Assignment-II (MA-101): Differential and Integral Calculus of Single Variable

1. State and Prove Taylor's theorem with Lagrange's form of remainder.

2. The function $f(x) = \sin(x)$ is approximated by Taylor's polynomial of degree three about the point $x = 0$.

3. Obtain the fourth degree Taylor's polynomial to $f(x) = e^{2x}$ about $x = 0$.

4. Obtain the Taylor's series expansion of the following

(a) a^x (b) $\log(1 - x)$.

5. Calculate the value of $f(11/10)$ where $f(x) = x^3 + 3x^2 + 15x - 10$ using Taylor's series.

6. Find the Curvature and radius of curvature of the following curves.

(i) $x^2 + y^2 = a^2$.

(ii) $x = a(t - \sin(t))$ and $y = a(1 - \cos(t))$.

7. Trace the following curves.

(i) $y = c\cos(x/c)$ (ii) $x = 2(\theta + \sin\theta)$, $y = 2(1 - \cos\theta)$ (iii) $r = 3(1 + \cos\theta)$ (iv) $r = 2\cos 2\theta$

(v) $(x^2 + y^2)^2 = 4(x^2 - y^2)$

8. Find the length of the curve $y = (\frac{x}{2})^2$ from $x = 0$ to $x = 2$.

9. Find the arc length of the loop of the curve $9ay^2 = (x - 2a)(x - 5a)^2$.

10. Find the surface area of the solid generated by revolving the part of the lemniscate $r^2 = 2a^2\cos(2\theta)$, $0 \leq \theta \leq \pi/4$, about the $x - axis$.

11. The area of cardioid $r = a(1 + \cos\theta)$ includes between $\theta = -\pi/2$ and $\theta = \pi/2$ is rotated about the line $\theta = \pi/2$. Show that volume generated is $2(2 + 5\pi/8)a^3\pi$ is rotated about the line $\theta = \pi/2$.

12. Find the area of the surface generated by revolving about $x - axis$ the top half of cardioid $r = 1 + \cos(\theta)$.

13. Find the length of the curve $x = a(t - \sin(t)), y = a(1 - \cos(t))$ from $t = 0$ to $t = 2\pi$.

14. Find the length of the polar curve $r = \cos\theta + \sin\theta$.

15. Find the volume of the reel shaped solid formed by the revolution of the part of the parabola $y^2 = 4ax$ cut off by the latus rectum about y-axis.