pressure and its Measurment

1. Fluid pressure at a point

consider a small area dA in large mass of fluid. If fluid is stationary, then force extered by surrounding fluid on the area dA will always be perpendicular to the Surface dA.

Then intensity of pressure or simply pressure is p

$$p = \frac{dF}{dA}$$

If force is F is uniformly distributed over area (A), then pressure at any point is given by

$$p = \frac{F}{A}$$

unit of pressure is in S.I unit N/m^2

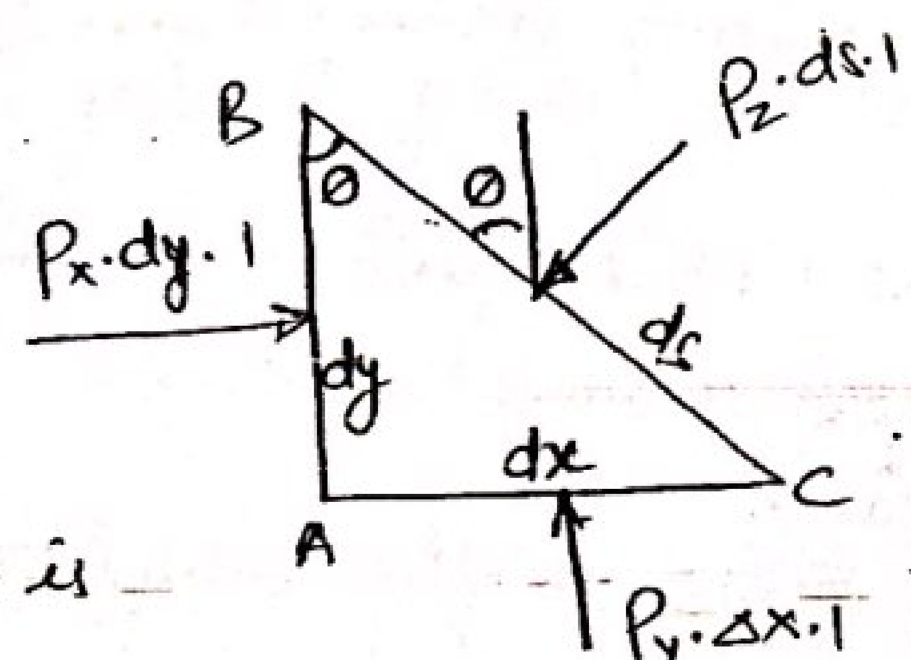
$$1\ N/m^2 = 1 \text{ pascal } (1\ pa)$$

$$1\ bar = 10^5\ pa = 10^5\ N/m^2$$

2. PASCAL'S LAW

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.

~~let~~ consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in figure.



Let width of the element perpendicular to plane of paper is unity. let P_x, P_y and P_z are the pressures acting on face AB, AC and BC respectively. Let $\angle ABC = \theta$

Resolving the forces in x-direction

$$P_x\, dy - P_z\, ds \cos\theta = 0$$

$$P_x\, dy - P_z\, dy = 0 \quad [\because dy = ds\cos\theta]$$

$$P_x = P_z \quad \text{—— (1)}$$

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Similarily, resolving the forces in y-direction, we get

$$P_y\, dx^{\times 1} - P_z\, ds \cos(90-\theta)^{\times 1} - \cancel{\frac{dx\,dy}{2} \times 1 \times \rho \times g} = 0$$

[ρ = density of fluid]

$$P_y\, dx - P_z\, ds \sin\theta$$

$$P_y\, dx - P_z\, dx \quad [\because dx = ds\sin\theta]$$

[Here $\frac{dx\,dy}{2} \times \rho \times g$ is very small so it is neglected]

$$\Rightarrow P_y = P_z \quad \text{--- (2)}$$

from eqn (1) & (2)

