ELECTRICAL SCIENCES

UNIT — I - 2.

UNIT 2

Series and Parallel Connections

- Two or more circuit elements that carry the same (not merely equal) current are said to be connected in series.

- The circuit elements across which the same (not merely equal) potential difference exists are said to be connected in parallel.

Series Combination of Resistances

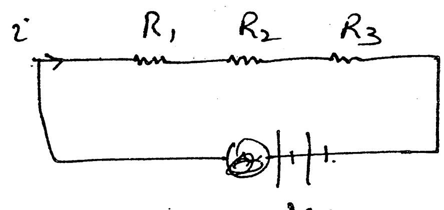


Fig A

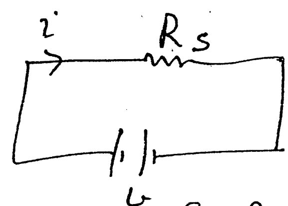


Fig B

The same current flows through R_1, R_2 and R_3 so they are connected in series.

For Fig A

$$iR_1 + iR_2 + iR_3 - v = 0$$

$$\Rightarrow \quad v = iR_1 + iR_2 + iR_3 = i(R_1 + R_2 + R_3) \quad \text{—①}$$

For Fig B

$$v = iR_s \quad \text{—②}$$

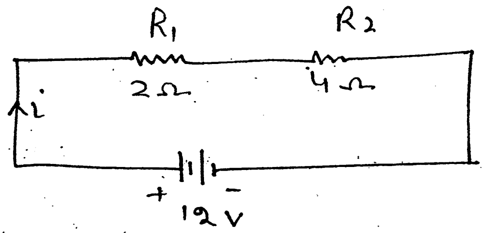
From ① and ② $\quad R_s = R_1 + R_2 + R_3$

For n resistances connected in series

$$R_s = R_1 + R_2 + \cdots + R_n = \sum_{j=1}^{n} R_j$$

Example: How much current flows through a 4Ω resistor connected in series with a 2Ω resistor, and the combination connected across a 12-V source? What is the voltage across each resistor?

Soln.



$$i = \frac{v}{R_1+R_2} = \frac{12}{2+4} = \underline{2\,A}$$

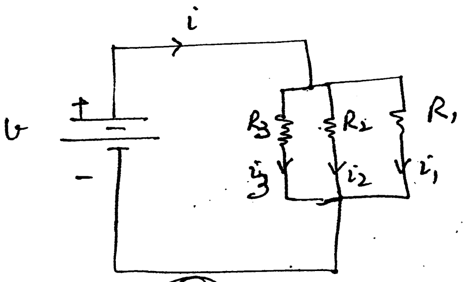
$$V_1 = iR_1 = 2\times 2 = \underline{4\,V} = v\frac{R_1}{R_1+R_2} = \frac{12\times 2}{2+4} = \underline{4}$$

$$V_2 = iR_2 = v\frac{R_2}{R_1+R_2} = 2\times 4 = \underline{8\,V}$$

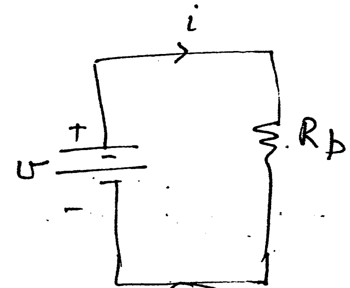
$$= \frac{12 \cdot 4}{2+4} = \underline{8\,V}$$

Note: The battery voltage 12 V is divided between R_1 and R_2 in proportion to their resistances. Such a circuit is known as potential divider.

Parallel Combination of Resistances



(A)



(B)

For (A) $\quad i = i_1 + i_2 + i_3$

$$i_1 = \frac{v}{R_1} \qquad i_2 = \frac{v}{R_2} \qquad i_3 = \frac{v}{R_3}$$

$$i = i_1 + i_2 + i_3 = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} = v\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] \quad -(1)$$

For (B) $$i = \frac{v}{R_p} \quad -(2)$$

From (1) and (2)

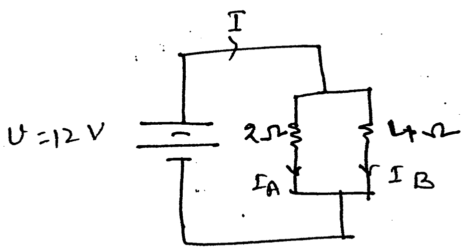
$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

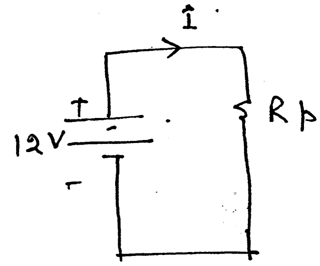
For n Resistances connected in parallel

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \; \cdots\cdots \; \frac{1}{R_n} = \sum_{i=1}^{n} \frac{1}{R_i}$$

— The reciprocal of the equivalent resistance of a number of resistances connected in parallel is equal to the sum of the reciprocals of the individual resistances.

Example: Calculate the current supplied by the battery in the circuit shown and how it is divided in two parallel branches.





Soln:

$$R_p = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{2 \times 4}{2+4} = \frac{4}{3}\,\Omega$$

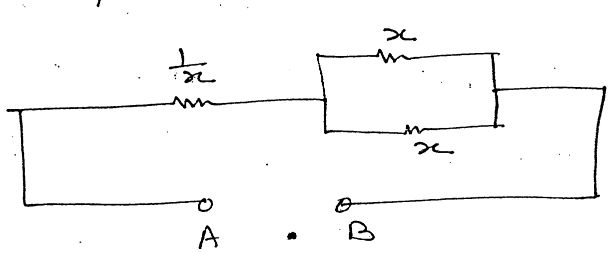
$$I = \frac{V}{R_p} = \frac{12}{4/3} = \underline{9\text{ A}}$$

$$I_A = \frac{12}{2} = \underline{6\text{ A}}$$

$$I_B = \frac{12}{4} = \underline{3\text{ A}}$$

Note: The current I is divided in the inverse ratio of the values of resistances. Such a circuit is called <u>Current divider circuit</u>.

Example: Find the value of x such that the resistance of the combination as seen from the points A and B is minimum.



Soln: The equivalent resistance can be written as

$$R_{eq} = \frac{x}{2} + \frac{1}{x} = \frac{x^2+2}{2x}$$

For R_{eq} to be minimum its derivative w.r.t x should be zero.

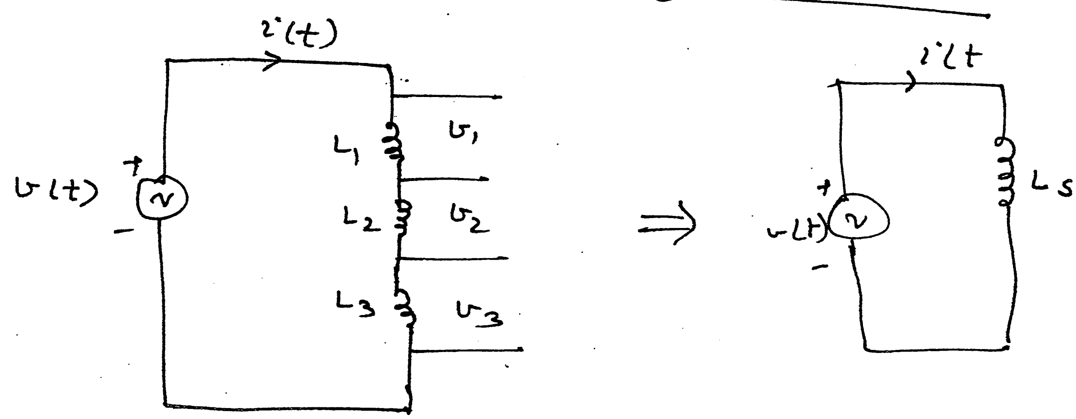
Therefore,

$$\frac{dR_{eq}}{dx} = \frac{(2x)(2x) - (x^2+2)(2)}{4x^2} = 0$$

$$\Rightarrow \frac{4x^2 - 2x^2 - 4}{4x^2} = 0$$

$$\Rightarrow 2x^2 - 4 = 0 \quad \Rightarrow x^2 = 2 \quad \text{or} \quad \underline{\underline{x = \sqrt{2}\ \Omega}}$$

