Third Assignment
B.Tech 2nd semester(Code: MA-102)

Special Functions

Q 1. Classify the singular point of the following equations:

(a) $(1-x^2)y'' + 2xy' + n(n+1)y = 0$

(b) $x^3(x-2)y'' + x^3y' + 6y = 0$

Q 2. Find the power series solution of the following differential equations.

(a) $y'' + (x-1)y' + y = 0$ about $x = 2$.

(b) $(1-x^2)y'' + 2xy' + y = 0$, $y(0) = 1$, $y'(0) = 1$ about $x = 0$.

(c) $y'' - y = 0$, $y(0) = 2$, $y'(0) = 0$ about $x = 0$.

Q 3. Find the two linear independent solution of the equations $2x^2y'' + xy' - (x^2+1)y = 0$

Q 4. Find the series solution about $x = 0$ of the equations $xy'' + y' - xy = 0$.

Q 5. Find the series solution about $x = 0$ of the given differential equation $x^2y'' + x^3y' + (x^2-2) = 0$.

Q 6. Solve the equation

$$x(1-x)\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

Q 7. Prove that the legendre's polynomial is given by

$$P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$$

Q 8. Using the recurrence $(n+1)P_{n+1}(x) = (2n+1)P_n(x) - nP_{n-1}(x)$, evaluate $P_2(1.5)$ and $P_3(2.1)$

Q 9. Prove that the given recurrence relation for $P_n(x)$;

$$P_n'(x) = xP_{n-1}'(x) - nP_{n-1}(x)$$

Q 10. Prove the following orthogonality properties of Special function:

(a)

$$\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & m \neq n \\ \frac{2}{2n+1} & m = n \end{cases}$$

(b) For $\nu \geq 0$ and $n = 1, 2, \cdots \ldots$

$$\int_{0}^{1} x\, J_\nu(j_n x)\, J_{-\nu}(j_m x)\, dx = \begin{cases} 0 & m \neq n \\ \frac{1}{2}[J_{\nu+1}(j_n)]^2 & m = n \end{cases}$$

where j_n and j_m are the zeros of $J_\nu(x)$

Q 11. Prove that the Bessel's function of the first kind satisfy the following recurrence relation:

(a) $xJ_\nu'(x) = \nu J_\nu(x) - \nu J_\nu(x)$

(b) $2\nu J_\nu(x) = x\left(J_{\nu-1}(x) + J_{\nu+1}(x)\right)$

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