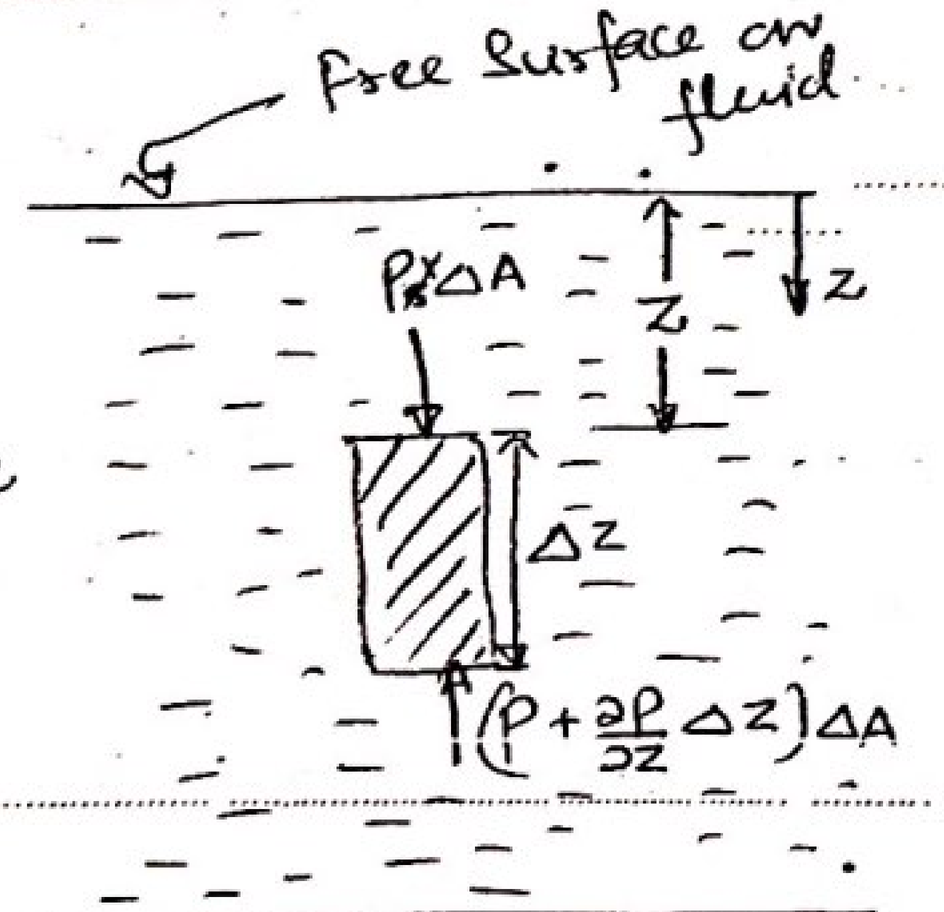
$$\boxed{P_x = P_y = P_z}$$

The above equation shows that the pressure at any point in x, y and z directions is equal. So pressure at any point is same in all directions.

PRESSURE VARIATION IN A FLUID AT REST

The pressure at any point in a fluid is obtained by the Hydrostatic Law which states that the rate of increase of pressure in a vertical downward direction must be equal to the specific weight of fluid at that point.

Consider an small element shown in figure. 2



Forces on a fluid element

fig.-2

Let ΔA = cross-sectional area of element

ΔZ = Height of fluid element

p = pressure on AB

Z = Distance of fluid surface from free surface.

For equilibrium of fluid element

$$p\Delta A - \left(p + \frac{\partial p}{\partial z}\Delta z\right)\Delta A + \rho g(\Delta A \Delta z) = 0$$

$$-\frac{\partial p}{\partial z}\Delta z\, \Delta A + \rho g \Delta A \Delta z = 0$$

$$\boxed{\frac{\partial p}{\partial z} = \rho g = w} \quad \text{--- (1)}$$

$w = \rho g$ = weight density of fluid

Above eqⁿ (1) states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is Hydrostatic law.

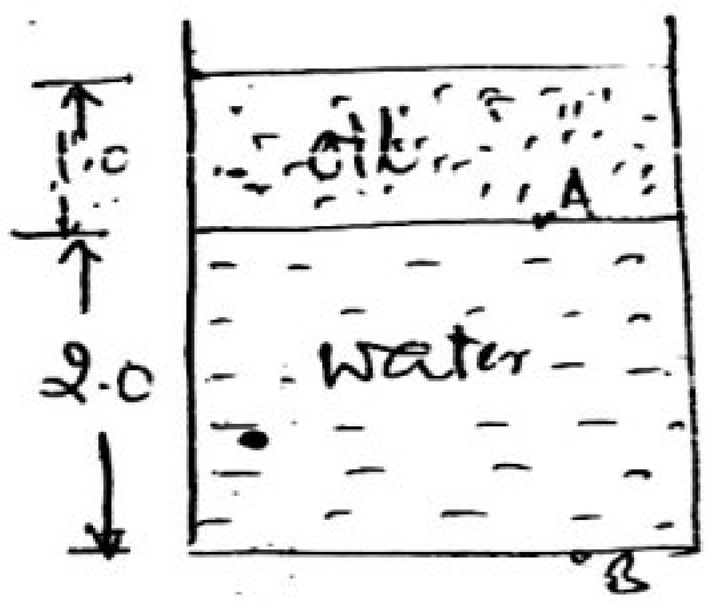
From above eqⁿ (1):

$$\int dp = \int \rho g \, dz = \rho g z$$

$$\boxed{p = \rho g z}$$

$$z = \frac{p}{\rho g}$$ z is called pressure head.

Q. An open tank contains water upto a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth of 1 m. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.



Solution: Given height of water, $z_1 = 2$ m

" " oil, $z_2 = 1$ m

Sp. gr. of oil $S_o = 0.9$

Density of water $\rho_1 = 1000 \; kg/m^3$

Density of oil $\rho_2 = 0.9 \times 1000 = 900 \; kg/m^3$

Pressure intensity at

1. interface i.e at A, $p = \rho_2 g \times 1.0$

$= 900 \times 9.81 \times 1.0 = 8829 \; N/m^2$

$= \frac{8829}{10^4} = 0.8829 \; N/cm^2$ Ans.