Series Combination of Inductances



$i(t)$ varies with time.

$$v(t) = v_1 + v_2 + v_3$$

$$= L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

$$= (L_1 + L_2 + L_3)\frac{di}{dt} \quad \text{(1)}$$

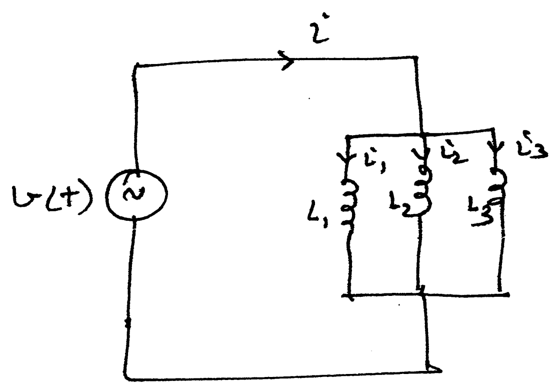
and $\quad v(t) = L_S \dfrac{di}{dt} \quad$ (2)

from (1) and (2)

$$L_S = L_1 + L_2 + L_3$$

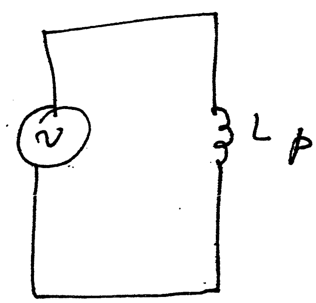
$$L_S = L_1 + L_2 - \cdots L_n = \sum_{i=1}^{n} L_i$$

Parallel Combination of Inductances



(A)

$\equiv$



(B)

$$i = i_1 + i_2 + i_3$$

$$v = \frac{L\,di}{dt} \quad \text{or} \quad i = \frac{1}{L}\int_0^t v\,dt$$

For Fig (A)

$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_0^t v(t)\,dt \quad \text{— (1)}$$

For Fig (B)

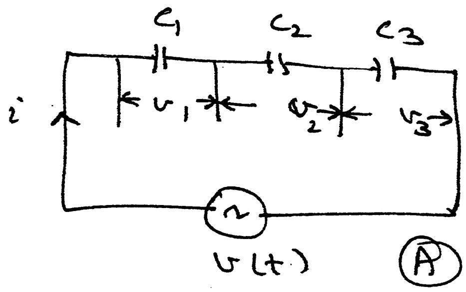
$$i = \frac{1}{L_p}\int_0^t v(t)\,dt \quad \text{—— (2)}$$

From (1) and (2)

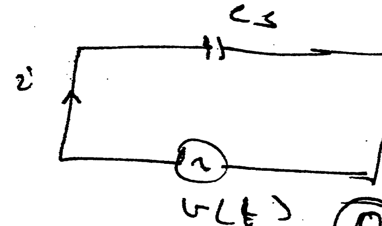
$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$\Rightarrow \quad \frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} - \cdots \frac{1}{L_n} = \sum_{i=1}^{n} \frac{1}{L_i}$$

Series Combination of Capacitances



(A)



(B)

for (A)

$$v(t) = v_1 + v_2 + v_3$$

$$v_1 = \frac{1}{C_1}\int_0^t i\,dt \qquad v_2 = \frac{1}{C_2}\int_0^t i\,dt \qquad v_3 = \frac{1}{C_3}\int_0^t i\,d$$

$$v(t) = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)\int_0^t i\,dt \qquad (1)$$

for (B)

$$v(t) = \frac{1}{C_S}\int_0^t i\,dt \qquad (2)$$

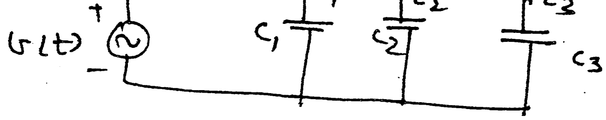
from (1) and (2)

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow \quad \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} \;---\; \frac{1}{C_n} = \sum_{i=1}^{n}\frac{1}{C_i}$$

stiffness or elastance $= S = \frac{1}{C}$

Parallel combination of Capacitances



(A)

For (A)

$$i_1 = C_1 \frac{dv}{dt} \qquad i_2 = C_2 \frac{dv}{dt} \qquad i_3 = C_3 \frac{dv}{dt}$$

$$i = i_1 + i_2 + i_3$$

$$= C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt}$$

$$= (C_1 + C_2 + C_3) \frac{dv}{dt} \qquad (1)$$

For (B)

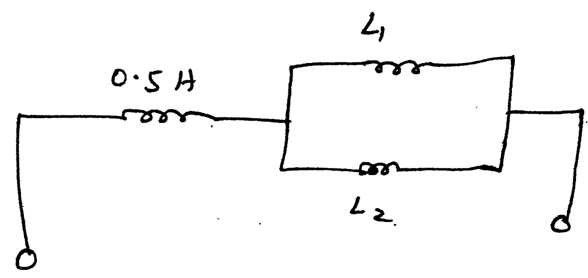
$$i = C_p \frac{dv}{dt} \qquad (2)$$

From (1) and (2)

$$C_p = C_1 + C_2 + C_3 = \sum C_i$$

$$= C_1 + C_2 - - - - C_n = \sum_{i=1}^{n} C_i$$

Example: Three inductances are connected as shown. Given that $L_1 = 2L_2$. Find L_1 and L_2 such that the equivalent inductance of the combination is 0.7 H.



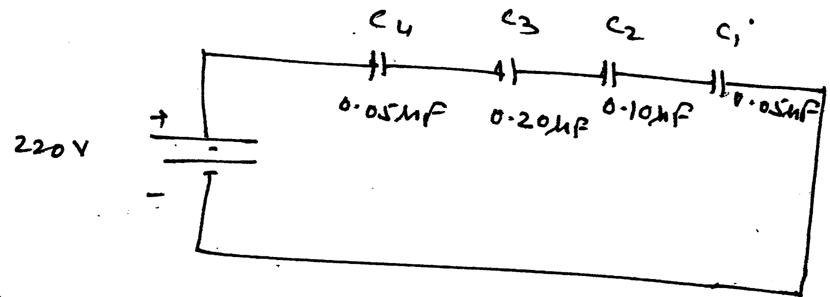
Soln

$$L_{eq} = 0.5 + \frac{L_1 L_2}{L_1 + L_2} = 0.7$$

$$\Rightarrow \frac{2L_2 L_2}{2L_2 + L_2} = 0.2 \qquad \frac{2L_2^2}{3L_2} = 0.2$$

$\underline{L_2 = 0.3H}$ $\qquad$ $\underline{L_1 = 0.6\,H}$

* **Example:** Determine the equivalent capacitance of the network shown and voltage drop across each capacitor.



Soln.

