Failure of Classical Physics
OR
Need of Quantum Physics

1. Classical physics can explain the motion of ~~naic~~ macroscopic particles or celestial bodies

2. Classical physics cannot explain the motion of sub-atomic particles. Eg: electrons, protons etc.
3. Cannot explain compton, photoelectric effect & black body radiation.

Wave Function and Probability

$\psi \longrightarrow$ Wave function
$|\psi|^2 \longrightarrow$ Probability of finding the particle within a given Volume dV at given position 'x' at time 't'.

Conditions for physically acceptable wave function:-
(i) Wave function should be continuous and single valued
(ii) $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$ (partial derivatives of ψ w.r.t $x, y,$ & z) should be continuous and single-valued

(iii) ψ should be normalized i.e ψ must go to 0 for $x \to \pm\infty$, $y \to \pm\infty$, $z \to \pm\infty$.

De-broglie Hypothesis

→ It states that moving particle ~~has~~ also also has wave-like properties or matter waves ~~are~~ of ~~a~~ particles are ~~a~~ with other waves.

→ Any de-broglie wave moving in +ve x- is represented by

$$y = a\sin(\omega t - kx)$$

Solution of plane wave equation:-

$$\boxed{\nabla^2 u = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}}$$

Phase Velocity:- Phase Velocity is the velocity of the wave for which the phase of the wave is constant.

i.e. $\omega t - kx = C$ ↳ constant

$\Rightarrow \omega t = kx + C$

$\Rightarrow \boxed{\omega dt = k dx}$

$\frac{\omega}{k} = \frac{dx}{dt}$ → Phase Velocity

$\Rightarrow \boxed{u = \frac{\omega}{k}}$

Wave Packet : Group Velocity & Wave Velocity

$$y = A\cos(kx - \omega t)$$

~~Differe~~ $kx - \omega t = \text{constant}$

Diff. w.r.t t,

$$k\frac{dx}{dt} + x\frac{dk}{dt} - \omega\frac{dt}{dt} - t\frac{d\omega}{dt} = 0$$

$$\Rightarrow k\frac{dx}{dt} + x\frac{dk}{dt} - \omega - t\frac{d\omega}{dt} = 0$$

If k & ω are independent of time,

$$k\frac{dx}{dt} - \omega = 0$$

$$\Rightarrow \boxed{v_p = \frac{\omega}{k}}$$ //.

$$v_p = \frac{2\pi\nu}{2\pi/\lambda} \Rightarrow$$

$$\boxed{v_p = \nu\lambda}$$ //.

$$\boxed{v_g = \frac{d\omega}{dk}}$$ //. , v_g = group Velocity

Group Velocity equals particle Velocity :-

$$\frac{1}{2}mv^2 = E - V$$

E = total E

V = potentia E

$$v = \sqrt{\frac{2(E-V)}{m}}$$

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2(E-V)}{m}}}$$

$$\Rightarrow \quad \lambda = \frac{h}{m}\sqrt{\frac{m}{2(E-V)}}$$

$$V_g = \frac{d\omega}{d\lambda} = \frac{d(2\pi\nu)}{d\left(\frac{2\pi}{\lambda}\right)}$$

$$= \frac{d\nu}{d\left(\frac{1}{\lambda}\right)}$$

$$\frac{1}{V_g} = \frac{d\left(\frac{1}{\lambda}\right)}{d\nu} = \frac{d\left[\frac{m}{h}\sqrt{\frac{2(E-V)}{m}}\right]}{d\nu}$$

$$= \frac{1}{h}\frac{d}{d\nu}\left[\sqrt{2m\,(\overset{h\nu}{E}-V)}\,\right]$$

$$= \frac{1}{h}\left[2m(E-V)\right]^{-1/2}(2mh)$$

$$\Rightarrow$$

$$\frac{1}{V_g} = \frac{\cancel{h}}{\cancel{h}}\frac{\cancel{2m}\;2m}{\sqrt{2m(E-V)}}$$

$$\frac{1}{V_g} = \sqrt{\frac{2m}{2(E-V)}} = \frac{1}{V}$$

$$V_g = V \;//.$$

Relation between Phase Velocity & Group Velocity

For dispersive medium, there is variation of v_p with wavelength

$$V_p = \frac{\omega}{k}, \quad V_g = \frac{d\omega}{dk}$$

$$= \frac{d\omega}{d\left(\frac{2\pi}{\lambda}\right)} = \frac{-\lambda^2}{2\pi}\frac{d\omega}{d\lambda}$$

$$= \frac{-\lambda^2}{2\pi}\frac{d}{d\lambda}\left(\frac{2\pi v_p}{\lambda}\right)$$

$$= -\lambda^2\left(\frac{-v_p}{\lambda^2} + \frac{1}{\lambda}\frac{dv_p}{d\lambda}\right)$$

$$\boxed{V_g = v_p - \lambda\frac{dv_p}{d\lambda}}$$

Uncertainity Principle

① $\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}$

② $\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$ $\quad$ ΔE = energy, Δt = time