ii) At the bottom, $p = \rho_2 g z_2 + \rho_1 g z_1$

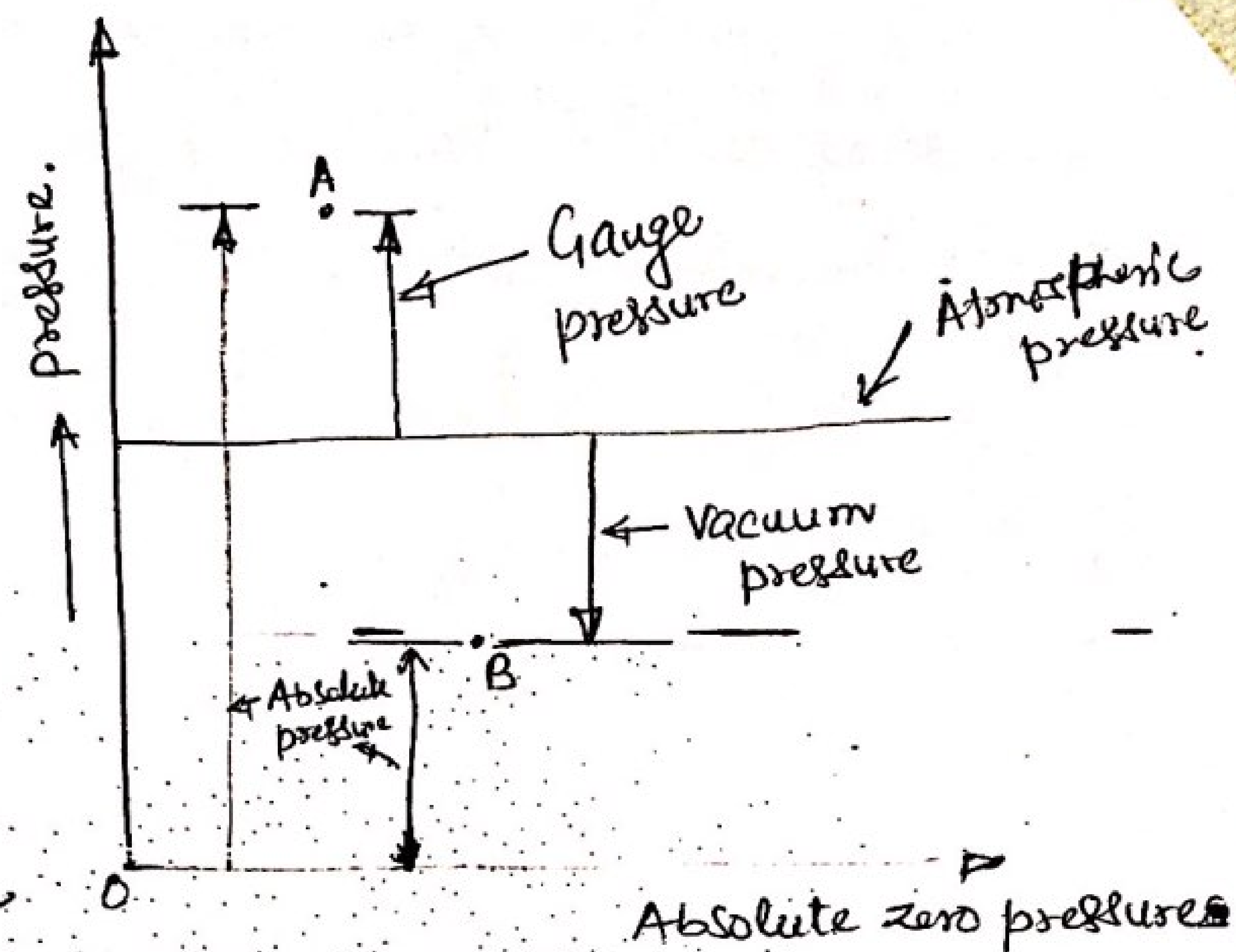
$= 900 \times 9.81 \times 1. + 1000 \times 9.81 \times 2.0$

$= 28449 \; N/m^2 = \frac{28449}{10^4} \; N/cm^2$

$= 2.8449 \; N/cm^2$

ABSOLUTE, GAUGE, ATMOSPHERIC AND VACUUM PRESSURES

1. Absolute pressure ; pressure which is measured with reference to absolute vacuum pressure.



Relationship between pressures

2. Gauge pressure ; pressure which is measured with help of pressure measuring instruments, in which the atmospheric pressure is taken as datum. The atmospheric pressure on scale is marked as zero.

3. vacuum pressure ; pressure below the atmospheric pressure

Absolute pressure = Atmospheric pressure + Gauge pressure

$$P_{abs} = P_{atm} + P_{gauge}$$

Vacuum pressure = Atmospheric pressure − Absolute pressure

Note : (I) Atmospheric pressure at sea level at $15^{\circ}C$ is $101.3\ kN/m^2$

(II) The atmospheric pressure head 760 mm of mercury or 10.33 m of water.

MEASURMENT OF PRESSURE

The pressure of a fluid is measured by the following devices:

1. Manometers

2. Mechanical Gauges.

Manometers ! devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as (a) Simple manometers (b) Differential manometers.

