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2K20/B17/33

ASSIGNMENT NO:→2.

Ques1 Find the General solution of the following

a) $(4D^4-4D^3-23D^2+12D+36)y=0$

A.E is $4m^4-4m^3-23m^2+12m+36=0$

$(m-2)^2\left(m+\frac{3}{2}\right)^2=0$

$\therefore m=2,2,-\frac{3}{2},-\frac{3}{2}$

Thus, $y=(C_1+C_2x)e^{2x}+(C_3+C_4x)e^{\frac{-3x}{2}}$

b) $y''+4y'+5y=0$

$(D^2+4D+5)y=0$

A.E is $m^2+4m+5=0$

$m=-2,\pm i$

$y=e^{-2x}(C_1\cos x+C_2\sin x)$

$\because$ P.I. $=0$

c) $(D^2+5D+6)y=\cos x+e^{-2x}$

A.E. is $m^2+5m+6=0 \Rightarrow m=-2,-3$

C.F. $=C_1e^{-2x}+C_2e^{-3x}$

P.I. $=\dfrac{\cos x}{D^2+5D+6}+\dfrac{e^{-2x}}{D^2+5D+6}$

P.I.$_1$ $=\dfrac{\cos x}{5D+5}$ $(\because D^2=-1)$

$=\dfrac{1}{5}\left[\dfrac{(D-1)\cos x}{D^2-1}\right]$

$=\dfrac{1}{10}(\sin x+\cos x)$

P.I.$_2$ $=D\to-2$, $\therefore$ Denominator $=0$

$\therefore$ Differential & Multiply x

$=\dfrac{xe^{-2x}}{2D+5}$ $(D\to-2)$

$=xe^{-2x}$

P.I. $=\dfrac{\sin x+\cos x}{10}+xe^{-2x}$

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$y = C.F. + P.I.$

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$\therefore$ The general solution will be

$$y = C_1 e^{-2x} + C_2 e^{-3x} + \frac{1}{10}(\sin x + \cos x) + xe^{-2x}$$

Ques 2.) Find the complete solution of the equation

$$\frac{d^4y}{dx^4} + \frac{5d^3y}{dx^3} + 6\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 8y = e^{-2x} + 2e^{-x} + 3e^x - 3$$

$$y(D^4 + 5D^3 + 6D^2 - 4D - 8) = e^{-2x} + 2e^{-x} + 3e^x - 3$$

$$A.E = m^4 + 5m^3 + 6m^2 - 4m - 8 = 0$$

$$\therefore m = -2, -2, -2, 1$$

$$C.F. \Rightarrow (C_1 + C_2 x + C_3 x^2) e^{-2x} + C_4 e^x$$

$$P.I. = \frac{1}{D^4 + 5D^3 + 6D^2 - 4D - 8} \times (e^{-2x} + 2e^{-x} + 3e^x - 3)$$

$$P.I._1 = \frac{e^{-2x}}{D^4 + 5D^3 + 6D^2 - 4D - 8}$$

$$f(D) = D^4 + 5D^3 + 6D^2 - 4D - 8$$

$$f(-2) = 16 - 40 + 24 + 8 - 8 = 0$$

$$f'(D) = 4D^3 + 15D^2 + 12D - 4$$

$$f'(-2) = 0$$

$$f''(-2) = 0$$

$$f''') = -18$$

$$\therefore P.I._1 = \frac{x^3 e^{-2x}}{-18}$$

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$P.I._2 = \frac{2e^{-x}}{-2}$ $P.I._4 = \frac{-3}{-8} = \frac{3}{8}$

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$P.I._3 = \frac{3e^x}{D^4+5D^3+6D^2-4D-8}$ $f(1)=0$, $f'(1)=27$

$P.I._3 = \frac{3xe^x}{27}$

$\therefore P.I. = -\frac{x^3e^{-2x}}{18} - e^{-x} + \frac{xe^x}{9} + \frac{3}{8}$

$y = C.F. + P.I. = \left[C_1 + C_2x + C_3x^2 - \frac{x^3}{18}\right]e^{-2x} - e^{-x} + \frac{3}{8} + \left[C_4 + \frac{x}{9}\right]e^x$

Ques3.) Solve $(D^2+36)y=0$ when $x=0, y=2$ and $x=\frac{\pi}{2}, \frac{dy}{dx}=3$.