③ $\Delta J \cdot \Delta \theta \geq \frac{\hbar}{2}$

ΔJ = angular momentum
$\Delta \theta$ = angle

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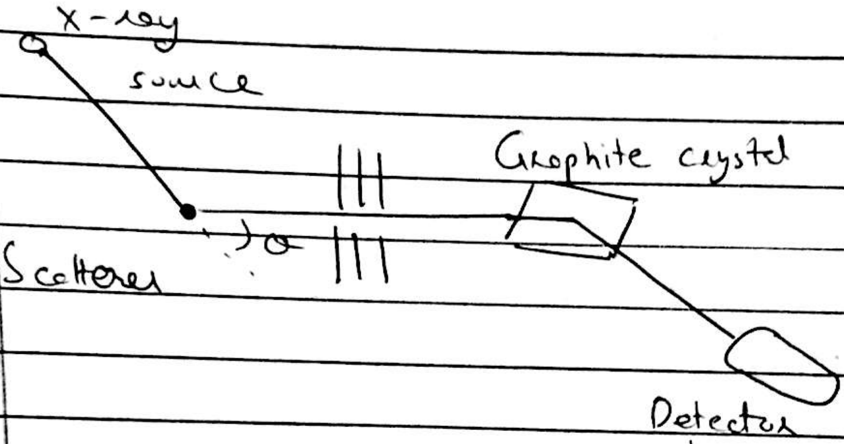
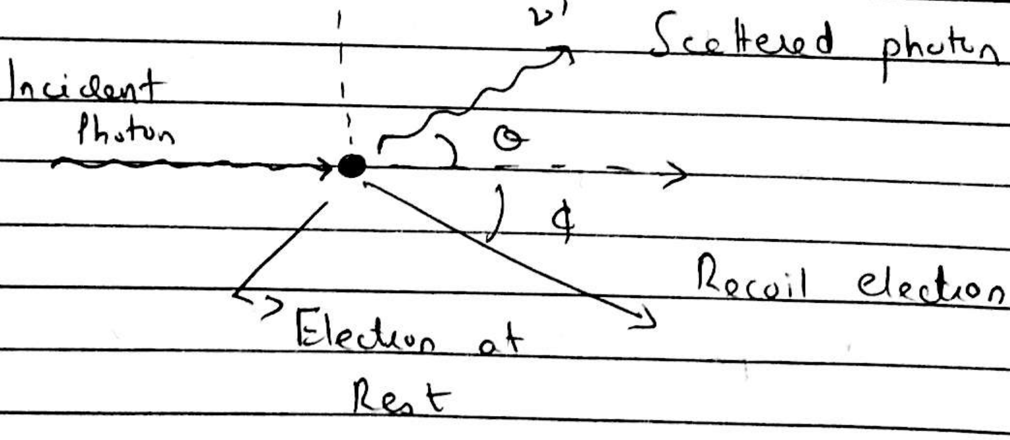
Compton's Experimental Arrangement

X- Rays of monochromatic wavelength λ fall on a graphite block, and intensity of the scattered light is measured at different angles. (λ & λ')





$$\frac{h\nu}{c} - \frac{h\nu'}{c}$$

By conservation of momentum,

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + \cancel{p_e \sin\phi}\ p_e\cos\phi \quad \text{--- (1)}$$

$$0 = \frac{h\nu'}{c}\sin\theta - p_e\sin\phi \quad \text{--- (2)}$$

$$h\nu = h\nu' + K_e$$

$$k_e = h(\nu - \nu') \quad -(5)$$

Eq (1) & (2) can be written as:

$$p_e c \cos\phi = h\nu - h\nu' \cos\theta \quad -(3)$$

$$p_e c \sin\phi = h\nu' \sin\theta \quad -(4)$$

Squaring & adding (3) & (4),

$$p_e^2 c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu' \cos\theta$$

from eq (5),

$$k_e + m_o c^2 = h(\nu - \nu') + m_o c^2$$

$$E_e = (c^2 p_e^2 + m_o^2 c^4)^{1/2}$$

$$\Rightarrow (c^2 p_e^2 + m_o^2 c^4)^{1/2} = h(\nu - \nu') + m_o c^2$$

Squaring this, we get

$$h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu' \cos\theta = h^2(\nu - \nu')^2 + 2h(\nu - \nu') m_o c^2$$