2. Mechanical Gauges? Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by spring or dead weight. The commonly used pressure gauges are :

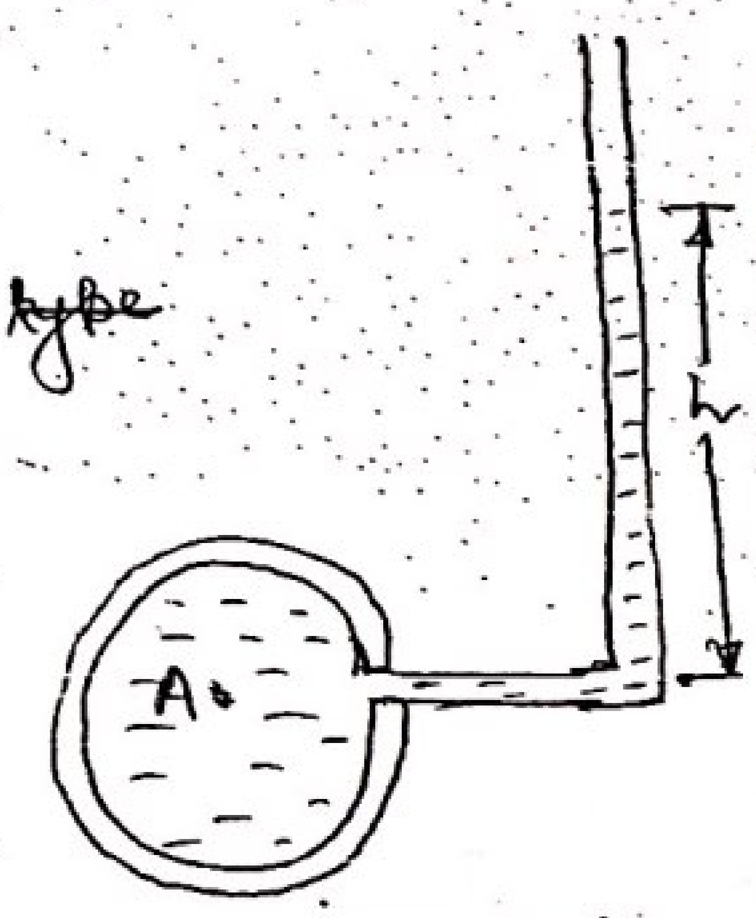
(a) Diaphragm pressure gauge . (b) Bourdon tube pressure gauge
(c) Dead-weight pressure gauge, and (d) Bellows pressure gauge.

SIMPLE MANOMETER'S

Simple manometer consists of a glass tube having one of its end connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are

1. piezometer
2. U-tube manometer.

1. piezometer: It is simplest ~~type~~ of form of manometer used for measuring gauge pressures. One end of this manometer is connected to point where pressure is to be measured and other end open to atmosphere as shown in figure. The rise of liquid gives the pressure head at that point.

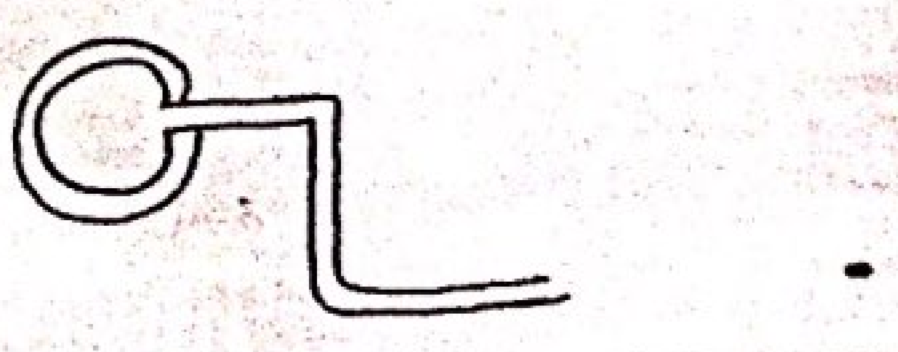


piezometer

The pressure at A $= \rho \times g \times h \ N/m^2$

$\rho =$ density of liquid.

2. U-tube manometer: It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to atmosphere as shown in figure. The tube consists generally mercury or any other liquid whose specific gravity is greater than the specific gravity of liquid whose pressure to be measured.



