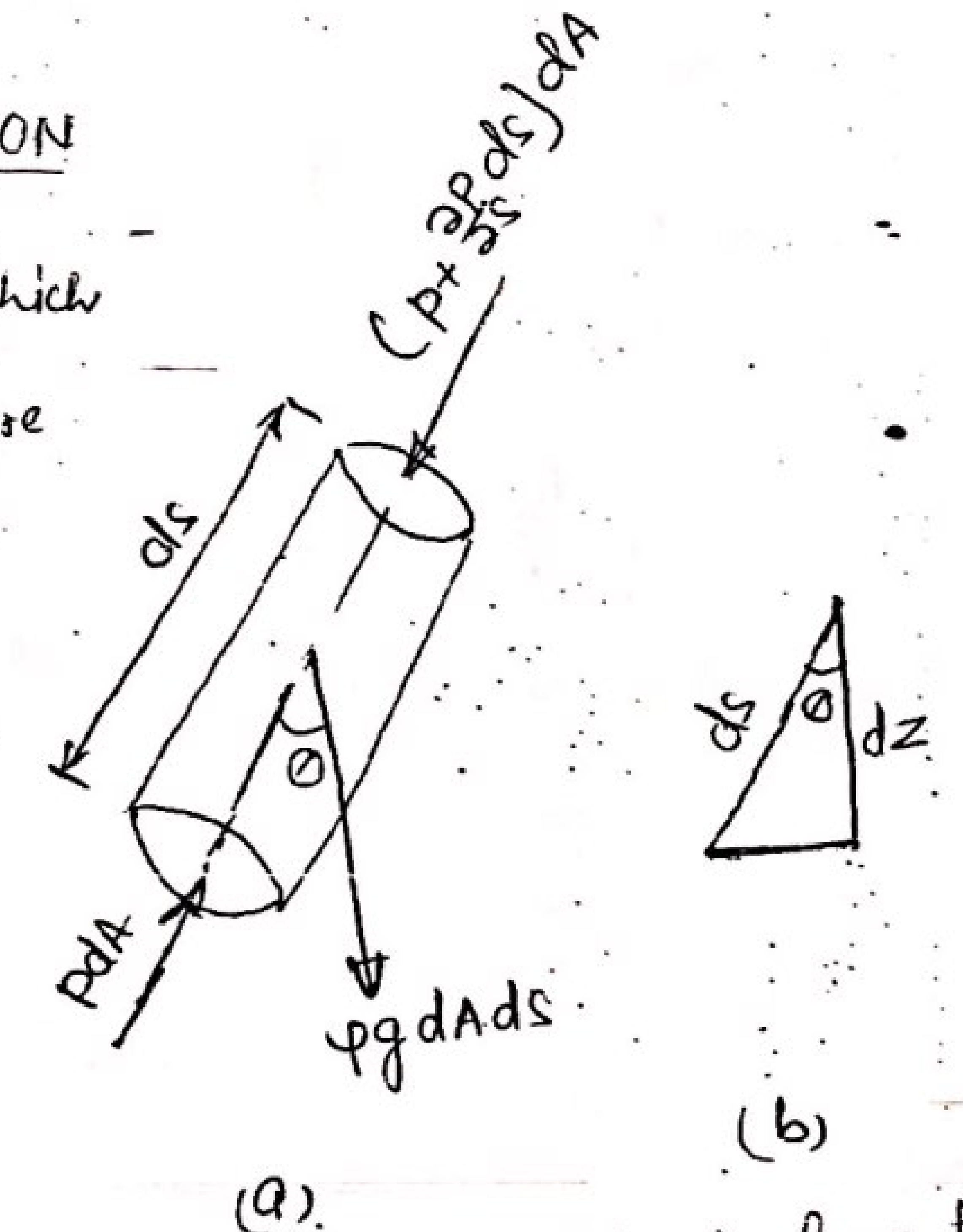
DYNAMICS OF FLUID FLOW

1) Dynamics of fluid flow is the study of fluid motion with the forces causing flow.

2) EULAR'S EQUATION OF MOTION

This is a equation of motion in which the forces due to gravity and pressure are considered.

Consider a stream line in which flow is taking place in s-direction as shown in figure (a).
Consider a cylindrical element of cross section dA and length ds.



fig(1) forces on a fluid element

So

$$p\,dA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times a_s \quad \text{—— (1)}$$

a_s = acceleration in direction of s.

