Basic Numerical Techniques for Astrophysics and Cosmology

1 Ordinary Differential Equations

1.1 Problem 1

Write a program using Euler's Method and using the 2^{nd} and 4^{th} order Runge-Kutta Method to find the phase space trajectory of a particle of unit mass in a 1-D potential $V(x) = (x^2 - 1)^2$.

- Show the trajectory for the initial conditions (x, P_x) = (1.0, 0.1), (-1.0, 0.1), (1.0, 10.0).
- Check the conservation of energy for different step sizes, for both the Euler Method and the 2^{nd} and 4^{th} Order Runge-Kutta method. Determine the step size where you have at least 1% accuracy in the energy over one whole period of the particle.

1.2 Problem 2

Write a program to follow the motion of an electron in an electric field $E(x, t)$ and a magnetic field $B(x, t)$. Numerically determine the trajectory of an electron which starts at the origin with velocity $v = (1.0, 1.0, 1.0)$ m/sec for the following field configurations:

- Uniform magnetic field 10^{-4} Tesla along the z axis.
- Uniform magnetic field 10^{-6} Tesla along the z axis and an uniform electric field 1V/m along the Y axis.
- Uniform Electric field 5×10^{-3} V/m ($|\vec{B}| = 0$) along the X-axis. Use first the Euler method and then the Leap frog method.

1.3 Problem 3

Solve the two dimensional trajectory of a projectile fired from a cannon shell located at the origin using the Euler method. Assume the initial projected speed is 700 m/s, and using different firing angles starting from 20° to 60° with an interval of 5°. Neglect the effects of the air resistance. Plot the trajectories of the projectile for different firing angles. Also plot the range of the projectile against the firing angles and show numerically that the maximum range of the projectile corresponds to a firing angle of 45°. Compare the numerical results with the analytical solutions obtained for the range and the duration of the projectile.

1.4 Problem 4

Extend the above problem to include the effect of the air resistance by assuming a value of $b/m = 4 \times 10^{-5}\text{m}^{-1}$, where the air drag force is expressed as $F_{\text{drag}} = bv^2$. Plot the trajectories for a given firing angle for the case of no air resistance and with air resistance.
Repeat this for different firing angles. Plot the range of the projectile against the different firing angles and find numerically the firing angle which corresponds to the maximum range of the projectile.