1-2. LIMIT

(1) $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$ $\quad x \in R^+, \ a \in R^+, \ n \in R$

(2) $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ $\qquad$ (3) $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1$

(4) $\lim_{n \to \infty} \frac{1}{n} = 0$ $\qquad$ (5) $\lim_{n \to \infty} r^n = 0, \quad |r| < 1$

(6) $\lim_{n \to \infty} (a + ar + ar^2 + \cdots + ar^n) = \frac{a}{1 - r}, \quad |r| < 1$

(7) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$ $\qquad$ (8) $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$

(8) $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a \quad (a > 0)$ $\qquad$ (9) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

(10) For a polynomial function, $P(x)$

$\lim_{x \to a} P(x) = P(a)$

Hence, polynomial is a continuous function

(11) If $f, g : (a, b) \to R$ are continuous at $c \in (a, b)$, then $f \pm g$, $f \times g$ and if $g(c) \neq 0$, then f/g are also continuous at c.

NOTE FOR OBJECTIVE QUESTION

If f and g are continuous at c, it does not necessarily follow that $f \pm g$, etc. are continuous at c. For this it is necessary that $f \pm g$ should be defined in some neighbourhood of c. For instance,

$f : [-1, 1] \to R, \ f(x) = \sqrt{1 - x^2}$ is continuous at $x = 1$.

$g : \{x \mid |x| \geq 1\} \to R, \ g(x) = \sqrt{x^2 - 1}$ is continuous at $x = 1$.

Here, $D_f \cap D_g = \{-1, 1\}$. So $f + g$ is defined only for $\{-1, 1\}$. As $f + g$ is not defined in <u>any</u> neighbourhood of 1, it is not continuous at 1.

(12) $e^x = 1 + x + x^2/2! + x^3/3! + \cdots + x^n/n! + \cdots \infty = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$e = 1 + 1 + 1/2! + 1/3! + \cdots + 1/n! + \cdots \infty$

$\therefore e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!} = \sum_{n=2}^{\infty} \frac{x^{n-2}}{(n-2)!}$ & so on

$e^{-x} = 1 - x + x^2/2! - x^3/3! + \cdots \infty$

$\therefore e^x + e^{-x} = 2\,(1 + x^2/2! + x^4/4! + \cdots \infty)$

Thus, $1 + x^2/2! + x^4/4! + \cdots \infty = \frac{1}{2}(e^x + e^{-x})$

and $x + x^3/3! + x^5/5! + \cdots \infty = \frac{1}{2}(e^x - e^{-x})$

(13) If $-1 < x \leq 1$, then $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \infty$

$x = 1$ gives $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots \infty$

For $-1 < x < 1$, Similarly, $\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots \infty$

$\therefore x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots \infty = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

(14) <u>Binomial series</u>

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots \infty$, $n \in Q - N$, $|x| < 1$

When n is a positive integer, R.H.S. is a finite series containing $n+1$ terms.

$(1+x)^{p/q} = 1 + \frac{p}{1}\left(\frac{x}{q}\right) + \frac{p(p-q)}{2!}\left(\frac{x}{q}\right)^2 + \cdots \infty$

$(1+x)^{-p/q} = 1 - \frac{p}{1}\left(\frac{x}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2 - \cdots \infty$

$(1-x)^{p/q} = 1 - \frac{p}{1}\left(\frac{x}{q}\right) + \frac{p(p-q)}{2!}\left(\frac{x}{q}\right)^2 - \cdots \infty$

$(1-x)^{-p/q} = 1 + \frac{p}{1}\left(\frac{x}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2 + \cdots \infty$

$p \in Z,\ q \in N - \{1\}$

3. DIFFERENTIATION

3. DIFFERENTIATION

(1) For $y = f(x)$, $\dfrac{dy}{dx} = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{t\to x} \frac{f(t) - f(x)}{t - x}$$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

