# A.C CIRCUITS

## UNIT ~~III~~ II

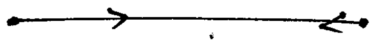
## ELECTRICAL SCIENCE

Prof. N. K. Jain.

First Year First Year First Semester

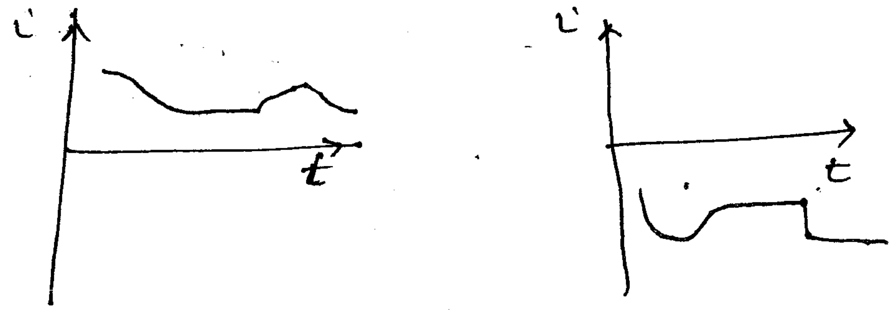
## Some Definitions

direction of Current



### Unidirectional Current

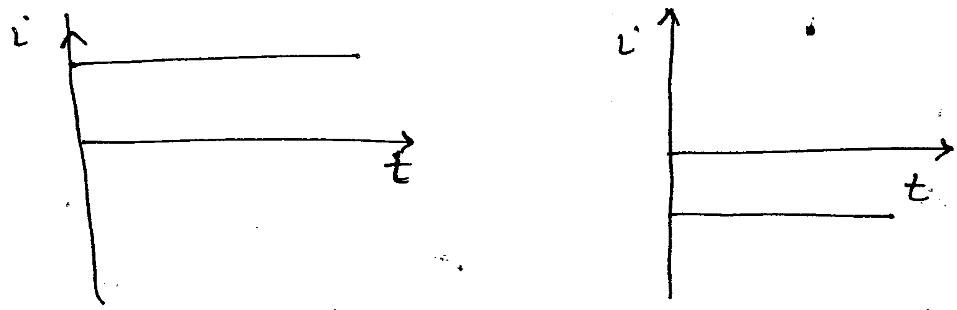
If the average current flow over a specified period of time through a conductor always remains ~~con~~ in one direction only, it is called unidirectional current




(A)

### Direct Current (D.C.)

If the magnitude of the current remains constant, it is called Direct Current (D.C).



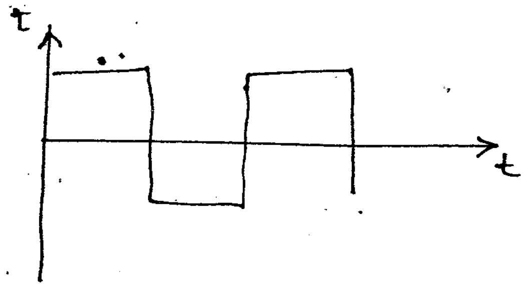

(B)

### Pulsating Current

It is a ~~direct cu~~ unidirectional current with current magnitude varies with time Fig (A).

### Alternating Current

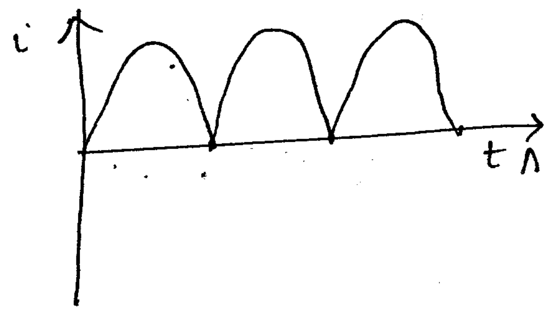
A current which keeps on changing its direction continually is called an alternating current. The +ve and -ve parts are similar



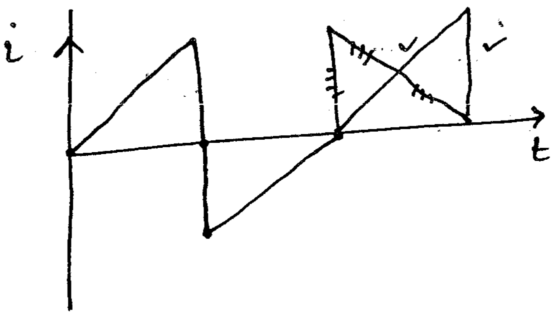

C

## Periodic Current

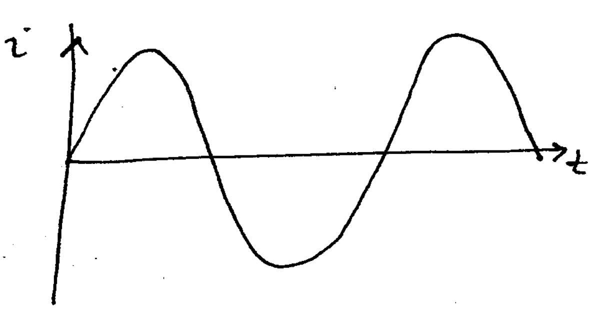
If the ~~magnitude~~ magnitude and direction of the current varies periodically, it is called periodic current




periodic unidirectional ~~direc~~ current




Periodic Alternating current (Triangular)




— do —

(Sinusoidal)

## ~~Rasa~~ Pulsating Current

If the +ve and -ve halves of an alternating current are not similar, it said to be pulsating current

## ~~Maximum Power Transfer Theorem~~ 

There fore,

For D.C.

1. D.C. flows in one direction i.e. it doesn't change its sign with time.
2. Any D.C. source has 2 terminals. One is marked +ve and the other as −ve. Conventional current flows from +ve terminal to the −ve terminal of the source (for outside the source).

For A.C

1. AC changes its magnitude and sign periodically.
2. The wave shape of AC may have any shape, but for electrical supply sinusoidal wave form is used.

The Advantages of Sinusoidal Wave form

1. Many phenomena occuring in nature are of sinusoidal nature e.g. simple pendulum and guitar string.
2. The sinusoidal function has an interesting mathematical characterstic.

— Its integral and derivatives are both sinusoids.

— So in a linear circuit the response to a sinusoid forcing function would also be sinusoid. This makes the mathematical analysis easier.

3. The application of sinusoidal voltage to appropriately designed coils results in a <u>revolving magnetic field</u> which has capacity to do work.

The majority of electric mototors work on this principle.

4. - The knowledge of the response of electric circuits to sinusoidal functions has mathematical advantage. ~~the~~

- We know that any periodic function can be represented in terms of an infinite series of sinusoids using <u>Fourier series</u>

- This means, response of an electric circuit to a <u>non-sinusoidal</u> forcing function can be obtained through a repeated application of the sinusoidal theory.

5. • When a sinusoidal voltage across a coil, the current also remains sinusoidal.
   • For other waveforms it is not true.

6. - If input to a transformer is sinusoidal, output also remains sinusoidal

- This means the sinusoidal voltage generated by an Electric Supply company reaches the consumer

7. AC generaters has comparatively lesser cost than DC generaters for the same rating.

8. Much higher voltages can be generated in AC as compared to DC

9. Use of Transformers is possible only in AC

10. AC motors are cheaper than DC motors in initial cost as well as in maintenance

11. Switchgears for AC are simpler than that required for a dc system.

12. However, DC transmission has its own advantages. That is why HVDC transmission is being adopted now a days.

## Generation of Alternating EMF

## Story of Faraday and Cavandish.

According to Faraday's laws an emf is induced (generated) whenever there is a relative motion ~~in~~ ~~the mag~~ between the magnetic field and the conductor.

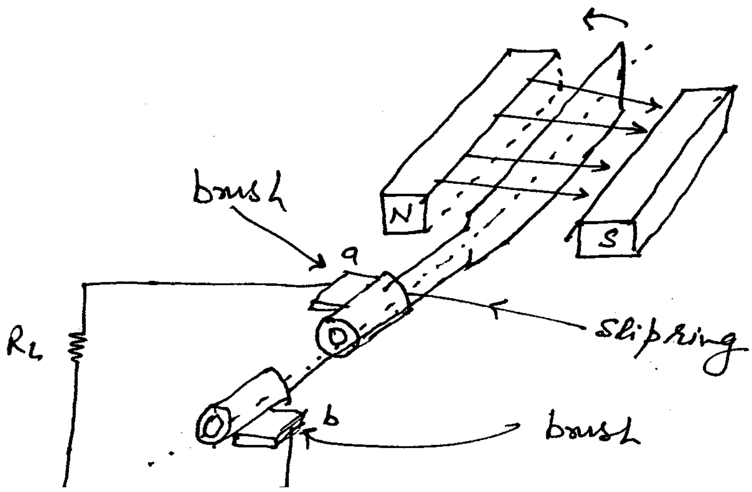
This can be ~~made~~ manipulated in two ways.

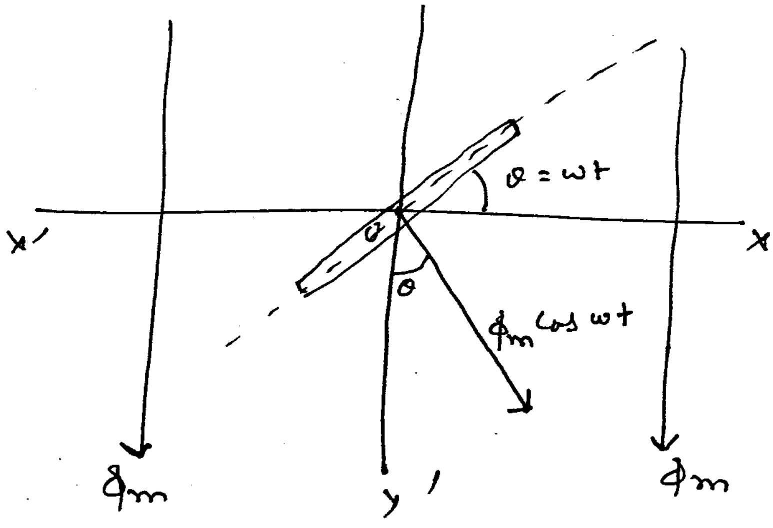
1. rotating a coil in a stationary magnetic field
2. Rotating a magnetic field with in a stationary coil.