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

$$= \frac{1}{0.05} + \frac{1}{0.10} + \frac{1}{0.20} + \frac{1}{0.05} = 55$$

$$C_T = \frac{1}{55} = 0.0182\,\mu F$$

The charge transferred to each capacitor is

$$Q = C_T V = 0.0182 \times 10^{-6} \times 220 = 4 \times 10^{-6}\,C$$

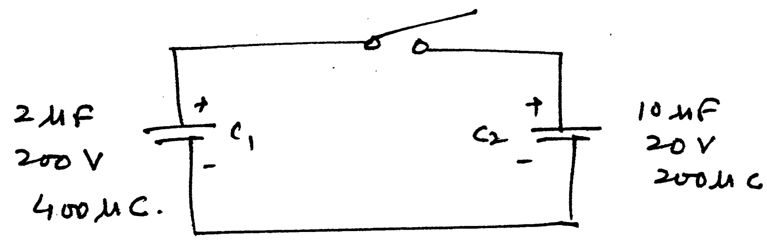
$$V_1 = \frac{Q}{C_1} = \frac{4\times10^{-6}}{0.05\times10^{-6}} = \underline{80V}$$

$$V_2 = \frac{Q}{C_2} = \frac{4\times10^{-6}}{0.10\times10^{-6}} = \underline{40V}$$

$$V_3 = \frac{Q}{C_3} = \frac{4\times10^{-6}}{0.20\times10^{-6}} = \underline{20V}$$

$$V_4 = \frac{Q}{C_4} = \frac{Q}{C_4} = \frac{4\times10^{-6}}{0.05\times10^{-6}} = 80V$$

Example: Two capacitors are charged as shown. After switch is closed, what voltage exists across the capacitors?



Soln: Following points can be noted after switch is closed

1. Voltage polarities across the capacitors are in the same direction

2. Total charge available is sum of charges on two individual capacitors i.e. (200 + 400) = 600 μC.

3. After closing the switch, the capacitors are connected in parallel. The equivalent capacitance is equal to the sum of individual capacitances.

$$C_{eq} = C_1 + C_2 = 2 + 10 = 12 \mu F.$$

Therefore, the voltage across the ‖ combination

$$V = \frac{Q}{C} = \frac{600 \times 10^{-6}}{12 \times 10^{-6}} = \underline{50 V}$$

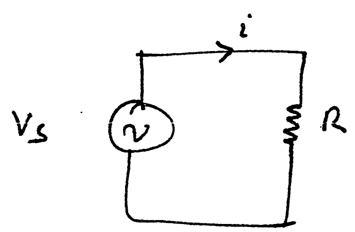
Comments on Series and Parallel Combination

1. Ideal voltage sources cannot be connected in parallel.

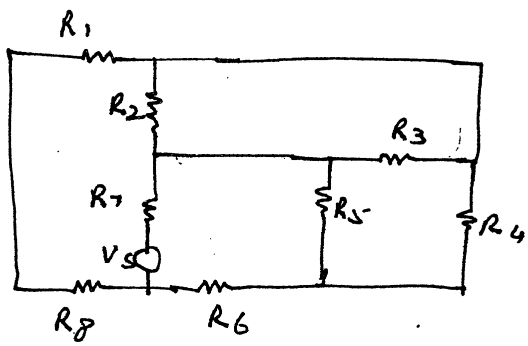
2. Ideal current ~~ckt~~ sources can not be connected in series

But practical Sources can be.

Identify the series and Parallel Connections



1. V_s and R are in series
2. " " " " parallel too.



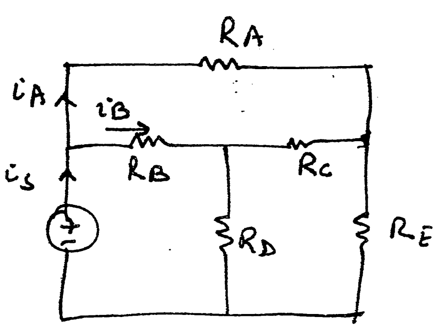
R_2 and R_3 in ||

R_1 and R_8 in series

V_s and R_7 " "

R_4 and R_5 no where

R_5 and R_6



Nothing in series

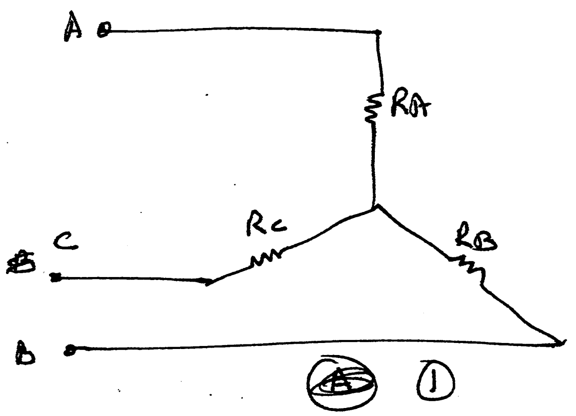
" " ||

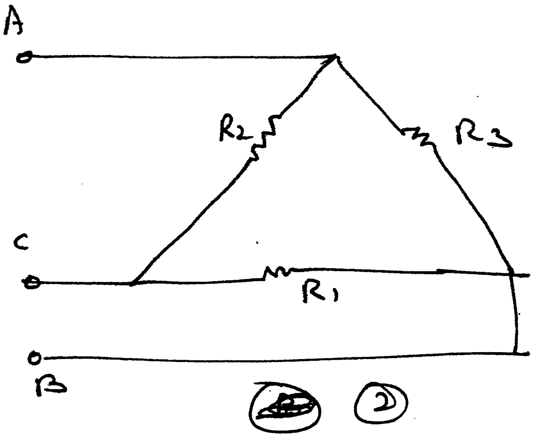
so series and parallel connections is not a compulsion.

Star - Delta Transformation

λ Δ

T Π





$\lambda - \Delta$ Transformation $\Rightarrow$ Find Δ for a given λ

$\Delta - \lambda$ " $\Rightarrow$ " λ " " Δ

For the two arrangements to be equivalent

— Resistance between any pair of terminals (AB, BC and CA) of the two arrangements should be same when the third terminal is left OPEN.

— Let A is kept open

— Resistance between B and C for fig ①

$$= R_B + R_C$$

and for fig ② R_2 and R_3 is series and the combination is in // with R_1

So

$$R_B + R_C = \frac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3} \quad (1)$$

Similarly we can write

$$R_C + R_A = \frac{R_2 (R_1 + R_3)}{R_2 + R_1 + R_3} \quad \text{--- (2)}$$

and

$$R_A + R_B = \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3} \quad \text{--- (3)}$$

Add eqs (1), (2) and (3)

$$\boxed{R_A + R_B + R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1 + R_2 R_3}} \quad \text{--- (4)}$$

Substracting eq. (1) from (4)

$$R_A = \frac{R_2 R_3}{R_1 + R_2 + R_3} \quad \text{--- (5)}$$

Further we can get

$$R_B = \frac{R_3 R_1}{R_1 + R_2 + R_3} \quad \text{--- (6)}$$

and

$$R_C = \frac{R_1 R_2}{R_1 + R_2 + R_3} \quad \text{--- (7)}$$

This transforms Δ into λ.

<u>Now let us derive relations for transforming λ into Δ</u>

multiply eq (5) & (6)

$$R_A R_B = \frac{R_1 R_2 R_3^2}{(R_1 + R_2 + R_3)^2} \quad \text{--- (8)}$$

Similarly we can get

$$R_B R_C = \frac{R_1^2 R_2 R_3}{(R_1 + R_2 + R_3)^2} \quad \text{— (9)}$$

and

$$R_C R_A = \frac{R_1 R_2^2 R_3}{(R_1 + R_2 + R_3)^2} \quad \text{— (10)}$$

Adding (8), (9) and (10)