(2) $\dfrac{d}{dx}(x^n) = nx^{n-1}$, if $n \in N$, then $x \in R$

if $n \in Z$, then $x \in R - \{0\}$

if $n \in R$, then $x \in R^+$

(3) $\dfrac{d}{dx}(\sin x) = \cos x \quad (x \in R)$

(4) $\dfrac{d}{dx}(\cos x) = -\sin x \quad (x \in R)$

(5) $\dfrac{d}{dx}(\tan x) = \sec^2 x$

(6) $\dfrac{d}{dx}(\sec x) = \sec x \tan x$

(7) $\dfrac{d}{dx}(a^x) = a^x \log_e a$

(8) $\dfrac{d}{dx}(e^x) = e^x$

(9) $\dfrac{d}{dx}(\log x) = \dfrac{1}{x}$

(10) $\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x} \log_a e$

(11) $\dfrac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(12) $\dfrac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

(13) $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$, $|x| < 1$

(14) $\dfrac{d}{dx}(\cos^{-1} x) = -\dfrac{1}{\sqrt{1-x^2}}$, $|x| < 1$

(15) $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$

(16) $\dfrac{d}{dx}(\cot^{-1} x) = -\dfrac{1}{1+x^2}$

(17) $\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{|x|\sqrt{x^2-1}}$, $|x| > 1$

(18) $\dfrac{d}{dx}(\operatorname{cosec}^{-1} x) = -\dfrac{1}{|x|\sqrt{x^2-1}}$ $(|x| > 1)$

(19) $\dfrac{d}{dx}(uvw) = uv\dfrac{dw}{dx} + uw\dfrac{dv}{dx} + vw\dfrac{du}{dx}$

(20) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

(21) To differentiate $y = [f(x)]^{g(x)}$ take log of both sides and then differentiate w.r.t. x.

4. APPLICATIONS OF THE DERIVATIVE

(1) The derivative $\frac{dy}{dx}$ is the rate of increase of y with respect to x. The rate of increase of the displacement s of a particle in linear motion with respect to time t is called its velocity v and expressed as $v = \frac{ds}{dt}$. Similarly, the rate of increase of the velocity v of a particle in linear motion with respect to time is called its acceleration a, and is expressed as $a = \frac{dv}{dt}$.

(2) When a change δx occurs in the value of x, the corresponding change in the value of the function, $y = f(x)$, is $\delta y \approx f'(x)\,\delta x$. Thus, the approximate value of $f(x+\delta x)$ is $f(x) + f'(x)\,\delta x$.

(3) For $y = f(x)$, $\left(\frac{dy}{dx}\right)_{(x,y)}$ = slope of tangent at $P(x,y)$

$$= \tan\theta \quad [\theta \in (0,\pi) - \{\pi/2\}]$$

where θ is the angle which the tangent at P makes with the positive direction of the X-axis.

(4) For $y = f(x)$, the equation of tangent at $P(x_1, y_1)$ on the curve is

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1,\, y_1)} (x - x_1)$$

and the equation of the normal at P is

$$y - y_1 = -\left(\frac{dx}{dy}\right)_{(x_1,\, y_1)} (x - x_1).$$

For parametric equations, $x = f(t)$ and $y = g(t)$, the equation of the tangent at the parametric point t_1 is given by

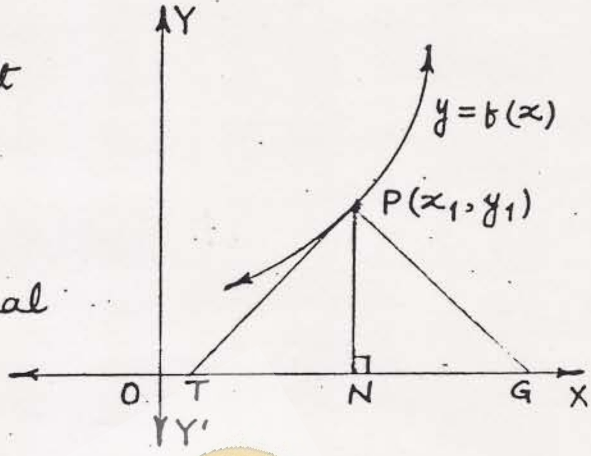
$$y - g(t_1) = \frac{g'(t_1)}{f'(t_1)}\,[x - f(t_1)]$$