- ① is adopted for small ac generators
- ② " " " large " "

The magnitude of the emf generated depends upon

1. the number of turns on the coil ($N$)
2. the strength of magnetic field ($\phi_m$)
3. the speed at which the coil/magnetic field rotates ($\omega$) radians/sec






— • Let the time be measured from the instant of coincidence of the plane of the coil with x-axis

— At this instant the maximum flux $\phi_m$ links with the coil.

— Let us consider an instant after a time of t seconds as shown in figure.

$\theta = \omega t$

Component of flux along ⊥ to the plane of coil

$= \phi_m \cos \omega t$

Flux linkages at this instant

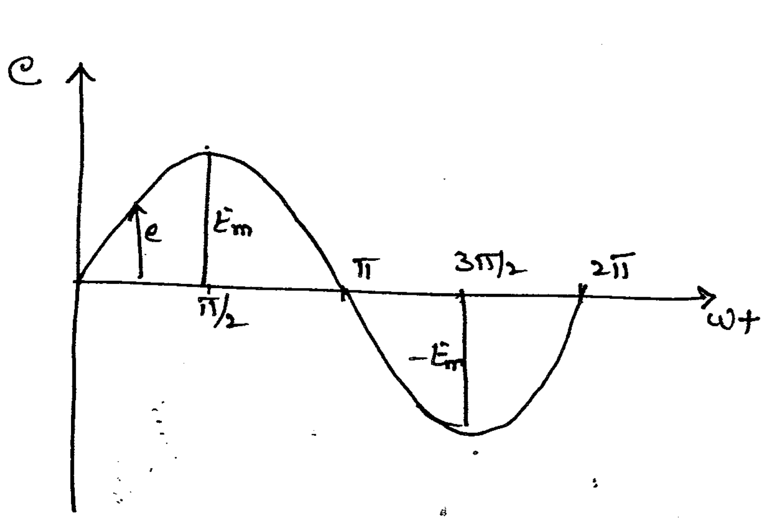
$= N \cdot \phi_m \cos \omega t$

emf induced $e = -\frac{d}{dt}\left[N \phi_m \cos \omega t\right]$

$= \phi_m N \frac{d}{dt}\left[-\cos \omega t\right] = \phi_m N \omega \sin \omega t$ —— (i)

$= E_m \sin \omega t$

Therefore, the following waveform can be drawn.




(A)

e = instantaneous value of emf $\omega t = 0$

Em = Maximum value of emf $\omega t = \pi/2$ & $3\pi/2$.

$\omega$ = angular velocity in radians/sec.

$f$ = frequency in cycles/sec.

in each rotation the radians covered = $2\pi$

$\therefore \quad \omega = 2\pi f$.

& $\quad e = E_m \sin \phi \omega t = E_m \sin 2\pi f t$

## Average Value

Arithmatically we define average value as

$$= \frac{\text{Sum of all values}}{\text{No. of values.}}$$

But in case of a waveform, we would like to define the average value as.

$$\frac{\text{the area under the waveform ~~divided by~~}}{\text{Length of one cycle.}}$$

The area under +ve half cycle (fig. A) is equal to the area under -ve half cycle in magnitude, but opposite in sign, So strictly speaking the total

For this reason, we will talk for the average value over the half cycle.

<u>For full cycle</u>

$$E_{av} = \frac{1}{2\pi}\int_0^{2\pi} e\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} e\, d(\omega t)$$

$$= \frac{1}{T}\int_0^{T} v . dt = 0 \text{ for full cycle.}$$

~~For half~~

$$= \frac{1}{2\pi}\int_0^{2\pi} E_m \sin\theta\, d\theta = \frac{E_m}{2\pi}\Big|-\cos\theta\Big|_0^{2\pi}$$

$$= -\frac{E_m}{2\pi}\left[-1-(-1)\right] = 0$$

<u>For half cycle</u>

$$E_{av} = \frac{1}{\cancel{2}\pi}\int_0^{\pi} e\, d\theta = \frac{1}{\cancel{2}\pi}\int_0^{\pi} E_m \sin\theta\, d\theta$$

$$= \frac{E_m}{\cancel{2}\pi}\int_0^{\pi}\sin\theta\, d\theta = \frac{E_m}{\cancel{2}\pi}\left[-\cos\theta\right]_0^{\pi}$$

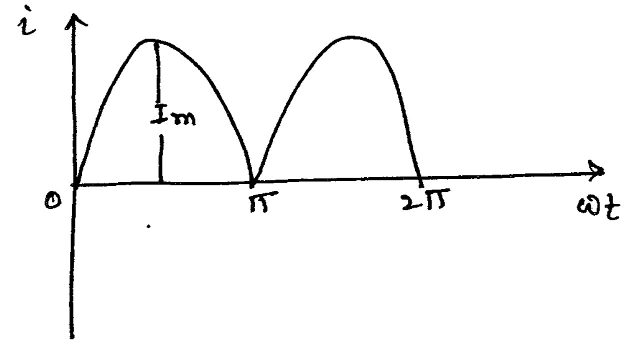
$$= \frac{E_m}{\pi}\left[-(-1)-(-1)\right] = \frac{E_m}{\pi}\left[1+1\right] = \frac{2E_m}{\pi}$$

$$\boxed{E_{av} = 0.637\, E_m}$$

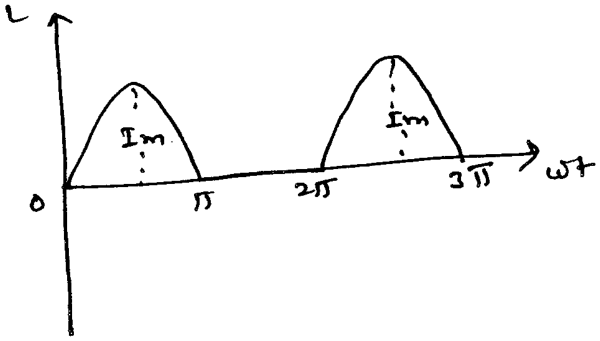
Similar expression can be derived for current.

PTO

Write down average values of the following waves by inspection.

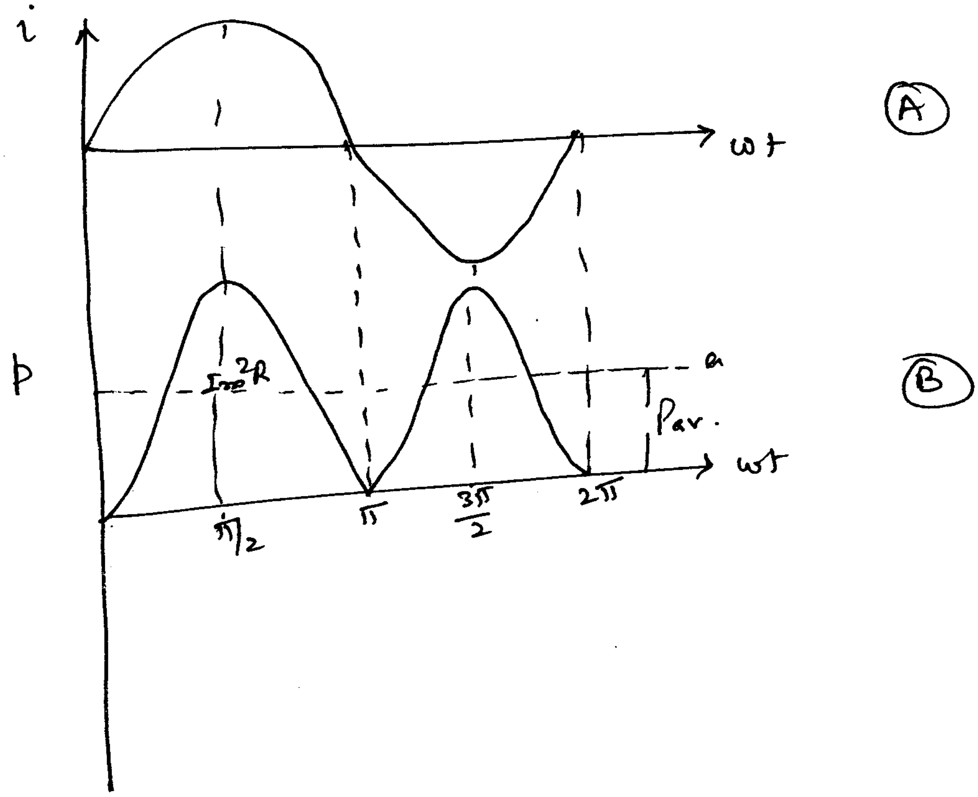



$I_{av} = 0.637\, I_m$



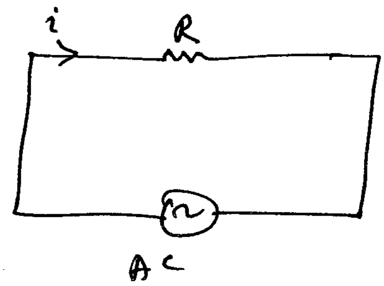

$I_{av} = 0.318\, I_m$.

