

UNIT X IV

Magnetic Circuits and Transformers

①

Ampere's Circuital Law

- This law gives a method to calculate the magnetic field due to a given current distribution.
- This states that the circulation $\oint \vec{B} \cdot d\vec{l}$ of the resultant magnetic field along a closed, plane curve is equal to μ_0 times the total current crossing the area bounded by the closed curve provided field inside the loop remains constant.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

μ_0 = Permeability of free space = $4\pi \times 10^{-7} \text{ wb/A-m}$.

Magnetomotive Force

- Interesting characteristic of lines of magnetic flux is that ~~the~~ each line is a closed loop.
- The complete closed path followed by any line (or group of lines) of magnetic flux is referred to as a magnetic circuit.
- electric current is due to emf
- By analogy we can say magnetic flux is due to mmf.
- This mmf is caused by the current flowing through one or more turns.

$$\text{This mmf} \propto I$$

$$\propto n$$

Ampere turns [

For a given current distribution...

(2)

$$\text{Magnetic field strength } H = \frac{NI}{l}$$

$$B = \text{Flux Density} = \mu H \text{ or } \mu_0 H \text{ if medium is air}$$

$$\text{otherwise } \mu = \mu_0 \mu_r$$

$$\text{for air } \mu_r = 1$$

$$\text{for nickel iron} = 1 \times 10^5$$

Reluctance of Magnetic Circuit

- Consider a ferromagnetic ring having a cross-sectional area A (sq. m), mean circumference l (m) and no. of turns N carrying a current I

$$\text{total flux } \phi = \text{flux density} \times \text{area}$$

$$= BA$$

- (1)

$$\text{m.m.f.} = \text{magnetic field strength} \times \text{length}$$

$$= H \cdot l$$

- (2)

$$\frac{\phi}{F} = \frac{BA}{H \cdot l} = \mu_0 \mu_r \frac{A}{l}$$

$$\phi = \frac{F}{(l / \mu_0 \mu_r A)}$$

- (3)

In an electric ckt.

$$I = \frac{E}{R}$$

(4)

Compare (3) and (4)

$$l / \mu_0 \mu_r A \Rightarrow R$$

$$\underline{\underline{=}} \text{ Reluctance } (S) \text{ (A/wb)}$$

(3)

Thus $S = \frac{1}{\mu_0 \mu_r} \frac{l}{A}$

$$R = \rho \frac{l}{A}$$

$$\therefore \frac{1}{\mu_0 \mu_r} \Rightarrow \rho$$

$$\mu_0 \mu_r \Rightarrow \frac{1}{\rho} \Rightarrow \sigma$$

magnetic flux $\phi = \frac{\text{MMF } F}{\text{reluctance } S}$

Comparison (Analogy) between Electric and Magnetic Circuit

Electric Circuit		Magnetic Circuit	
1.	EMF (E) volts	MMF (F) Amperes	
2.	-	magnetic field strength (ampere/m)	
3.	Current (I) ampere	magnetic flux (ϕ) weber	
4.	Electric Current density (J) amperes/m ²	magnetic flux density (B) Tesla Wb/m ²	
5.	$R = \rho \frac{l}{a} \Omega$	Reluctance $S = \frac{1}{\mu_0 \mu_r} \frac{l}{A}$ amperes/Wb.	
	$I = \frac{E}{R}$	$\therefore \phi = \frac{F}{S}$	

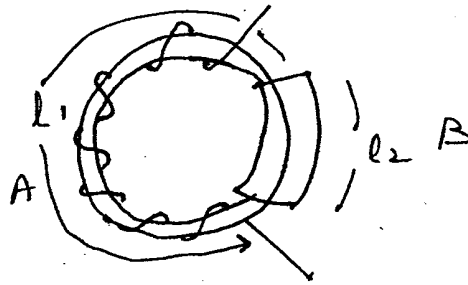
Differences

1. Immediately releases its energy as heat
2. Energy must be supplied to maintain flow of electricity

Stores energy in its field

Magnetic flux once set up doesn't require further

Composite Magnetic Circuit



— Consider the above magnetic ckt consisting of material A and B with ~~mean length~~ l_A & l_B , material A mean length l_A absolute permeability μ_A

B

 l_B μ_B

Then reluctances

$$S_A = \frac{l_A}{\mu_A A}$$

$$S_B = \frac{l_B}{\mu_B B}$$

$$\text{Total Resultant Reluctance} = S_A + S_B$$

$$= \frac{l_A}{\mu_A \cdot A} + \frac{l_B}{\mu_B \cdot B}$$

$$\text{Total flux } \phi = \frac{\text{mmf of coil}}{\text{Total reluctance}}$$

$$= \frac{NI}{\frac{l_A}{\mu_A A} + \frac{l_B}{\mu_B B}}$$

If material B happens to be air with $\mu_B = \mu_0$
 A " " steel.