and the equation of the normal is

$$y - g(t_1) = -\frac{f'(t_1)}{g'(t_1)}\,[x - f(t_1)].$$

4. APPLICATIONS OF THE DERIVATIVE

(5) For $y = f(x)$, the tangent and normal at $P(x_1, y_1)$ meet the X-axis at T and G respectively. $\overline{PN} \perp \overleftrightarrow{OX}$ (see fig.).



TN = length of subtangent

$$= \left| \frac{y_1}{y_1'} \right|$$

NG = length of subnormal

$$= | y\, y_1 | \quad y_1 y_1'$$

PT = length of tangent $= \left| \dfrac{y_1}{y_1'} \right| \sqrt{1 + y_1'^2}$

PG = length of normal $= |y_1| \sqrt{1 + y_1'^2}$

where, $y_1' = \left(\dfrac{dy}{dx} \right)_{(x_1, y_1)}$.

(6) The acute (or right) angle between the tangents to two curves at their point of intersection is called the angle between the given curves. First find the point of intersection by solving the equations of the curves simultaneously. Then find the slopes, m_1, m_2, of the tangents to the curves at their point of intersection. If $m_1 = m_2$, the angle between the curves is zero and the curves touch each other. If $m_1 m_2 = -1$, the angle between the curves is $\pi/2$ and the curves are said to be intersecting orthogonally. In other cases, the acute angle ϕ is given by the formula

$$\tan \phi = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| .$$

(7) Rolle's Theorem : If a real function f is continuous on $[a, b]$ and differentiable on (a, b) and if $f(a) = f(b)$, then there must be some $x \in (a, b)$ such that $f'(x) = 0$.

The conditions of Rolle's Theorem are not all necessary. One may find $f'(x) = 0$ for some $x \in (a, b)$ even if none of the conditions is satisfied. However, the conditions are sufficient for the theorem to hold.

(8) Mean Value Theorem : If f is continuous in $[a, b]$ and differentiable in (a, b), then we can always find $c \in (a, b)$ such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

These conditions are also sufficient and not necessary. Other forms of mean value theorem are

(i) for some θ, $0 < \theta < 1$, $f(a+h) = f(a) + h f'(a+\theta h)$,

(ii) $a = x$, $h = 0 \Rightarrow f(x) - f(0) = x f'(\theta x)$, $0 < \theta < 1$.

(9) (i) If $f'(x) > 0$, $f(x)$ is strictly increasing,

(ii) If $f'(x) < 0$, $f(x)$ is strictly decreasing

(iii) If for some $x \in D_f$; $f'(x) = 0$ and $f''(x) > 0$, $f(x)$ has a local minimum.

(iv) If for some $x \in D_f$, $f'(x) = 0$ and $f''(x) < 0$, $f(x)$ has a local maximum.

(v) For global extreme values of $f(x)$, which is continuous in the domain $[a, b]$, find values of x_i for which $f'(x_i) = 0$ and values of y_i where f is not differentiable. Then

global maximum of f in $[a, b]$ = max. $\{f(a), f(b), f(x_i), f(y_i)\}$

" minimum " " = min { " " " " }.

5 - 6. INDEFINITE INTEGRATION

5 – 6. INDEFINITE INTEGRATION

(1) $\int x^n\, dx = \dfrac{x^{n+1}}{n+1} + c, \quad n \neq -1,\ x \in R^+$

(2) $\int \frac{1}{x}\, dx = \log|x| + c$

(3) $\int a^x\, dx = \dfrac{a^x}{\log a} + c$ (4) $\int e^x\, dx = e^x + c$

(5) $\int \cos x\, dx = \sin x + c$ (6) $\int \sin x\, dx = -\cos x + c$

(7) $\int \sec^2 x\, dx = \tan x + c$ (8) $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