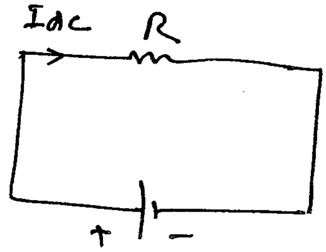
## Effective Value



The effective value of an ac current is defined as that value of a direct current which when allowed to through a given circuit for a given time produces the equal amount of heat as produced by the ac current when allowed to flow through the same circuit for the same time.

Consider




$i = I_m \sin \omega t$

$p = i^2 R$ wave form for p is shown fig (B).

The average value of p over a complete cycle be represented by Pav.

- We are interested to find $I_{dc}$ which gives power equal to Pav.

$i = I_m \sin \omega t$

$$p = i^2 R = R \; I_m^2 \sin^2 \omega t$$

$$= \frac{R I_m^2}{2}(1 - \cos 2\omega t) = \frac{R I_m^2}{2} - \frac{R I_m^2}{2} \cos 2\omega t$$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} p \, d\omega t = \frac{1}{2\pi}\int_0^{2\pi}\Big[\underbrace{\frac{R I_m^2}{2}}_{\text{constant}} - \underbrace{\frac{R I_m^2}{2}\cos 2\omega t}_{\int = 0}\Big] d\omega t$$

- The I term is constant & independent of ωt
- ∫ of II term ...

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} \frac{R I_m^2}{2} \cdot d\omega t - \frac{1}{2\pi}\int_0^{2\pi} \frac{R I_m^2}{2} \cos 2\omega t \, (d\omega t).$$

$$= \frac{1}{2\pi}\left[\frac{R I_m^2}{2}\right] \left|\omega t\right|_0^{2\pi} - \frac{1}{2\pi} \cdot \frac{R I_m^2}{2} \left[\frac{\sin 2\omega t}{2}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[\frac{R I_m^2}{2}\right]\left[2\pi - 0\right] - \frac{1}{2\pi} \cdot \frac{R I_m^2}{2}\left[\frac{\sin 4\pi - \sin 0}{2}\right]$$

$$= \frac{R I_m^2}{2}$$

for dc.

$$P_{dc} = I_{dc}^2 R = P_{av} = \frac{R I_m^2}{2}$$

$$I_{dc}^2 = \frac{I_m^2}{2}$$

$$I_{dc} = \sqrt{\frac{I_m^2}{2}} = \frac{1}{\sqrt{2}} I_m = 0.707 I_m . = I_{eff} = I_{rms}$$

$I_{rms}$ = root mean square

<u>Justification of rms</u>

$$p = i^2 R$$

$$P_{av} = \frac{1}{T}\int_0^T p \, dt = \frac{1}{T}\int_0^T i^2 R \, dt$$

$$= \underbrace{\left[\frac{1}{T}\int_0^T i^2 dt\right]}. R = I_{eff}^2 R.$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

root mean square

rms value > Average value except for retangular wave for which

rms value = Average value.

- V and I without any subscript means rms values
- $V_{rms}$, $V_{eff}$ are not necessary ~~as~~ except, where a confusion may exist.
- Any specification made without any reference means rms value.

Form Factor

$$= \frac{\text{effective value}}{\text{average value}}$$

$$= \frac{V_{rms}}{V_{av}} = \frac{0.707\ V_m}{0.637\ V_m} = 1.11.$$

Peak Factor

$$= \frac{V_{max}}{V_{rms}} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2} = 1.414$$

Example 1, Find the rms value of a resultant current in a wire, which carries simultaneously a direct current of 5A, and a sinusoidal a.c. with a peak value of 5A.

Soln: - Let I be the rms value of the resultant current.

- rms value is defined on the basis of ~~energy~~ consumption

$$\therefore \quad I^2Rt = 5^2Rt + \left(\frac{5}{\sqrt{2}}\right)^2 Rt$$

$$= \left[5^2 + \left(\frac{5}{\sqrt{2}}\right)^2\right] Rt$$

$$I^2 = 5^2 + \left(\frac{5}{\sqrt{2}}\right)^2 = 25 + 12.5 = 37.5$$

$$I = \sqrt{37.5} = \underline{6.12\,A}$$

<u>Alternatively</u>,

by definition of rms value

$$I = \sqrt{\frac{\cancel{25}5^2 + (5/\sqrt{2})^2}{\cancel{2}}} = \sqrt{25+12.7} = \sqrt{37.5}$$

$$= \underline{6.12\,A}$$

<u>Example 2</u>: Calculate the rms value of current given by $i = 10 + 5\cos(628t + 30^\circ)$

$$I_{rm} = \sqrt{10^2 + \left(\frac{5}{\sqrt{2}}\right)^2} = \sqrt{100 + 12.5}$$

$$= \sqrt{112.5} = \underline{10.61A}$$

<u>Example 3</u>: Compute the maximum and rms values of the following quantities

(i) $40\sin\omega t$ (ii) $(A+B)\sin(\omega t - N)$

(iii) $10\sin\omega t - 17.3\cos\omega t$.

<u>Soln</u>: (i)

$$Q_m = \underline{40}$$

$$Q_{rms} = \frac{40}{\sqrt{2}} = \underline{28.28}$$

(ii)

$$Q_m = \underline{A+B}$$

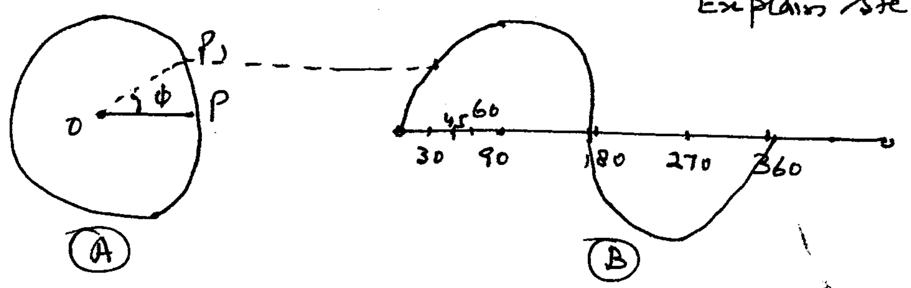
$$Q_{rms} = \underline{\underline{\frac{A+B}{\sqrt{2}}}}$$

(iii) <u>PTO</u>

## Phasor Diagrams

Rotating Bar

Explain step by step



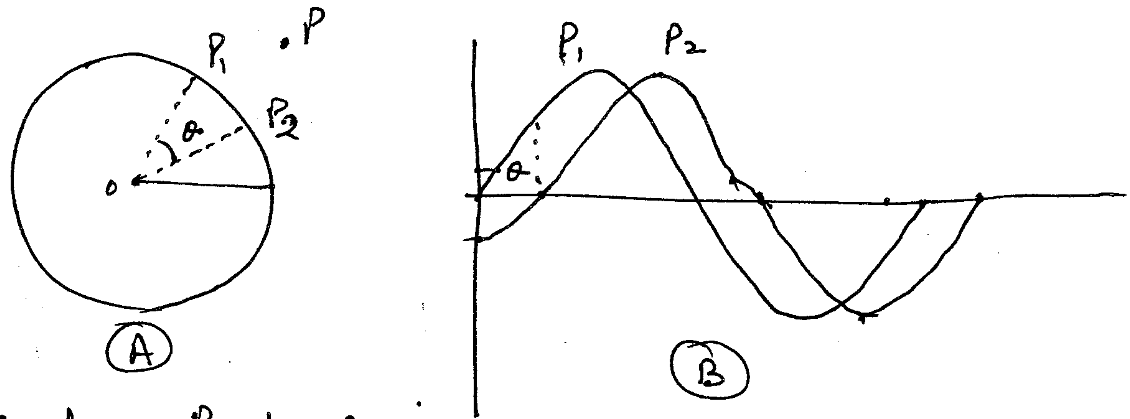

OP is phasor representation, OP gives max value.

$v = V_m \sin \omega t$

OP rotates at angular velocity of ω radians/sec.

fig. (A) and (B) give the same information.

Consider 2 phasors




$P_2$ lags $P_1$ by $\theta$ i.e.

if $v_1 = V_m \sin \omega t$

$v_2 = V_m \sin(\omega t - \theta)$

- If we do our work only with fig (A), it would be complete.
- We normally represent electrical data in terms of rms values.
- Nothing will be lost if $OP_1$ represents rms value in place of max. value.

Such representation is called phasor diagram.

- Phasor representation in form (A) is of a little meaning, if only one phasor.
- Phasor representation would serve better purpose if more than one phasor are involved.
- Phasor ~~As~~ diagrams represents the relative positions of phasors at an instant of time
- Phasors are rotating vectors.
- rms values are taken on phasor diagrams
- For drawing phasor diagrams we need to take a reference phasor.
- It is to be selected conveniently
- For <u>series</u> ~~curr~~ circuits <u>current</u> is taken as reference phasor for convenience
- For <u>parallel</u> circuits <u>voltage</u> is taken as reference phasor for convenience.

## Algebraic Operations on Phasors

- These are done similar to vectors which you have studied in your school.

- There are two forms in which phasors are represented

  - Polar Form : $A \angle \theta$
  - Rectangular form: $x + jy$ $\qquad j = \sqrt{-1}$

$$\left.\begin{array}{ll} A = \sqrt{x^2+y^2} & \theta = \tan^{-1}\left(\frac{y}{x}\right) \\ x = A\cos\theta & y = A\sin\theta . \end{array}\right\} \quad (A)$$

### Addition and substraction of Phasor

- For this the phasors should be represented in rectangular form

$$A \angle \theta_1 + B \angle \theta_1$$

$$(A\cos\theta_1 + jA\sin\theta_1) + (B\cos\theta_2 + jB\sin\theta_2)$$

$$= (A\cos\theta_1 + B\cos\theta_2) + j(A\sin\theta_1 + B\sin\theta_2)$$

### Multiplication of Phosors

- For this phasors should be represented in polar form

$$A \angle \theta_1 \times B \angle \theta_2 = A.B \angle \theta_1 + \theta_2$$

$$\frac{A \angle \theta_1}{B \angle \theta_2} = \left(\frac{A}{B}\right) \angle \theta_1 - \theta_2 .$$

- Change of phasors from polar form to rectangular form and vice versa is to be done frequently. This can be done using (A)

In calculator ...