$A.E. = m^2+36=0$

$m = \pm 6i$

$C.F. = e^{0x}(C_1\cos 6x + C_2\sin 6x)$

$\therefore y = C_1\cos 6x + C_2\sin 6x$

$y(2)=0 \Rightarrow C_1 = 2$ $\frac{dy}{dx}=3, x=\frac{\pi}{2} \Rightarrow C_2 = -\frac{1}{2}$

$\therefore y = 2\cos 6x - \frac{\sin 6x}{2}$

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Ques 4) Consider the equation $y''+y=0$

a.) Verify that the boundary value problem for the given equation with boundary conditions $y(0)=1$, $y(\frac{\pi}{2})=1$ has unique solution

b.) Verify that the boundary value problem for the given equation with boundary conditions $y(0)=1$, $y(\pi)=1$ has no solution.

c) Verify that the boundary value problem for the given equation with boundary conditions $y(0)=1$, $y(2\pi)=1$ has infinitely many solutions.

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$(D^2+1)y=0$ AE is $m^2+1=0$

$m = \pm i$

$\therefore P.I.=0$

$y = c_1\cos x + c_2\sin x$.

a.) $y(0)=1 \Rightarrow 1 = c_1 \cdot 1 + 0 \cdot c_2$ $\quad c_1 = 1$ $\quad \therefore y = \cos x + \sin x$

$y(\pi/2)=1 \Rightarrow 1 = 0 \cdot c_1 + 1 \cdot c_2$ $\quad c_2 = 1$

$\therefore$ It has unique solution

b.) $y(0)=1 \Rightarrow 1 = c_1 + 0 \cdot c_2$ $\quad c_1 = 1$ Not possible

$y(\pi)=1 \Rightarrow 1 = -c_1 + 0 \cdot c_2$ $\quad c_1 = -1$

$\therefore$ It has no solution

c.) $y(0)=1 \Rightarrow c_1 + 0 \cdot c_2 = 1$

$y(2\pi)=1 \Rightarrow c_1 + 0 \cdot c_2 = 1$ $\quad \because \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

$\therefore$ It has infinitely many solutions

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Ques6.) Find the complete solution of the following Cauchy Euler equations

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a.) $x^2\dfrac{d^2y}{dx^2}+5x\dfrac{dy}{dx}+4y=x\log x$

Put $x=e^z$

$z=\log x$ and let $D=\dfrac{d}{dz}$

$y(D^2+4D+4)=ze^z$

$y_c(z)=(c_1+c_2z)e^{-2z}$

$=(c_1+c_2\log x)x^{-2}$

$y_p(z)=\dfrac{ze^z}{(D^2+4D+4)}=e^z\left[\dfrac{z}{(D+1)^2+4(D+1)+4}\right]$

$=e^z\left(\dfrac{z}{D^2+6D+9}\right)$

$=\dfrac{e^z}{9}\left[\left(1+\dfrac{D^2}{9}+\dfrac{2D}{3}\right)^{-1}z\right]$

$=\dfrac{e^z}{9}\left[\left(1-\dfrac{2D}{3}-\dfrac{D^2}{9}\right)z\right]$

$=\dfrac{e^z}{9}\left(z-\dfrac{2}{3}\right)=\dfrac{x}{27}\left[3\log x-2\right]$

$y(x)=(c_1+c_2\log x)x^{-2}+\dfrac{x}{27}\{3\log x-2\}$

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b.) $x^2\frac{d^2y}{dx} - 3x\frac{dy}{dx} + 5y = x^2\sin[\log x]$ PARTH JOHRI 2K20/B17/33

$D = \frac{d}{dz}$, $x = e^z$

$x^2\frac{d^2}{dx^2} = D(D-1)$ $\qquad \frac{xd}{dx} = D$

$(D^2 - 4D + 5)y = e^{2z}\sin z$

$y_c(z) = e^{2z}[c_1\sin z + c_2\cos z]$

$y_c(x) = x^2[c_1\sin(\log x) + c_2\cos(\log x)]$

$y_p(z) = \frac{e^{2z}\sin z}{D^2 - 4D + 5} = e^{2z}\left[\frac{\sin z}{(D+2)^2 - 4(D+2) + 5}\right]$