(9) $\int \sec x \tan x\, dx = \sec x + c$ (10) $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$

(11) $\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + c \quad (a \neq 0)$

(12) $\int \dfrac{dx}{x^2-a^2} = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right| + c \quad (x \neq \pm a,\ a \neq 0)$

(13) $\int \dfrac{dx}{a^2-x^2} = \dfrac{1}{2a}\log\left|\dfrac{x+a}{x-a}\right| + c \quad (x \neq \pm a,\ a \neq 0)$

(14) $\int \dfrac{dx}{\sqrt{x^2 \pm a^2}} = \log\left|x + \sqrt{x^2 \pm a^2}\right| + c$

(15) $\int \dfrac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\dfrac{x}{a} + c \quad (a > 0)$

(16) $\int \dfrac{dx}{|x|\sqrt{x^2-a^2}} = \dfrac{1}{a}\sec^{-1}\dfrac{x}{a} + c \quad (|x| > |a| > 0)$

(17) $\int \tan x\, dx = \log|\sec x| + c$ (18) $\int \cot x\, dx = \log|\sin x| + c$

(19) $\int \sec x\, dx = \log|\sec x + \tan x| + c = \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$

(20) $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + c = \log\left|\tan\frac{x}{2}\right| + c$

(21) If $\int f(x)\, dx = F(x)$, then $\int f(ax+b)\, dx = \frac{1}{a}F(ax+b) + c$

5 - 6. INDEFINITE INTEGRATION

(22) $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$

(23) $\int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)} + c$

(24) For integrals of the form $\int \sin^m x \cos^n x\,dx$,

if (i) m is odd, put $\cos x = t$

(ii) n is odd, put $\sin x = t$

(iii) m, n are even, then use

$$\sin^2 x = \frac{1}{2}(1-\cos 2x);\ \cos^2 x = \frac{1}{2}(1+\cos 2x)$$

(25) For integrals of the form $\int \frac{dx}{a + b\cos x + c\sin x}$,

put $\tan\frac{x}{2} = t \quad \therefore \frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$ or, $dx = \frac{2dt}{1+t^2}$

Also, $\cos x = \frac{1-t^2}{1+t^2}$ and $\sin x = \frac{2t}{1+t^2}$

(26) For $\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}\,dx$ or, $\int \frac{ae^x + be^{-x}}{ce^x + de^{-x}}\,dx$

Let numerator = l (denominator)

+ m (differentiation of denominator)

Find values of constants l and m and integrate

(27) For $\int \frac{dx}{ax^2+bx+c}$ or, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$

Express ax^2+bx+c as $a[(x+\alpha)^2 \pm \beta^2]$ & integrate

(28) For $\int \frac{Ax+B}{ax^2+bx+c}\,dx$ or, $\int \frac{Ax+B}{\sqrt{ax^2+bx+c}}\,dx$

Let $Ax+B = m\,(\text{derivative of } ax^2+bx+c) + n$

$= m(2ax+b) + n$

Comparing, $m = A/2a$ and $n = B - mb$

Now, $\int \frac{Ax+B}{ax^2+bx+c}\,dx = m\int \frac{2ax+b}{ax^2+bx+c}\,dx + n\int \frac{dx}{ax^2+bx+c}$

$$= m \log|ax^2+bx+c| + n\int \frac{dx}{ax^2+bx+c}$$

then proceed to work out the second integral.

$\int \frac{Ax+B}{\sqrt{ax^2+bx+c}}\,dx$ can be solved similarly.

(29) <u>Important trigonometric substitutions</u>:

Integrand	Substitution	Integrand	Substitution
$\sqrt{a^2-x^2}$	$x = a\sin\theta$	$\sqrt{a+x}$ or $\sqrt{a-x}$	$x = a\cos 2\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$	$\sqrt{a-x}$	$x = a\sin^2\theta$
$\sqrt{x^2+a^2}$	$x = a\tan\theta$	$\sqrt{a+x}$	$x = a\tan^2\theta$
		$\sqrt{2ax-x^2}$	$x = 2a\sin^2\theta$

(30) For $\int \frac{(a+bx)^m}{(a'+b'x)^n}\,dx$ where $m \in N$ and $n \in Q$ substitute $a'+b'x = z$.