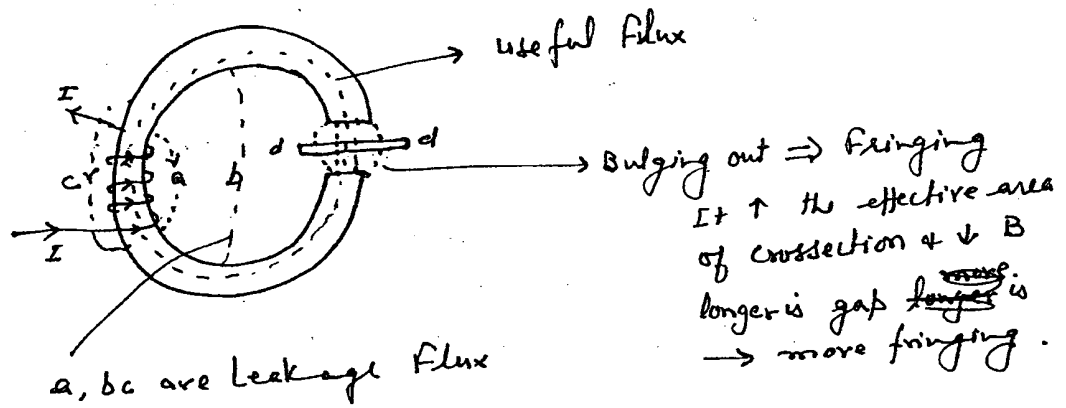
Then

$$S = \frac{l_A}{\mu_A \cdot A} + \frac{l_B}{\mu_0 \cdot B}$$

$$\mu_A \gg \mu_0$$

$$\text{So } S_A \ll S_B \text{ so } S \approx S_B$$

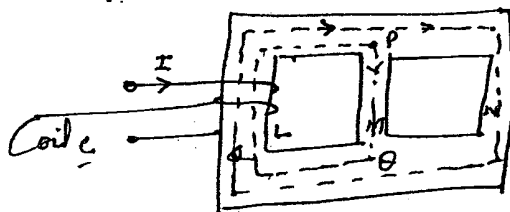
Magnetic Leakage and Fringing



Kirchhoff's Laws for Magnetic Circuits

First Law: The total magnetic flux towards a junction is equal to the total magnetic flux away from that junction.

Consider



$$\Phi_L = \Phi_m + \Phi_N$$

Second Law In a closed magnetic circuit, the algebraic sum of the product of the magnetic field strength and the length of each part of the circuit is equal to the resultant magnetomotive force.

$$\left. \begin{array}{l} \text{If magnetic field strength} \\ \text{for limb L be } H_L \\ \text{" M be } H_M \\ \text{" N be } H_N \end{array} \right\} \text{to } \left\{ \begin{array}{l} \text{Length of each} \\ \text{from Q via L to P } L_L \\ \text{" P via M to Q } L_M \\ \text{" P via N to Q } L_N \end{array} \right.$$

Then

$$\text{total mmf of Coil C} = H_L l_L + H_M l_M$$

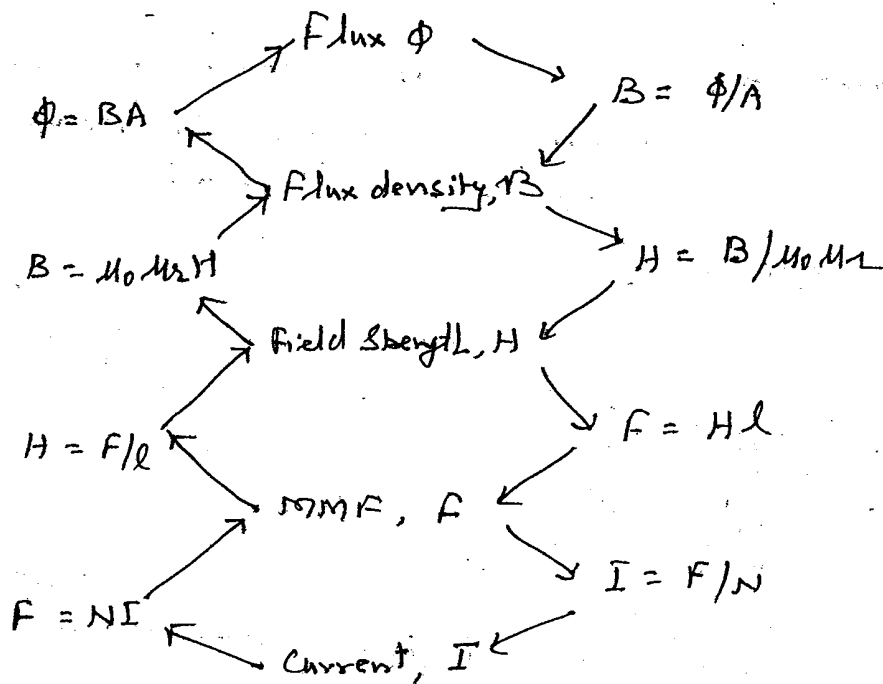
Also " " " = $H_L l_L + H_H l_H$

and $0 = H_M l_M - H_H l_H$

In general $\sum \text{mmf} = \sum Hl$

Hints to solve magnetic Circuits

1. — The form of solution in most magnetic circuits problems takes a standard form shown below



— In most problems on magnetic circuits, you are to start at or near one end of the above chain. ~~Go on down~~

— Go on doing the calculations, and move from one end to the other end.