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_1 R_2 R_3^2 + R_1^2 R_2 R_3 + R_1 R_2^2 R_3}{(R_1 + R_2 + R_3)^2}$$

$$= \frac{R_1 + \cancel{R_2} R_3}{\cancel{R_1}} = \frac{R_1 R_2 R_3}{R_1 + R_2 + R_3}$$

$$= \frac{R_2 R_3}{R_1 + R_2 + R_3} R_1 \quad \text{— (11)}$$

$$= R_A R_1$$

Therefore,

$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A} \quad (12)$$

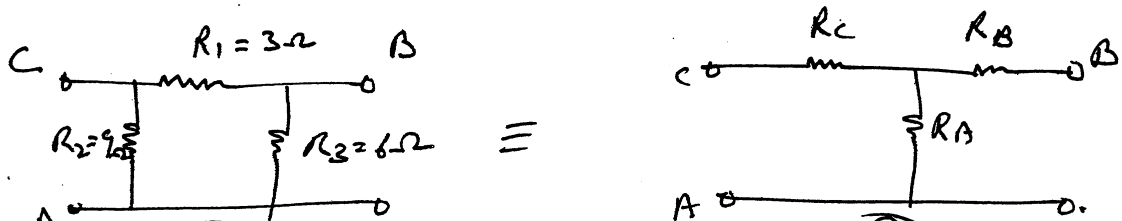
Similarly we can write

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B} \quad (13)$$

and

$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C} \quad (14)$$

Example: A π-section (Δ) of resistors is given in Fig. A where $R_1 = 3\Omega$, $R_2 = 9\Omega$ and $R_3 = 6\Omega$. Convert this π-section into an equivalent T-section (λ)



(A) (B)

Soln:

$$R_A = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{9 \times 6}{3 + 9 + 6} = \frac{54}{18} = \underline{3\Omega}$$

$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{3 \times 6}{18} = \underline{1\Omega}$$

$$R_C = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{3 \times 9}{18} = 1.5\Omega$$

Example: Fig. A shows a network. The number on each branch represents the value of resistance in ohms. Find the resistance between the points E and F.

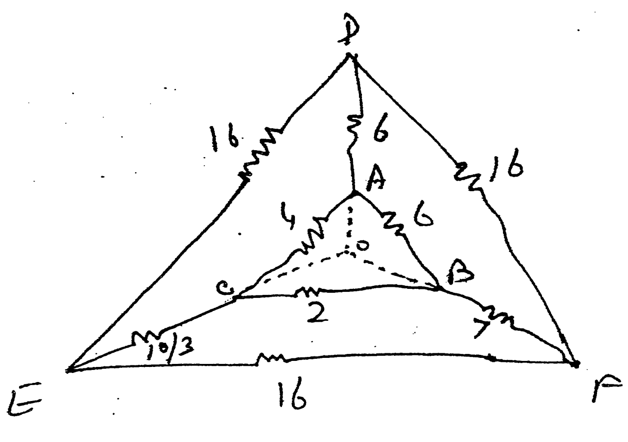


Fig (A)

Soln. The Δ represented by the points A, B and C can be converted into a ⅄ section, creating a new node O. The resistances in the branches of the newly created ⅄, say AO, BO and CO can be calculated as.

$$R_{AO} = \frac{4\times6}{2+4+6} = \frac{24}{12} = 2\,\Omega$$

$$R_{BO} = \frac{2\times6}{2+4+6} = \frac{12}{12} = 1\,\Omega$$

$$R_{CO} = \frac{2\times4}{2+4+6} = \frac{8}{12} = 2/3\,\Omega$$

Now, the network can be redrawn as follows.

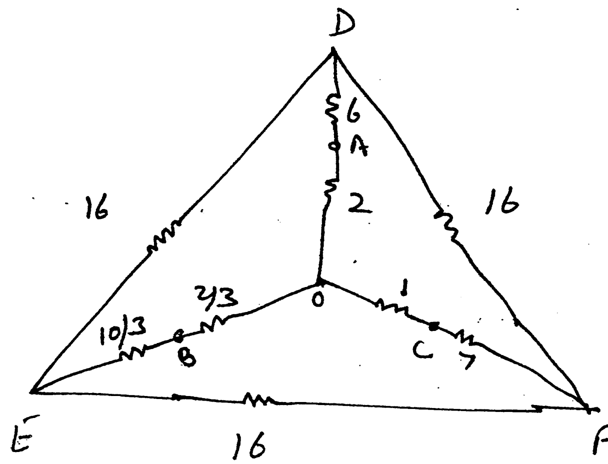
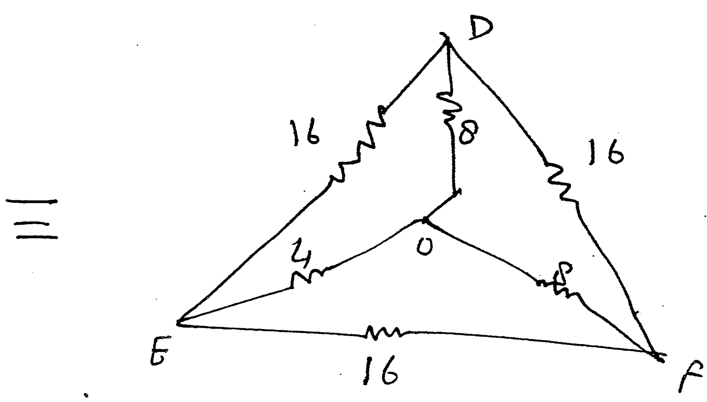


Fig. (B)



Note: The points A, B and C are lost

Fig (C)

Next, we transform the inner λ into Δ. The resulting network is shown in Fig. D.

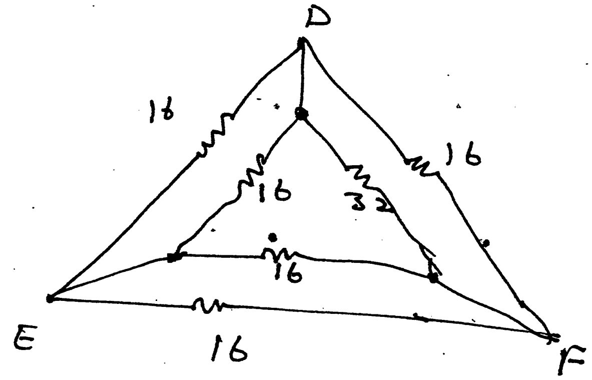


Fig (D)
(Point O is lost)

Now, the parallel resistances in the three branches of delta can be combined easily to give a network of Fig. (E)

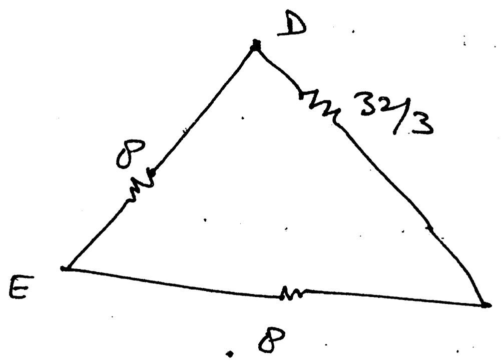


Fig. (E)
(Three points of inner Δ are lost)

The resistance between points E and F is parallel combination of R_{EF} with a series combination of R_{DE} and R_{DF}. Thus the required resistance is

$$R_{eq} = \frac{8(56/3)}{8+56/3} = \frac{8\times 56}{80} = \frac{56}{10} = \underline{5.6\,\Omega}$$

Example: Find the current drawn from the 5-volt battery in the network shown in Fig. A.

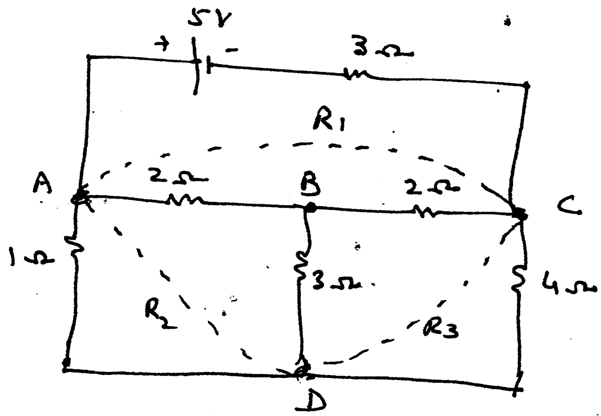


Fig (A)

Soln. – In Fig. (A) the branches AB, BC and BD form a λ section.