**Example 1:** Determine the phase difference of the phasor $i_1 = 40 \sin(100\pi t + 30^\circ)$ w.r.t $i_2 = 60 \sin(100\pi t)$ in terms of time and draw the phasor diagram to represent the two phasors.

Soln:

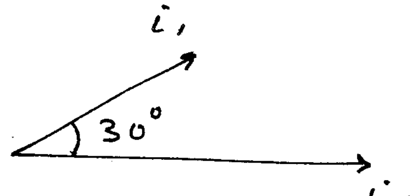
$$\omega = 2\pi f = 100\pi \qquad f = 50\ Hz$$

$$T = \frac{1}{f} = \frac{1}{50}\ sec.$$

The full revolution $(360^\circ)$ for either phasor is $(1/50)$ sec. The time taken to cover $30^\circ$ is

$$t = \frac{1}{50} \cdot \frac{30}{360} = \frac{1}{600}\ Sec = \underline{1.67\ mS}$$

Phasor $i_1$ leads $i_2$ by $30^\circ$. So phasor diagram is:




**Example 2:** A current $I = 4 + j3$ amperes is flowing through a resistor of $10\Omega$. How much power is being consumed by the resistor?

Soln:

$$I = 4 + j3 = \sqrt{4^2 + 3^2}\ \tan^{-1} 3/4$$

$$= 5 \angle 36.87^\circ$$

$$P = I^2 R = 5^2 . 10 = \underline{250\ W}$$

The power due to real component of current

$$P_1 = 4^2 . 10 = 16 . 10 = 160\ W$$

Powe due to imaginary component of current

$$P_2 = 3^2 . 10 = 9 . 10 = 90\ W.$$

Conclusion: The imaginary part of the current also carries real power.

Comment: 1. The concept of complex numbers i.e. real numbers and imaginary numbers is a mathematical concept and doesn't mean that imaginary numbers are for ~~imagina~~ imagination only. They don't have any real implication.

BUT

imaginary numbers are as real as real numbers.

2. Your imaginations/belief has real effect ~~on~~ in your life.

Example 3: Determine $10\angle 0^0 + 20\angle 60^0$

Soln:

$I_1 = 10\angle 0 = 10 + j0$

$I_2 = 20\angle 60 = 20\cos 60 + j20\sin 60$

$= 10 + j10\sqrt{3}$

$I_1 + I_2 = (10+10) + j10\sqrt{3} = 20 + j10\sqrt{3}$

$= 26.46\angle 40.9^0$

Example 4: Determine $I = \cancel{0}\, I_1 + I_2 + I_2$

$i_1 = 5\sin\omega t \qquad i_2 = 5\sin(\omega t + 30)$

$i_3 = 5\sin(\omega t - 120^\circ)$

**Soln:**

$$I_1 = \frac{5}{\sqrt{2}}\angle 0 = \frac{5}{\sqrt{2}} + j0 = 3.536 + j0$$

$$I_2 = \frac{5}{\sqrt{2}}\angle 30^\circ = \frac{5}{\sqrt{2}}\cos 30 + j\frac{5}{\sqrt{2}}\sin 30$$

$$= 3.06 + j\,1.768$$

$$I_3 = \frac{5}{\sqrt{2}}\angle -120^\circ = \frac{5}{\sqrt{2}}\left(\cos(-120^\circ) - j\sin(-120^\circ)\right)$$

$$= -1.768 - j\,3.06$$

$$I = I_1 + I_2 + I_3 = (3.536 + 3.06 - 1.768) + j$$
$$+ j\,(1.768 - 3.06)$$

$$= 4.828 - j\,1.292 = 5\angle \tan^{-1}\frac{-1.292}{4.828}$$

$$= \underline{5\angle -15^\circ}$$

**Example 5:** Determine $V/Z$

$$V = 100 - j50 \qquad Z = 10 + j15$$

**Soln:**

$$V = 100 - j50 = 111.8\angle -\tan^{-1}0.5 = 111.8\angle -26.6$$

$$Z = \cancel{111.8}\ 10 + j15 = 18.05\angle \tan^{-1}1.5 = 18.03\angle 56.3^\circ$$

$$\frac{V}{Z} = \frac{111.8\angle -26.6^\circ}{18.03\angle 56.3^\circ} = 6.2\angle -26.6 - 56.3$$

$$= 6.2\angle -82.9^\circ$$

$$= 6.2\left[\cos(-82.9^\circ) + j\sin(-82.9^\circ)\right]$$

$$= 6.2 \times 0.1236 + j\,6.2 \times (-0.9923)$$

$$= \underline{7.66 - j\,6.152}$$

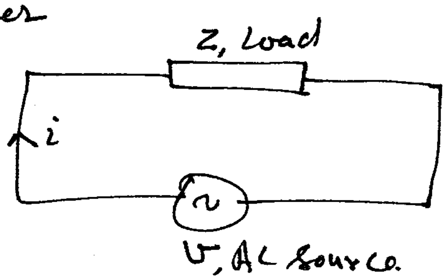
## Average and Effective Values of non-~~soi~~ sinusoidal Waveforms.

Do it your self

Have some practice of solving some problems.

## Power and Power Factor

~~Condi~~ Consider




$$p = vi = [V_m \sin \omega t][I_m \sin(\omega t - \theta)]$$

$$= V_m I_m \sin \omega t \sin(\omega t - \theta)$$

$$= \frac{V_m I_m}{2}\left[\cos(\omega t - (\omega t - \theta)) - \cos(\omega t + \omega t - \theta)\right]$$

$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\left[\cos\theta - \cos(2\omega t - \theta)\right]$$

$$= VI\left[\cos\theta - \cos(2\omega t - \theta)\right]$$

$$= \underbrace{VI\cos\theta}_{\text{Constant}} - VI\cos(2\omega t - \theta)$$

The average value of 2nd term $= 0$

$P = P_{av} = VI\cos\theta$

$P =$ real Power $= VI\cos\theta$

$VI =$ Apparant power

$$\cos\theta = \frac{P}{VI} = \frac{\text{real power}}{\text{Apparant power}}$$

In an inductive circuit I lags V and $\cos\theta$ is +ve

" " Capacitive " I leads V " " " −ve

This is convention.

Example : In an electric network, the instantaneous voltage and current are given as

$v = 55 \sin \omega t$ and $i = 6.1 \sin(\omega t - \pi/5)$

Determine $P_{av}$, $P_a$, $p$ at $\omega t = 0.3$, p.f.

Soln:

$$\theta = \pi/5$$

$$V = \frac{55}{\sqrt{2}} = 38.89 \text{ V}$$

$$I = \frac{6.1}{\sqrt{2}} = 4.31 \text{ A}$$

$$P_{av} = VI\cos\theta = 38.89 \times 4.31 \cos(\pi/5)$$

$$= 167.62 \times 0.809 = \underline{135.6 \text{ W}}$$

Apparent power $P_a = VI = 38.89 \times 4.31$

$$= \underline{167.62 \text{ VA}}$$

$$p = VI\cos\theta - V \cdot I\cos(2\omega t - \theta)$$

$$= 135.6 - 167.62\cos(2 \times 0.3 - \pi/5)$$

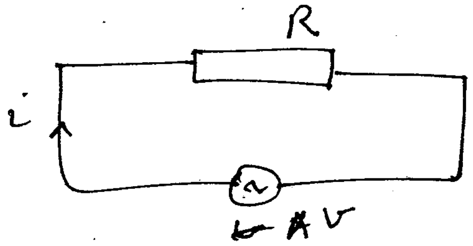
$$= 135.6 - 167.55 = \underline{-31.95 \text{ VA}}$$

$$pf = \cos\theta = \underline{0.809}$$

# Steady State Response to Sinusoids

## Resistive Circuit

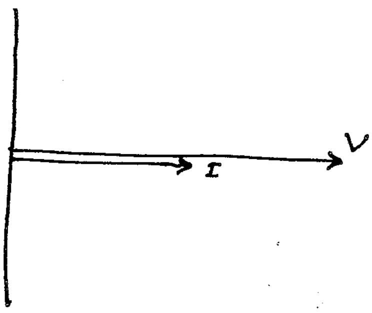
Consider




$i = \frac{v}{R}$ ; $I_m = \frac{V_m}{R}$ ; $I = \frac{V}{R}$

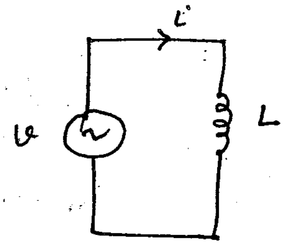
$$i = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t$$

$\therefore$ I & V are in phase




## Inductive Circuit

$i = I_m \sin \theta = I_m \sin \omega t$




induced emf

$$e = - L \frac{di}{dt} = - L \frac{d}{dt} ( I_m \sin \omega t)$$

$$= - L I_m \omega \cos \omega t = - \omega L I_m \cos \omega t$$

$$= \omega L I_m \sin ( \omega t - \pi/2)$$

So, ~~[illegible]~~ whole of the applied voltage $v$ is absorbed in neutralizing the induced voltage $e$.

