Total no. of pages :2 | Roll No. B1/112

1st SEMESTER | B.Tech (All groups)

END SEMESTER EXAMINATION | NOV 2015

MA – 101 Mathematics-I

Time : 3 hrs | Max. Marks: 50

Note: Attempt all questions selecting *two parts* from each question. All questions carry equal mark. Assume missing data, if any.

1 (a) State and prove integral test for convergence of the infinite series.

(b) Discuss the convergence of series

$$1+\frac{1!}{2}x+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+----$$

(c) Test the convergence of the series

$$\frac{\sin x}{1^3}-\frac{\sin 2x}{2^3}+\frac{\sin 3x}{3^3}+---$$

2 (a) Compute the value of $\cos 32^0$ upto four decimal places using Taylor series.

(b) Find the area common to two cardioids $r=a(1+cos\theta)$ and $r=a(1-cos\theta)$.

(c) Find the surface area of the solid generated by revolving one arch of the cycloid $x=a(\theta+\sin\theta)$, $y=a(1-cos\theta)$ about the tangent at the vertex.

3 (a) If $u=\operatorname{cosec}^{-1}\left(\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right)^{1/2}$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{12}\left(\frac{13}{12}+\frac{\tan^2 u}{12}\right).$$

(b) If z is a function of x and y, where $x=e^{u}+e^{-v}$ and $y=e^{-u}-e^{v}$, show that

$$\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}.$$

(c) Using Lagrange's multiplier method to find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

4.
(a) Evaluate $\int_0^a\int_{x/a}^{\sqrt{(x/a)}}(x^2+y^2)dxdy$ by changing the order of integration.

(b) Using suitable multiple integral to find the volume bounded by the *xy*-plane, the cylinder $x^2+y^2=1$ and the plane $x+y+z=3$.

(c) Prove the followings.

(i) $\int_0^{\pi/2}\left(\sqrt{\tan\theta}+\sqrt{\sec\theta}\right)d\theta=\frac{1}{2}\Gamma(1/4)\left[\Gamma(3/4)+\frac{\sqrt{\pi}}{\Gamma(3/4)}\right]$,

(ii) $\frac{\beta(m+1,n)}{\beta(m+n)}=\frac{m}{(m+n)}$.

5. (a) A vector field is given by $\bar{F}=2xyz^3\hat{\imath}+x^2z^3\hat{\jmath}+3x^2yz^2\hat{k}$ Show that it is irrotational ,and hence find its scalar potential.

(b) Evaluate $\iint_S \bar{r}.\,d\bar{s}$ where $\bar{r}$ is the position vector, and S is the surface $x^2+y^2+z^2=9$.

(c) Verify Stokes' Theorem for the function $\bar{F}=x^2\hat{\imath}+xy\hat{\jmath}$ integrated round the square in the plane z=0 and bounded by the lines x=0, y=0, and x=a, y=a.