– This can be converted into Δ as shown in ~~Fig (B)~~ dotted lines.

$$R_1 = \frac{2\times 3 + 3\times 2 + 2\times 2}{3} = \frac{16}{3}\,\Omega$$

$$R_2 = \frac{2\times 3 + 3\times 2 + 2\times 2}{2} = \frac{16}{2}\,\Omega = 8\,\Omega$$

$$R_3 = \frac{2\times 3 + 3\times 2 + 2\times 2}{2} = \frac{16}{2} = 8\,\Omega$$

The resulting network is shown in Fig. (B)

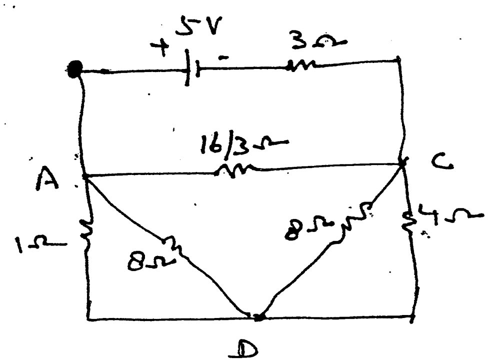


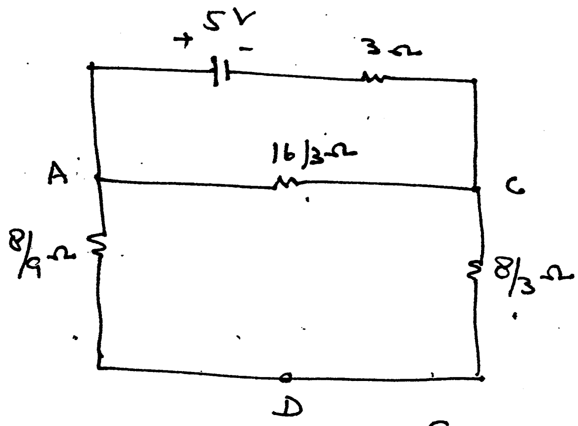
Fig. (B)
(The point B is lost)

Note :

Across points A and D 1Ω and 8Ω are in ||

" " C + D 8Ω + 4Ω " ||

So,



$$\frac{8\times 1}{8+1} = \frac{8}{9}\,\Omega$$

$$\frac{8\times 4}{8+4} = \frac{32}{12} = \frac{8}{3}\,\Omega$$

Fig. (C)

≡

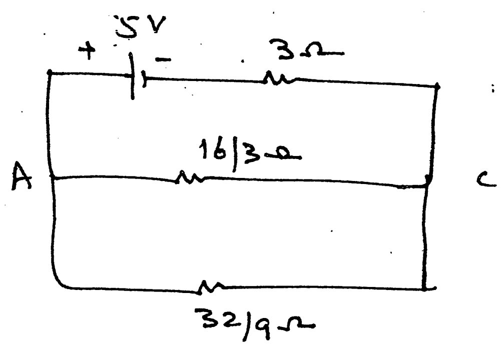


Fig. (D)

$$\frac{8}{3} + \frac{8}{9} = \frac{24+8}{9} = \frac{32}{9}$$

≡

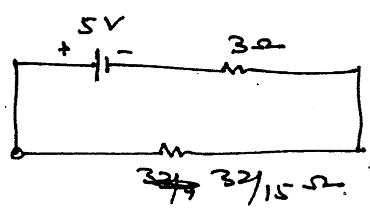


Fig (E)

$$\frac{16/3 \times 32/9}{16/3 + 32/9} = \frac{16\times 32}{80\times 3} = \frac{32}{15}\,\Omega$$

≡

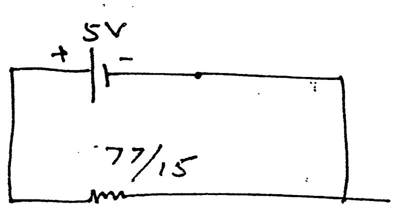


Fig (F)

$$R_{eq} = 3 + \frac{32}{15} = \frac{45+32}{15} = \frac{77}{15}\,\Omega$$

$$I = \frac{5}{R_{eq}} = \frac{5\times 15}{77} = \frac{75}{77} = 0.974\ A$$

Kirchhoff's Laws

- These laws enables us to calculate
  - currents flowing in various branches of the network
  - and voltages at various points of the network.

Kirchhoff's Current Law

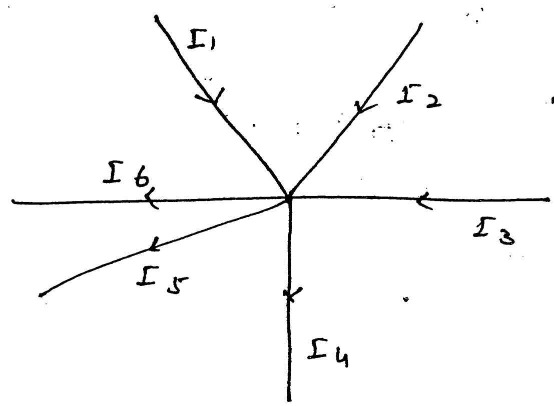
It states that algebraic (phasor) sum of currents meeting at a junction of conductors is zero.

Alternatively,

The sum of currents flowing away from a junction is equal to the sum of currents flowing towards the junction.

- currents entering into the junction are taken as +ve
- Currents leaving the junction are taken as −ve

or vice-versa



$$I_1 + I_2 + I_3 + I_4 - I_5 - I_6 = 0$$

$$\sum_{j=1}^{6} I_j = 0$$

OR $$I_1 + I_2 + I_3 + I_4 = I_5 + I_6$$

This law is a <u>restatement</u> of principle of <u>Conservation of Charge.</u>

Which means

- that accumulation of electric-charge at a junction is not possible.

OR

the amount of charge entering a junction <u>at any instant</u> must be the same as the amount of charge leaving the junction.

<u>Kirchhoff's Voltage Law (KVL)</u>

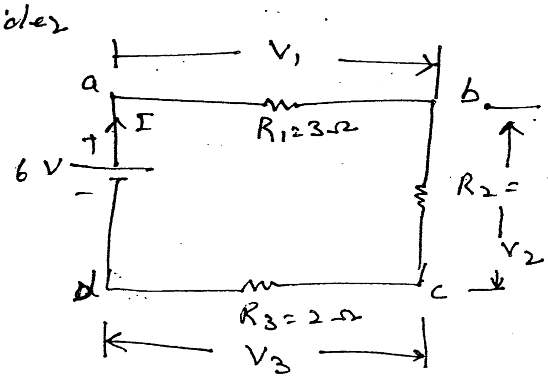
It states that at any instant the algebraic (phasor) sum of voltages around a closed circuit/loop is zero.

$$\sum_{j=1}^{k} v_j = 0 \qquad \text{—— (1)}$$

$$\Rightarrow \quad v_1 + v_2 - - - - + v_k = 0 \qquad \text{—— (2)}$$

This law is a restatement of law of <u>conservation of energy.</u>

Consider



Energy supplied by battery in a unit time = EI

This energy is dissipated in 3 resistors

So,

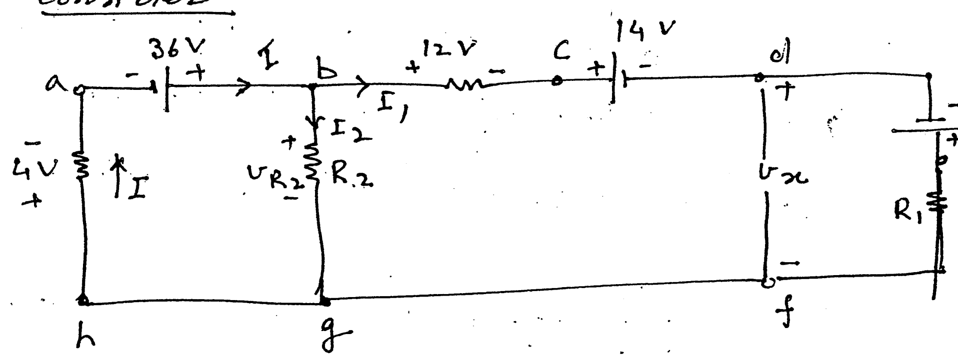
$$EI = I^2R_1 + I^2R_2 + I^2R_3$$

$$\Rightarrow \quad E = IR_1 + IR_2 + IR_3 \qquad \text{—— (3)}$$

This means

In any closed circuit the algebraic sum of the products of current and resistance in each of the conductors is equal to the algebraic sum of emfs of the batteries.

