RELATIVITY

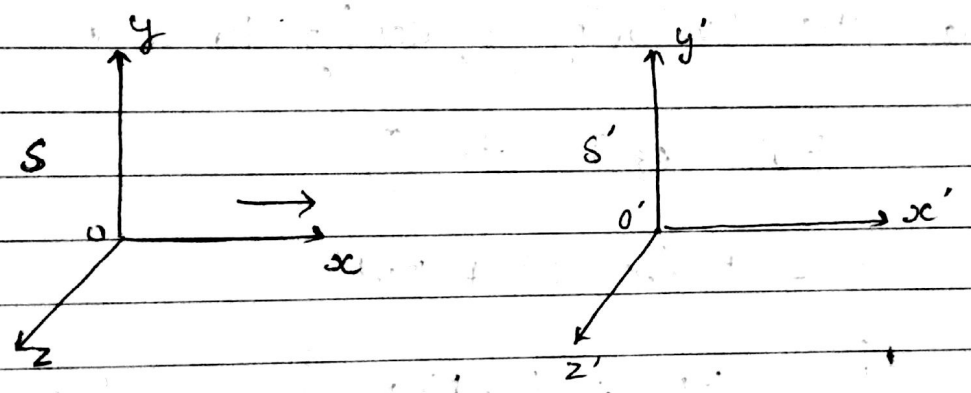
[to clear ambiguity the frame of reference]

frame of reference [Cartesian → position
space → position and time

motion of time → relative

Gallilean Transformation

$x' = x - vt$	$V'_x = V_x - V$	$a'_x = a_x$	$F'_x = F_x$
$y' = y$	$V'_y = V_y$	$a'_y = a_y$	$F'_y = F_y$
$z' = z$	$V'_z = V_z$	$a'_z = a_z$	$F'_z = F_z$



Special theory of relativity

The theory of relativity deals with the consequences of lack of universal frame of reference

POSTULATES of ~~the~~ special theory of relativity

→ This laws of physics are same in all inertial frame of physics

→ The special of light in free space has the same value in all interial frame of reference.

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LORENTZ transformation

$$x' = k(x - vt) \quad - (1)$$
$$x = k(x' + vt') \quad - (2)$$
$$y' = y \quad - (3)$$
$$z' = z \quad - (4)$$

Where k is a factor that does not depend upon x, t but may be the function of v.

(.) The choice of eq (i) follows from several condition
→ k is linear in x and x'
→ k should be simple
→ It has the possibility of reducing to x-vt

put eq ① in ②

$$x = k^2(x - vt) + kvt'$$

$$\boxed{t' = kt + \left(\frac{1-k^2}{kv}\right)x} \quad - (5)$$

The second postulates of relativity gives us a way to evaluate k at the instant t=0 the origin of two frame of reference are the in same phase and similarly when t'=0

Suppose that a flare is sett off at the common origin of s & s' at t=t'=0 and the observer in each system measure the speed with which the flare lights spread out, both observer must find the same speed 'c'.

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$$x = ct \quad - (6)$$
$$x' = ct' \quad - (7)$$

put (1) & (5) in (7)

$$k(x - vt) = ckt + \left(\frac{1-k^2}{kv}\right) cx$$

Solving for x,

$$x = \frac{ckt + vkt}{k - \frac{(1-k^2)}{kv} c}$$

$$= ct \left[\frac{k + \frac{vk}{c}}{k - \left(\frac{1-k^2}{kv}\right) c} \right]$$

$$x = ct \left[\frac{1 + v/c}{1 - \left(1/k^2 - 1\right) c/v} \right]$$

from eq (6)

$$\frac{1 + v/c}{1 - \left(1/k^2 - 1\right) c/v} = 1$$

$$\boxed{k = \sqrt{1 - \frac{v^2}{c^2}}} \quad - (8)$$

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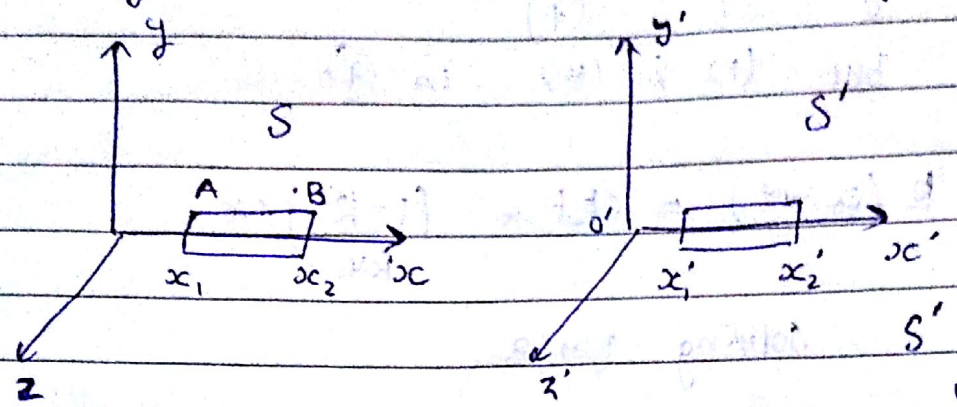
Length Contraction



