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Roll No. XB6/CE/1050

FIRST SEMESTER

B.Tech.(Group A & B)

END SEMESTER EXAMINATION

NOVEMBER-2010

AM- 101 MATHEMATICS-I

Time: 3 Hours

Max. Marks : 70

Note :	Answer ***ALL*** questions taking ***TWO*** parts out of the three set in each question. Assume suitable missing data, if any.

1[a] Discuss the convergence of the series

$$1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\cdots\ldots\ldots\ldots\ldots(x>0).$$

[b] State a test to check for the convergence of an alternating series. Examine the convergence/absolute convergence of the series

$$\frac{1}{2^3}-\frac{1}{3^3}(1+2)+\frac{1}{4^3}(1+2+3)-\cdots\ldots\ldots\ldots\ldots\ldots\ldots$$

[c] (ii) Show that the radius of curvature at any point of the cardiod $r=a(1-cos\theta)$ varies as $\sqrt{r}$.

(iii) Expand $e^{asin^{-1}x}$ upto the term containing x^4.

2[a] The area lying inside the cardiod $r=2a(1-cos\theta)$ and outside the parabola $r(1+cos\theta)=2a$ is revolved about the initial line $\theta=0$. Find the volume of the solid generated.

[b] If $u=sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, then prove that

$$x^2\frac{\partial^2u}{\partial x^2}+2xy\frac{\partial^2u}{\partial x\partial y}+y^2\frac{\partial^2u}{\partial y^2}=-\frac{sinucos2u}{4cos^3u}$$

[c] A balloon is in the form of a right circular cylinder of radius 1.5 meter and length 4 meter and is surmounted by hemispherical ends. If the radius is increased by 0.01 meter and length by 0.05 meter, find the percentage change in the volume of balloon.

3[a] Using Lagrange's multipliers method find the smallest and the largest distances between the points P and Q such that P lies on the plane $x+y+z=2a$ and Q lies on the sphere $x^2+y^2+z^2=a^2$ where a is any constant.

[b] Expand $f(xy) = tan^{-1}xy$ in powers of $(x-1)$ and $(y-1)$ using Taylor's series expansion upto second degree terms and then compute f(1.1, 0.8).

[c] Using double integration calculate the area which is inside the cardiod $r = 2(1+\cos\theta)$ and outside the circle r =2.

4[a] Change the order of integration and hence evaluate the integral

$$\int_0^{a/\sqrt{2}}\int_0^{x} x\,dy\,dx + \int_{a/\sqrt{2}}^{a}\int_0^{\sqrt{a^2-x^2}} x\,dy\,dx$$

[b] Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ using

(i) Cylindrical co-ordinates.

(ii) Spherical polar co-ordinates.

[c] Define beta and gamma functions. Derive the relation between the two. using these functions evaluate the integral $\int_0^{\pi/2} \sin^7\theta \cos^5\theta d\theta$.

5[a] Define gradient of a scalar field. What does its direction and magnitude represent? Find the angle between the two surfaces

$x^2+y^2+z^2=9$, and $z=x^2+y^2-3$

at (2, -1, 2).

[b] Prove that $div(r^n\vec{r}) = (n+3)r^n$. Hence show that $\vec{r}/r^3$ is solenoidal. Is $\vec{r}/r^3$ irrotatinal also? Verify your answer. Here $\vec{r}$ is the position vector of a point P(x, y, z) and $r=|\vec{r}|$.

[c] If $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, evaluate $\iint_S \vec{F}.\hat{N}\,dS$ where S is the surface of the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$.