Consider



$$I = I_1 + I_2 \qquad \text{—— (4)}$$

If $v_{R_2} = ?$, Then for loop abgha

$$-36 + v_{R_2} + 4 = 0$$

$$v_{R_2} = \underline{32\,V}$$

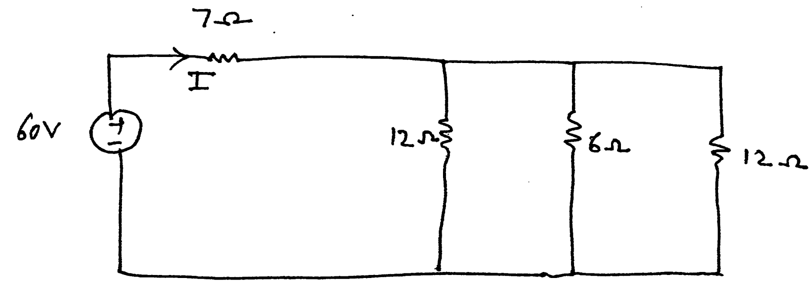
If $v_x = ?$. Then for the loop habcdfgh

$$4 - 36 + 12 + 14 + v_x = 0$$

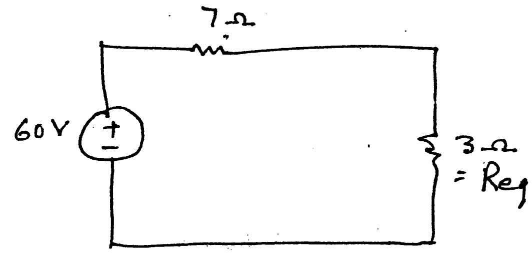
$$v_x = \underline{6\,V}$$

Note: h, g and f form a single node (junction)

Example : Find the current supplied by the 60-V source in the given network.



Soln:



$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{6} + \frac{1}{12}$$

$$= \frac{1+2+1}{12} = \frac{4}{12}$$

$$= \frac{1}{3}$$

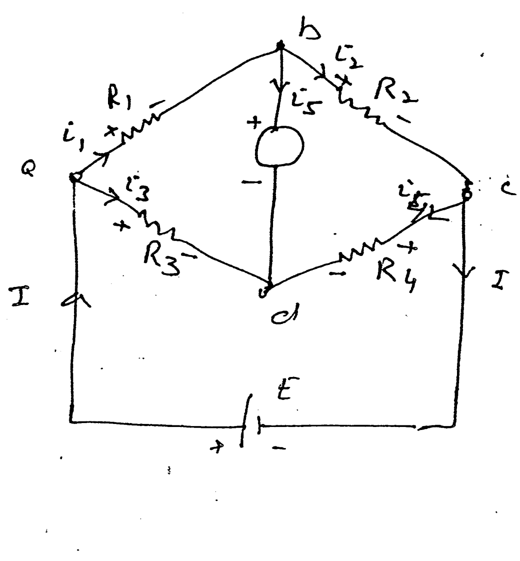
$$R_{eq} = 3\Omega$$

$$I = \frac{60}{10} = \underline{6\ A}$$

Example : For the bridge circuit shown, write

(i) the Kirchhoff's current law equations at nodes a, b, c and d

(ii) the Kirchhoff's voltage law equations around loops abda, bcdb and adca.



Soln. <u>Kirchhoff's Current Law equations</u>

Node a: $I = i_1 + i_3$

b: $i_1 = i_2 + i_5$

c: $i_2 = I + i_4$

d: $i_3 + i_4 + i_5 = 0$

<u>Kirchhoff's voltage law equations</u>

Loop abda: $i_1 R_1 + i_5 R_5 - i_3 R_3 = 0$

Loop bcdb: $i_2 R_2 + i_4 R_4 - i_5 R_5 = 0$

Loop adca: $i_3 R_3 + i_4 R_4 - E = 0$

<u>Example</u>: For the network of above example consider $i_5 = 0$.

(i) Determine R_4 if $R_1 = 10\Omega$, $R_2 = 20\Omega$ and $R_3 = 30\Omega$

(ii) Calculate the current supplied by the battery if $E = 9V$

<u>Soln</u>: (i) Since $i_5 = 0$, we have

$$\left.\begin{aligned} i_1 &= i_2 \\ i_3 &= -i_4 \end{aligned}\right\}$$

and $V_b = V_d$

Thus $i_1 R_1 = i_3 R_3$ and $i_2 R_2 = -i_4 R_4$

$i_1 R_2 = i_3 R_4$

From these equations, we obtain

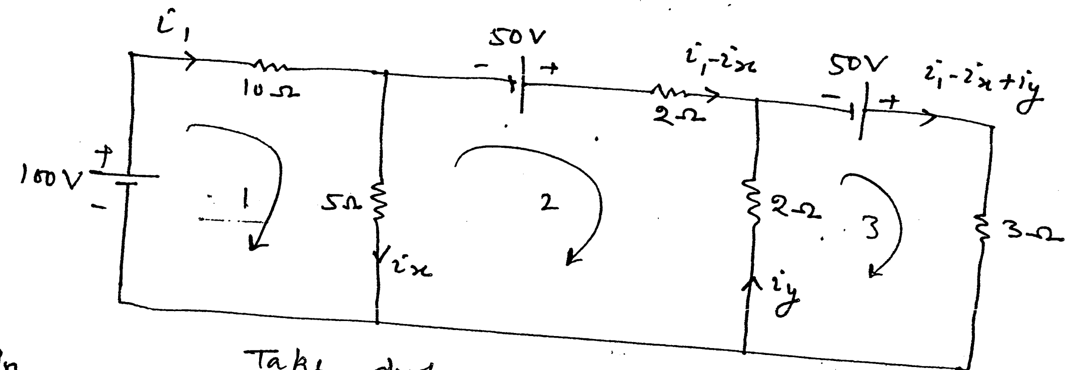
$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow R_4 = \frac{R_2 R_3}{R_1} = \frac{20 \times 30}{10} = \underline{60\Omega}$$

(ii) For $i_s = 0$, The equivalent resistance across battery is

$$R_e = \frac{(10+20)(30+60)}{(10+20)+(30+60)} = \frac{30 \times 90}{120} = \frac{90}{4} = 22.5\,\Omega$$

$$I = \frac{E}{R_e} = \frac{9}{22.5} = \underline{0.4\,A}$$

<u>Example</u> Determine the currents i_x and i_y in the network shown



<u>Soln</u> Take drop as +ve
rise as −ve

For mesh 1 $\quad 10 i_1 + 5 i_x - 100 = 0 \quad$ —— (1)

mesh 2 $\quad 2(i_1 - i_x) - 2 i_y - 5 i_x - 50 = 0 \quad$ —— (2)

