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① Soln $\vec{w}(t) = (3t\hat{i} + 5t^2\hat{j} + 6\hat{k}) \cdot (t^2\hat{i} - 2t\hat{j} + t\hat{k})$

(a) $\vec{w}(t) = \vec{F} \cdot \vec{G}$

$\frac{d\vec{w}(t)}{dt} = \vec{F} \cdot \frac{d\vec{G}}{dt} + \frac{d\vec{F}}{dt} \cdot \vec{G}$ using this we have

$\Rightarrow (3t\hat{i} + 5t^2\hat{j} + 6\hat{k}) \cdot (2t\hat{i} - 2\hat{j} + \hat{k}) + (3\hat{i} + 10t\hat{j} + 0\hat{k}) \cdot (t^2\hat{i} - 2t\hat{j} + t\hat{k})$

$= (6t^2 - 10t^2 + 6) + (3t^2 - 20t^2 + 0)$

$= (6t^2 - 10t^2 + 6 + 3t^2 - 20t^2) \Rightarrow -21t^2 + 6$ Ans.

(b) $\vec{w}(t) = (t\hat{i} + e^t\hat{j} - t^2\hat{k}) \times (t^2\hat{i} + \hat{j} + t^3\hat{k})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t & e^t & -t^2 \\ t^2 & 1 & t^3 \end{vmatrix} = \hat{i}(e^t \cdot t^3 + t^2) - \hat{j}(t^4 + t^4) + \hat{k}(t - e^t \cdot t^2)$$

$= \hat{i}(t^3 \cdot e^t + t^2) - 2t^4\hat{j} + (t - e^t \cdot t^2)\hat{k}$

$\vec{w}(t) = (t^3 \cdot e^t + t^2)\hat{i} - 2t^4\hat{j} + (t - e^t \cdot t^2)\hat{k}$

$\frac{d\vec{w}(t)}{dt} = (t^3 \cdot e^t + e^t \cdot 3t^2 + 2t)\hat{i} - 8t^3\hat{j} + [1 - (t^2 \cdot e^t + e^t \cdot 2t)]\hat{k}$

$= (t^3 \cdot e^t + e^t \cdot 3t^2 + 2t)\hat{i} - 8t^3\hat{j} + (1 - e^t(t^2 + 2t))\hat{k}$ Ans.

soln

(2) $\nabla^2(\log r) = \frac{1}{r^2}$ (To show)

(a) let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$

$\nabla(\nabla \log r) = \nabla^2(\log r)$

$(\nabla \log r) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\log(x^2 + y^2 + z^2)^{\frac{1}{2}}\right)$

$= \frac{1}{2}\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\log(x^2 + y^2 + z^2)\right)$

$(\nabla \log r) = \frac{1}{2}\left[\frac{2x\hat{i} + 2y\hat{j} + 2z\hat{k}}{(x^2 + y^2 + z^2)}\right] = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{(x^2 + y^2 + z^2)}$

$$\nabla(\nabla \log r) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\frac{x\hat{i}+y\hat{j}+z\hat{k}}{x^2+y^2+z^2}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2+z^2}\right) + \frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2+z^2}\right) + \frac{\partial}{\partial z}\left(\frac{z}{x^2+y^2+z^2}\right)$$

$$= \frac{x^2+y^2+z^2 - x\cdot 2x}{(x^2+y^2+z^2)^2} + \frac{x^2+y^2+z^2-y\,2y}{(x^2+y^2+z^2)^2} + \frac{(x^2+y^2+z^2) - z\cdot 2z}{(x^2+y^2+z^2)^2}$$

$$= \frac{\cancel{(x^2+y^2+z^2)}}{(x^2+y^2+z^2)^{\cancel{2}}} = \frac{1}{(x^2+y^2+z^2)} = \frac{1}{r^2}$$ proved.