Since, $\nu = c/\lambda$, and $\nu' = c/\lambda'$,

$$\frac{h^2c^2 \cos\theta}{\lambda\lambda'} - \frac{h^2c^2}{\lambda\lambda'} + h\left(\frac{c}{\lambda} - \frac{c}{\lambda'}\right) m_o c^2 = 0$$

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$$h(1-\cos\theta) = c(\lambda' - \lambda) m_0$$

$$\Rightarrow \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\boxed{\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)}$$ //

Compton Shift.

Also, $\boxed{\frac{h}{m_0 c}}$ // is called the compton wavelength.

$$\frac{h}{m_0 c} = \frac{6.6 \times 10^{-34}}{9 \times 10^{-31} \times 3 \times 10^8} = \cancel{0.0246}\ 0.02426 A^0$$

$1-\cos\theta\ (max) = 2$

hence,

$$\Delta\lambda(max) = 0.048 \approx 0.05 A^0$$

Compton Effect cannot be observed for visible light Rays.

Direction of Recoil Electron:—

$$\tan\phi = \frac{h\nu' \sin\theta}{h\nu - h\nu' \cos\theta}$$

$$\boxed{\tan\phi = \frac{\lambda \sin\theta}{\lambda' - \lambda\cos\theta}}$$ //

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⇒ Kinetic Energy of Recoil Electron

$$k_e = (m - m_0)c^2 = h\nu - h\nu'$$

$$= \frac{hc}{\lambda} - \frac{hc}{\lambda'}$$

$$\left[\lambda' = \lambda + \frac{h}{m_0 c}(1 - \cos\theta) \right]$$

$$\nu' = \frac{\nu}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos\theta)} = \frac{\nu}{1 + x(1 - \cos\theta)}$$

$$x = \frac{h\nu}{m_0 c^2} \cancel{(1 - \cos\theta)}$$

$$k_e = h\nu - h\nu'$$

$$= h\nu \left[1 - \frac{1}{1 + x(1 - \cos\theta)} \right]$$

$$\cancel{k_e = h\nu \left[\frac{x(1 - \cos\theta)}{1 + x - x\cos\theta} \right]} \qquad x = \frac{h\nu}{m_0 c^2}$$

$$\boxed{k_e = h\nu \left[\frac{x(1 - \cos\theta)}{1 + x(1 - \cos\theta)} \right]}$$

$$x = \frac{h\nu}{m_0 c^2}$$

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Q. Prove that $v_g = \frac{c}{\sqrt{2}}$, when compton wavelength is equal to de-Broglie wavelength.

A. $\lambda_c = \frac{h}{m_0 c}$, $m = \frac{m_0}{\sqrt{1 - v_g^2/c^2}}$

Also, $\lambda_d = \frac{h}{m v_g}$

$$\frac{\cancel{h}}{m v_g} = \frac{\cancel{h}}{m_0 c}$$

$\Rightarrow$ $m_0 c = m v_g$

$$\cancel{m_0} c = \frac{\cancel{m_0} v_g}{\sqrt{1 - v_g^2/c^2}}$$

$\Rightarrow$ $1 - \frac{v_g^2}{c^2} = \frac{v_g^2}{c^2} \Rightarrow 1 = \frac{2 v_g^2}{c^2}$

$\Rightarrow$ $\frac{c^2}{2} = v_g^2 \Rightarrow \boxed{v_g = \frac{c}{\sqrt{2}}}$ Hence, Proved.