2. The series parallel combinations of reluctances can be carried out as in case of Resistances.

~~so some problems are~~

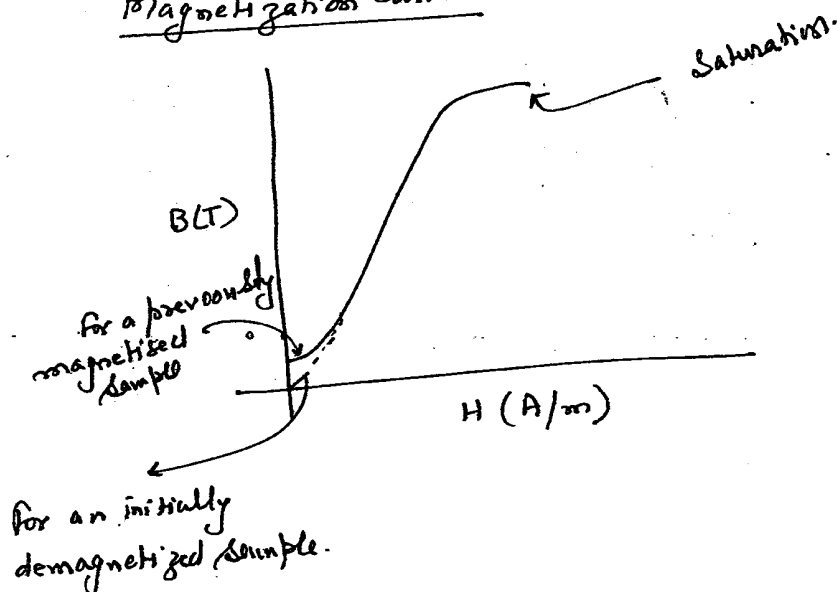
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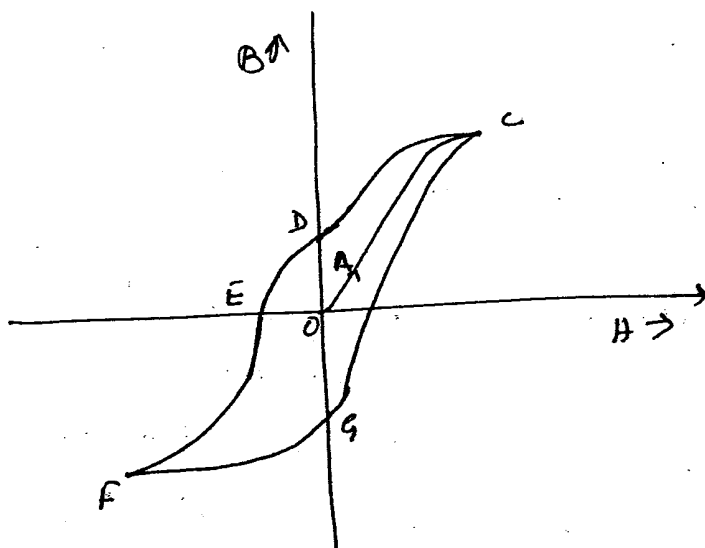
Do some problems on magnetic Circuits yourself.

Air Gap in magnetic Circuits

- The reluctance of air gap is $\uparrow\uparrow$ as compared to that of magnetic material.
- For this reason air gap puts an extra demand on the magnetizing current to create a given amount of flux density.
- Following Then why should we have air gaps in magnetic circuits.
- There two reasons
- First, is to permit part of magnetic circuit to move (e.g. motors & relays)
- Second, To make the magnetisation characteristic of the circuit more linear.

Magnetization Curve





In general

$$\begin{cases} OD, OG = \text{Residual magnetism / flux density} \\ OE = \text{Coercive force.} \end{cases}$$

If C and F are at saturation.

Then $OE = \text{Coercivity.}$

$OD = \text{Remanance / Retentivity}$

Hysteresis Loss

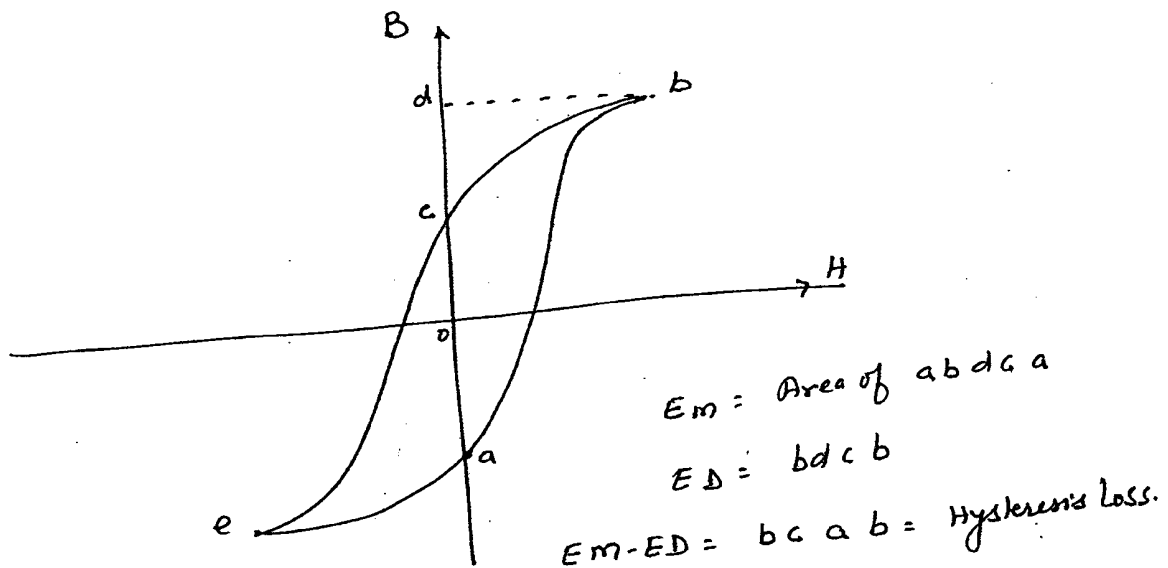
- It occurs in AC application.
- A ferromagnetic material
 - while mag being magnetised stores energy (E_m)
 - on being demagnetised stores releases energy (E_d)

Due to Hysteresis

$$E_d < E_m$$

- The difference ($E_m - E_d$) is dissipated as heat and is called Hysteresis Loss
- The Area of hysteresis loop indicates the hysteresis loss i.e. Loss.

(9)



Hysteresis loss

$$P_h = v f (K_h B_m^n)$$

B_m = Value of maximum flux density

n : $1.5 \leq n \leq 2.5$ depending upon material used.

K_h : Constant and depends on material

v = volume of material used

f = frequency.

Eddy Current Loss

- It arises due to the circulating currents that are formed found to exist in closed paths within the body of a ferromagnetic material.
- If Ferromagnetic material used is a good conductor of electricity, R is small so high values eddy currents would mean \uparrow eddy current loss.
- For this reason the magnetic core is made of thin laminations to $\uparrow R$ and so to \downarrow eddy currents.

