentum and angular momentum are conserved.

Reaction Cross-Section:-

→ Probability of single projectile particle will interact with an individual target nucleus is expressed in reaction cross-section.

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$n =$ no. of particles / area

$N =$ number of reactions

$$\sigma = \frac{N}{n} = \frac{\text{Number of reactions}}{\text{flux}}$$

order of $10^{-28} m^2$

$1 \text{ barn} = 10^{-28} m^2$

$N(\text{single target nuclei}) = \sigma n \times 1$

$N(\text{multiple target nuclei}) = \sigma n N_{target}$

Q- value of a Nuclear Reaction :-

$$x + X \longrightarrow y + Y$$

$$K_x + m_x c^2 + K_X + M_X c^2$$
$$= K_y + m_y c^2 + K_Y + M_Y c^2$$

$$\Rightarrow (K_y + K_Y) - (K_x + K_X) = [(m_x + M_X) - (m_y + M_y)]c^2$$

~~$Q = \Delta K$~~

$$Q = K_f - K_i = (m_i - m_f)c^2$$

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→ The gain in Kinetic Energy in a nuclear reaction is called the Q-value of the nuclear reaction & is denoted by Q.

→ If $Q > 0$, reaction is said to be exothermic. If $Q < 0$, reaction is said to be endothermic.

Nuclear Models

① Liquid Drop Model

② Shell Model

Liquid Drop Model

(i) the density of the nuclear matter is independent of the size of the nucleus

(ii) binding energy per nucleon is constant

→ Approximates the nucleus as a sphere with uniform density.

$R \propto A^{1/3}$, ~~A~~ Surface $A \propto A^{2/3}$, $V \propto A$

$d =$ constant

Volume Energy

$$\boxed{E_v = a_1 A}$$

$a_1 =$ constant

$E_v \propto A$,

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Surface Energy:-

Some ~~nucleotides are on the~~

$$\boxed{E_s = -a_2 A^{2/3}}$$

→ It is very significant for the lighter nuclei since a great fraction of nucleons are on the surface.

Coulomb Energy:-

The ~~Coto.~~ coulomb energy E_c of a nucleus is the work that must be done to bring together Z protons from infinity to a spherical aggregate of the size of the nucleus.

$$V = \frac{-e^2}{4\pi\varepsilon_0 r} \quad \text{(pair of protons)}$$

for $\frac{Z(Z-1)}{2}$ protons,

$$E_c = \frac{Z(Z-1)}{2} V$$

$$= -\frac{Z(Z-1)}{8\pi\varepsilon_0} e^2 \left(\frac{1}{r}\right)_{av}$$

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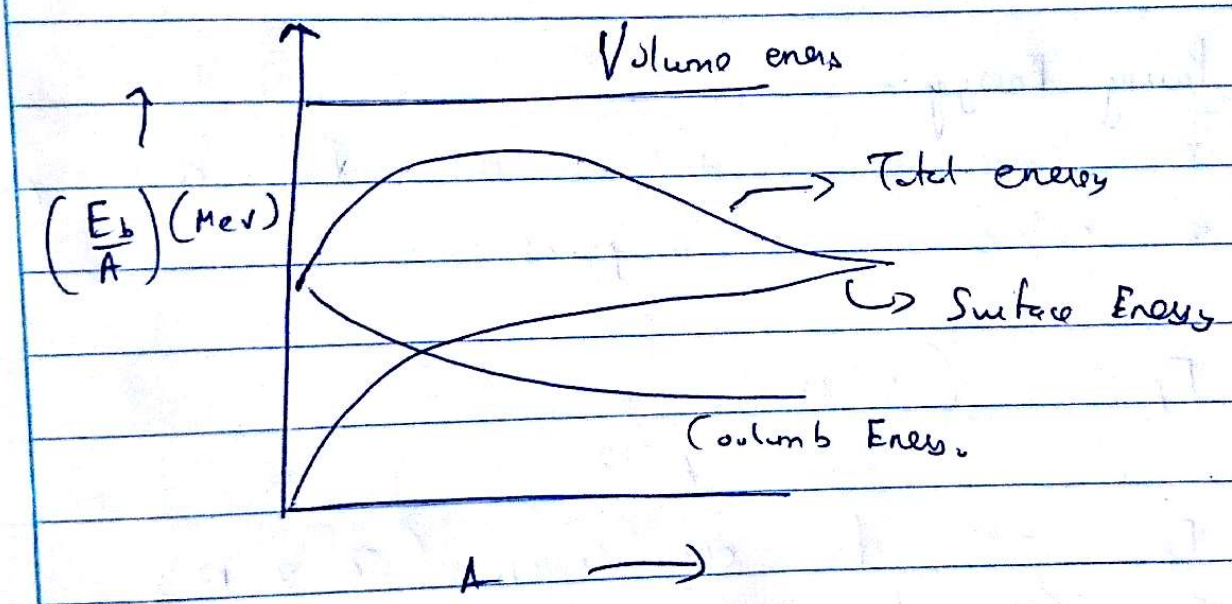
2) $$\boxed{E_c = -a_3 \frac{Z(Z-1)}{A^{1/3}}}$$, since $R \propto A^{1/3}$.