Operators

$$\psi(x,t) = e^{i(kx - \omega t)}$$

$$\frac{\partial \psi}{\partial x} = e^{i(kx - \omega t)} ik = ik\, \psi(x,t)$$

$$\frac{\partial^2 \psi}{\partial x^2} = e^{i(kx - \omega t)} i^2 k^2 = i^2 k^2 \psi(x,t)$$

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$$\frac{\partial\psi}{\partial t} = e^{i(kx-\omega t)}(-i\omega) = \underline{-i\omega\psi(x,t)}$$

$$\hbar\frac{\partial\psi}{\partial x} = \hbar ik\, e^{i(kx-\omega t)} \Rightarrow p\psi = -i\hbar\frac{\partial\psi}{\partial x}$$

$$\Rightarrow \boxed{\hat{p} = -i\hbar\frac{\partial}{\partial x}} \longrightarrow \underline{\text{Momentum Operator}} \quad - (1)$$

Also,

$$\hbar\frac{\partial\psi}{\partial t} = \underbrace{-i\omega\hbar}_{-iE}\, e^{i(kx-\omega t)}$$

$$\Rightarrow E\psi = i\hbar\frac{\partial\psi}{\partial t} \Rightarrow \boxed{\hat{E} = i\hbar\frac{\partial}{\partial t}} \quad - (2)$$

$\longrightarrow$ Energy Operator

$$p^2 = (-i\hbar)^2\frac{\partial^2}{\partial x^2}$$

$$K.E = \frac{p^2}{2m} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$$

We know,

$$K.E + P.E = T.E$$

$$\Rightarrow T + V = E$$

$$\Rightarrow T\psi + V\psi = E\psi$$

Expectation Values

→ Expectation values are basically mean values in the case of quantum mechanics.

$$\langle x \rangle = \frac{\int_{-\infty}^{\infty} x |\psi|^2 dx}{\int_{-\infty}^{\infty} |\psi|^2 dx}$$

→ If ψ is a normalised function,

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1$$

So, $$\langle x \rangle = \int_{-\infty}^{\infty} x |\psi|^2 dx$$

→ Similarly, for any other function also we can do this i.e.

$$\langle G(x) \rangle = \int_{-\infty}^{\infty} G(x) |\psi|^2 dx$$

→ This isn't possible for $E(x)$ & $p(x)$ since no such function exists due to Uncertainty principle.

Operators

→ It is a rule by which given any function, we can find another function.

$$\boxed{AB\psi \neq BA\psi}$$

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Physical Quantity	Symbol	Operator
Position	$q(r)$	$q(x, y, z, t)$
Energy	E	$i\hbar \frac{\partial}{\partial t}$
Kinetic Energy	T	$-\frac{\hbar^2 \nabla^2}{2m}$
Momentum	p	$-i\hbar \nabla$ $\left(-i\hbar \frac{\partial}{\partial x}, -i\hbar \frac{\partial}{\partial y}, -i\hbar \frac{\partial}{\partial z}\right)$
Velocity	v	$-\frac{i\hbar}{m} \nabla$ $\left(-\frac{i\hbar}{m} \frac{\partial}{\partial x}, -\frac{i\hbar}{m} \frac{\partial}{\partial y}, -\frac{i\hbar}{m} \frac{\partial}{\partial z}\right)$
Hamiltonian	H	$i\hbar \frac{\partial}{\partial t}$

SCHRODINGER EQUATION: TIME DEPENDENT FORM

$$\psi = Ae^{-i\omega(t - x/v)}$$

$\omega = 2\pi\nu$, ~~$\nu\nu$~~ $v = \lambda\nu$,

$$\psi = Ae^{-2\pi i\left(\nu t - \frac{x}{\lambda}\right)}$$

$E = h\nu = 2\pi\hbar\nu$ ~~x~~ $\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}$

~~$\psi = Ae^{-i(}$~~

$$\psi = Ae^{-(i/\hbar)(Et - px)} \quad - (1)$$

Diff twice w.r.t x,

$$\frac{\partial^2\psi}{\partial x^2} = \frac{-p^2}{\hbar^2}\psi = p^2\psi$$

~~$= \frac{-\hbar^2}{} \partial^2\psi$~~

$$= -\hbar^2 \frac{\partial^2\psi}{\partial x^2}$$

Diff (1) w.r.t t,

$$\frac{\partial\psi}{\partial t} = -\frac{iE\psi}{\hbar} \Rightarrow E\psi = -i\hbar\frac{\partial\psi}{\partial t}$$

$$E = \frac{p^2}{2m} + U(x,t)$$

Multiplying by ψ on both sides,

$$E\psi = \frac{p^2\psi}{2m} + U\psi$$

↳ Total Energy

$$\Rightarrow \quad i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + U\psi$$

↳ Potential Energy

In 3-D,

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right] + U\psi}$$ //.