For steady flow

$$a_s = \frac{dv}{dt} = \frac{\partial v}{\partial s}\frac{ds}{dt} = v\frac{\partial v}{\partial s} \qquad \left[v = \frac{ds}{dt} \right]$$

So eq^n (1) becomes

$$-\frac{\partial p}{\partial s} ds\,dA - \rho g\, dA\, ds = \rho\, dA\, ds\, v\frac{\partial v}{\partial s}$$

Dividing by $\rho\, ds\, dA$

$$-\frac{\partial p}{\rho \partial s} - g\cos\theta = v\frac{\partial v}{\partial s}$$

$$\frac{\partial p}{\rho \partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$$

$$\cos\theta = \frac{dz}{ds}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{v\partial v}{\partial s} = 0$$

or $$\boxed{\frac{\partial p}{\rho} + g\,dz + v\,dv = 0}$$ Eular's equation of motion

BERNOULLI'S EQUATION FROM EULER'S EQUATION

Bernoulli's equation is obtained by integrating Euler's equation of motion

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = \text{Constant}.$$

If flow is incompressible, ρ is constant

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{Constant}$$

or $$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\boxed{\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{Constant}}$$ Bernoulli's Equation

$\frac{p}{\rho g}$ = pressure head. ie pressure energy per unit weight

$\frac{v^2}{2g}$ = Kinetic head ie kinetic energy per unit weight

z = potential head ie potential energy per unit weight

Assumptions: (i) The flow is ideal. ie viscosity is zero (ii) The flow is steady (iii) flow is incompressible (iv) flow is irrotational.

Q(1) Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross section, which is 5 m above the datum line.

Sol^n Given Diameter of pipe $D = 5cm = 0.5m$

pressure $p = 29.43\ N/cm^2 = 29.43\times10^4\ N/m^2$

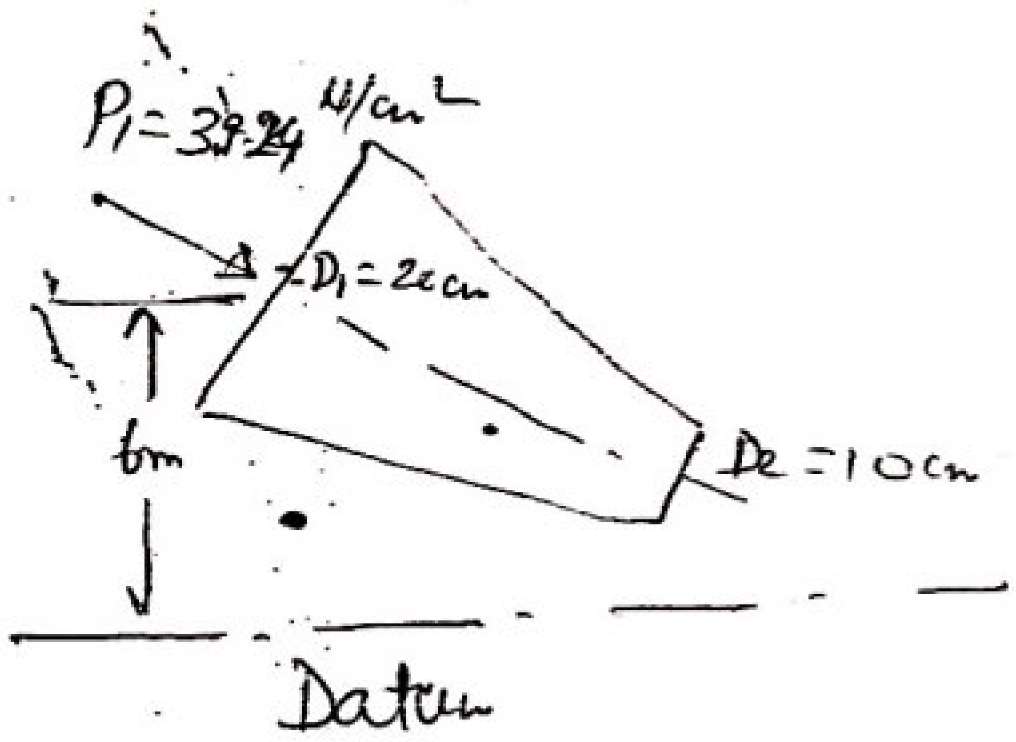
Velocity $v = 2.0\ m/s$

Datum head $z = 5m$

$$\text{Total head} = \frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{29.43\times10^4}{1000\times9.81} + \frac{2^2}{2\times9.81} + 5 = 35.204\,m$$

Ans.

Q(2) The water is flowing through a pipe having diameters 20 cm and 10 cm at section 1 and 2 respectively. The rate of flow through pipe is 35 liters/s. The section 1 is 6 m above datum and 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm². Find the intensity of pressure at section 2.



Given

At section 1, $D_1 = 20cm = 0.2m$

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.0314\,m^2$

$P_1 = 39.24\ N/cm^2$

$= 39.24\times10^4\ N/m^2$

$Z_1 = 6.0\,m$

At section 2 $D_2 = 0.1\,m$, $A_2 = \frac{\pi}{4}(0.1)^2 = 0.00785\,m^2$

$Z_2 = 4m$, $P_2 = ?$

Rate of flow. $Q = 35\ lit/s = \frac{35}{1000}\ m^3/s$

$Q = A_1V_1 = A_2V_2$

$V_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114\ m/s$

$V_2 = \frac{Q}{A_2} = \frac{0.035}{0.00785} = 4.456\ m/s$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{39.24\times10^4}{1000\times9.81} + \frac{(1.114)^2}{2\times9.81} + 6 = \frac{P_2}{1000\times9.81} + \frac{(4.456)^2}{2\times9.81} + 4$$

$P_2 = 40.27\ N/cm^2$..