Electromagnetic Induction and Force

- If flux ϕ passes through a coil of N turns it is said to cause flux linkages

$$\lambda = N \phi \quad \text{wb-T}$$

- If λ changes the emf is induced in the coil given by

$$e = - \frac{d\lambda}{dt} = -N \frac{d\phi}{dt}$$

- The $-$ sign is due to Lenz's law which states that

the emf ~~will~~ would be induced in a direction which would tend to cause a current flow in the coil so as to oppose the change in flux. (which is the cause of emf induction).

The change in flux can occur in two/^{three} ways

1. when coil is fixed and flux changes with time (statically induced emf) transformers
2. when B is constant with time and stationary in space, but coil moves relative to B (dynamically induced emf) DC Generators
3. when B is constant with time but moves in space & coil is fixed. (AC alternators).

(11)

Dynamically induced emf is

$$e = |\vec{v} \times \vec{B}| l = Blv \sin \theta \text{ volts}$$

B = flux density (T)

l = conductor length

v = speed (m/s) at which conductor cuts the flux

θ = angle between flux direction and the conductor speed.

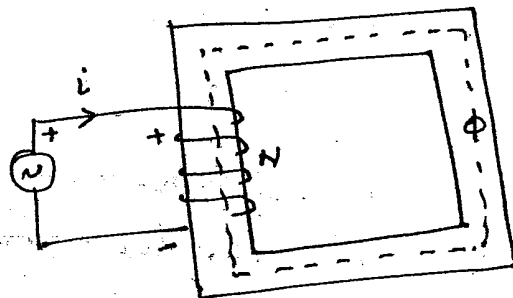
In electrical m/c $\theta = 90^\circ$,

$$\therefore e = Blv \text{ volts}$$

Fleming's L.H Rule
" R.H Rule.

Inductances : Self and Mutual

SELF
Consider



Φ is induced by i in coil and links with this coil only.

Φ = Self Flux

L = Self Inductance

$$e = \frac{d\lambda}{dt} = \frac{d\lambda}{di} \cdot \frac{di}{dt}$$

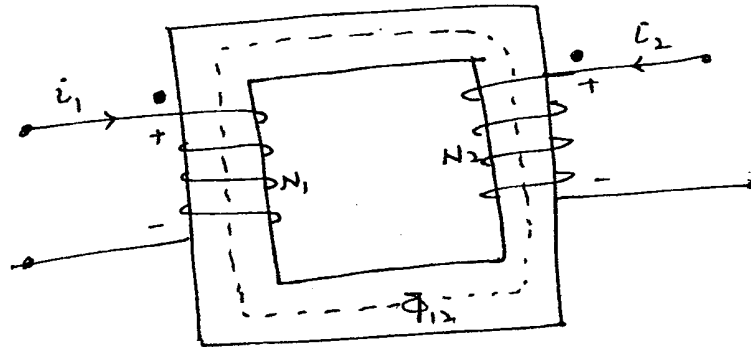
$$= \left(\frac{d\lambda}{di} \right) \frac{di}{dt}$$

$$= L \frac{di}{dt}$$

$$L = \frac{d\lambda}{di} = \text{Self Inductance}$$

Mutual Inductance

Consider



The flux produced by one coil links with the other coil too.

The Mutual Inductance between the coils is defined as

$$L_{12} \text{ (or } M_{12}) = \frac{\lambda_{12}}{i_1}$$

$$L_{21} \text{ (or } M_{21}) = \frac{\lambda_{21}}{i_2}$$

λ_{12} = flux linkage of coil 1 due to current in coil 2

λ_{21} = flux linkage of coil 2 due to the current in coil 1.

For a bilateral circuit

$$M_{12} = M_{21}$$

In linear magnetic circuits

$$M = k \sqrt{L_1 L_2}$$

k = coefficient of coupling = 1 for tight coupling

Dot Convention

- This defines the sign in mutual inductance
- Current flowing into the dotted terminal of each coil produces mutual flux in the same direction in the core.

Coupled Circuits

L_1, L_2 = Self Inductances of two coils

M = Mutual Inductance between two coils $= k \sqrt{L_1 L_2}$

k = coefficient of coupling

for tight coupling $k=1$

Flux linkages of coils can be expressed as

$$\lambda_1 = L_1 i_1 + M i_2$$

$$\lambda_2 = M i_1 + L_2 i_2$$

Induced emfs of the coils can then be written as

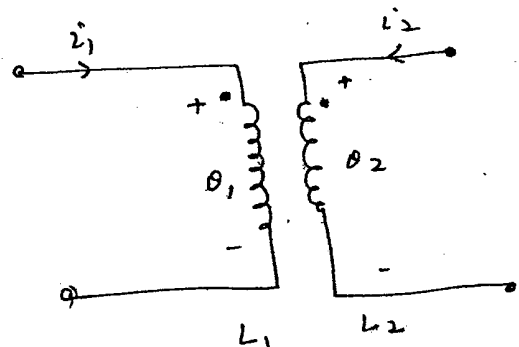
$$e_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$e_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

