

## Theoretical Part.

### 1. Subproblems :-

The main aim is to find  $MIN(n)$ .  
We can solve this task by dividing it into 'i' fragments.

→ The last line segment approximates the points  $P_i \dots P_n$ .

→ The rest of line segments constitute the solution for  $P_1 \dots P_{i-1}$ .

Therefore, the optimal solution for  $n$ -two can be built by breaking it into two fragments of 'i'-length.

Assume the line segment  $L_0$  is the last segment of the optimal solution for  $P_1 \dots P_j$  and it gives the approx. values  $P_i \dots P_j$  and the rest of the segments of this soln are different from the set that approximates  $P_1 \dots P_{i-1}$ . This set is non-empty.

We combine this soln for  $P_1 \dots P_{i-1}$  with  $L_0$  and get a better soln  $P_1 \dots P_j$  with

this contradicts our assumption for optimal solution.

Hence, set constituting the optimal solution for  $P_1 \dots P_j$  is optimal substructure.



2. Best case: (Base case).

Base case  $\Rightarrow \text{MIN}(0) = 0$   
Approximating 0 points costs 0.

We build other sub-problems based on this base case.

3. Recurrence Relation.

$$\text{MIN}(j) = \min_{1 \leq i \leq j} \{ e(i, j) + 1 + \text{MIN}(i-1) \}$$

$\Rightarrow$  Trying all the possibilities for the least square line segment.

For each  $i$  we find the least square approximation of  $P_i \dots P_j$ .

This costs  $e(i, j) + 1 + \text{MIN}(i-1)$ .

$e(i, j) \Rightarrow$  sum of squared errors.

1  $\Rightarrow$  cost of creating a new line.

$\text{MIN}(i-1) \Rightarrow$  cost for approximating the set of remaining points.

$\Rightarrow$

4. Pseudocode.

$MIN[0] = 0;$

for  $j: 1$  to  $n$   $MIN[j] = +infinity;$

for  $i: 1$  to  $j-1$   $MIN[j] = \min(a, b)$

return  $MIN[n];$

$a \rightarrow MIN[j];$

$b \rightarrow e(i, j) + i + MIN[i-1];$

5. Time Complexity.

$n \rightarrow$  Total number of points.

each problem is calculated in  $O(n)$

we have  $n$ -sub problems.  
therefore,  $O(n^2)$ .