$= e^{2z}\left[\frac{\sin z}{D^2+1}\right]$

$= e^{2z}\,\mathrm{Img}\left[\frac{e^{iz}}{D^2+1}\right] = e^{2z}\,\mathrm{Img}\left[\frac{e^{iz}}{(D+i)^2+1}\right]$

$= e^{2z}\,\mathrm{Img}\left[\frac{ze^{iz}}{2i}\right]$

$= \frac{-e^{iz}}{2} z\cos z$

$y(x) = x^2(c_1\sin(\log x) + c_2\cos(\log x)) - \frac{x^2}{2}\log x\cos(\log x)$

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Ques 7)

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$$\frac{d^2y}{dx^2} + y = 0$$

$$(D^2+1)y = 0$$

$y = c_1 \cos x + c_2 \sin x$

$y(0) = 1 \Rightarrow c_1 = 1$

$y(\pi) = 0 \Rightarrow -c_1 = 0 \Rightarrow c_1 = 0$

But $0 \neq 1$

LHS $\neq$ RHS <u>No solution</u> Ans

<u>Ques 8.)</u> Let $y_1 = e^x \sin x$ & $y_2 = e^x \cos x$ be soln of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

$y_1' = e^x \sin x + e^x \cos x$

$y_1'' = e^x(\sin x + \cos x) + e^x(\cos x - \sin x)$

$= 2e^x \cos x$

$y_1'' - 2y_1' + 2y_1 = 0 \Rightarrow y_1 = e^x \sin x$ is soln

$y_2' = e^x(\cos x - \sin x)$

$y_2'' = -2e^x(\sin x)$

$y_2'' - 2y_2' + 2y_2 = 0 \Rightarrow y_2 = e^x \cos x$ is soln

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = -e^{2x} \neq 0$$

So, y_1 & y_2 are linearly independent soln of $y'' - 2y' + 2y = 0$

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$(D^2-2D+2)y=0$

$y(x)=e^x(c_1\sin x+c_2\cos x)$

$y'(x)=e^x(c_1\sin x+c_2\cos x)+e^x(c_1\cos x-c_2\sin x)$

$y(0)=2 \Rightarrow c_2=2$

$y''(0)=-3 \Rightarrow c_1+c_2=-3 \Rightarrow c_1=-5$

Soln $= e^x(2\cos x-5\sin x)$

Ques 9.) $y_1=\sin x \qquad y_2=\cos x$

$y_1'=\cos x \qquad y_2'=-\sin x$

$y_1''=-\sin x \qquad y_2''=-\cos x$

$y_3=\sin x-\cos x$

$y_3'=\cos x+\sin x$

$y_3''=-\sin x+\cos x$

For $y''+y=0$

$y_1''+y_1=0 \Rightarrow \sin x$ is soln

$y_2''+y_2=0 \Rightarrow \cos x$ is soln

$y_3''+y_3=0 \Rightarrow \sin x-\cos x$ is soln

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$$W(y_1, y_2, y_3) = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix}$$

$$= \begin{vmatrix} \sin x & \cos x & \sin x - \cos x \\ \cos x & -\sin x & \cos x + \sin x \\ -\sin x & -\cos x & \cos x - \sin x \end{vmatrix}$$

$$= 0$$

So, y_1, y_2 & y_3 are linearly dependent.

Ques10.) $\frac{d^2y}{dx^2} - y = e^x$

$(D^2-1)y = e^x$

$y_c(x) = c_1 e^x + c_2 e^{-x}$

$y_p(x) = \frac{e^x}{D^2-1} = e^x \frac{1}{(D+1)^2-1} = \frac{e^x}{D(D+2)}$

$= \frac{xe^x}{2}$

$y(x) = c_1 e^x + c_2 e^{-x} + \frac{xe^x}{2}$

$y'(x) = c_1 e^x - c_2 e^{-x} + \frac{e^{x}}{2} + \frac{xe^x}{2}$

$y(0) = 0 \Rightarrow c_1 + c_2 = 0$

$y'(0) = 3/2 \Rightarrow c_1 - c_2 = 1$

$c_1 = 1/2, \; c_2 = -1/2$

$\left(\sin h(x) = \frac{e^x - e^{-x}}{2}\right)$

$y(x) = \frac{e^x - e^{-x}}{2} + \frac{xe^x}{2}$ $\quad y(x) = \sin h(x) + \frac{xe^x}{2}$

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