$\therefore \quad v = -e = \omega L I_m \cos \omega t = \omega L I_m \sin (\omega t + \pi/2)$

$\therefore$ Applied voltage leads the current by $\pi/2$

OR

Current lags the applied voltage by $\pi/2$

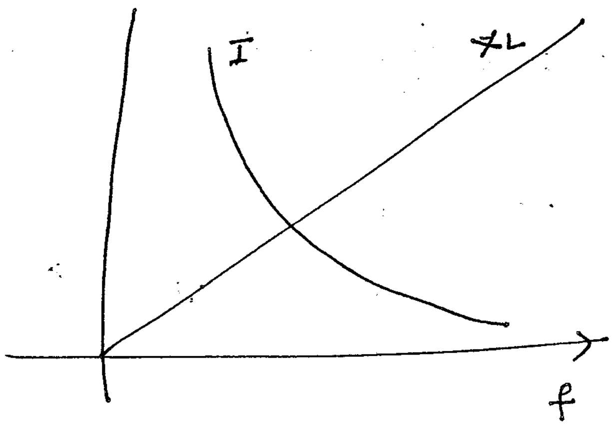
from above

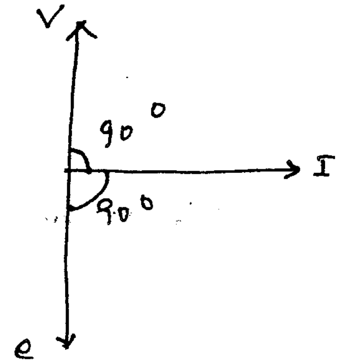
$$V_m = \omega L I_m$$

$$\frac{V_m}{I_m} = \omega L = \frac{V}{I} = \text{Inductive reactance}$$
$$= X_L \ \Omega$$

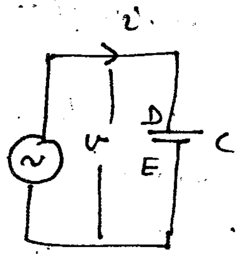
$$X_L = \omega L = 2\pi f L$$

$$I = \frac{V}{X_L} = \frac{V}{2\pi f L}$$




## Capacitive Circuit




$$i = C\frac{dv}{dt}$$

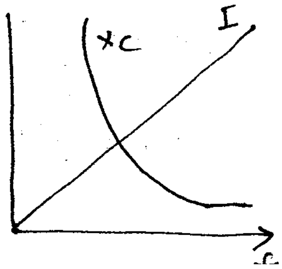
$v = V_m \sin \omega t = V_m \sin \theta$.

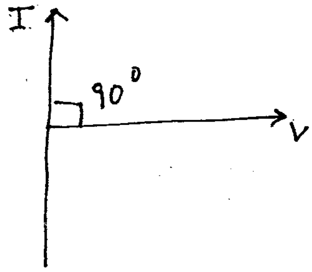
$$i = C\frac{dv}{dt} = C.\frac{d}{dt}(V_m \sin \omega t)$$

$$= C V_m \omega \cos \omega t = \omega C V_m \cos \omega t$$

$$= \omega C V_m \sin(\omega t + \pi/2)$$

$$\frac{V}{I} = \frac{1}{\omega C} = \frac{1}{2\pi f C} = X_C = \text{Capacitive Reactance}$$



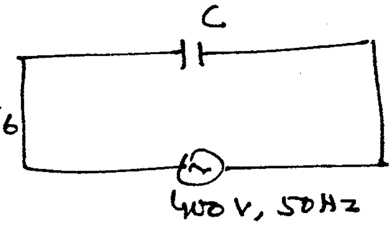



**Example:** A 30 μF capacitor is connected across a 400V, 50 Hz supply. Calculate

(a) reactance of the capacitor

(b) current in the circuit.

**Soln:**

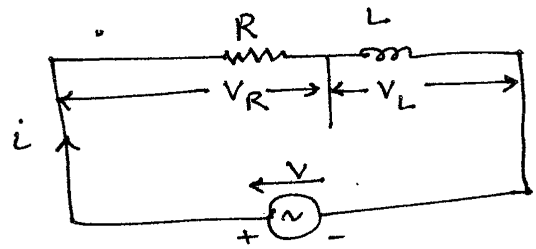



$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\times\pi\times 50\times 30\times 10^{-6}}$$

$$= \underline{106.2\ \Omega}$$

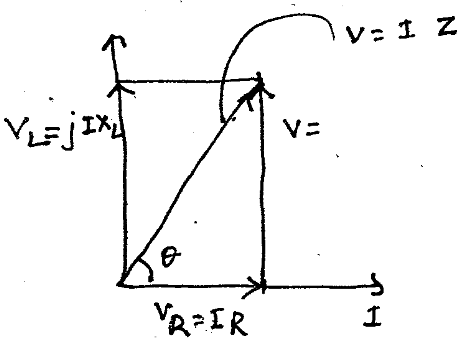
$$I = \frac{V}{X_C} = \frac{400}{106.2} = \underline{3.77\text{A}}$$

**Series RL Circuit** (reactive power + complex impedance)

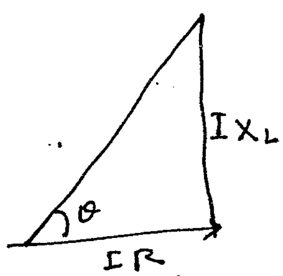



Bold letters means phasor in books.
$\bar{V}$ here means phasor

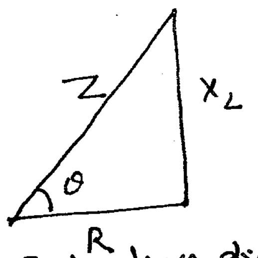
$$\bar{V} = \bar{V}_R + \bar{V}_L$$




Phasor diagram




Voltage diagram




Impedance diagram

$$\bar{I} = I\angle 0 \qquad \bar{V}_R = \bar{I}R \qquad \bar{V}_L = \bar{I}.jX_L = j\bar{I}X_L = j\bar{I}\omega L$$

$$\bar{V} = \bar{I}R + j\bar{I}\omega L = \bar{I}(R + j\omega L)$$

$$\bar{I} = \frac{\bar{V}}{R + j\omega L} = \frac{\bar{V}}{\bar{Z}}$$

$$\bar{Z} = R + j\omega L$$

$$Z = \sqrt{R^2 + \omega^2 L^2} \qquad \theta = \tan^{-1}\frac{\omega L}{R} = \sin^{-1}\frac{\omega L}{Z}$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V\angle 0}{Z\angle\theta} = \frac{V}{Z}\angle -\theta$$ (If V is taken as reference)

Example : For the circuit shown below

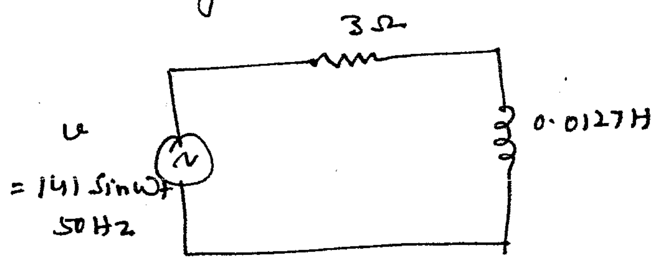
(i) Calculate the rms value of the steady state current as well as the reactive phase angle.

(ii) Write the expression for the instantaneous current

(iii) Find the average power dissipated in the circuit

(iv) Calculate power factor

(v) Draw the phasor diagram.




Soln:

(i) $\bar{V} = V\angle 0^\circ = \frac{V_m}{\sqrt{2}}\angle 0 = \frac{141}{\sqrt{2}}\angle 0 = 100\angle 0$

$= 100 + j\,0$

$\bar{Z} = R + j\omega L = 3 + j\,2\pi f L = 3 + j\,2\pi \times 50 \times 0.027$

$= 3 + j4 = 5\angle 53.1^\circ\ \Omega$

$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{100\angle 0}{5\angle 53.1} = 20\angle -53.1^\circ$ A

effective value of current = 20 A

lags by the $\bar{V}$ by $53.1^\circ$.

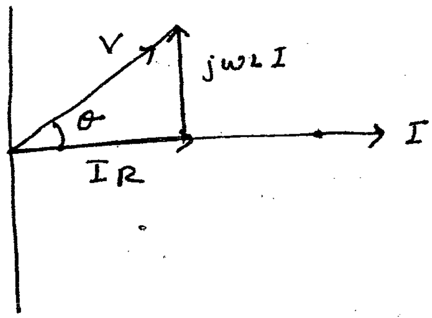
(ii) $i = \underline{20\sqrt{2}\,\sin(\omega t - 53.1^\circ)}$

(iii) $P_{av} = VI\cos\theta = 100 \times 20 \times \cos(53.1^\circ)$

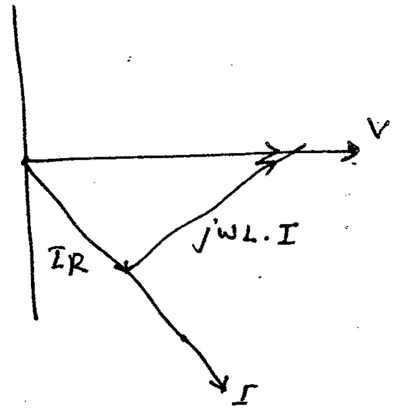
$= \underline{1200\ W}$

(IV) $pf = \cos\theta = \cos(53.1^\circ) = \underline{0.6\ \text{lagging}}$

(V) <u>Phasor diagram</u>




(A)




(B)

<u>Reactive Power</u>

$VI$ = Apparant Power

$VI\cos\theta$ = Real Power

If $\bar{V} = V\angle\theta$ and $\bar{I} = I\angle\phi$

$P = VI\cos(\theta - \phi)$ = Real power

$= VI\,Re\left[e^{j(\theta-\phi)}\right] = Re\left[\underbrace{(Ve^{j\theta})}_{I}\underbrace{(Ie^{-j\phi})}_{II}\right]$