SCHRODINGER'S EQUATION: Steady State form
(Time Independent)

$$\psi(t) = A e^{-(i/\hbar)(Et - px)}$$

$$= A e^{-i(E/\hbar)t} e^{+(ip/\hbar)x}$$

$$= \psi e^{-(iE/\hbar)t}$$

Substituting this in Schrodinger's 3-D eq,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\left[\partial \nabla^2\right] + U\psi$$

we get,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}\left[E - U\right]\psi = 0$$

$$\Rightarrow \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2m}{\hbar^2}\left[E - U\right]\psi = 0$$

Eigen Values & Eigen Functions

→ Values of E_n for which Schrodinger's eq^n can be solved are Eigen values & corresponding functions are Eigen functions (ψ_n)

$$\boxed{E_n = \frac{-me^4}{32\pi^2\varepsilon_0^2\hbar^2}\left(\frac{1}{n^2}\right) \quad n = 1, 2, 3 \ldots}$$

Degenerate & Non-degenerate Eigenfunctions

→ If corresponding to a single eigenvalue, there are a number of eigenfunctions, these eigenfunctions are known as Degenerate eigenfunctions, and number of such functions is its degeneracy

Total degeneracy

$$= \sum_{l=0}^{n-1}(2l+1) = \frac{2(n)(n-1)}{2} + n$$

$$= \boxed{n^2}$$

Particle in a Box

→ Motion of particle is restricted in $x = 0$ & $x = l$

→ V = infinite on both sides of the box and $V = 0$ inside it.

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$$

General solution of this is

$$\psi = A\sin\sqrt{\frac{2mE}{\hbar}}x + B\cos\sqrt{\frac{2mE}{\hbar}}x$$

$$k = \sqrt{\frac{2mE}{\hbar}}$$

At $x=0$, $\psi=0$,

$$0 = B\cos$$

At $x = L$, $\psi = 0$

$$0 = A\sin\sqrt{\frac{2mE}{\hbar}}L$$

So,

$$\sin\sqrt{\frac{2mE}{\hbar}}L = \cancel{n\pi}\ 0$$

$$\Rightarrow \sqrt{\frac{2mE}{\hbar}}L = n\pi$$

$$\Rightarrow \boxed{E_n = \frac{n^2\pi^2\hbar^2}{2mL^2},\quad n = 1, 2, 3 \ldots}$$

Wave function of Particle in a Box

$$\psi_n = A\sin kx$$

$$\psi_n = A\sin\sqrt{\frac{2mE_n}{\hbar}}x$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