For AC steady state analysis

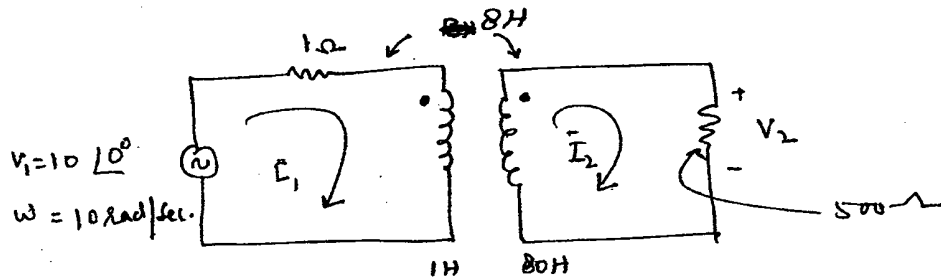
$$\bar{E}_1 = j\omega L_1 \bar{I}_1 + j\omega M \bar{I}_2$$

$$\bar{E}_2 = j\omega M \bar{I}_1 + j\omega L_2 \bar{I}_2$$



Example For the circuit given find

$$\bar{I}_2, \bar{V}_2 \text{ and } \bar{V}_2 / \bar{V}_1$$



Soln: Writing the mesh equation for two meshes

$$(1 + j10) \bar{I}_1 - j10 \times 8 \bar{I}_2 = 10 \angle 0^\circ \quad (\text{Mesh 1}) \quad \text{---(1)}$$

$$\Rightarrow (1 + j10) \bar{I}_1 - j80 \bar{I}_2 = 10 \angle 0^\circ$$

$$-j10 \times 8 \bar{I}_1 + (500 + j10 \times 80) \bar{I}_2 = 0 \quad (\text{Mesh 2})$$

$$j80 \bar{I}_1 - (500 + j800) \bar{I}_2 = 0 \quad \text{---(2)}$$

$$\bar{I}_1 = \frac{(500 + j800) \bar{I}_2}{j80} = (10 - j6.25) \bar{I}_2$$

Put in (1)

$$(1 + j10) (10 - j6.25) \bar{I}_2 - j80 \bar{I}_2 = 10 \angle 0^\circ$$

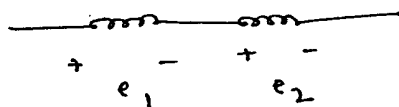
$$\bar{I}_2 = \frac{1.135 \angle -10.7^\circ}{1} \text{ Ans.}$$

$$\bar{V}_2 = 500 \bar{I}_2 = \frac{68 \angle -10.7^\circ}{1} \text{ Ans}$$

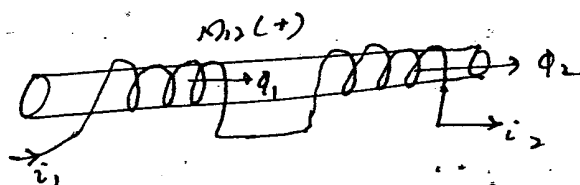
$$\frac{\bar{V}_2}{\bar{V}_1} = \frac{6.8 \angle -10.7^\circ}{1} \text{ Ans}$$

15 (1)

$$M = M_{12}(+)$$



$e_1 + e_2$ = induced voltage



ϕ_1 & ϕ_2 are in the same direction

$$e_1 = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt}$$

if $i_1 = i_2 = i$

$$e_1 = L_1 \frac{di}{dt} + M_{12} \frac{di}{dt}$$

$$= (L_1 + M_{12}) \frac{di}{dt}$$

Also $e_2 = (L_2 + M_{12}) \frac{di}{dt}$

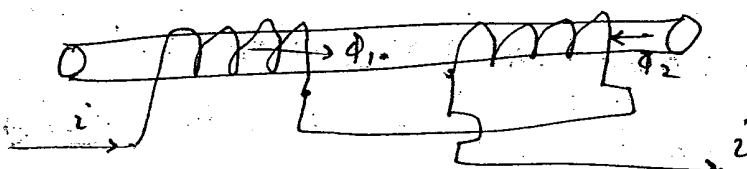
$$e_T = e_1 + e_2 = (L_1 + M_{12}) \frac{di}{dt} + (L_2 + M_{12}) \frac{di}{dt}$$

$$= (L_1 + L_2 + M_{12} + M_{12}) \frac{di}{dt}$$

$$L_T(+) = L_1 + L_2 + 2 M_{12}$$

(1)

But if coils are connected in the opposite direction



ϕ_1 & ϕ_2 are in opposite direction

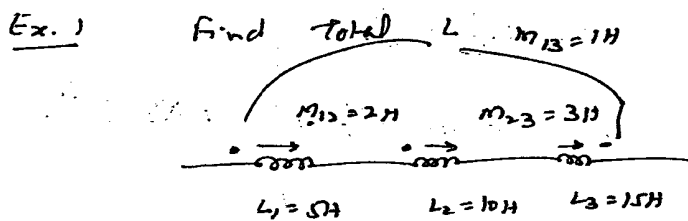
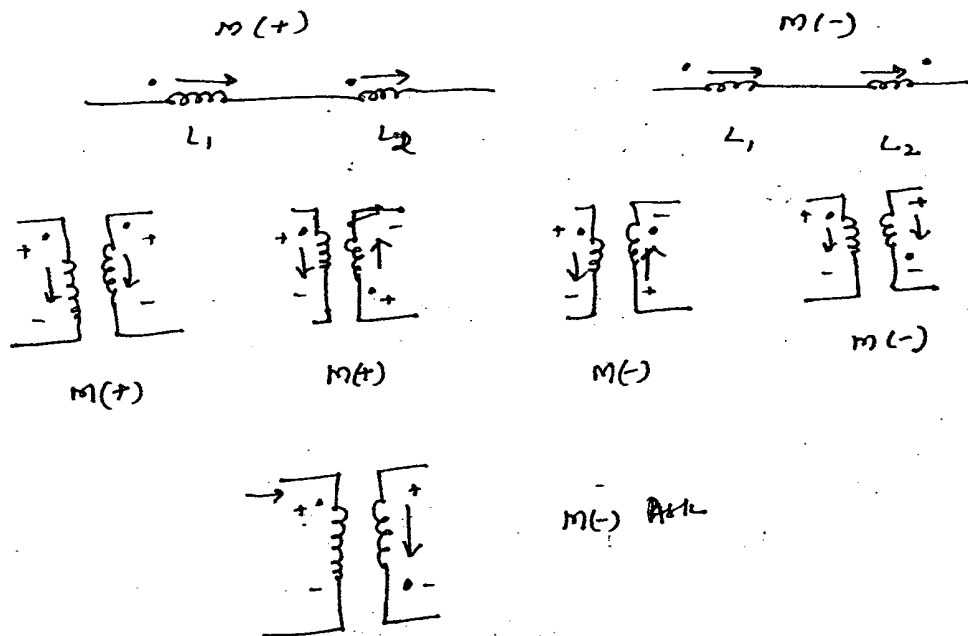
$$L_T(-) = L_1 + L_2 - 2 M_{12}$$

