ASSIGNMENT-1

UNIT- 1: INFINITE SERIES

(B.Tech. Semester - I)

1. Discuss the convergence and divergence of Geometric Series.
2. Using limit comparison test, examine the convergence of the series:

$$\frac{\sqrt{2}-1}{3^3-1}+\frac{\sqrt{3}-1}{4^3-1}+\frac{\sqrt{4}-1}{5^3-1}+\cdots$$

Answer: Convergent.

3. Discuss the convergence of p – series using
   i) Comparison test
   ii) Integral test
4. Verify whether the infinite series converges or diverges:

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\cdots$$

Answer: Convergent.

5. Examine the convergence of the following series:

$$\sum_{n=1}^{\infty}\frac{\sqrt{n+1}-\sqrt{n}}{n^p}$$

Answer: Convergent if $p > {}^1/_2$ and divergent if $p \le {}^1/_2$.

6. Discuss the convergence of the series:
   a. $1+1+\frac{2^p}{2!}+\frac{3^p}{3!}+\frac{4^p}{4!}+\cdots (p>0)$ Answer: Convergent.
   b. $\sum_{n=1}^{\infty}\frac{n!}{5^n}$ Answer: Divergent.
   c. $\sum_{n=1}^{\infty}\frac{n!}{n^n}$ Answer: Convergent.
7. Show that the series

$$\sum\frac{3.6.9\ldots 3n}{7.10.13\ldots(3n+4)}x^n\ ,x>0$$

Converges for $x \le 1$, and diverges for $x > 1$.

8. Find the nature of the series

$$\sum\frac{(n!)^n}{(2n!)}x^n,\qquad (x>0)$$

Answer: Divergent.

9. State and prove Integral test and hence determine all the values of p for which the series $\sum e^{-np}$ converges.

Answer: The series converges for all $p > 0$.

10. Determine if the following series converges or diverges:

$$1+\frac{1}{2\sqrt[3]{2}}+\frac{1}{3\sqrt[3]{3}}+\frac{1}{4\sqrt[3]{4}}+\cdots$$

Answer: Convergent.

11. Test for the convergence of the series

$$\frac{x}{1+x}-\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}-\cdots \quad (0<x<1).$$

Answer: Convergent.

12. Test the convergence for the following series:

$$\frac{1}{a.1^2+b}+\frac{1}{a.2^2+b}+\frac{1}{a.3^2+b}+\ \ldots$$ where a and b are real numbers.

Answer: Convergent.

13. Discuss the convergence of the series

$$\frac{1}{1^p}+\frac{1}{3^p}+\frac{1}{5^p}+\ \ldots$$

Answer: Convergent for $p>1$ and divergent for $p\leq 1$.

14. Examine the convergence of the following series:

$$\sum_{n=1}^{\infty}\frac{1}{(an+\mathrm{b})^{\mathrm{p}}}$$

Answer: Converges for $p>1$and divergent for $p\leq 1$.

15. Test the convergence of the series $\sum_{n=1}^{\infty}\frac{x^{n-1}}{1+x^n}$; $x>0$

Answer: Converges for $0<x<1$and divergent for $x\geq 1$.

16. Discuss the convergence of the series:

(a) $\frac{1^2 2^2}{1!}+\frac{2^2 3^2}{2!}+\frac{3^2 4^2}{3!}+\ \ldots\ldots\ldots\ldots$ Answer: convergent.

(b) $\frac{1}{3}+\frac{1\cdot 2}{3\cdot 5}+\frac{1\cdot 2\cdot 3}{3\cdot 5\cdot 7}+\frac{1\cdot 2\cdot 3\cdot 4}{3\cdot 5\cdot 7\cdot 9}+\ \ldots\ldots$ Answer: convergent.

17. Discuss the convergence or divergence of the following series :

$$\frac{x^2}{1}+\frac{2^2\,x^4}{3\cdot 4}+\frac{2^2\cdot 4^2\,x^6}{3\cdot 4\cdot 5\cdot 6}+\frac{2^2\cdot 4^2\cdot 6^2\,x^8}{3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8}+\ \ldots$$

Answer: converges for $x^2\leq 1$, diverges for $x^2>1$.

18. State and prove Leibnitz test and hence determine whether the following series converges

$$\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$$

Answer: Convergent.

19. Define Absolute Convergence and Conditional Convergence of a series $\sum_{n=1}^{\infty} a_n$.

Test the convergence and absolute convergence of the series:

a. $1 - \frac{1(1+2)}{2^3} + \frac{1(1+2+3)}{3^3} - \frac{1(1+2+3+4)}{4^3} + \ldots$ Answer: Conditionally convergent.

b. $\sum_{n=1}^{\infty} (-1)^{n-1} \text{Sin} \frac{1}{n}$ Answer: Conditionally convergent.

20. Test the convergence and absolute convergence of the series:

$$\sum_{n=1}^{\infty} (-1)^n \frac{n+2}{2^n + 5}$$

Answer: Absolutely convergent.

21. Show that the series $\sum_{n=1}^{\infty} (1 - \text{Cos} \frac{\pi}{n})$ converges.