HYDROSTATIC FORCES ON SURFACES

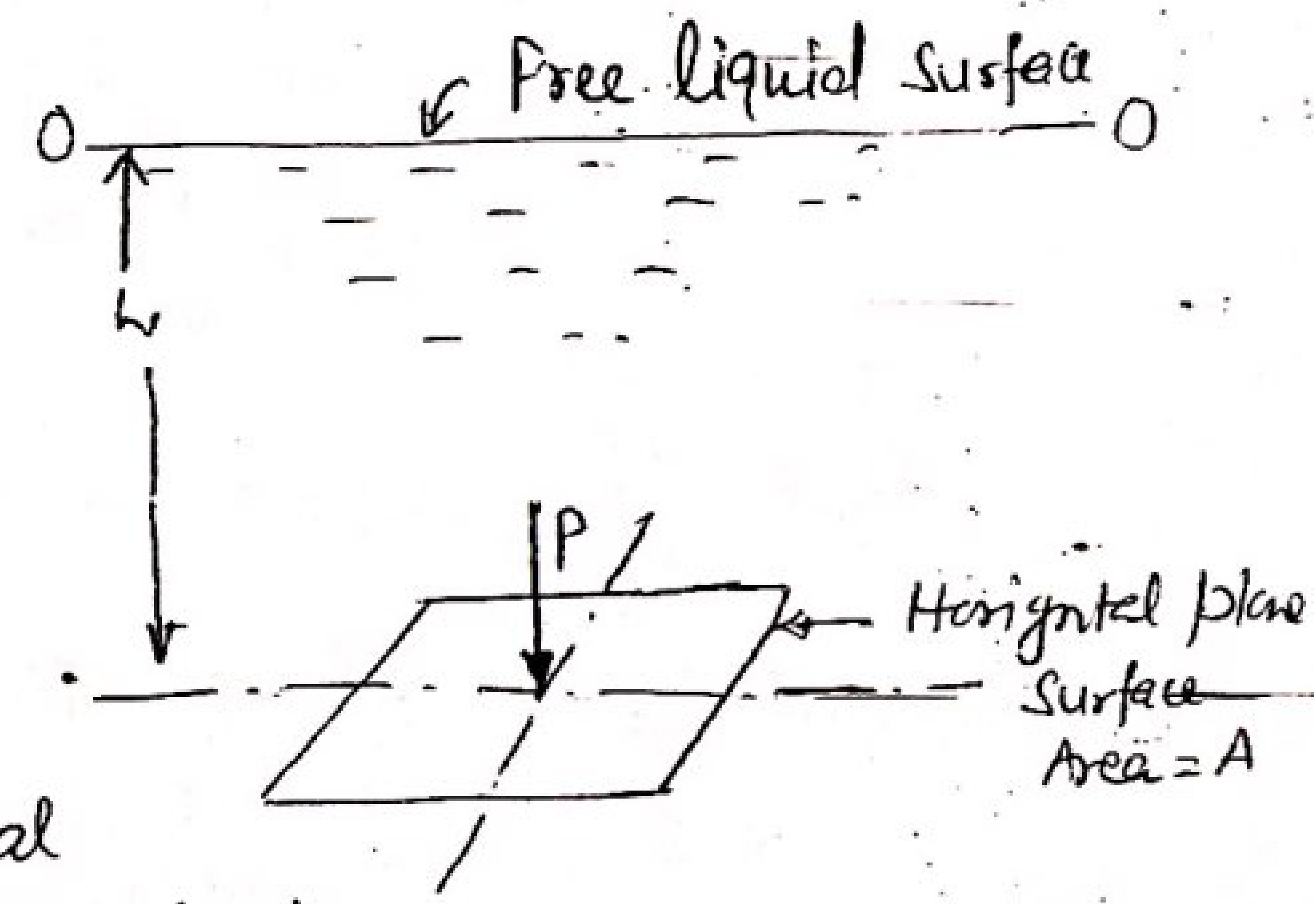
1.) TOTAL PRESSURE AND CENTRE OF PRESSURE

When a static mass of fluid comes in contact with a surface either plane or curved, a force is exerted by fluid on the surface. This force is known as total pressure. Since for a fluid at rest no tangential force exists, the total pressure acts in the direction normal to the surface. The point of application of total pressure on the surface is known as centre of pressure.

2.) TOTAL PRESSURE ON A PLANE SURFACE

(a) Total pressure on a Horizontal plane surface

Since every point on the surface is at the same depth below the free surface of liquid



Total pressure $P = pA = whA$ —(1)

p = pressure intensity

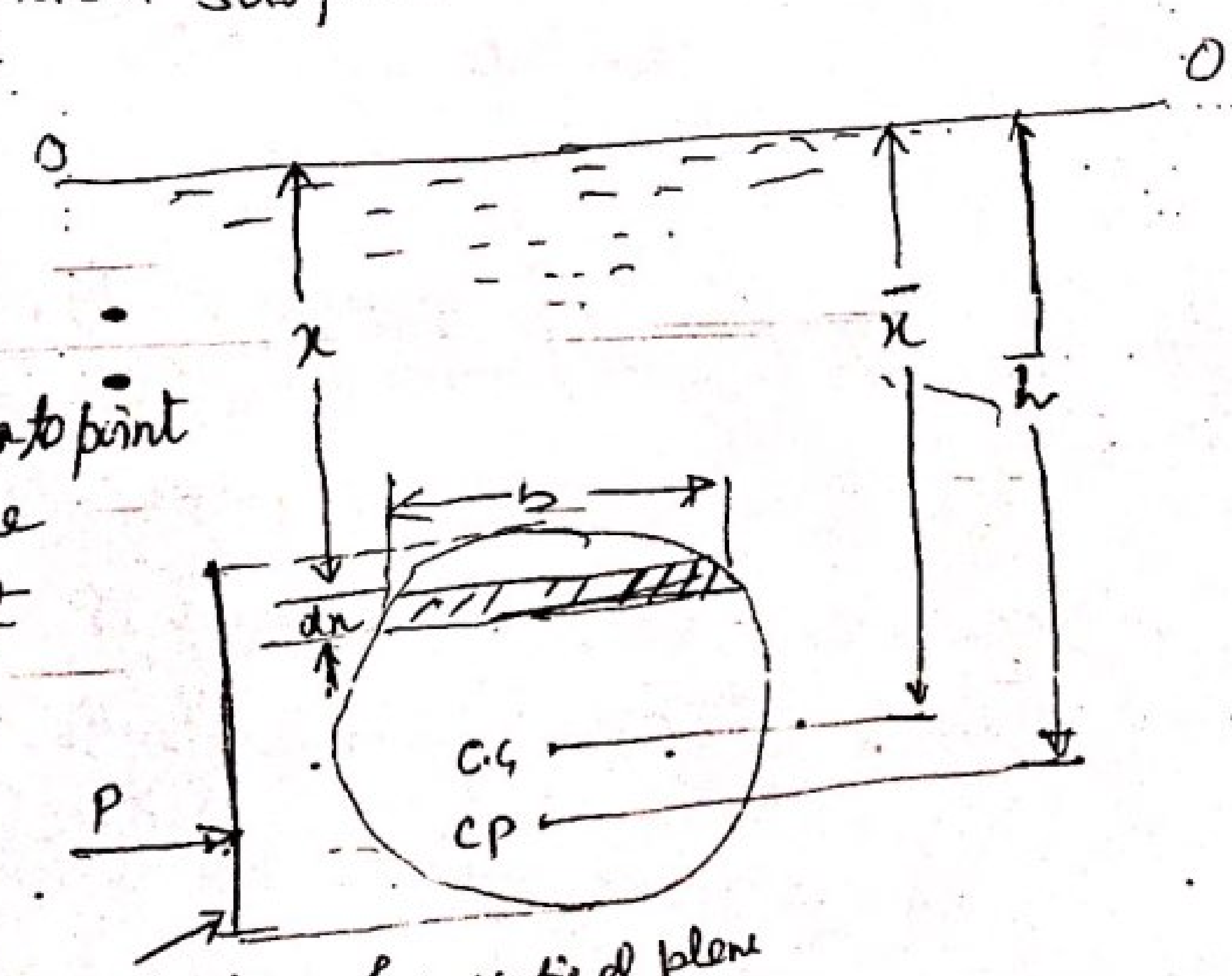
The direction of this force is normal to the surface, as such it is acting towards the surface in the vertical downward direction at the centroid of the surface.