S' moving with velocity 'v'

$$l_0 = x_2 - x_1$$

$$l = x_2' - x_1'$$

$$x_1 = \frac{x_1' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \& \quad x_2 = \frac{x_2' + vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x_2 - x_1 = \frac{x_2' - x_1'}{\sqrt{1 - v^2/c^2}}$$

$$l_0 = \frac{l}{\sqrt{1 - v^2/c^2}}$$

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$l = \frac{l_0}{k}$$

case when $v \to c$; $l \to 0$

when $v <<< c$, $\frac{v}{c} = 0 \Rightarrow l = l_0$

The length of a rod as measured in a frame of reference is called proper length and it could be same in all stationary frame of reference

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It is only in the moving ref. frame that it will differ depending upon relative vel. w.r.t observer.

RECIPROCITY of length contraction

$$l_0 = x_2' - x_1'$$

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}} \quad \& \quad x_2' = \frac{x_2 - vt}{\sqrt{1 - v^2/c^2}}$$

$$l_0 = x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}}$$

$$l_0 = \frac{l}{\sqrt{1 - v^2/c^2}} \Rightarrow l = l_0\sqrt{1 - \frac{v^2}{c^2}}$$

Ques 1. If a rod travells with a speed $v = 0.8c$ its length, calculate the percentage of contraction?

Ques 2. A rocket ship is 100 meter long on the ground when it is in flight, its length 99 m to an observer on ground. What is its speed?

INVERSE LORENTZ Transformation

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - v^2/c^2}}$$

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Consequences of lorentz transf:

length contraction Time dialation

Every clock appears to run at its fastest rate when it is at rest, relative to the observer but appears to be slow down by the factor $\sqrt{1-v^2/c^2}$ relating to an observer moving at a velocity 'v'

$$t_1 = \frac{t_1' + \frac{v}{c^2}x'}{\sqrt{1-\frac{v^2}{c^2}}} \qquad t_2 = \frac{t_2' + \frac{v}{c^2}x'}{\sqrt{1-v^2/c^2}}$$

$$\Delta t = t_2 - t_1$$

$$= \frac{t_2' - t_1'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \boxed{\Delta t = \frac{t_0}{\sqrt{1-v^2/c^2}}}$$

$t_0 = t_2' - t_1'$ → proper time

In other words the clock in frame s' appears to run slower compare to a clock in stationary frame this effect is known as time dialation.

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frame of reference of length $[l_0, \bullet]$

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Qu1. A spacecraft is moving relative to the earth and observer on the earth finds that between 1 pm to 2 pm according to her clock, 3601 second ellapses on the space crafts clock. What is the spacecraft speed relative to the earth.

Qu2. A stationary μ-meson decay is 2.2×10^{-8} s. What will be length of its path if it is moving towards the earth with velocity 0.99c?

Qu 3. A clock showing correct time when at rest, loses one hour in a day when it is moving. What is its velocity?

Qu 4 A stationary μ-meson has lifetime 2×10^{-6} s
(a) What is mean life when particle is travelling with a speed of 2.994×10^{10} cm/s.
(b) How far does it go during one mean life
(c) What distance would be without relavistic effect?

Qu5. A clock keeps correct time. What speed should it be moved relative to an observer so that it may appear to lose 4 min in 24 hrs?

Qu6. A meterstick makes an angle 30° wrt axis of O' in frame S'. What should be value of 'v' if meterstick makes an angle of 45° wrt x-axis of O in frame S?

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Einstein's Mass-Energy Relation

According in classical mechanics energy is defined in terms of work (force × distance) and the force is rate of change of momentum

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv)$$

Acc' to theory of relativity mass & velocity are variable, thus

$$F = m\frac{dv}{dt} + v\frac{dm}{dt} \quad (1)$$

When a particle is displaced through a distance 'dx' by applying force 'F' then increase in K.E 'dK' is given as :

$$dK = F\,dx \quad (2)$$

Put (1) in (2)

$$dK = m\frac{dv}{dt}(dx) + v\frac{dm}{dt}(dx)$$

$$dK = mv\,dv + v^2\,dm \quad (3)$$

Variation in mass is given as

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m^2 = \frac{m_0^2 c^2}{(c^2 - v^2)}$$

$$m^2c^2 - m^2v^2 = m_0^2c^2 \quad (4)$$

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differentiating (4)

$$(2m\,dm)c^2 - v^2(2m\,dm) - m^2(2v\,dv) = 0$$

$$mv\,dv + v^2 dm = c^2 dm \quad \text{— (5)}$$

Put (5) in (3)

$$dk = c^2 dm \quad \text{— (6)}$$

Now considering that initially body is at rest & by applying force it acquires velocity 'v' Mass of body ↑ses from m_0 to m. Total K.E acquired by body is :