(b) $\vec{\nabla}^2 r^n = n(n+1)r^{n-2}$

Soln $\nabla^2 r^n = \nabla(\nabla r^n)$

$$(\nabla r^n) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(r^n)$$

$$\Rightarrow \left(n r^{n-1}\cdot\frac{\partial r}{\partial x}\cdot\hat{i} + \hat{j}\cdot n r^{n-1}\cdot\frac{\partial r}{\partial y} + \hat{k}\cdot n r^{n-1}\frac{\partial r}{\partial z}\right)$$

$$\Rightarrow n\left(r^{n-1}\frac{\partial r}{\partial x}\hat{i} + \hat{j}\, r^{n-1}\frac{\partial r}{\partial y} + \hat{k}\, r^{n-1}\frac{\partial r}{\partial z}\right)$$

we know $r = \sqrt{x^2+y^2+z^2}$

$r^2 = x^2+y^2+z^2$

$\cancel{2}r\cdot\frac{\partial r}{\partial x} = \cancel{2}x \Rightarrow \frac{\partial r}{\partial x} = \left(\frac{x}{r}\right)$

similarly $\frac{\partial r}{\partial y} = \frac{y}{r}$

$\frac{\partial r}{\partial z} = \frac{z}{r}$

$$\Rightarrow n\left(\hat{i}r^{n-1}\cdot\frac{x}{r} + \hat{j}\cdot r^{n-1}\cdot\frac{y}{r} + \hat{k}\cdot r^{n-1}\cdot\frac{z}{r}\right)$$

$$\Rightarrow n\left(x r^{n-2}\hat{i} + y r^{n-2}\hat{j} + z r^{n-2}\hat{k}\right) \quad \text{——— } (1)$$

$$\Rightarrow n r^{n-2}(x\hat{i}+y\hat{j}+z\hat{k}) = n r^{n-2}\cdot\vec{r}$$

using (1) and operating ∇ to obtain $\nabla(\nabla r^n)$

$$\Rightarrow \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(n x r^{n-2}\hat{i} + n y r^{n-2}\hat{j} + n z r^{n-2}\hat{k}\right)$$

using product rule

$$\Rightarrow n\left(r^{n-2} + x\cdot(n-2)r^{n-3}\cdot\frac{\partial r}{\partial x} + r^{n-2} + y(n-2)r^{n-3}\cdot\frac{\partial r}{\partial y} + r^{n-2} + z(n-2)r^{(n-3)}\cdot\frac{\partial r}{\partial z}\right)$$

using values of $\frac{\partial r}{\partial x}, \frac{\partial r}{\partial y}$ & $\frac{\partial r}{\partial z}$

$\Rightarrow 3nr^{n-2} + n(n-2)r^{n-3}\left[x\cdot\frac{\partial r}{\partial x} + y\frac{\partial r}{\partial y} + z\frac{\partial r}{\partial z}\right]$

$\Rightarrow 3nr^{n-2} + n(n-2)r^{n-3}\left[x\cdot\left(\frac{x}{r}\right) + y\left(\frac{y}{r}\right) + z\left(\frac{z}{r}\right)\right]$

$= 3nr^{n-2} + n(n-2)r^{n-4}\left[x^2+y^2+z^2\right]$

$\Rightarrow 3nr^{n-2} + n(n-2)\,r^{\cancel{n-4}}\cdot r^2 \qquad [r^2 = x^2+y^2+z^2)$

$\Rightarrow r^{n-2}\left[3n + n^2 - 2n\right] = r^{n-2}\left[n^2+n\right] = n(n+1)r^{n-2}$ proved.

(3) $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

(a) $\text{grad } r = \frac{\vec{r}}{r}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ Taking L.H.side

$|\vec{r}| = r = \sqrt{x^2+y^2+z^2}$

$\text{grad } r = \nabla r = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(x^2+y^2+z^2\right)^{\frac{1}{2}}$

$= \frac{1\times\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2+z^2}}\hat{i} + \frac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2+z^2}}\hat{j} + \frac{\cancel{2}z}{\cancel{2}\sqrt{x^2+y^2+z^2}}\hat{k}$

$= \left(\frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}}\right) = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r}$ proved.