So, $\psi_n = A \sin \frac{n\pi \cancel{\hbar}}{L \cancel{\hbar}} x$

$$\psi_n = A \sin \frac{n\pi x}{L}$$

To normalize it,

$$\int_{-\infty}^{\infty} |\psi_n|^2 dx = 1$$

$$\Rightarrow \int_0^L A^2 \sin^2 \frac{n^2\pi^2 x^2}{L^2} dx = 1$$

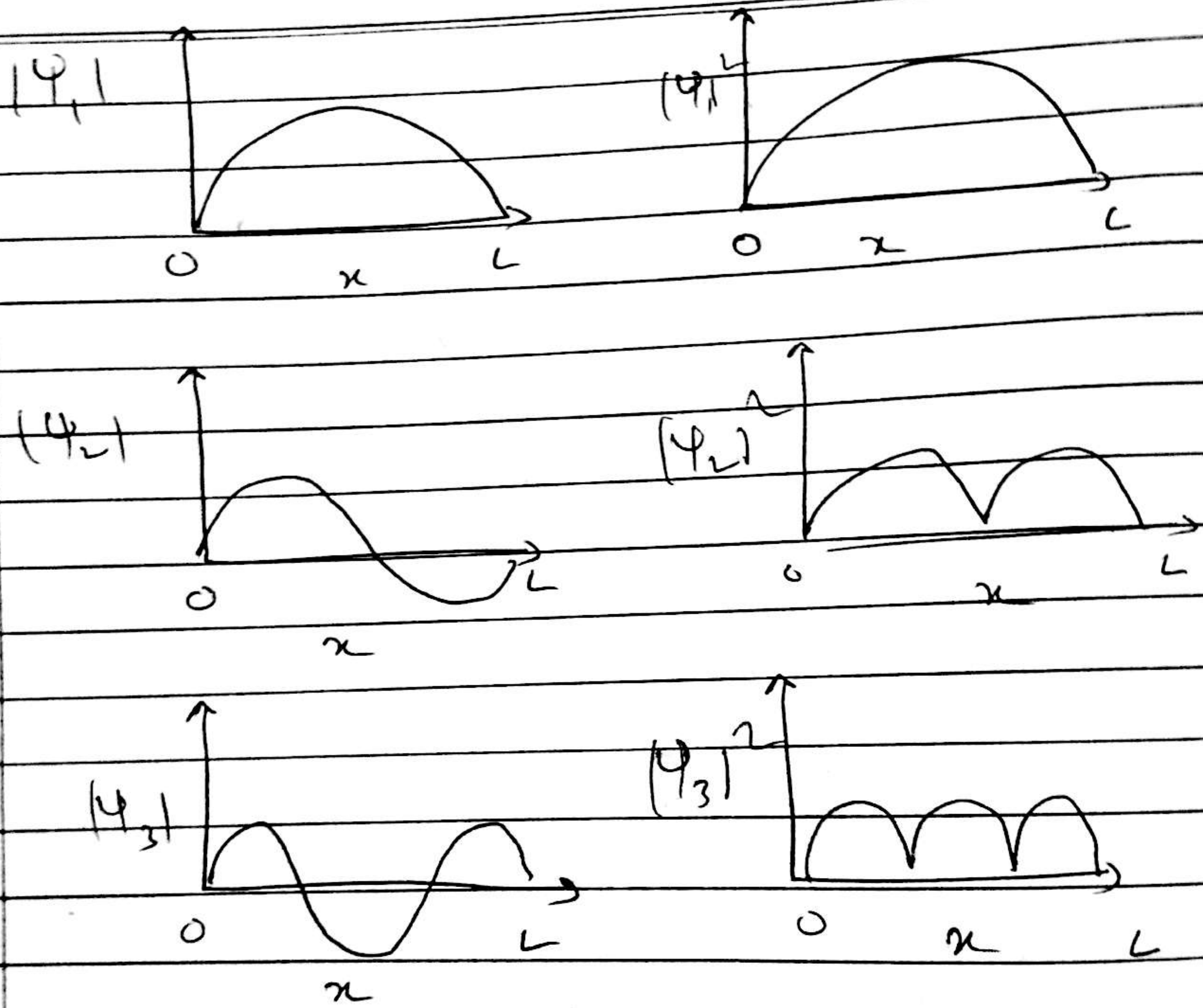
$$\Rightarrow A^2 \left(\frac{L}{2}\right) = 1 \Rightarrow A^2 = \frac{2}{L}$$

$$\Rightarrow A = \sqrt{\frac{2}{L}}$$

So,

$$\boxed{\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}} \quad n = 1, 2, 3 \ldots$$

Here, n = number of loops in graph.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Heisenberg Uncertainity Principle :-

$$\Delta x \cdot \Delta p_x \geq \frac{h}{4\pi}$$

$$\boxed{\Delta x \, m \, \Delta v_x \geq \frac{h}{4\pi}}$$

$\Delta x \rightarrow$ uncertainity in position

$\Delta p_x \rightarrow$ uncertainity in momentum

$$\cancel{\Delta x = \sqrt{\langle x^2 \rangle}}$$

$$\boxed{\Delta x = \sqrt{\langle x^2 \rangle - (\langle x \rangle)^2}} \,//.$$

$$\boxed{\Delta p_x = \sqrt{\langle p_x^2 \rangle - (\langle p_x \rangle)^2}} \,//$$

ASSIGNMENT.

Q1 Show that electron cannot exist inside the nucleus.

Q2 Show that proton & neutron can exist inside the nucleus.

Q3. Make two slides on Photoelectric effect.

Harmonic Oscillator

~~P.E = 1/2 kx²~~

→ This is a system in which particle ~~oscill~~ oscillates with frequency 'ν' under the effect of harmonic ~~oscillator~~ potential $\frac{1}{2}kx^2$ (P.E)

In Schrödinger eqn :-

putting P.E = $\frac{1}{2}kx^2$,

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

$$\Rightarrow \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} \cancel{+ \frac{1}{2}kx^2} = \psi(E-V)$$

$$\Rightarrow \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - \frac{1}{2}kx^2\right)\psi = 0$$

//

$$\Rightarrow \cancel{\psi_{(x)} = A_n H_n(\xi) e^{-\xi^2/n}}$$

$$\Rightarrow \boxed{\psi(x) = A_n H_n(\beta)\, e^{-\beta^2/2}}$$

//

$$\beta = 2x$$

$$\alpha = \sqrt{\frac{m\omega}{\hbar}} \quad , \quad \omega = \sqrt{\frac{k}{m}}$$

→ $H_n(\beta)$ is Hermite polynomial.