Total binding energy,

$$E_b = E_v + E_s + E_c$$

$$= a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}}$$

$$\boxed{\frac{E_b}{A} = a_1 - \frac{a_2}{A^{1/3}} - a_3 \frac{Z(Z-1)}{A^{4/3}}}$$



Asymmetry Energy:-

ΔE = (Number of new neutrons). (increase in energy per new neutron)

$$= \frac{1}{2}(N-Z) \cdot \frac{1}{2}(N-Z)\frac{\varepsilon}{2}$$

$$= \frac{\varepsilon'}{8}(N-Z)^2 \qquad N = A-Z$$

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$$\cancel{\Delta E = \frac{\varepsilon}{8}(A-2Z)^2}$$

$$\boxed{\Delta E = \frac{\varepsilon}{8}(A-2Z)^2}$$

$$E_a \propto \frac{1}{A},$$

$$E_a = -\Delta E = -a_4\left(\frac{(A-2Z)^2}{A}\right)$$

(asymmetric energy)

$a_4 =$ Constant

Pairing Energy:-

This correction is due to the fact that protons & neutrons exist in pairs.

$$E_p = (\pm, 0)\frac{a_5}{A^{4/3}}$$

$E_p =$ +ve for even-even (Z & N)

$E_p = 0$ for odd-even, even-odd

$E_p =$ −ve for odd-odd (Z & N)

$$\boxed{E_b = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A}(\pm, 0)\frac{a_5}{A^{4/3}}}$$

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Shell Model

→ This model shows the fluctuation in any property of the nucleus based on the energy levels of the nucleons.

→ Atoms having 2, 10, 18, 36, 54, 86 e^- in their shells are exceptionally stable. These numbers are sometimes called atomic magic numbers.

→ Similarly, 2, 8, 20, 28, 50, 82 & 126 are nuclear magic numbers that account for the exceptional stability of nuclei having values of N & Z as these.

* Alpha - Decay

→ $$^{A}_{Z}X \xrightarrow{\alpha\text{-decay}} {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + Q$$

(α - particle)

Here, $Z \geq 82$ (unstable)

→ $$Q = [M_x - (M_y + M_\alpha)]\, c^2 \quad — (1)$$

So, $$[M_x - (M_y + M_\alpha)]\, c^2 = \frac{1}{2} M_\alpha V_\alpha^2 + \frac{1}{2} M_y V_y^2 \quad — (2)$$

Also, $$M_\alpha V_\alpha = M_y V_y \quad — (3)$$

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Substitute value,

$$Q = \frac{1}{2} M_\alpha V_\alpha^2 + \frac{1}{2} M_y \frac{M_\alpha^2 V_\alpha^2}{M_y^2}$$

$$= \frac{1}{2} M_\alpha V_\alpha^2 \left(1 + \frac{M_\alpha}{M_y} \right)$$

$$= K_\alpha \left(\frac{M_\alpha}{M_y} + 1 \right)$$

$$\frac{M_\alpha}{M_y} = \frac{4}{A-4}$$

$$\frac{M_\alpha}{M_{\alpha y}} + 1 = \frac{4}{A-4} + 1 = \frac{4 + A - 4}{A-4} = \frac{A}{A-4}$$

$$Q = K_\alpha \times \frac{A}{A-4}$$

$$\Rightarrow \boxed{K_\alpha = \frac{A-4}{A} Q}$$

As $A >>> 4$, $K_\alpha \simeq Q$.

Almost all KE is that of α-particle.

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