(31) For $\int \frac{dx}{x^m(a+bx)^n}$, where m and $n \in N$, or even when m and n are fractions, but $(m+n) \in N - \{1\}$, substitute $a+bx = zx$.

(32) For $\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$ $(a \neq 0,\ p \neq 0)$, put $px+q = \frac{1}{z}$

(33) For $\int \sqrt{\frac{ax+b}{cx+d}}\,dx$, express as $\int \frac{(ax+b)\,dx}{\sqrt{(ax+b)(cx+d)}}$ and integrate

5 - 6. INDEFINITE INTEGRATION

(34) For $\int \frac{p(x)}{q(x)}\,dx$, where $p(x)$ and $q(x)$ are polynomials.

If $q(x)$ is a polynomial of degree 2 or if of higher degree but factorizable so that no factor is a polynomial of degree higher than 2, and if $p(x)$ is of degree higher than that of $q(x)$, then perform synthetic division and proceed with integration using method of partial fraction, if necessary.

(35) For $\int \sin^p x \cos^q x\,dx$ where $p+q$ is a negative even integer, whatever p and q may be, substitute $\tan x = z$.

Try $\int \frac{\sin^2 x}{\cos^6 x}\,dx$ and $\int \frac{dx}{\sin^{1/2} x \cos^{7/2} x}$.

(36) Any integral powers of tan and cot and even positive powers of sec and cosec can be readily integrated.

Odd positive powers of sec and cosec are to be integrated using the method of integration by parts.

(37) Rule of integration by parts :

$$\int uv\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$$

For selection of u, remember ILATE giving order of preference for selection of u, where I - inverse, L - logarithmic, A - algebraic, T - trigonometrical and E - exponential functions.

5 - 6. INDEFINITE INTEGRATION

(38) $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

(39) $\int \sqrt{x^2 \pm a^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\log\left|x+\sqrt{x^2\pm a^2}\right| + c$

(40) $\int e^x [f(x) + f'(x)]\, dx = e^x f(x) + c$

(41) For $\int \frac{p(x)}{q(x)}\, dx$ where $p(x)$ and $q(x)$ are polynomials.

If degree of $p(x) \geq$ degree of $q(x)$, then express $p(x)/q(x)$ as $r(x) + p_1(x)/q(x)$ so that degree of $p_1(x) <$ degree of $q(x)$. Then

(i) if $q(x)$ resolves into linear non-repeated factors. $ax+b$, $cx+d$, $ex+f$, then find A, B, C from

$$\frac{p_1(x)}{q(x)} \equiv \frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{ex+f}$$

and then integrate;

(ii) if $q(x)$ resolves into linear but repeated factors, then find A, B, C from

$$\frac{p_1(x)}{q(x)} \equiv \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3}$$

$$\text{or, } \equiv \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{cx+d}$$

and then integrate;

(iii) if the factors of $q(x)$ are of the type $(ax+b)$ and (cx^2+dx+e), then find A, B, C from

$$\frac{p_1(x)}{q(x)} \equiv \frac{A}{ax+b} + \frac{Bx+C}{cx^2+dx+e}$$

(42) For $\int \frac{dx}{(x-a)^m (x-b)^n}$, $m, n \in N$ and $a \neq b$

put $x - a = z(x-b)$.

Try $\int \frac{dx}{(x-1)^2(x-2)^3}$

7-8-9. DEFINITE INTEGRALS, APPLICATIONS OF INTEGRATION, DIFFERENTIAL EQUATIONS

7-8-9. DEFINITE INTEGRALS, APPLICATIONS OF INTEGRATION, DIFFERENTIAL EQUATIONS

(1) If $f(x)$ is continuous for $x \in [a,b]$ and $\int f(x)dx = F(x)+c$, then $\int_a^b f(x)dx = [F(x)]_a^b = F(b)-F(a)$.