(b) Total pressure on a vertical surface

Let A = total area of surface

$\bar{x}$ = Centroid of surface

In this case. Since the depth of liquid varies from point to point on the surface, the ~~pressure inten~~ pressure intensity is not constant over the entire surface. As such the total pressure on the surface may be determined by dividing the entire surface into a number of small strips and computing the total pressures on each of these strips.



End view of a vertical plane

Total pressure on a vertical plane Surface

The summation of these total pressures on all the small strips will give the total pressure on the entire plane surface.

Consider on the plane surface a horizontal strip of thickness dx and width 'b' lying at a vertical depth x below the free surface of the liquid. Since the thickness of strip is very small, for this strip the pressure intensity may be assumed to be constant equal to $p = wx$

Area of strip $dA = b\,dx$

Total pressure on strip

$$dp = p\,dA = wx(b\,dx)$$

Total pressure on entire surface is

$$P = \int dP = \int wx\,(b\,dx)$$

But $\int x\,(b\,dx)$ = Sum of the first moments of the areas of the strips about ~~an~~ axis OO

$$\int x(b\,dx) = A\bar{x}$$

$\bar{x}$ = Centroid of the surface area from the axis OO.

$$\text{so } \boxed{P = wA\bar{x}} \quad \text{——— (1)}$$

Centre of pressure for vertical plane surface

Let $\bar{h}$ be the vertical depth of the centre of pressure for the plane surface immersed vertically. Then moment of total force P about axis OO is equal to $(P\bar{h})$.

Total pressure of the strip ie. $dP = wx(b\,dx)$ and its moment about OO is $(dP)x = wx^2 b\,dx$

The sum of the moments of the total pressures on all strips becomes $\int (dP)x = w\int x^2 (b\,dx)$

By using "principle of moments", which states that the moment of resultant of a system of forces about ~~the~~ an axis is equal to sum of the moments of the components about the same axis, the amount of total pressure about axis OO is

$$P\bar{h} = w\int x^2 (b\,dx) \quad \text{——— (1)}$$

$\int x^2 (b\,dx)$ represent the second moment of the areas of the strips about axis OO, which ~~is equal to moment of inertia~~ I_o ~~for~~ of the plane surface about axis OO. That is

$$I_o = \int x^2 (b\,dx) \quad \text{——— (2)}$$

from eqn. (1) & (2)

$$P\bar{h} = w I_o$$

$$\bar{h} = \frac{w I_o}{P} = \frac{w I_o}{w A\bar{x}} = \frac{I_o}{A\bar{x}} \quad \text{——(3)}$$

from parallel axis theorem $I_o = I_G + A\bar{x}^2$

I_G = moment of inertia of the area about an axis passing through the centroid of area and parallel to axis OO.