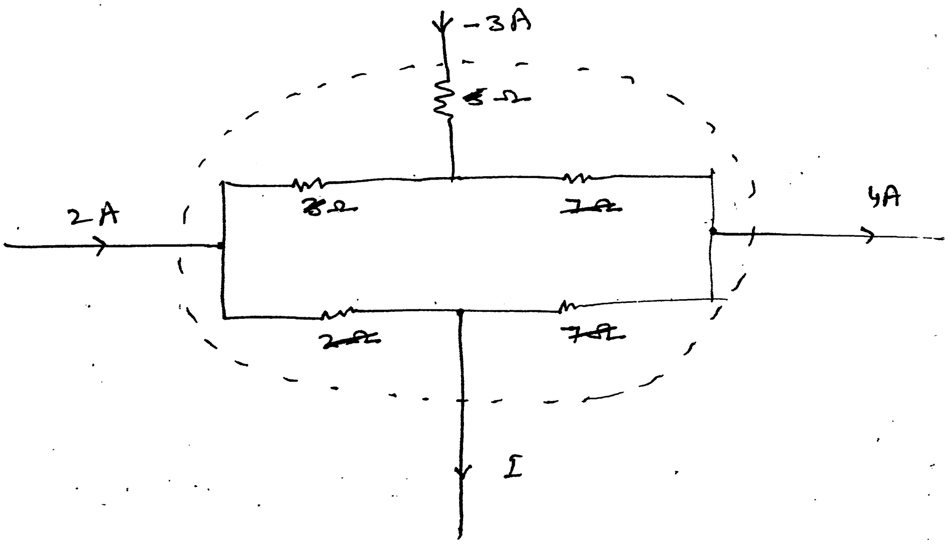
mesh 3 $\quad 3(i_1 - i_x + i_y) + 2 i_y - 50 = 0 \quad$ —— (3)

Solving these three simultaneous equations for i_1, i_x and i_y we get

$$i_x = \underline{-3.07\ A} \quad \text{and} \quad i_y = \underline{0.51\,A}$$

−ve sign for i_x implies that the actual current flows in a direction opposite to the assumed direction of i_x.

Example: Determine the value of I



$$2 - 3 - I - 4 = 0 \qquad I = \underline{-5A}$$

NETWORK THEOREMS

Need for Network Theorems

Kirchhoff's Laws are sufficient and capable of solving the network fully.

But

fully only and not partly.

. If we have to solve the given network partly, Network Theorems can simplify the problem with reduced computational effort.

- Some theorems have universal application
- Some can be applied to linear impedances only

R, L, C are linear, diodes are not.

Loop or Mesh Analysis

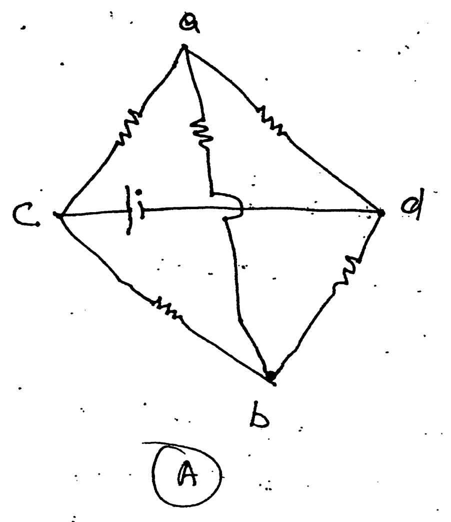
Node: ~~A point~~ It is a point at which three or more elements have a common connection.

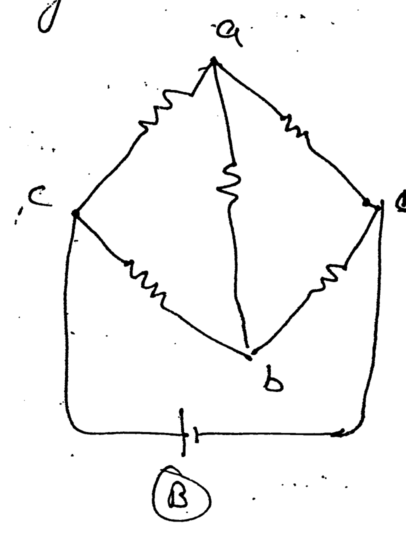
Path: If we trace a circuit passing through various nodes, the set of nodes and elements traced is known as a path

Loop: If we start at a node and trace the circuit and return back to the starting node without repeating any element/node, the path is called a loop/closed path.

— Mesh analysis is applicable to planner networks only

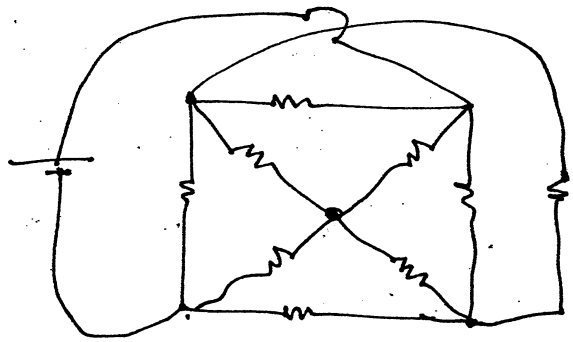
Planner Network: A network is said to be planner if it can be drawn on a sheet of paper without crossing lines.





Network (A) seems to be non-planner, but can be drawn as in (B), so is a planner network

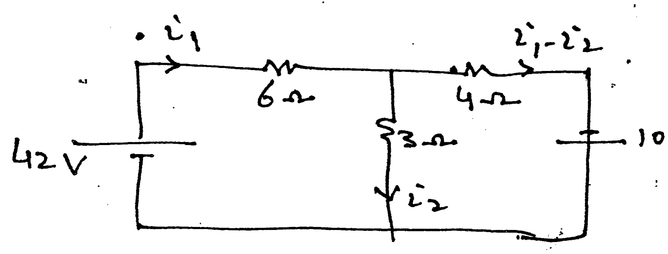
Example of a non-planner network



Mesh: It is a loop which doesn't contain any other loop within it.

- Once a network has been drawn neatly in a planner form, it often has the appearance of a multi-panned window.
- Each pane in the window may be considered as mesh.

Example: Using mesh analysis determine the currents flowing through each ~~of the~~ branch of circuit shown.



Soln. Clearly there are two panes, therefore two meshes.

Writing KVL for left hand mesh.

$$42 - 6i_1 - 3i_2 = 0$$

$$6i_1 + 3i_2 = 42 \quad \text{———} \quad (1)$$

For Right hand mesh.

$$3i_2 - 4(i_1 - i_2) + 10 = 0$$

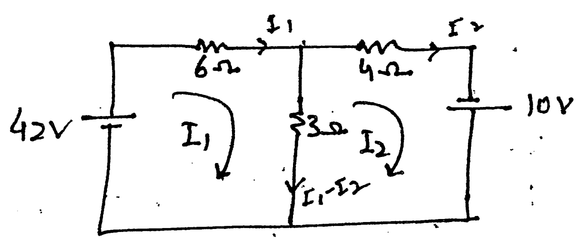
$$4i_1 - 7i_2 = 10 \qquad \text{—— (2)}$$

We can easily solve equs (1) and (2) to give

$i_1 = \underline{6A}$ and $i_2 = \underline{2A}$

Concept of Mesh Currents

- By definition, a mesh current is that current which flows around the perimeter of a mesh.
- Reconsider the network of previous example.



- I_1 and I_2 are mesh currents
- both have been assumed clockwise, but it is not necessary. However, it proves to be more convenient.
- branch currents have physical identity and can be measured
- mesh currents are intermediate variables, introduced/assumed for simplifying the process of solving the network.