(2)

From eqs (1) and (2)

$$M = \frac{1}{4} [L_T(+) - L_T(-)] \quad \text{--- 3}$$

The manner in which coils are wound/connected can be represented by the dot (.) convention.

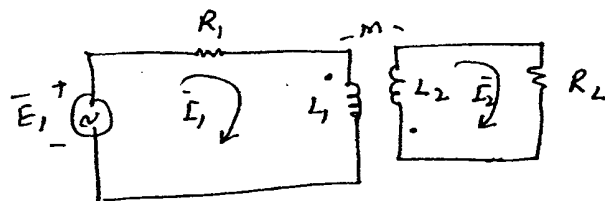


$$\left. \begin{aligned} \text{coil 1: } & L_1 + M_{12} - M_{13} \\ \text{coil 2: } & L_2 + M_{12} - M_{23} \\ \text{coil 3: } & L_3 - M_{23} - M_{13} \end{aligned} \right\}$$

$$L_T = L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{13}$$

$$L_T = 5 + 10 + 15 + 2(2) - 2(3) - 2(1) = 34 - 8 = \underline{26H}$$

Ex 2 Write the mesh eq = as



Loop 1

$$\bar{E}_1 - \bar{I}_1 \bar{R}_1 - \bar{I}_1 \bar{X}_{L1} - \bar{I}_2 \bar{X}_m = 0$$

Loop 2

$$- \bar{I}_2 \bar{X}_{L2} - \bar{I}_1 \bar{X}_m - \bar{I}_2 \bar{R}_L$$

TRANSFORMERS

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- It is an electric machine with no moving part
- It is used to transfer electric power from one circuit to another
 - at the same frequency
 - at different voltages and different current values

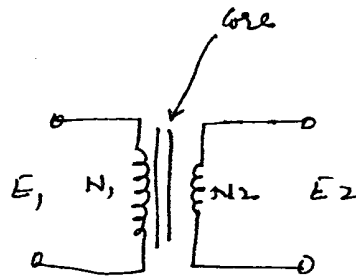
Parts

- | | |
|-----------------------------|------------------------|
| 1. Windings - 2 copper | 3. Tank with oil |
| 2. Core - magnetic material | 4. Bushings/Terminals. |
| Windings are insulated | 5. Conservator |
| | 6. Breather |
| | 7. Buchholz relay. |
- from core - paper and varnish.
 - from layer to another layer - paper
 - from one turn to another turn - varnish

Applications

1. Generator Transformer
2. For Transmission
3. For Distribution
4. In electronic circuits for power supply.
5. Impedance matching
6. Instrumentation : CT & PT.

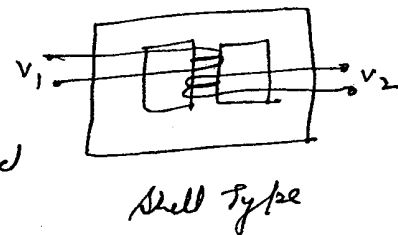
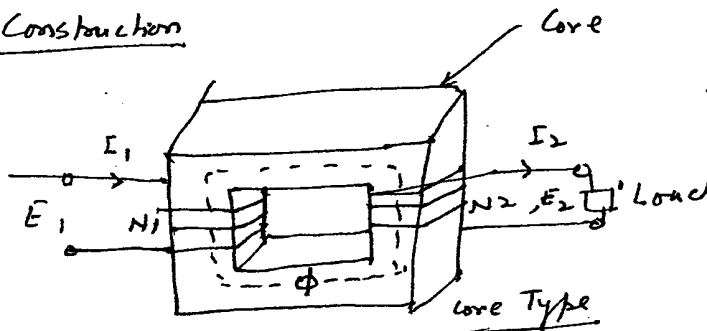
Symbol



$N_1 > N_2$ step down $N_2 < N_1$

$N_1 < N_2$ step up $N_2 > N_1$

Construction



EMF EQUATION

magnetic flux due to applied voltage E ,

$$\phi = \phi_m \sin \omega t$$

ϕ_m = peak value of flux

ω = angular frequency = $2\pi f$,

the induced emf $= -N \frac{d}{dt} (\phi_m \sin \omega t)$

$$e = -N_1 \frac{d\phi}{dt} = -\omega N_1 \phi_m \sin(\omega t + \pi/2) \cos \omega t$$

$$= N_1 \omega \phi_m \sin(\omega t - \pi/2)$$

peak value of induced emf = $2\pi f N \phi_m$

the rms value

$$E = \frac{1}{\sqrt{2}} \cdot 2\pi f N \phi_m = 4.44 f N \phi_m$$

Example: The primary windings of a 50 Hz step-down transformer has 480 turns and fed from 6400 V supply.

Find

- peak value of the flux in core
- secondary voltage if secondary windings have 20 turns

Soln.

$$\phi_m = \frac{E}{4.44 f N_1} = \frac{6400}{4.44 \times 50 \times 480} = 0.06 \text{ wb Ans}$$

$$E_2 = 4.44 f N_2 \phi_m = 4.44 \times 50 \times 20 \times 0.06 = 266.4 \text{ V Ans}$$

Transformation Ratio

$$E_1 = 4.44 f N_1 \phi_m$$

$$E_2 = 4.44 f N_2 \phi_m$$

Divide.

