Part II [Vatic]

◦ Matching Solar Loss with Equivalent Wind Loss

We consider an electricity system with solar, wind, and multiple thermal generating units (coal, combined-cycle gas, steam turbines, etc.), for a winter day (January 19) and a summer day (July 19).

We will study two forms of system stress:

1. A **Cloudy Day Scenario**: a uniform 75% reduction in solar generation (and equivalent wind shock).

2. A **Daytime Demand Shock (Summer only)**: a 40% increase in load from 10:00–18:00 due to a heat wave.

Let S_{base} = total daily solar MWh (baseline), W_{base} = total daily wind MWh (baseline). A 75% reduction in solar means

$$\Delta S = 0.75\, S_{\text{base}}.$$

Suppose wind is reduced by a fraction p, so that lost wind energy is

$$\Delta W = p\, W_{\text{base}}.$$

Using the simulator's data for January 19 and July 19, compute the numerical value of p such that same amount of production is removed from the market.

For each day, and for each of the following scenarios, run the simulation for:

a) Baseline

b) Cloudy Day (25% solar reduction)

c) Wind-equivalent scenario (wind reduced by your computed percentage p)

Record the total system CO_2 emissions (metric tons) generated by the simulator. Compare the CO_2 increase from removing solar and removing the equivalent amount of wind.

◦ Summer Daytime Demand Shock (10:00–18:00)

Due to extreme heat wave and increased AC usage, a 40% increase in load is applied between 10:00 and 18:00 on July 19. Run the simulator with the modified summer load.

Report the total CO_2 (metric tons), thermal dispatch by technology, available headroom for nuclear, coal, combined cycle, etc.

Using the price outputs, discuss the changes between:

a) Baseline July 19 price curve

b) Shocked July 19 price curve

Explain the differences, using dispatch and headroom results, including which sources' headroom may fall to zero under the load shock.

Part II [Intensity Control]

You are advising a power-generation consortium on optimal capacity expansion. Total installed capacity is denoted by s, demand by x, and the spot price is given by

$$P(s,x) = \sqrt{\frac{x}{\max(s,\varepsilon)}} - 1,$$

for a small numerical constant $\varepsilon > 0$.

The consortium can invest in N different technologies. For technology j:

- A one-step capacity increment is δ_j (measured in units of s).
- Investment is controlled via an intensity $\lambda_j(t,s,x) \geq 0$.
- The instantaneous cost of intensity λ is of the form

$$C_j(\lambda) = \rho_j \lambda + \frac{1}{\beta_j}\lambda^{\beta_j}, \qquad \beta_j > 1,\ \rho_j > 0.$$

The consortium maximizes the discounted expected present value of profits over a finite horizon $[0,T]$, with discount rate r and demand-growth parameter λ_X. The associated Hamilton–Jacobi–Bellman (HJB) equation is solved numerically on a grid in (s,x)-space using the Python code that is provided to you in the file `hjbSolver.py`.

- Use the provided Python code to generate the requested plots and numerical summaries.
- Answer the economic and conceptual questions based on these outputs.

○ Base Case Analysis

In the base case, there are four technologies:

$$\begin{array}{llll}
\delta_{\mathrm{A}} = 3.0, & \delta_{\mathrm{B}} = 2.0, & \delta_{\mathrm{C}} = 5.0, & \delta_{\mathrm{D}} = -1.0, \\
\beta_{\mathrm{A}} = 2.0, & \beta_{\mathrm{B}} = 2.0, & \beta_{\mathrm{C}} = 2.0, & \beta_{\mathrm{D}} = 2.0, \\
\rho_{\mathrm{A}} = 0.2, & \rho_{\mathrm{B}} = 0.2, & \rho_{\mathrm{C}} = 0.2, & \rho_{\mathrm{D}} = 0.2.
\end{array}$$

The planning horizon and market parameters are

$$T = 5, \qquad r = 0.02, \qquad \lambda_X = 4.$$

The code discretizes s and x on a regular grid with $I = 50$ supply nodes and $J = 50$ demand nodes, minimum capacity level $s_{\min} = 0.1$, and steps $\Delta s = 1$, $\Delta x = 1$.

1. **Run the base-case script.** Using the provided code (with the base-case parameters above), solve the HJB and obtain $V(0,s,x)$ and $\lambda_j^*(0,s,x)$ for $j \in \{\text{A,B,C,D}\}$.

2. **Plot the value function surface.** Create a 3D surface plot of $V(0, s, x)$ with x on one axis, s on the other, and V on the vertical axis.

   a) Describe how $V(0, s, x)$ varies with s holding x fixed: is the value function increasing, decreasing, or non-monotone in capacity? What does this say about the value of additional capacity?

   b) Describe how $V(0, s, x)$ varies with x holding s fixed.

   c) Comment on any curvature you observe (e.g., diminishing returns) and provide an economic interpretation.

3. **Cross-sections in demand.** Using the code, extract cross-sections of the optimal intensities at $x \approx 10$ and $x \approx 30$ (i.e., fix x and plot $\lambda_j^*(0, s, x)$ as a function of s for each technology j).

   a) At $x \approx 10$, which technologies are deployed first?

   b) At $x \approx 30$, how does the ordering of technologies change, if at all? Are higher-δ technologies favored at higher demand levels?

○ Policy Implications and Subsidy Design

Suppose Technology A (with $\delta_A = 3$) is interpreted as a representative *renewable* technology that policymakers would like to promote. In the base case of Part 1, Technology A has parameters $\beta_A = 2.0$ and $\rho_A = 0.2$. We consider three policy scenarios that change only these two parameters for Technology A, holding all other technologies fixed:

- **Baseline:** no subsidy (same as Part 1).
- **Subsidize ρ:** lower fixed-cost term for Tech A to $\rho_A = 0.1$.
- **Subsidize β:** lower convexity for Tech A to $\beta_A = 1.6$.
- **Subsidize both:** moderate reductions, $\rho_A = 0.15$ and $\beta_A = 1.8$.

1. **Solve the HJB under each policy scenario.** Using the provided code and the above parameter changes for Technology A, recompute the HJB solution for:

$$\text{Baseline}, \quad \text{“Subsidize } \rho\text{”}, \quad \text{“Subsidize } \beta\text{”}, \quad \text{“Subsidize both”}.$$

   For each scenario, obtain the surface $\lambda_A^*(0, s, x)$ for Technology A.

2. **Pointwise subsidy effectiveness comparison at selected states.** Construct a small table that, for each appropriate test points, lists the baseline λ_A^* and the absolute and percentage changes under each subsidy scenario.

3. **Connection to real-world renewable policies.** Provide a brief discussion (a couple sentences) comparing your findings to actual renewable-energy support schemes, which real-world policies behave more like reducing ρ or β?