KVL for mesh 1 is

$$42 - 6I_1 - 3(I_1 - I_2) = 0$$

$$\Rightarrow \quad 9I_1 - 3I_2 = 42 \quad \text{— (1)}$$

For mesh 2

$$-3(I_2 - I_1) - 4I_2 + 10 = 0$$

$$-3I_1 + 7I_2 = 10 \quad \text{— (2)}$$

We can solve (1) and (2) using Crammer's rule or otherwise

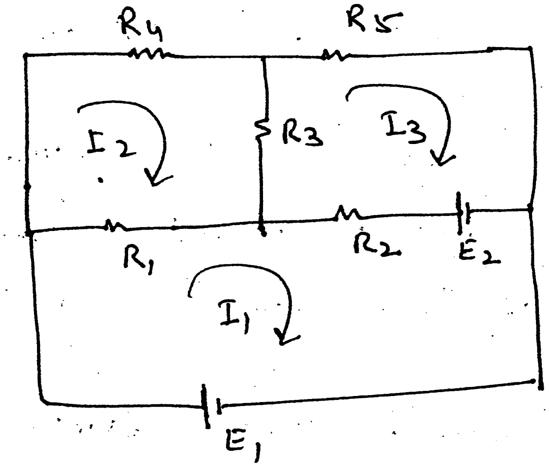
$$I_1 = \frac{\begin{vmatrix} 42 & -3 \\ 10 & 7 \end{vmatrix}}{\begin{vmatrix} 9 & -3 \\ -3 & 7 \end{vmatrix}} = \frac{294 + 30}{63 - 9} = \frac{324}{54} = \underline{6A}$$

$$I_2 = \frac{\begin{vmatrix} 9 & 42 \\ -3 & 10 \end{vmatrix}}{\begin{vmatrix} 9 & -3 \\ -3 & 7 \end{vmatrix}} = \frac{90 + 126}{54} = \frac{216}{54} = \underline{4A}$$

Current through 6Ω $= I_1 = \underline{6A}$

" 3Ω $= I_1 - I_2 = 6 - 4 = \underline{2A}$

" 4Ω $= I_2 = \underline{4A}$



For mesh 1, 2 and 3 we can write KVL equations

$$(I_1 - I_2) R_1 + (I_1 - I_3) R_2 + E_2 - E_1 = 0$$

$$I_2 R_4 + (I_2 - I_3) R_3 + (I_2 - I_1) R_1 = 0$$

$$I_3 R_5 - E_2 + (I_3 - I_1) R_2 + (I_3 - I_2) R_3 = 0$$

These equations can be rearranged as

$$\left.\begin{array}{llll} (R_1 + R_2) I_1 & -R_1 I_2 & -R_2 I_3 & = E_1 - E_2 \\ -R_1 I_1 & (R_4 + R_3 + R_1) I_2 & -R_3 I_2 & = 0 \\ -R_2 I_1 & -R_3 I_2 & (R_5 + R_2 + R_3) I_3 & = E_2 \end{array}\right\} \text{(A) (1)}$$

For sake of symmetry we replace the symbols for resistances as follows

$R_{11} = R_1 + R_2 =$ the self resistance of mesh 1 and is sum of all resistances found in mesh 1

$R_{12} = R_1 =$ Mutual resistance between meshes 1 and 2 $= R_{21}$

$R_{13} = R_2 =$ Mutual Resistance between mesh 1 and 3 $= R_{31}$

$R_{22} = R_4 + R_3 + R_1 =$ the self resistance of mesh 2

$R_{23} = R_3 =$ the mutual resistance of mesh 2 and 3 $= R_{32}$

$R_{33} = R_5 + R_2 + R_3$ = the self resistance of mesh 3.

$E_{11} = E_1 - E_2$ = the sum of voltage sources in mesh 1, producing current in the same direction as the assumed mesh current

$E_{22} = 0$ = Sum of voltage sources of mesh 2

$E_{33} = E_2$ = the sum of " " " 3.

The eq (1) can be rewritten as

$$R_{11} I_1 - R_{12} I_2 - R_{13} I_3 = E_{11}$$
$$-R_{21} I_1 + R_{22} I_2 - R_{23} I_3 = E_{22}$$
$$-R_{31} I_1 - R_{32} I_2 + R_{33} I_3 = E_{33}$$

$$\Rightarrow \begin{bmatrix} +R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_{11} \\ E_{22} \\ E_{33} \end{bmatrix}$$

$$R\,I = E$$

About R

1. It is a symmetrical matrix about the major diagonal
2. Diagonal elements are +ve and heavier as compared to off-diagonal elements
3. off-diagonal elements are -ve
4. R is known as Resistance Matrix
5. Mutual resistance between 2 meshes is zero, if there is no resistance common to them.

Procedure for Mesh Analysis

1. This procedure is for network containing only independant Voltage sources, however, the presence of a practical current source can always be converted into a practical voltage source.

2. Make sure that network is planar network

3. Each voltage source should have its reference polarities marked

4. The resistance values are available. Instead if conductance values ~~are~~ (G) are available, convert using $R = 1/G$.

5. Assuming the network has m meshes, assign clockwise mesh currents in each mesh $I_1, I_2 ---- I_m$

6. Write the KVL equations (Resistance matrix) by mere inspection.

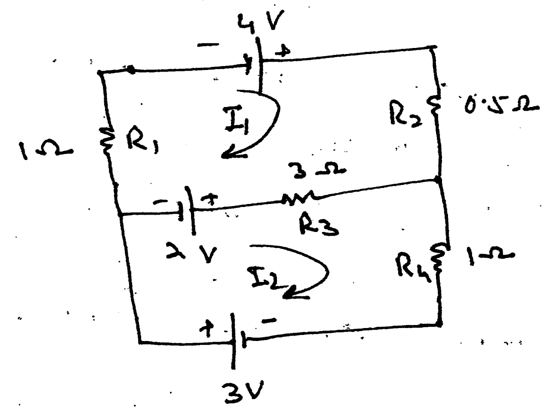
7. Check the symmetry of the R matrix

8. Write down the RHS of equations for voltage source.

9. Solve these equations by cramer's rule or Guass Elimination method.

10. Determine branch currents ~~ex~~ and voltages.

Example: Determine the currents in the various resistances of the network shown



Soln: There are two meshes.
Mark the mesh currents I_1 and I_2

KVL equations

$$\begin{bmatrix} 1+0.5+3 & -3 \\ -3 & 3+1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 4-2 \\ 2+3 \end{bmatrix}$$

$$\begin{bmatrix} 4.5 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

These can be solved to obtain

$I_1 = \underline{2.55A}$ and $I_2 = \underline{3.167A}$

Next

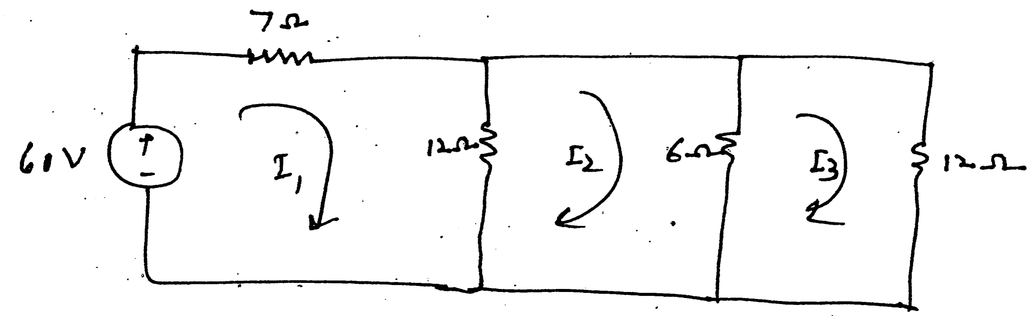
Current through $R_1 = I_1 = \underline{2.55\ A}$

" " $R_2 = I_1 = 2.55\ A$

" " $R_3 = I_1 - I_2 = 2.55 - 3.167 = -\underline{0.617}\ A$

" " $R_4 = I_2 = \underline{3.167\ A}$

Example: Find the current drawn from the source in the network shown using mesh current method



Soln: There are three meshes, Mark the mesh currents

Write KVL equations.

$$\begin{bmatrix} 19 & -12 & 0 \\ -12 & 18 & -6 \\ 0 & -6 & 18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ 0 \end{bmatrix}$$

Using Cramer's Rule.

$$I_1 = \frac{\left|\begin{bmatrix} 60 & -12 & 0 \\ 0 & 18 & -6 \\ 0 & -6 & 18 \end{bmatrix}\right|}{\begin{vmatrix} 19 & -12 & 0 \\ -12 & 18 & -6 \\ 0 & -6 & 18 \end{vmatrix}} = \underline{6\ A}$$