$$\int dk = \int_{m_0}^{m} c^2 dm$$

$$\boxed{KE = c^2(m - m_0)} \quad \text{— (7)}$$

↑ses in KE due to ↑se in mass

we know total energy of system

= its K.E + its rest mass energy

$$E = K.E + m_0c^2$$

$$= c^2(m - m_0) + m_0c^2$$

$$\boxed{E = mc^2}$$

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① Relativistic K.E of a body

$$K = c^2(m - m_0)$$

$$m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

$$K = m_0c^2\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right]$$

(a) If $v << c$

$$K = m_0c^2\left[\left(1-\frac{v^2}{c^2}\right)^{1/2} - 1\right]$$

$$\Rightarrow m_0c^2\left[\cancel{1} + \frac{v^2}{2c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots \cancel{-1}\right]$$

$$\Rightarrow m_0c^2 \times \frac{v^2}{2c^2} \Rightarrow \left(\frac{1}{2}m_0v^2\right)$$

(b) If $\boxed{v = c}$

$$K.E = m_0c^2[\infty - 1]$$

$\boxed{K = \infty}$ ⇒ If particle could move with velocity of light, it would have possed ∞ K.E

② Relavistic momentum & force

$$p = mv = \frac{m_0v}{\sqrt{1-v^2/c^2}}$$

$$F = \frac{dp}{dt} = \frac{d}{dt}\left[\frac{m_0v}{\sqrt{1-v^2/c^2}}\right]$$

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(3) Relation b/w Energy & Momentum

Total energy and momentum are conserved in an isolated system

So these quantities are more fundamental as compare to velocity of K.E

$$\boxed{E^2 = m_0^2 c^4 + p^2 c^2}$$

$$E = mc^2$$

$$E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$$

$$E^2 = \frac{m_0^2 c^4}{(1 - v^2/c^2)} \quad \text{--- (1)}$$

$$p = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

$$p^2 = \frac{m_0^2 v^2}{(1 - v^2/c^2)}$$

$$p^2 c^2 = \frac{m_0^2 c^2 v^2}{(1 - v^2/c^2)} \quad \text{--- (2)}$$

(1) - (2)

$$E^2 - p^2 c^2 = \frac{m_0^2 c^4}{(1 - v^2/c^2)} - m_0^2 c^2 v^2$$

$$= \frac{m_0^2 c^4 \left(1 - \frac{v^2}{c^2}\right)}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\therefore \boxed{E^2 - p^2 c^2 = m_0^2 c^4}$$ Hence proved

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Because mc^2 is invariant $\therefore E^2 - p^2c^2$ will also be same in all frames of reference.

(4) Rest Mass of Photon

$$P = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

Photons travells with speed of light. acc to quantum theory of radiation. momentum is given by

$$P = \frac{h}{\lambda}$$

$$\frac{h}{\lambda} = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

$$m_0 = \frac{h}{v\lambda}\left[\sqrt{1 - \frac{v^2}{c^2}}\right]$$

For photon $v = c$

$$\boxed{m_0 = 0}$$

Rest mass of photon is zero
existence of ~~rest~~ massless particle that posses energy and momentum can be achieved by travelling with speed of light.

In classical mechanics particle must have mass to have energy and momentum but in relativistic mechanics this requirement does not hold.

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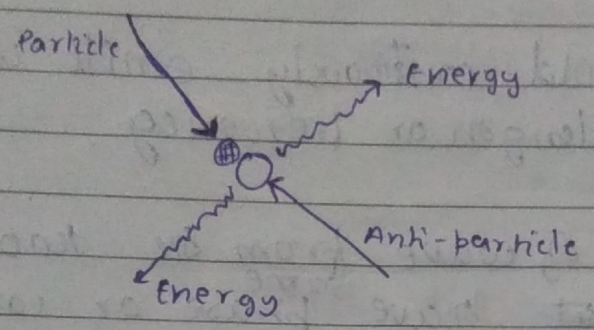
EXPERIMENTAL VERIF. OF $\boxed{E = mc^2}$

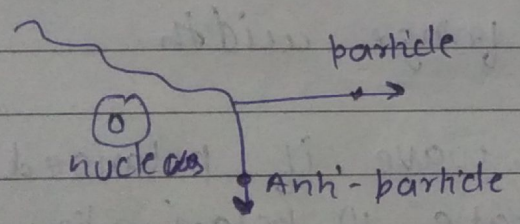
(1) Pair Annihilation



$$e^- + e^+ \rightleftharpoons \gamma + \gamma$$

When e^- & positron come close together they annihilate each other and equivalent amount of energy is produced in form of ~~pair~~ γ ray photon

(2) Pair Production



When radiant energy comes near a charged nucleous, particle & antiparticle is created. Energy is converted into mass

When photon of energy 1.02 mev comes near the nucleous, under its intense electric field nucleous splits in the particle e^- & e^+

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