(2) $\int_a^b f(x)dx = \int_a^b f(t)dt$ (3) $\int_a^a f(x)\,dx = 0$

(4) $\int_a^b f(x)\,dx = \int_a^c f(x) + \int_c^b f(x)\,dx \quad (a<c<b)$

(5) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ (6) $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)dx$

(7) $\int_a^b [f(x) + g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

(if f and g are continuous on $[a,b]$.)

(8) While using the method of substitution, do not forget to change the limits

(9) If f is integrable on a given interval, then

(i) $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$ if $f(x)$ is even function

$= 0$ if $f(x)$ is odd function

(ii) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$; (iii) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(iv) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

$= 2\int_0^a f(x)\,dx$, if $f(2a-x) = f(x)$

Find $\int_0^\pi \log\sin x\,dx$

(10) While substituting $x = \phi(t)$, ϕ and ϕ' should be continuous and ϕ' should have constant sign.

(11) Definite integral as limit of a sum:

Let $y = f(x)$ be a continuous, increasing and non-negative function defined on $[a, b]$. Then,

$$\int_a^b f(x)\,dx = \lim_{\substack{n \to \infty \\ \text{or, } h \to 0}} h \sum_{i=1}^{n} f(a + ih) \quad \text{where, } h = \frac{b-a}{n}.$$

(12) Let $y = f(x)$ be continuous on $[a, b]$ and $f(x) \geq 0$ for all $x \in [a, b]$ or $f(x) \leq 0$ for all $x \in [a, b]$. Then the area of the region bounded by $y = f(x)$, X-axis and lines $x = a$ and $x = b$ is given by

$$A = \left| \int_a^b f(x)\,dx \right| = \left| \int_a^b y\,dx \right|$$

(13) Let g be continuous function on $[c, d]$.
Let $g(y) \geq 0$, $\forall\, y \in [c, d]$ or $g(y) \leq 0$ $\forall\, y \in [c, d]$.
Then the area of the region bounded by $x = g(y)$, Y-axis and lines $y = c$ and $y = d$ is given by

$$A = |I| \quad \text{where } I = \int_c^d g(y)\,dy = \int_c^d x\,dy$$

(14) Let the graph of $y = f(x)$ intersect X-axis at $(c, 0)$ only and $a < c < b$. Let $f(x) \geq 0$ $\forall\, x \in [a, c]$, $f(x) \leq 0$ $\forall\, x \in [c, b]$.
Then the area of the region bounded by $y = f(x)$, $x = a$, $x = b$ and X-axis is given by $A = |I_1| + |I_2|$ where, $I_1 = \int_a^c f(x)\,dx$, $I_2 = \int_c^b f(x)\,dx$.

(15) If the curves $y = f_1(x)$ and $y = f_2(x)$ intersect only for $x = a$ and $x = b$, then the area enclosed by them is given by,

$$A = |I| \quad \text{where } I = \int_a^b [f_1(x) - f_2(x)]\,dx$$

Note : If the given curves have equations $x = g_1(y)$ and $x = g_2(y)$ and if they intersect only at $y = c$ and $y = d$ then,

$$A = |I| \text{ where } I = \int_c^d [g_1(y) - g_2(y)]\, dy$$

Let $y = f(x)$ be continuous function on $[a, b]$.

(16) If the region bounded by $y = f(x)$, X-axis, $x = a$, $x = b$ is revolved completely about X-axis, the volume of solid of revolution V is given by $V = \int_a^b \pi y^2 dx$.

(17) If the region bounded by $x = g(y)$, Y-axis and lines $y = c$ and $y = d$ is revolved completely about Y-axis. the volume of solid of revolution V. is given by $V = \int_c^d \pi x^2 dy$.

(18) If the region between $y = f_1(x)$ and $y = f_2(x)$ is revolved about X-axis, the volume of solid generated,

$$V = \pi \int_a^b [(f_1(x))^2 - (f_2(x))^2]\, dx$$

where a, b are x-coordinates of points of intersection of $y = f_1(x)$ and $y = f_2(x)$.