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = \text{Transformation Ratio} \\ = \text{Turn Ratio} = K$$

For an Ideal Transformer

There are no losses.

$$E_1 I_1 = E_2 I_2$$

$$\frac{I_1}{I_2} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{1}{K}$$

Impedance Ratio

The impedance of Z_2 as seen from primary is Z_2'

$$Z_2' = \frac{V_1}{I_1} = \frac{V_1}{I_1} \frac{V_2 I_2}{V_2 I_2} = \left(\frac{V_1}{V_2} \right) \left(\frac{I_2}{I_1} \right) \left(\frac{V_2}{I_2} \right)$$

$$= K \cdot K \cdot Z_2 = K^2 Z_2$$

(21)

(18)

Example: A 1-phase transformer has a core with cross-sectional area of 150 cm^2 . It operates at a maximum flux density of 1.1 Wb/m^2 from a 50 Hz supply. If the secondary winding has ~~resistance~~ 66 turns, determine output in kVA when connected to a load of 4Ω impedance. Neglect any voltage drop in the transformer.

Soln. $B_m = 1.1 \text{ Wb/m}^2$ $A = 150 \text{ cm}^2 = 0.015 \text{ m}^2$

$$\phi_m = B_m A = 1.1 \times 0.015 = 0.0165 \text{ Wb.}$$

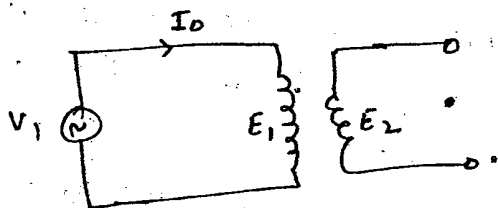
$V_2 \approx E_2$ as voltage drop is neglected.

$$V_2 = 4.44 f N_2 \phi_m = 4.44 \times 50 \times 66 \times 0.0165 = 241.76 \text{ V}$$

output current $I_2 = \frac{V_2}{Z_2} = \frac{241.6}{4} = 60.44 \text{ A}$

output VA = $\frac{241.76 \times 60.44}{1000} = 14.61 \text{ kVA Ans.}$

Transformer on no Load



$$\phi = \phi_m \sin \omega t$$

$$E_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d(\phi_m \sin \omega t)}{dt}$$

$$= -N_1 \omega \phi_m \cos \omega t = N_1 \omega \phi_m \sin(\omega t - \pi/2)$$

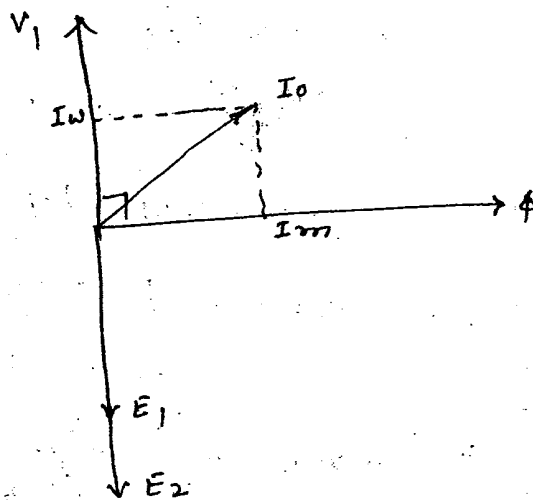
$$E_2 = N_2 \omega \phi_m \sin(\omega t - \pi/2)$$

$$V_1 = -E_1 = -N_1 \omega \phi_m \sin(\omega t - \pi/2) = N_1 \omega \phi_m \sin(\omega t + \pi/2)$$

This means

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1. E_1 & E_2 are in phase
2. V_1 leads ϕ by $\pi/2$
3. V_1 is in phase opposition to E_1
4. $V_2 = E_2$ no voltage drop considered as $r_2 = 0$



input power at no load = $V_1 I_0 \cos \phi_0$

reactive power = $V_1 I_0 \sin \phi_0$

Example: A 230V/110-V, 1-phase transformer takes an input of 350 VA at no load while working at no-load rated voltage. Core loss is 110 watts. Find iron loss component of no load current, the magnetising component of no load current and no load p.f.

Soln:

$$V_1 I_0 = 350 \text{ VA}$$

$$I_0 = \frac{350}{230} = 1.52 \text{ A}$$

Core loss, input power at no load

$$P_i = V_1 I_0 \cos \phi_0$$

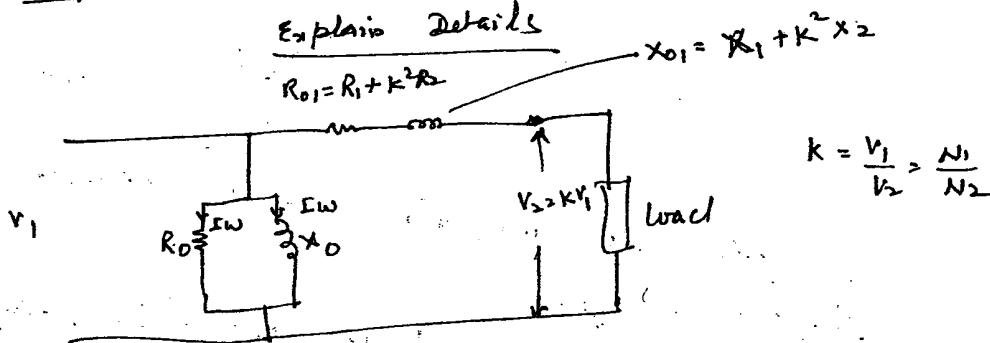
$$\cos \phi_0 = \frac{P_i}{V_1 I_0} = \frac{110}{350} = 0.314 \text{ Ans}$$

$$I_w = I_0 \cos \phi_0 = 1.52 \times 0.314 = 0.478 \text{ A Ans}$$

$$I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{(1.52)^2 - (0.478)^2} = 1.44 \text{ A Ans.}$$