(b) $\text{grad}\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$

Soln. $r = \sqrt{x^2+y^2+z^2}$

$\frac{1}{r} = \frac{1}{\sqrt{x^2+y^2+z^2}}$

$\text{grad}\left(\frac{1}{r}\right) = \nabla\left(\frac{1}{r}\right) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right)$

$= \frac{-\cancel{2}x\,\hat{i}}{\cancel{2}(x^2+y^2+z^2)\times\sqrt{x^2+y^2+z^2}} - \frac{\cancel{2}y\,\hat{j}}{\cancel{2}(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}} - \frac{\cancel{2}z\,\hat{k}}{\cancel{2}(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}$

$= \frac{-(x\hat{i} + y\hat{j} + z\hat{k})}{(x^2+y^2+z^2)^{1+\frac{1}{2}}} = \frac{-(x\hat{i} + y\hat{j} + z\hat{k})}{(x^2+y^2+z^2)^{3/2}}$ —— 1

$= \frac{-\vec{r}}{r^{3/2}}$ proved

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$r = (x^2+y^2+z^2)^{\frac{1}{2}}$

$r^3 = (x^2+y^2+z^2)^{3/2}$ putting in 1

(c) $\nabla r^n = n r^{n-2} \vec{r}$

L.H.side.

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\nabla r^n = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(r^n)$

$= \left(n r^{n-1} \cdot \frac{\partial r}{\partial x}\hat{i} + n r^{n-1} \cdot \frac{\partial r}{\partial y}\hat{j} + n r^{n-1} \frac{\partial r}{\partial z}\hat{k}\right)$ —— ①

since $r^2 = x^2 + y^2 + z^2$ diff. w.r.t. x partially

$\cancel{2}r \cdot \frac{\partial r}{\partial x} = \cancel{2}x$

$\left(\frac{\partial r}{\partial x}\right) = \left(\frac{x}{r}\right)$

similiarly

$\frac{\partial r}{\partial y} = \left(\frac{y}{r}\right)$

$\frac{\partial r}{\partial z} = \left(\frac{z}{r}\right)$

Substituting there in ① we have

$= \left(n r^{n-1} \cdot \frac{x}{r}\hat{i} + n r^{n-1} \cdot \frac{y}{r}\hat{j} + n r^{n-1} \cdot \frac{z}{r}\hat{k}\right)$

$= n r^{n-2} (x\hat{i} + y\hat{j} + z\hat{k})$

$= n r^{n-2} \vec{r}$ proved.

ques. 4th part ① on page ④

(4) ① curl (grad u) = 0

grad u = $\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) u$

grad u $= \left(i\frac{\partial u}{\partial x} + \hat{j}\frac{\partial u}{\partial y} + k\frac{\partial u}{\partial z}\right) = \vec{\nabla} u$

curl of grad of u = curl grad u)

w.k.T. curl of $\vec{F}$ is $\vec{\nabla} \times \vec{F}$, so,

curl of grad u is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \end{vmatrix}$$

$\hat{i}\left(\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial y}\right)\right) - \hat{j}\left(\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}\right)\right) + \hat{k}\left(\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)\right.$

$\Rightarrow \hat{i}\left(\frac{\partial^2 u}{\partial y \cdot \partial z} - \frac{\partial^2 u}{\partial z \cdot \partial y}\right) - \hat{j}\left(\frac{\partial^2 u}{\partial x \partial z} - \frac{\partial^2 u}{\partial z \cdot \partial x}\right) + \hat{k}\left(\frac{\partial^2 u}{\partial x \cdot \partial y} - \frac{\partial^2 u}{\partial y \cdot \partial x}\right)$

since $\frac{\partial^2 u}{\partial y \cdot \partial z} = \frac{\partial^2 u}{\partial z \cdot \partial y}$ similiarly for others as well

We get Value = 0 R.H.side proved.

⑤ $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

To show

$\text{div}\left(\frac{r}{r^3}\right) = 0$

$r = (x^2+y^2+z^2)^{\frac{1}{2}}$

$r^3 = (x^2+y^2+z^2)^{3/2}$

$\left(\frac{r}{r^3}\right) = r^{-2} = \frac{1}{r^2} = \frac{1}{x^2+y^2+z^2}$.

$\text{div}.\left(\frac{r}{r^3}\right) = \vec{\nabla}.$

⑤ (b) $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$r = (x^2+y^2+z^2)^{\frac{1}{2}}$ — ①

$\text{grad}\left(\frac{1}{r}\right) = \nabla\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right)$

$\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right)$

$\Rightarrow -\frac{\cancel{2}x}{\cancel{2}(x^2+y^2+z^2)^{3/2}}\hat{i} - \frac{\cancel{2}y\hat{j}}{\cancel{2}(x^2+y^2+z^2)^{3/2}} - \frac{\cancel{2}z\hat{k}}{\cancel{2}(x^2+y^2+z^2)^{3/2}}$