Similarly, we can write formula for volume when revolution takes place about Y-axis.

(19) $F(x, y, y_1, y_2, \dots y_n) = 0$ where y_1, y_2, y_n, etc are derivatives of first, second, n th order is called a differential equation provided there is at least one derivative term in the equation. Its order is the order of the highest derivative. Its degree is the degree of the highest order derivative after the equation is freed from radicals and fractions.

(20) A family of curves may be given by an equation which may not be a differential equation. To convert it into a differential equation, note down different constants in the equation. Decide how many of these are arbitrary constants. Arbitrary constants are those which give different members of the family of curves on varying their values. If there are n arbitrary constants, differentiate the given equation n times to get n equations. These, alongwith the original equation make $n+1$ equations. Eliminate n arbitrary constants from these $n+1$ equations to obtain the required differential equation.

(21) The differential equation of the type $f(x)dx + g(y)dy = 0$ is called separable variable type and its solution is

$$\int f(x)\,dx + \int g(y)\,dy = c.$$

(22) Solve homogeneous differential equation of the type $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ using the substitution $y = vx$.

(23) The differential equation of the type $(ax+by+c)\,dx + (a'x+b'y+c')\,dy = 0$ can be converted into homogeneous type taking $x = x'+h$ and $y = y'+k$ where, (h, k) is given by $ah+bk+c=0$ and $a'h+b'k+c'=0$. For equations of the type $(ax+by+c)\,dx + (ax+by+c')\,dy = 0$, put $ax+by = v$ and solve.

(24) Linear differential equation of the type $\frac{dy}{dx} + Py = Q$ where P, Q are functions of x can be solved by multiplying both sides with $e^{\int P dx} = e^{Px}$ (as P = const. is in XII std. course) and noting that $e^{Px}\frac{dy}{dx} + Py\,e^{Px} = \frac{d}{dx}\left(y e^{Px}\right)$.

10-11-12-13. PROBABILITY, COMPUTING METHODS

	Description of an event in words	Set theoretic notation of event
(1)	A is an event	$A \subset U$
(2)	Event A does not occur	A' or $\bar{A}$
(3)	Occurrence of event A ⇒ occurrence of event B	$A \subset B$
(4)	Impossible event	ϕ
(5)	Certain event	U
(6)	A and B are mutually exclusive events	$A \cap B = \phi$
(7)	A and B are exhaustive events	$A \cup B = U$
(8)	Only B out of events A and B occurs	$B - A$ or $B \cap A'$
(9)	Only one out of events A and B occurs	$(A-B) \cup (B-A)$ OR $(A \cap B') \cup (B \cap A')$ OR $A \Delta B$
(10)	Events A and B occur together	$A \cap B$
(11)	At least one out of A and B occurs	$A \cup B$
(12)	Only A out of events A, B and C occurs	$A - (B \cup C)$ OR $A \cap B' \cap C'$
(13)	Only A and B out of events A, B and C occur	$A \cap B \cap C'$

(14) For impossible event ϕ, $P(\phi) = 0$.

(15) If A is any event $P(A') = 1 - P(A)$

(16) If A and B are events and $A \subset B$, then

$P(B-A) = P(B) - P(A)$ and $P(A) \leq P(B)$

(i) For event A, $0 \leq P(A) \leq 1$.

(ii) For any events A and B, $P(A \cap B') = P(A) - P(A \cap B)$

(17) If A and B are events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(18) If A, B, C are events, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

(19) By classical definition of probability (under assumption of equiprobable elementary events),

$$\text{Probability of an event} = \frac{\text{No. of outcomes favourable to the occurrence of the event}}{\text{total number of outcomes in the sample space}}.$$

(20) $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ $[P(B) > 0]$

(21) $P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$

$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2)$

(22) If B_1 and B_2 are mutually exclusive and exhaustive events, $P(B_1) > 0$ and $P(B_2) > 0$ and A occurs under the influence of any one of the two events B_1 and B_2, then

$$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2)$$

(23) Bayes' Rule:

Let B_1 and B_2 be mutually exclusive and exhaustive events. If $P(B_i)$ and $P(A|B_i)$ for $i = 1, 2$ are given and $P(A) \neq 0$, then

$$P(B_i/A) = \frac{P(B_i)P(A|B_i)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2)}, \quad i = 1, 2$$

The theorem can be extended to three mutually exclusive and exhaustive events.