I Factor $Ve^{j\theta}$ = phasor Voltage

II Factor $Ie^{-j\phi}$ = complex conjugate of $\bar{I}$

$= *$ $\quad I \quad -j\phi$

Therefore,

$$P = \text{real power} = \text{Re}[\bar{V}\bar{I}^*]$$

$S$ = Complex Power (VA)

$$\bar{S} = \bar{V}\bar{I}^* = VI\, e^{j(\theta-\phi)}$$

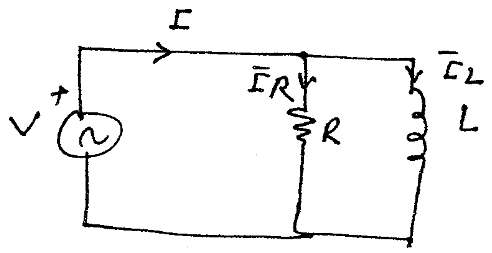
$$= P + jQ$$

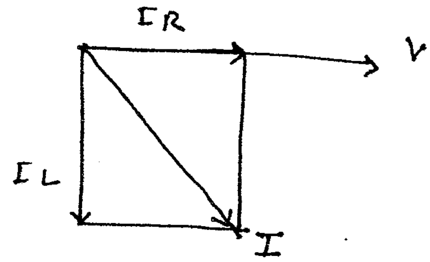
$P$ = Real Power (W)

$Q$ = Reactive Power (VAR)

$$Q = VI \sin(\theta - \phi)$$

## Parallel RL Circuit




$$\bar{I}_R = \frac{\bar{V}}{R}, \quad \bar{I}_L = \frac{\bar{V}}{jX_L} = \frac{\bar{V}}{j\omega L}$$

$$\bar{I} = \bar{I}_R + \bar{I}_L = \bar{V}\left[\frac{1}{R} + \frac{1}{j\omega L}\right] = \bar{V}\bar{Y}$$

$$\bar{Y} = \text{Complex admittance} = \frac{1}{R} - j\frac{1}{\omega L}$$

$$= G + jB$$

$G$ = conductance

$B$ = Susceptance

$$G = \frac{1}{R} \quad \text{and} \quad B = -\frac{1}{\omega L}$$

$$\bar{I} = \bar{V}\bar{Y} = \bar{V}(G + jB) = V\left(\frac{1}{R} - j\frac{1}{\omega L}\right)$$

$$\theta = \tan^{-1}\frac{-1/\omega L}{1/R} = \tan^{-1}\left(\frac{-R}{\omega L}\right)$$

**Example:** For $Z = 6 + j8$

Determine conductance.

**Soln.** $Y = G + jB = \dfrac{1}{Z} = \dfrac{1}{6+j8}$

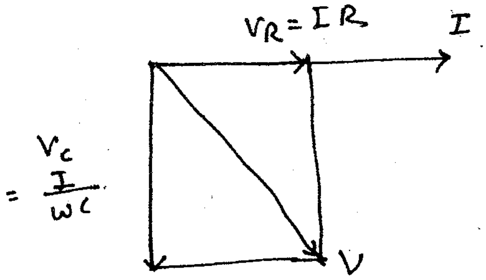
$$= \frac{6-j8}{(6+j8)(6-j8)} = \frac{6-j8}{6^2+8^2}$$

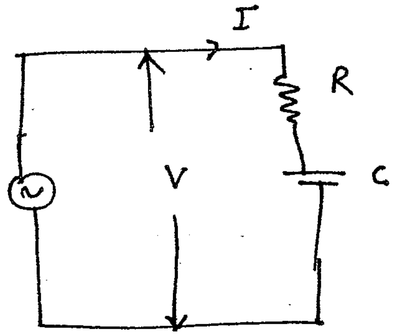
$$= \frac{6-j8}{100} = \frac{6}{100} - j\frac{8}{100}$$

$$= (.06 - j\,0.08)\ \mho$$

So $G = \underline{0.06\ \mho}$

## Series RC Circuit




$$\bar{V} = \bar{I}R + I\left[\frac{1}{jwC}\right] = \bar{I}\left[R + \frac{1}{jwC}\right] = \bar{I}\bar{Z}$$

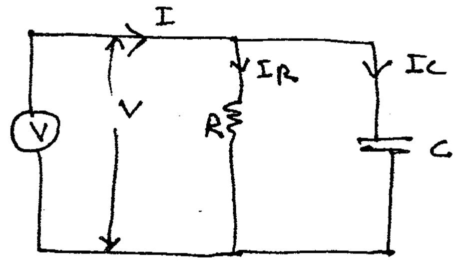
$$\bar{Z} = R - j\frac{1}{wC} = \sqrt{R^2 + \frac{1}{w^2C^2}}\ \angle -\tan^{-1}\frac{1}{wCR}$$

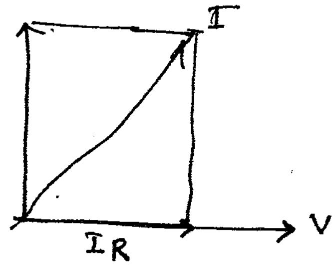
$$\bar{I} = I\angle\theta = \frac{V\angle 0}{\sqrt{R^2 + 1/w^2C^2}}\ \angle \tan^{-1}(1/wCR)$$

$$I = \frac{V}{\sqrt{R^2 + 1/w^2C^2}} \qquad \theta = \tan^{-1}(1/wRC)$$

$$i = I_m \sin(wt + \theta) = \frac{V_m}{\sqrt{R^2 + 1/w^2C^2}} \sin\left(wt + \tan^{-1}(1/wCR)\right)$$

## Parallel RC Circuit




$$\bar{I}_R = \frac{\bar{V}}{R} \qquad I_C = \frac{\bar{V}}{\frac{1}{j} X_C} = \frac{V}{1/j\omega C} = Vj\omega C$$

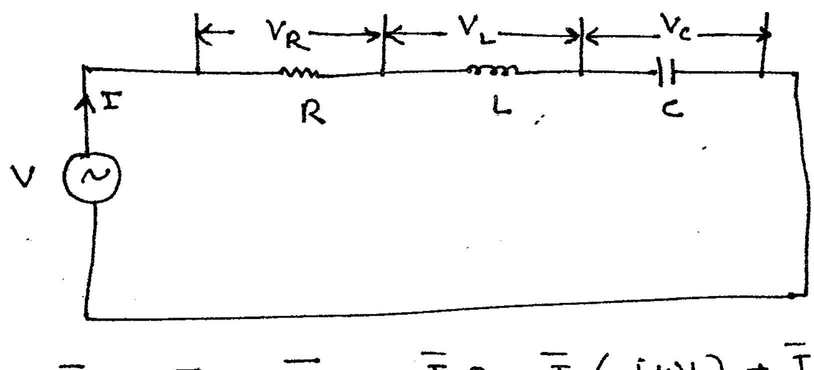
$$\bar{I} = \bar{I}_R + \bar{I}_C = \frac{\bar{V}}{R} + \bar{V}(j\omega C) = \bar{V}\left[\frac{1}{R} + j\omega C\right] = \bar{V}\bar{Y}$$

$\bar{Y}$ = Complex ~~impedance~~ Admittance = $G + jB$

$$G = \frac{1}{R} \qquad B = \omega C$$

$$\theta = \tan^{-1}\frac{B}{G} = \tan^{-1}\frac{\omega C}{1/R} = \tan^{-1}\omega CR$$

## Series RLC Circuits




$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C = \bar{I}R + \bar{I}(j\omega L) + \bar{I}(1/j\omega C)$$

$$= \bar{I}\left[R + j\omega L + \frac{1}{j\omega C}\right] = \bar{I}\left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right] = \bar{I}\bar{Z}$$

$$\bar{Z} = R + j\left[\omega L - \frac{1}{\omega C}\right] = \sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2} \angle \tan^{-1}\frac{(\omega L - 1/\omega C)}{R}$$

$$\bar{I} = I\angle\theta = \frac{V\angle 0^\circ}{\sqrt{R^2 + (\omega L - 1/\omega C)^2} \angle \tan^{-1}\frac{(\omega L - 1/\omega C)}{R}}$$

$$i = \sqrt{2}\, I \sin(\omega t + \theta)$$

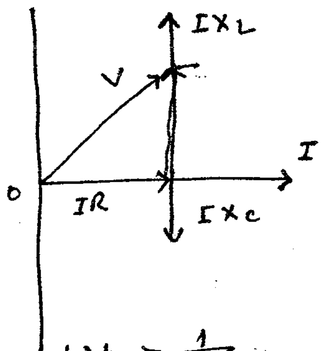
$$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \; ; \quad \theta = -\tan^{-1}\frac{(\omega L - 1/\omega C)}{R}$$

$\bar{I}$ may lag, lead or in phase with $V$

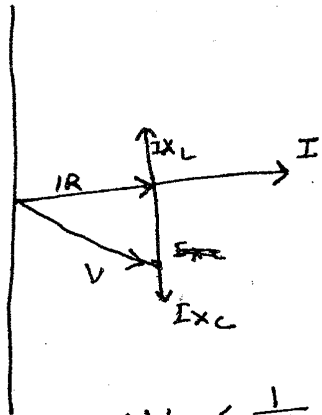
(a) $\omega L > \frac{1}{\omega C}$ $\qquad \theta$ is $-ve$ $\qquad \bar{I}$ lags $\bar{V}$

(b) $\omega L < 1/\omega C$ $\qquad \theta$ is $+ve$ $\qquad \bar{I}$ leads $\bar{V}$