So eqn (3) becomes

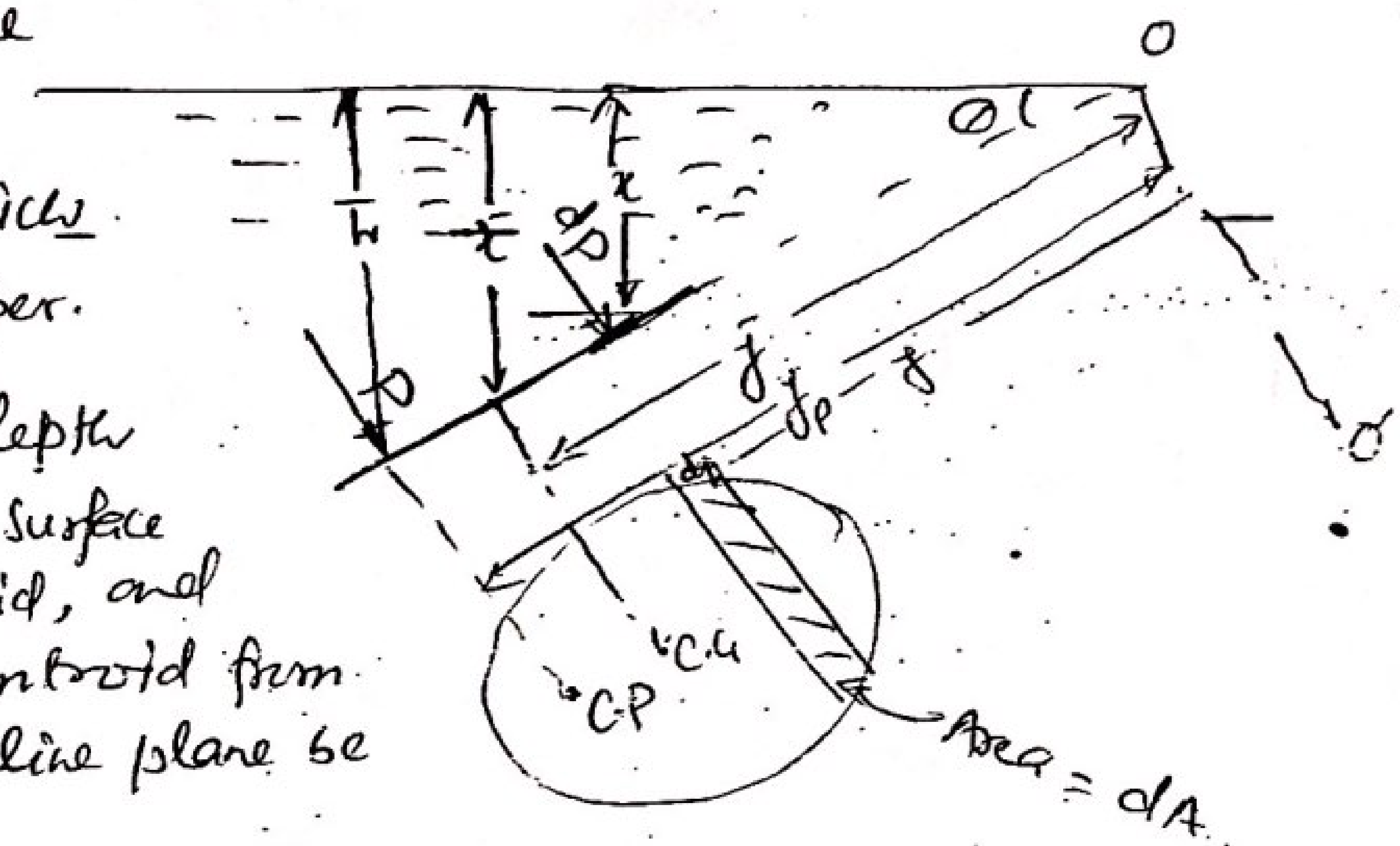
$$\bar{h} = \frac{I_G + A\bar{x}^2}{A\bar{x}} = \bar{x} + \frac{I_G}{A\bar{x}}$$

$$\boxed{\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}}}$$

Here $\bar{h} > \bar{x}$ so centre of pressure i.e centre of pressure always below the centroid of the area.

TOTAL PRESSURE ON INCLINED PLANE SURFACE

Consider a plane surface of arbitrary shape and total area A, wholly submerged in a static mass of liquid of specific weight w. The surface is held inclined such that the plane of surface makes an angle θ with the horizontal as shown in figure



The intersection of this plane with free surface of liquid is represented by axis OO', which is normal to plane of paper.

Let $\bar{x}$ be the vertical depth of the centroid of the plane surface below the free surface of liquid, and inclined distance of the centroid from axis OO' measured along incline plane be $\bar{y}$.

Consider a plane surface, a small strip of area 'dA' lying at a vertical depth of x and its distance from axis OO' is y. For this strip pressure intensity may be assumed constant equal to $p = wx$

∴ total pressure on the strip is $dP = wx(dA)$

$$x = y\sin\theta$$

$$dP = w(y\sin\theta)\,dA$$

Total pressure on entire surface is

$$P = w\sin\theta \int y\,dA$$

Again $\int y\,dA$ = sum of first moments of areas of the strips about axis OO',

$$\int y\,dA = A\cdot\bar{y}$$

A = Total area of incline plane
$\bar{y}$ = distance of centroid from axis OO'.

$$P = wA\bar{y}\sin\theta$$

$$\boxed{P = wA\bar{x}} \quad \because \bar{x} = \bar{y}\sin\theta$$

Above equation indicating that for a plane surface wholly submerged in a static mass of liquid and held eigh either vertical or inclined, the total pressure is equal to the product of the pressure intensity and centroid of the area and the area of plane surface.

Centre of pressure for inclined surface

As shown in figure, let $\bar{h}$ be the vertical depth of the centre of pressure for the inclined surface below the free surface of the liquid and its inclined distance from axis OO' is y_p.