(24) If $P(A \cap B) = P(A)P(B)$, $P(A) > 0$, $P(B) > 0$, then A and B are said to be independent events.

(25) A_1, A_2, A_3 are pairwise independent if $P(A_i) > 0$, $i = 1, 2, 3$; $P(A_1 \cap A_2) = P(A_1)P(A_2)$; $P(A_2 \cap A_3) = P(A_2)P(A_3)$; and $P(A_3 \cap A_1) = P(A_3)P(A_1)$.

If, in addition, $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$, then they are mutually independent.

(26) Mathematical expectation of random variable X is called an expected value of X or mean of X. It is denoted by the symbol $E(X)$ or u_x.

$$E(X) = \sum_{i=1}^{n} x_i\, p(x_i) \qquad E(X^2) = \sum_{i=1}^{n} x_i^2\, p(x_i)$$

(27) Variance, σ_x^2 or $V(X) = E(X^2) - [E(X)]^2$

$$= \sum_{i=1}^{n} x_i^2 p(x_i) - \left[\sum_{i=1}^{n} x_i\, p(x_i)\right]^2$$

(28) σ_x is called standard deviation.

(29) For $Y = aX + b$, $E(Y) = E(aX+b) = a\,E(X) + b$.

$$\sigma_Y^2 = V(Y) = a^2 V(X) = a^2 \sigma_X^2$$

$$\sigma_Y = \sqrt{V(Y)} = |a|\,\sigma_X$$

(30) For $Z = aX^2 + bX + c$,

$$E(Z) = E(aX^2 + bX + c) = a\,E(X^2) + b\,E(X) + c$$

(31) <u>Bernoulli trial</u> : Suppose a random experiment has two possible outcomes, namely, success (S) and failure (F). If the probability p $(0 < p < 1)$ of getting success at each of the n trials (or repetitions) of this experiment is constant, then the trials are called Bernoulli trials. Bernoulli trials are mutually independent with constant probability, p, of success (S) or, q, of failure (F) at each trial. $q = 1 - p$.

If x denotes number of successes in a sequence of n Bernoulli trials of a random experiment having a constant probability, p, of success, then

$$p(x) = \binom{n}{x} p^x q^{n-x} \qquad x = 0, 1, 2 \ldots n \;\;\&\;\; q = 1 - p.$$

(32) The mean and variance of binomial distribution with parameters n and p are

$$E(X) = np \quad \text{and} \quad \sigma_x^2 = npq = np(1-p)$$

(33) Components of computer : (i) Input unit, (ii) Memory unit, (iii) Control unit, (iv) Arithmetic and Logical unit, (v) Output unit. The control unit and arithmetic and logical unit together is called central Processing Unit (CPU).

(34) The time duration of each generation of computer and electronic parts used in its construction are tabulated below.

Generation	Time duration	Electronic component
First	1949 – 1955	Vacuum tubes.
Second	1956 – 1965	Transistors
Third	1966 – 1975	Integrated circuit
Fourth	1976 – till today	Large scale integrated circuits, Micro-chips.

(35) Some important languages used in computers are

(i) BASIC – (Beginner's All Purpose Symbolic Instruction Code)

(ii) COBOL – (Common Business Oriented Language)

(iii) FORTRAN – (Formula Translator)

(iv) PASCAL – (Language named after Blaise Pascal, the French mathematician)

(36) In computer terminology, binary digits 0 and 1 are called bits. A sequence of eight bits is called a byte.

(37) RAM (Random Access Memory)
ROM (Read Only Memory)
} For detail, refer pg. 300 of Text-book.