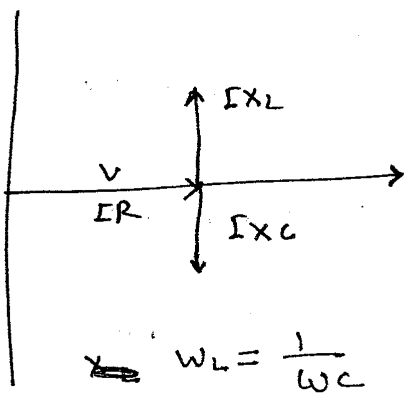
(c) $\omega L = \frac{1}{\omega C}$ $\qquad \theta = 0$ $\qquad \bar{I}$ in phase with $\bar{V}$ (Resonance)




$\omega L > \frac{1}{\omega C}$
$X_L > X_C$

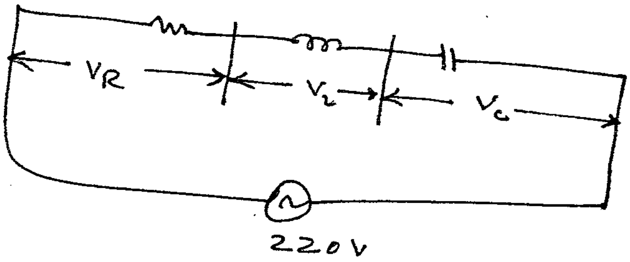



$\omega L < \frac{1}{\omega C}$
$X_L < X_C$




$\omega_L = \frac{1}{\omega C}$
$X_L = X_C$

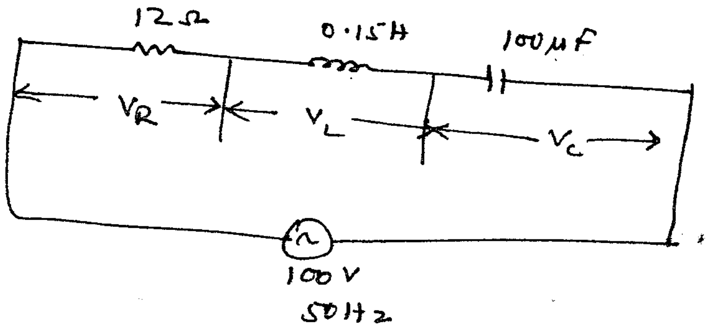
## Example 1




If $V_L = 50V$ $V_c = 50V$ find $V_R = ?$

Soln : $V_R = 220V$

## Example 2




Calculate :

(a) impedance (b) current (c) $V_R$, $V_L$ & $V_c$

(d) phase difference between $\bar{V}$ and $\bar{I}$

Soln : (a) $R = 12\,\Omega$

$X_L = \omega L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.1$

$X_c = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.85$

$Z = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{12^2 + (47.1 - 31.85)^2} = \underline{19.4\,\Omega}$

(b) $I = V/Z = \frac{100}{19.4} = \underline{5.15\ A}$

~~(iii)~~ (c) $V_R = IR = 5.15 \times 12 = \underline{61.8\,V}$

$V_L = I X_L = 5.15 \times 47.1 = \underline{242.5\,V}$

$V_c = I X_c$

~~(iv)~~
(d) Phase difference between $\bar{V}$ and $\bar{I}$

$$\phi = \cos^{-1}\frac{\dot{V}_R}{V} = \cos^{-1}\left[\frac{61.8}{100}\right] = 51^\circ 50'$$

Also
$$\phi = \tan^{-1}\frac{(X_L - X_C)}{R} = \tan^{-1}\frac{(47.1 - 31.85)}{12}$$

$$= \tan^{-1} 1.271 = 51^\circ 50'$$

Since $X_L > X_C$ $\bar{I}$ lags $\bar{V}$.

## Resonance in Series Circuits

Definitions

1. Circuit p.f. is unity
2. $\bar{V}$ and $\bar{I}$ are in phase
3. $X$ is zero
4. B is zero.

Consider the circuit at p-36

$$Z = R + jX_L - jX_C = R + j(X_L - X_C)$$

If $X_L - X_C = 0 \longrightarrow$ resonance

The conditions of resonance are represented as

$$f_0,\ Z_0,\ X_{C0},\ X_{L0}$$

Therefore: At Resonance

$$X_{L0} - X_{C0} = 0 \qquad X_{L0} = X_{C0}$$

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

~~fo~~ $Z_0 = R + j0$ & $I_0 = \frac{V}{R}$

and

$$V_R = I_0 R = \frac{V}{R} R = V$$

$$V_L = I_0 X_{L0} = \frac{V}{R} X_{L0} = V \frac{X_{L0}}{R} = VQ$$

$$V_C = I X_{C0} = \frac{V}{R} X_{C0} = V \frac{X_{C0}}{R} = VQ$$

But, $X_{L0} = X_{C0}$

$\therefore \quad V_L = V_C$ and $V_R = V$

Quality Factor (of a coil)

- It is measure of effectiveness of a coil
- It is ~~defi~~ defined as ratio of energy stored to the energy dissipated per cycle.

$$Q = \frac{\text{Stored Power}}{\text{Dissipated power}} = \frac{I^2 X_L}{I^2 R} = \frac{X_L}{R} = \tan\theta$$

$I^2 X_L$ = Reactive Power in the ~~cir~~ coil
= Power stored

Here R means the resistance of the coil and ~~not~~ may/may not ~~doesn't~~ include any resistance outside the coil.

- Quality factor is used as a figure of merit for the coil

- ↑ Q means better coil.

$Q < 10$ low-Q-coils

$Q > 10$ high-Q-coils

$Q \simeq 200-300$ are used in electronic circuits.

Comments: If Q of a coil is

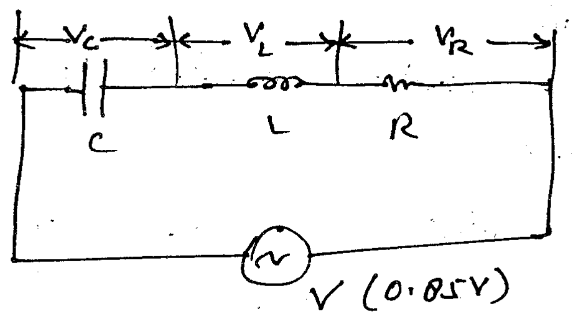
Example: The voltage applied to the series circuit given below. The Q of the coil is 50 and the value of capacitor is 320 pF. The resonant frequency is 175 kHz. Find:

(a) L

(b) Current in circuit

(c) Voltage across capacitor under resonance

(d) Draw phasor diagram.




Soln:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$L = \frac{1}{(2\pi)^2 . C . f_0^2}$$

$$= \frac{1}{4(\pi^2) . 320\times10^{-12} \times (175)^2} = \underline{2.58 \text{ mH}}$$

$$X_{L0} = X_{C0} = 2\pi f_0 L = 2\pi(175\times10^3)\times 0.00258$$

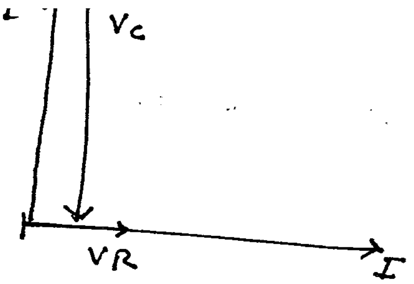
$$= 2836.85\,\Omega$$

$$R = \frac{X_{L0}}{Q} = \frac{2836.85}{50} = \underline{56.74\,\Omega}$$

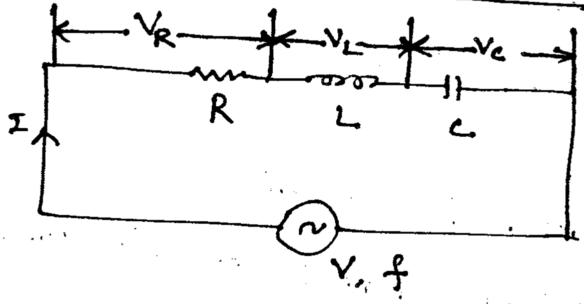
~~Z of~~ $Z_0 = (56.74 + j0)\,\Omega$

$$I_0 = \frac{V}{R} = \frac{0.85}{56.74} = 0.01498A = \underline{14.98 \text{ mA}}$$

$$V_c = QV = 50\times 0.85 = 42.5V = V_L$$




## Effects of Variation of Frequency




Let R, L, C, and V are fixed

f is variable

Purpose, here is to study variation of $X_L$ and $X_C$ with frequency

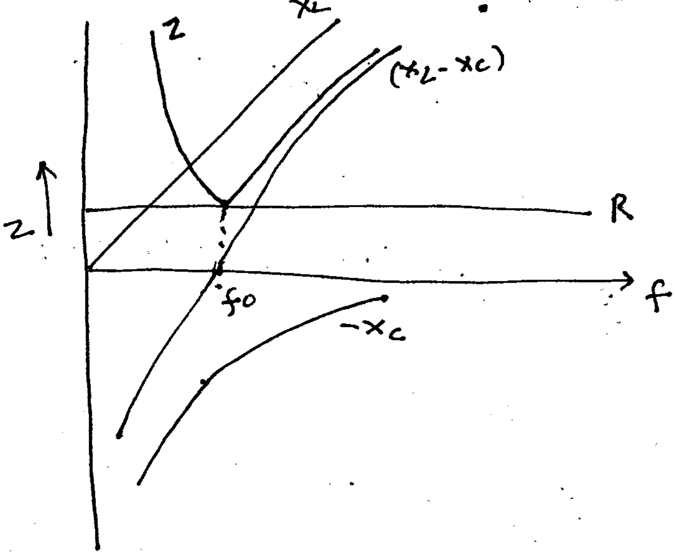
+j is associated with $X_L$, $X_L$ is taken as +ve