$\Rightarrow -\left[\frac{x\hat{i}+y\hat{j}+z\hat{k}}{(x^2+y^2+z^2)^{3/2}}\right] = \frac{-\vec{r}}{(r)^3} = -\frac{\vec{r}}{r.r^2} = -\left(\frac{\vec{r}}{r}\right)\cdot\frac{1}{r^2} = -\frac{\hat{r}}{r^2}$ Ans

(a) prove

$\text{div}\left(\frac{r}{r^3}\right) = 0$

wrong ques as $\left(\frac{r}{r^3}\right)$ is a scalar quantity where $r = (x^2+y^2+z^2)^{\frac{1}{2}}$ and we can calculate divergence for only vector functions.

(6) $\phi = xy^2 + 2yz - 8$

normal to surface is given by gradient of scalar function ϕ

$$\left(\hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial}{\partial y}\phi + \hat{k}\frac{\partial\phi}{\partial z}\right) = \left(y^2\hat{i} + (2yx+2z)\hat{j} + 2y\hat{k}\right)$$

gradient of ϕ at $(3,-2,1) = (4\hat{i} + (2\times 2\times 3 + 2)\hat{j} - 4\hat{k})$

$= 4\hat{i} - 10\hat{j} - 4\hat{k}$

now lets call vector as $\vec{n} = 4\hat{i} - 10\hat{j} - 4\hat{k}$

unit vector $= \dfrac{4\hat{i} - 10\hat{j} - 4\hat{k}}{\sqrt{16+16+100}} = \dfrac{4\hat{i} - 10\hat{j} - 4\hat{k}}{\sqrt{132}} = \dfrac{(2\hat{i} - 5\hat{j} - 2\hat{k})}{\sqrt{33}}$ Ans.

(7) $z = \sqrt{x^2+y^2}$

squaring both sides

$z^2 = x^2 + y^2$

$\phi = x^2 + y^2 - z^2$

normal vector to surface ϕ

grad $\phi = (2x\hat{i} + 2y\hat{j} - 2z\hat{k})$

grad ϕ at $(3,4,5)$

$= 6\hat{i} + 8\hat{j} - 10\hat{k} = \vec{n}$ (say)

normal unit vector $= \dfrac{6\hat{i} + 8\hat{j} - 10\hat{k}}{\sqrt{100+64+36}} = \dfrac{6\hat{i} + 8\hat{j} - 10\hat{k}}{10\sqrt{2}} = \dfrac{3\hat{i} + 4\hat{j} - 5\hat{k}}{5\sqrt{2}}$ Ans

Equation of tangent plane is given by

$$(x-x_1)\frac{\partial f}{\partial x} + (y-y_1)\frac{\partial f}{\partial y} + (z-z_1)\frac{\partial f}{\partial z} = 0$$

$$\frac{\partial\phi}{\partial x} = 2x, \quad \frac{\partial\phi}{\partial y} = 2y, \quad \frac{\partial\phi}{\partial z} = -2z$$

$(x-3)2x + (y-4)\cdot 2y + (z-5)\cdot -2z = 0$

now at pt. $(3,4,5)$

$6(x-3) + 8(y-4) + (z-5)(10) = 0$

$6x + 32y - 10z - 18 - 32 + 50 = 0$

$\boxed{6x + 32y - 10z = 0}$ ← tangent plane.

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(8) $F(x,y,z) = x^2y^3 + xy \longrightarrow$ scalar function

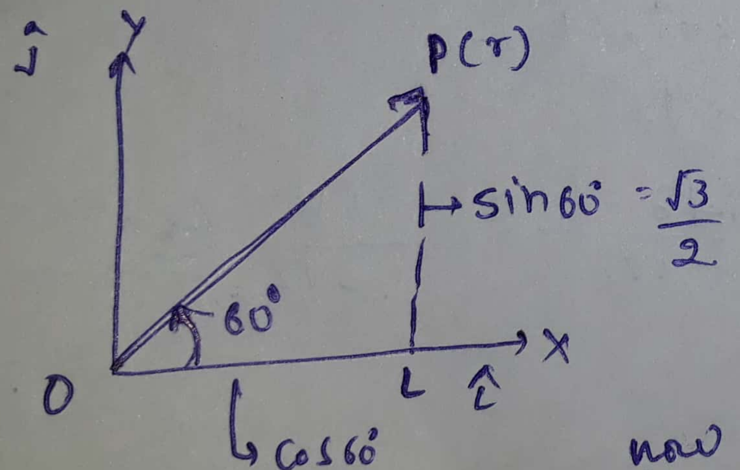
gradient of $F(x,y,z)$

$$\nabla F = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^2y^3 + xy)$$

$$\nabla F = (y^3 \cdot 2x + y)\hat{i} + (3y^2 \cdot x^2 + x)\hat{j}$$

$$\nabla F]_{\text{at }(2,1)} = (4+1)\hat{i} + (3\times 4 + 2)\hat{j}$$

$$= \underline{\underline{5\hat{i} + 14\hat{j}}}$$ gradient of F at (2,1)



given $\overrightarrow{OP}$ is unit vector

$|\overrightarrow{OP}| = 1$

$OL = 1 \times \cos 60° = \frac{1}{2}$ unit

so, $\boxed{\overrightarrow{OP} = \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}} = \hat{OP}$

now directional derivative $= \left(5\hat{i} + 14\hat{j}\right)\cdot\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$

$= \frac{5}{4} + \frac{14\sqrt{3}}{4} = \frac{5 + 14\sqrt{3}}{4}$

(4) (11) div (curl $\vec{r}$) = 0 (4)

$\rightarrow$ curl of $\vec{r}$ = $\vec{\nabla} \times \vec{r}$

let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\text{curl of } \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$\text{curl of } \vec{r} \Rightarrow \hat{i}\left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right) - \hat{j}\left(\frac{\partial z}{\partial x} - \frac{\partial x}{\partial z}\right) + \hat{k}\left(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y}\right)$$

now. divergence of curl of $\vec{r}$

divergence of $\vec{F}$ is $\vec{\nabla} \cdot \vec{F}$,

similiarly divergence of curl of $\vec{r}$

$$\left[\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\right] \cdot \left[\left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right)\hat{i} + \left(\frac{\partial x}{\partial z} - \frac{\partial z}{\partial x}\right)\hat{j} + \left(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y}\right)\hat{k}\right]$$

$$\Rightarrow \cancel{\frac{\partial^2 z}{\partial x \cdot \partial y}} - \cancel{\frac{\partial^2 y}{\partial x \cdot \partial z}} + \cancel{\frac{\partial^2 x}{\partial y \cdot \partial z}} - \cancel{\frac{\partial^2 z}{\partial y \partial x}} + \cancel{\frac{\partial^2 y}{\partial z \cdot \partial x}} - \cancel{\frac{\partial^2 x}{\partial z \cdot \partial y}}$$

$= 0$ = Right hand side proved

since $\frac{\partial^2 y}{\partial z \cdot \partial x} = \frac{\partial^2 y}{\partial x \cdot \partial z}$

similiaritiary for others as well

(9) $F(x,y,z) = xy^2 + 4xyz + z^2$

$$\nabla\vec{F} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(xy^2 + 4xyz + z^2)$$

$$= (y^2 + 4yz)\hat{i} + (2yx + 4xz)\hat{j} + (4xy + 2z)\hat{k}$$

gradient of F at (1,2,3)

$$= (4 + 4\times2\times3)\hat{i} + (2\times2\times1 + 4\times1\times3)\hat{j} + (4\times1\times2 + 2\times3)\hat{k}$$

$$\vec{\nabla F} = 28\hat{i} + 16\hat{j} + 14\hat{k}$$

let $\vec{a} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

$$\hat{a} = \frac{3\hat{i} + 4\hat{j} - 5\hat{k}}{\sqrt{9+16+25}} = \frac{3\hat{i} + 4\hat{j} - 5\hat{k}}{5\sqrt{2}}$$

we know directional derivative of F in the direction of $\vec{a}$ is

given by $\nabla F \cdot \hat{a}$

$$= (28\hat{i} + 16\hat{j} + 14\hat{k}) \cdot \frac{(3\hat{i} + 4\hat{j} - 5\hat{k})}{5\sqrt{2}}$$

$$= \frac{(84 + 64 - 70)}{5\sqrt{2}} = \frac{148 - 70}{5\sqrt{2}} = \frac{78}{5\sqrt{2}} \text{ Ans.}$$