Total pressure on the strip shown in figure is $[w (y \sin\theta) dA]$ and its moment about axis OO' = $(dP)y = (w \sin\theta\, y^2 dA)$

~~From~~ principle of moments

$$P\, y_p = w \sin\theta \int y^2 dA \quad \text{——(1)}$$

$\int y^2 dA$ = Sum of second moments of the areas of the strips about axis OO' which is equal to moment of inertia I_o of the plane surface about axis OO'.

$$I_o = \int y^2 dA$$

$$y_p = \frac{w \sin\theta\, I_o}{P}$$

From parallel axis theorem $I_o = I_G + A\bar{y}^2$

I_G = moment of inertia of the area an axis passing through the centroid of area and parallel to axis OO'.

$$y_p = \frac{w \sin\theta\,(I_G + A\bar{y}^2)}{w A (\bar{y} \sin\theta)}$$

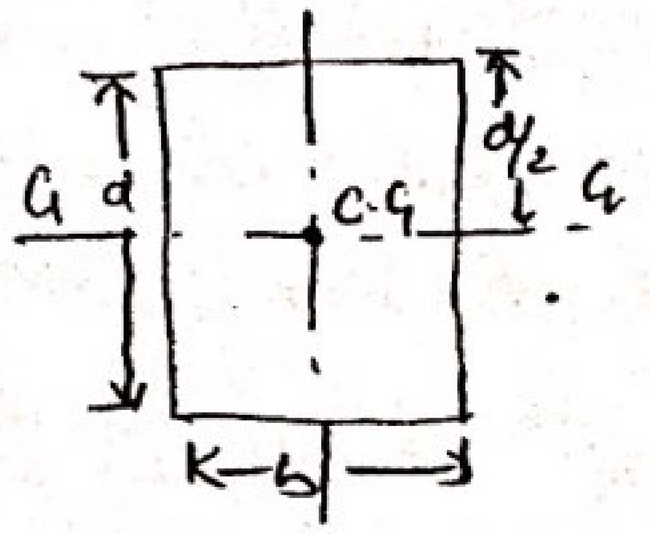
$$y_p = \bar{y} + \frac{I_G}{A\bar{y}}$$

$$y_p = \frac{\bar{h}}{\sin\theta} \quad \text{and} \quad \bar{y} = \frac{\bar{x}}{\sin\theta}$$

$$\boxed{\bar{h} = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}}$$

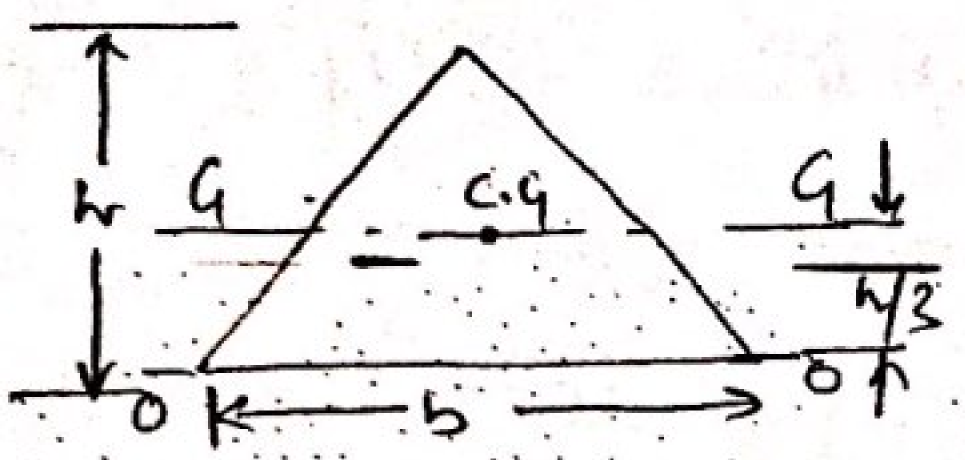
Moment of Inertia

(a) Rectangle



$$I_G = \frac{bd^3}{12}$$

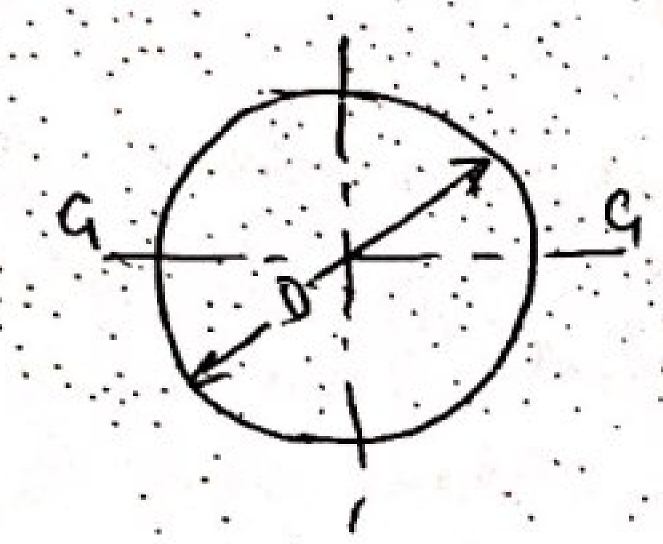
b) Triangle



$$I_G = \frac{bh^3}{36}$$

$$I_{O-O} = \frac{bh^3}{12}$$

c) Circle



$$I_G = \frac{\pi D^4}{64}$$

Q A vertical gate closes a horizontal tunnel 5 m high and 3 m wide running full with water. The pressure at bottom of the gate is 2×10^4 kg/m^2. Determine the total pressure on the gate and the ~~position of the~~ position of the centre of pressure. Ans $2{,}625 \times 10^2$ kg, 17.619 m