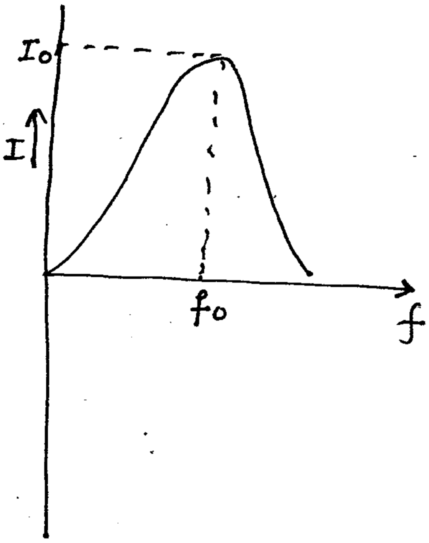
−j " " " $X_C$, $X_C$ " " −ve

$$X_L = 2\pi f L$$

$$X_C = \frac{1}{2\pi f C}$$

Net reactance $X = X_L - X_C$



at $f < f_0$ circuit is capacitive $I$ leads $V$

$f > f_0$ " " Inductive $I$ lags $V$

$f = f_0$ " " Resistive. $I$ in phase with $V$

$$\bar{Z} = \sqrt{R^2 + (X_L - X_C)^2} \angle \tan^{-1} \frac{X_L - X_C}{R}$$

$$\bar{Z}_0 = R + j0 = R$$

$$I_0 = \frac{V}{Z_0} = \frac{V}{R}$$

$I$ is maximum at $f_0$

<u>Next,</u>

Consider $f_1$ and $f_2$ where

$$|X_L - X_C| = R$$

At $f_1$ circuit is capacitive $X_L - X_C = -R$

$f_2$ " " Inductive. $X_L - X_C = R$

Now, the impedances at $f_1$ and $f_2$

$$Z_1 = \sqrt{R^2 + (-R)^2} \angle \tan^{-1} \frac{-R}{R} = \sqrt{2}\, R \angle -45^0$$

$$Z_2 = \sqrt{R^2 + R^2} \angle \tan^{-1} \frac{R}{R} = \sqrt{2}\, R \angle 45^0$$

<u>Currents</u>

$$I_1 = \frac{V}{\sqrt{2}\, R \angle -45^0} = \frac{V}{\sqrt{2}\, R} \angle 45^0 = 0.707\, I_0 \angle 45^0$$

$$I_2 = \frac{V}{\sqrt{2}\, R \angle 45^0} = \frac{V}{\sqrt{2}\, R} \angle -45^0 = 0.707\, I_0 \angle -45^0$$

This means at $f_1$ as well as $f_2$ the current falls to $0.707\, I_0$. This concept can be used to define <u>Bandwidth</u>.

$$BW = f_2 - f_1$$

$f_1$ = lower cut-off frequency

$f_2$ = upper cut off frequency

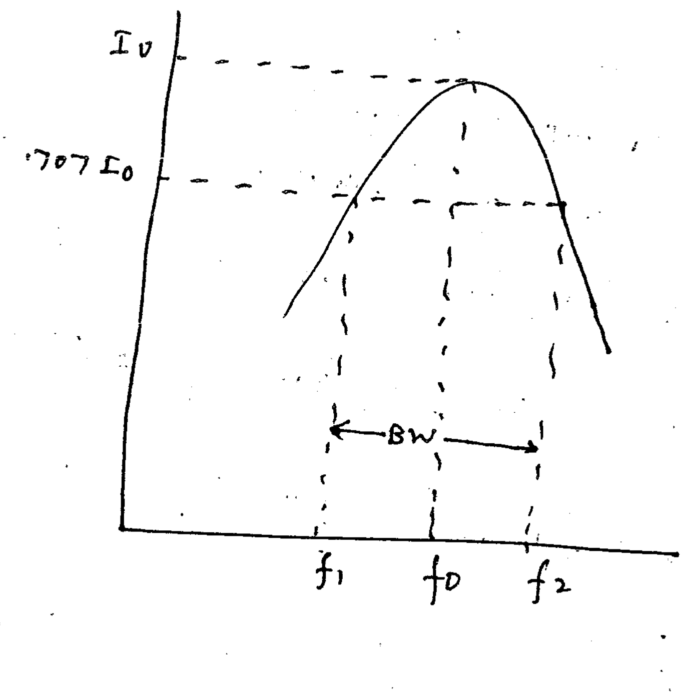
— At $f_1$ and $f_2$ current reduces to $I_0/\sqrt{2}$

$$P_0 = I_0^2 R$$

$$P_1 = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = \frac{I_0^2}{2} R = P_0/2$$

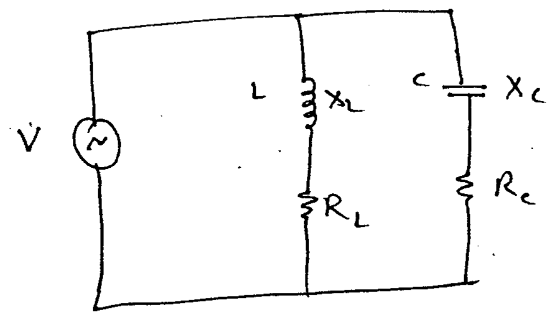
$$P_2 = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = \frac{I_0^2 R}{2} = P_0/2$$

So, $f_1$ and $f_2$ ~~are~~ ~~also~~ known as <u>half power frequencies</u>.



# Resonance in Parallel AC Circuits

Consider :




$$\bar{X}_C = \frac{1}{j\omega C} = -j\left(\frac{1}{\omega C}\right)$$

$$Z_L = R_L + jX_L \qquad Y_L = \frac{1}{R_L + jX_L}$$

$$Y_L = \frac{R_L - jX_L}{(R_L + jX_L)(R_L - jX_L)} = \frac{R_L - jX_L}{R_L^2 + X_L^2}$$

And

$$Z_C = R_C - jX_C \qquad Y_C = \frac{1}{R_C - jX_C}$$

$$Y_C = \frac{R_C + jX_C}{(R_C - jX_C)(R_C + jX_C)} = \frac{R_C + jX_C}{R_C^2 + X_C^2}$$

$$Y_T = \cancel{Y_L +} Y_L + Y_C = \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_C + jX_C}{R_C^2 + X_C^2}$$

Collecting the real and imaginary terms.

$$Y_T = \left[\frac{R_C}{R_C^2 + X_C^2} + \frac{R_L}{R_L^2 + X_L^2}\right] + j\left[\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2}\right]$$

At resonance the reactance part (j part) = 0

$$\therefore \frac{X_{C0}}{R_C^2 + X_{C0}^2} - \frac{X_{L0}}{R_L^2 + X_{L0}^2} = 0$$

$$\Rightarrow \quad f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - (L/C)}{R_C^2 - (L/C)}}$$

- In most of the applications C can be considered lossless i.e. $R_c = 0$.
- If it is so.

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2-(L/c)}{-(L/c)}} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1-R_L^2C/L}$$

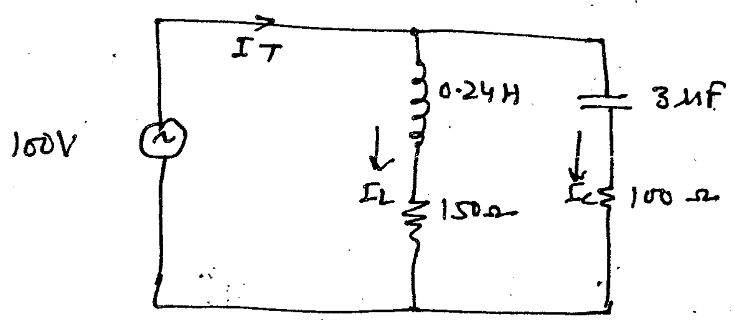
If $R_L$ is also $= 0$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

<u>Example:</u> For the circuit given below, determine

(a) Source current $\bar{I}_T$

(b) input impedance $\bar{Z}_T$




<u>Soln:</u>

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2-(L/c)}{R_c^2-(L/c)}} = \underline{170\ Hz}$$

$$X_{L0} = 2\pi f_0 L = 2\pi \times 170 \times 0.24 = 256\,\Omega$$

$$X_{C0} = \frac{1}{2\pi f_0 C} = \frac{1}{2\pi \times 170 \times 3\times10^{-6}} = 312\,\Omega$$

$$\bar{I}_L = \frac{V}{R_L + jX_{L0}} = \frac{100}{150+j256} = (0.170 - j0.291)\,A$$

(Do it on calculators & report now)

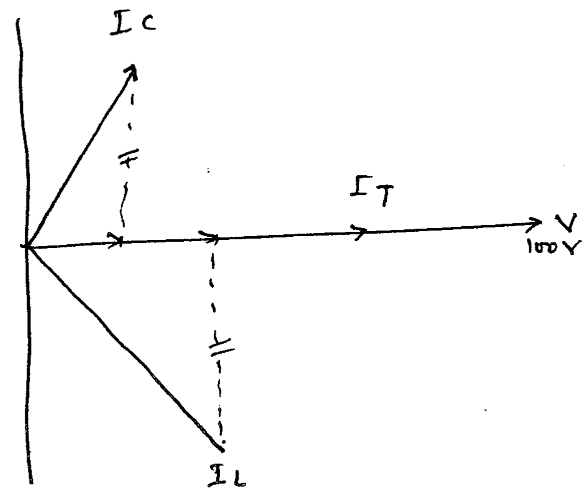
$$\bar{I}_C = \frac{V}{R_c - jX_{C0}} = \frac{100}{100-j312} = (0.093 + j0.291)\,A$$

Total current

$$\bar{I}_T = \bar{I}_L + \bar{I}_C = (0.170 - j\,0.291) + (0.0393 + j\,0.291)$$

$$= (0.263 + j0)\ A =$$

$$\bar{Z}_T = \frac{V}{I_T} = \frac{100}{0.263 + j0} = \underline{\underline{(380 + j\,0)}}\ A$$




Example: