MID SEMESTER EXAMINATION **September-2012**

AM-101 MATHEMATICS-I

Time: 1Hour 30 Minutes *Max. Marks : 20*

Note : Answer *ALL* questions by selecting any *TWO* from each question.
Assume suitable missing data, if any.

1[a] State integral test and apply to test the convergence of infinite series $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$.

[b] Test the convergence of any two of the following infinite series.

(i) $\sum n^{\ln x}$

(ii) $\frac{1}{2}x + \left(\frac{2}{3}\right)^4 x^2 + \left(\frac{3}{4}\right)^9 x^3 + \cdots\cdots\cdots\cdots + \infty, \quad x > 0.$

(iii) $\sum \frac{n^p}{(n+1)^q}, \quad (p, q > 0)$

[c] State Leibnitz's test for the convergence of alternating series and hence test the convergence of the series $\sum \frac{Cos n\pi}{n^2+1}$.

3, 3

2[a] Use Taylor's series expansion to evaluate sin31° correct to four decimal places (cos30° = 0.8660).

[b] State Maclaurin's series expansion and hence obtain the expansion of the function $log(1 + sinx)$ up to x^5.

[c] Define absolute convergence and conditionally convergent of a series with suitable examples. 3½, 3½

3[a] Establish the formula to find the radius of curvature of $y = f(x)$ at any point (x, y) and hence find the radius of curvature of the curve $y = e^x$ at the point where it crosses the y-axis.

[b] If ρ is the radius of curvature at any point P on the parabola $y^2 = 4ax$ and S is its focus, then show that ρ^2 varies as $(SP)^3$.

[c] Prove that the curvature of a circle is constant whereas it is zero for straight line at any of its point. 3½, 3½