Equivalent ckt. of Transformer

Explain Details



Also

$$R_{02} = R_2 + \frac{1}{k^2} R_1$$

$$X_{02} = X_2 + \frac{1}{k^2} X_1$$

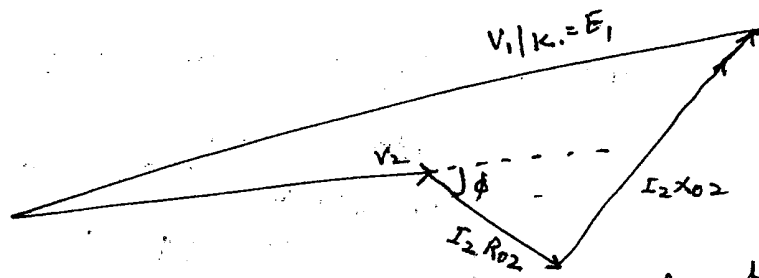
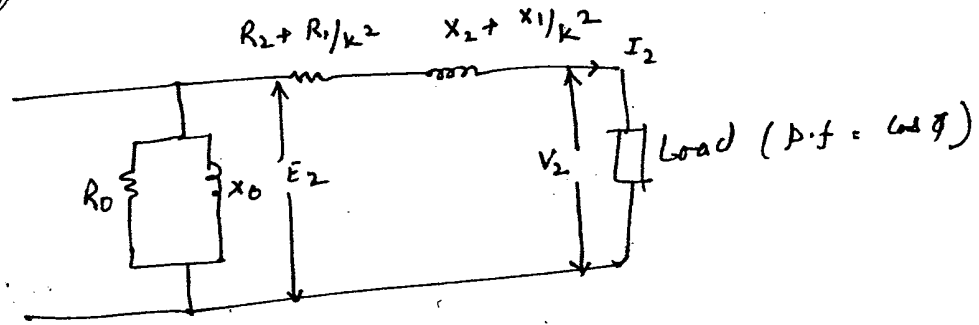
Explain OC & SC Test

& η calculations.

Voltage Regulation

$$\% \text{ Reg} = \frac{E - V}{V} \times 100$$

Equivalent Ckt. of Transformer referred to Secondary. (24)



With this diagram considering similar triangles you can find

$$\% \text{ Regulation} = \frac{I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi}{V_2} \times 100\%$$

Condition for Zero Regulation

$$I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi = 0$$

$$\tan \phi = -\frac{R_{02}}{X_{02}}$$

i.e. leading p.f.

Condition for maximum Regulation

$$\frac{d}{d\phi} (I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi) = 0$$

$$\tan \phi = \frac{X_{02}}{R_{02}}$$

Condition for Maximum η

$$\eta = \frac{V_2 \cos \phi_2}{V_2 \cos \phi_2 + I_2 R_{02} + P_i / I_2}$$

η is maximum when ~~losses~~ are minimum

$$\text{i.e. } \frac{d(I_2 R_{02} + P_i / I_2)}{dI_2} = 0$$

$$R_{02} - \frac{P_i}{I_2^2} = 0$$

$$\text{or } I_2^2 R_{02} = P_i$$

i.e. Copper losses = Iron losses.

Distribution Transformer

but at less load

It operates all the time. There fore W_0 should be ↓. So it designed to have max. η at low load say 70%.

$$I_2^2 = \frac{W_0}{R_{02}} \quad \text{if } W_0 \text{ is } \downarrow \quad I_2^2 \text{ is also } \downarrow$$

Generator Transformer

It operates at full load so Cu losses should be low. It doesn't operate all the time. It may be designed to have η_{\max} at full load.

Auto Transformer

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- The operating principle of an auto transformer is same as that of conventional two-winding transformer.

Differences

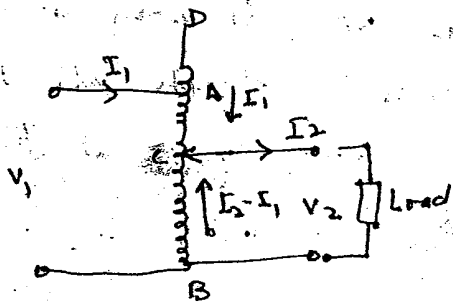
- It is a single-winding transformer.
- The same winding serve the purpose of primary as well as the secondary winding.
 - Therefore, the windings are not electrically isolated.

Construction

Two Types

There is a continuous winding with taps brought out at convenient points to determine by the desired secondary voltages.

Instead of bringing out the taps, there is variable/sliding contact at which variable voltage can be obtained.



Important:

- The winding (primary is AB)
- It is extended to D, so as to obtain a secondary voltage $V_2 > V_1$ upto 15% of V_1 .
- Secondary voltage V_2 is obtained between BC.

If No. of turns $AB = N_1$

" " $BC = N_2$

Then
$$\frac{V_2}{V_1} = \frac{E_{BC}}{E_{AB}} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = k \text{ transformation ratio}$$

k can have a value from 0 to 1.15.

Advantages

1. Continuously variable voltage is obtained
2. Less Cu required and η is \uparrow than that of two winding transformers.

Limitations

- The objective remains to obtain a variable voltage.
- The power ratings are ~~too~~ low.

Applications

1. Starting of induction motors and synchronous motors
2. Testing and learning laboratories
3. Regulating transformers (tap changing transformers)
4. furnace transformers