xR37LR/38

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SECOND SEMESTER | **B. TECH. [ALL]**

END SEMESTER EXAMINATION | ***MAY-2011***

AM-111 MATHEMATICS-II

Time: 3 Hours | ***Max. Marks : 70***

Note : Answer ***ALL*** questions selecting any ***TWO*** from each question.
Assume suitable missing data, if any.

1[a] Discuss the consistency of the system of linear equations. Test the consistency of the following system of linear equations and solve, if consistent, using rank method.

$$x + 2y + z = 3;$$
$$2x + 3y + 2z = 5;$$
$$3x - 5y + 5z = 2;$$
$$3x + 9y - z = 4.$$

3.5, 3.5

[b] Write any two properties of eigen values and eigen vectors and hence find the eigen vectors for the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

2, 5

[c] Define diagonalization of a matrix. Show that the matrix

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

is diagonalizable and obtain the digaonalizing matrix.

1,6

2[a] Solve the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x tanx$$

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[b] A mechanical system with two degrees of freedom satisfies the equations

$$2\frac{d^2x}{dt^2}+3\frac{dy}{dt}=4; \qquad 2\frac{d^2y}{dt^2}-3\frac{dx}{dt}=0$$

Solve for x and y when x, y, $\frac{dx}{dt}$ and $\frac{dy}{dt}$ all vanish at $t = 0$.

[c] A body executes damped forced oscillations given by

$$\frac{d^2x}{dt^2}+2k\frac{dx}{dt}+b^2x=e^{-kt}\sin\omega t.$$

Solve the above equation for both cases when $\omega^2 \neq b^2-k^2$ and $\omega^2 = b^2-k^2$.

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3[a] Solve in series the differential equation

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$$

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[b] Show that

(i) $J_{5/2}(x)=\sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\sin x-\frac{3}{x}\cos x\right)$

(ii) $J_4(x)=\left(\frac{48}{x^3}-\frac{8}{x}\right)J_1(x)+\left(1-\frac{24}{x^2}\right)J_0(x)$

3.5, 3.5

[c] State and prove Rodrigue's formula and hence find the Legendre polynomial $P_4(x)$.

4[a] Find the Laplace transform of the following functions

(i) $\int_0^t \frac{e^{-2t}\sin t}{t}\,dt$ (ii) $t^2e^{-3t}\sin 2t$

3.5, 3.5

[b] Find the Inverse Laplace transform of the functions.

(i) $\frac{(s+2)}{(s^2+4s+5)^2}$

(ii) $\frac{1}{(s+1)(s+9)^2}$ (using Convolution theorem)

3.5, 3.5

[c] Using Laplace transform to solve

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \mu(t-\pi)cos(t-\pi)$$

where $y(0) = 1$, $y'(0) = 0$. and $\mu(t-\pi)$ is the unit step function.

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5[a] Find the Fourier series of the function defined in (-2, 2) as

$$f(x) = 2, \qquad -2 \le x \le 0$$
$$= x, \qquad 0 < x < 2.$$

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[b] Find the Fourier Transform of $f(x)$, where

$$f(x) = 1 - x^2, \qquad |x| < 1$$
$$= 0 \quad , \qquad |x| > 1.$$

and use it to evaluate

$$\int_0^\infty \left(\frac{xcosx - sinx}{x^3}\right) cos\frac{x}{2}dx.$$

3.5, 3.5

[c] The following values of y give the displacement in inches of a certain machine part for the rotation x of the flywheel. Expand y in the form of a Fourier series upto second harmonic.

x:	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$	π
y :	0	9.2	14.4	17.8	17.3	11.7	0

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