→ A_n is normalisation constant.

→ $$H_n(\beta) = (-1)^n e^{-\beta^2} \frac{d^n}{d\beta^n} (e^{-\beta^2})$$

→ $$\int_{-\infty}^{\infty} e^{-\beta^2} H_n^2(\beta) = \sqrt{\pi}\, 2^n n!$$

→ $$A_n^2 \int_{-\infty}^{\infty} e^{-\beta^2} H_n^2(\beta) = 1$$

→ $$A_n = \sqrt{\frac{1}{\sqrt{\pi}\, 2^n n!}}$$

→ $$A_n^2 \sqrt{\pi}\, 2^n n! = 1$$

Linear Operators

1. $$\hat{A}(f+g) = \hat{A}f + \hat{A}g$$

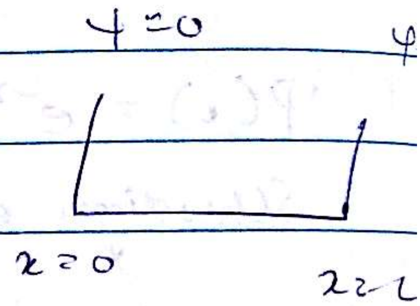
2. $$\hat{A}(cf) = c\hat{A}f$$

Q2. For particle in a box

$0 \le x \le 2L$,

Find energy eigen values and wave function.

A2. $\psi(x) = A\cos kx + B\sin kx$



for, $x = 0$, $\psi = 0$

$\Rightarrow \quad 0 = A \Rightarrow \underline{A = 0}$

for, $x = 2L$, $\psi = 0$

$\Rightarrow \quad 0 = A\cos 2kL + B\sin 2kL$

$B\sin 2kL = 0$

$\sin 2kL = 0$

~~$\sin 2kL = \sin$~~

$2kL = n\pi$

$k = \dfrac{n\pi}{2L}$ — (1)

$\Rightarrow$ We know, $k^2 = \dfrac{2mE}{\hbar^2} \Rightarrow$

$\dfrac{2mE_n}{\hbar^2} = \dfrac{n^2\pi^2}{4L^2}$

$\Rightarrow \quad \boxed{E_n = \dfrac{\hbar^2 n^2 \pi^2}{8mL^2}}$

↳ Eigen value.

~~for~~. Also, we know for ~~co~~ $\psi(x)$ in a box,

$$\psi(x) = \sqrt{\frac{2}{Length}} \sin\left(\frac{n\pi x}{2L}\right)$$

$$\psi(x) = \frac{1}{\sqrt{L}} \sin\left(\frac{n\pi x}{2L}\right)$$

~~Q8~~. Energy Eigen Value for Harmonic Oscillator

$$\boxed{E_n = \left(n + \frac{1}{2}\right)\hbar\omega}$$

$n = 0 \rightarrow$ ground state.

For, $n = 0$,

$E_0 = \cancel{\frac{1}{2}}\ \frac{1}{2}\hbar\omega$, = zero point energy.

For, ~~$0 < x < 2L$~~

~~$n^2 = n_x^2 + n_y^2 + n_z^2$~~

$\rightarrow$ for harmonic ~~co~~ oscillator,

$n = n_x + n_y + n_z$.

Eg: $E = 7/2\ \hbar\omega$.

Here, $n + 1/2 = 7/2 \Rightarrow n = 3$.

$3 = n_x + n_y + n_z$

Hence, there can be many combinations of n_x, n_y, n_z.

So, number of possible combinations = no. of wave functions with $E = \frac{7}{2}\hbar\omega$.

$$\boxed{\text{Total no. of combinations} = \frac{(n+1)(n+2)}{2}}$$

↳ Harmonic Oscillator.

Also known as Degeneracy of H.O

f $n = 3$, degeneracy $= \frac{(3+1)(3+2)}{2} = \frac{4 \times 5}{2} = \underline{\underline{10}}$.