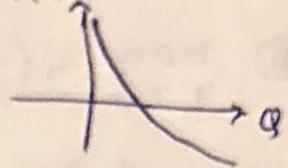


Parth Sathish Laturia

(Q.1) Given :-  $P(Q) = -\log Q + N \text{ player P}(Q)$ 

$$\Rightarrow (a) \lim_{Q \rightarrow 0^+} P(Q) = \lim_{Q \rightarrow 0^+} -\log Q \\ = \underline{\underline{\infty}}$$



$$(b) P(Q) < 0 \Rightarrow -\log Q < 0 \Rightarrow \log Q > 0 \Rightarrow Q > 1$$

$\Rightarrow$  saturation qty = 1

(c)  $N=1$ , to solve for  $q$  s.t.  ~~$q(-\log q - c)$~~  is max

$$\hat{q}^* = \arg \max_{q \geq 0} q(-\log q - c) \quad \hookrightarrow \text{Revenue}$$

$$\Rightarrow \text{let } f(q) = q(-\log q - c) \Rightarrow \hat{q}^* = \arg \max_{q \geq 0} f(q)$$

$$\Rightarrow \frac{df}{dq} \Big|_{q=q^*} = 0 \Rightarrow \hat{q}^* \left( -\frac{1}{q^*} \right) + (-\log q^* - c) \cdot 1 = 0$$

$$\Rightarrow 1 + \log q^* + c = 0$$

$$\Rightarrow \log q^* = -(c+1)$$

$$\Rightarrow \boxed{q^* = e^{-(c+1)}}$$

To verify that  $\frac{d^2f}{dq^2} \Big|_{q=q^*} > 0$ ,  $\frac{d^2f}{dq^2} = \frac{d}{dq} (-\log q - c - 1)$

$$= -\frac{1}{q^*} \Big|_{q=q^*} = -\frac{1}{e^{-(c+1)}} = -\frac{1}{e^{c+1}} < 0$$

$-q^* = e^{-(c+1)}$  is the qty at which Revenue is maximized

$$\Rightarrow P(q^*) = -\log q^* = -(-c-1) = \boxed{c+1}$$

(d)  $N=2$  but same cost

To find :-  $q_1^*$ ,  $q_2^*$ ,  $P(Q^*)$  ~~DP~~

Eq<sup>n</sup> solved :-

$$\textcircled{1} \max_{q_1 > 0} q_1 (-\log(q_1 + q_2) - c)$$

$$\textcircled{2} \max_{q_2 > 0} q_2 (-\log(q_1 + q_2) - c)$$

$$\frac{d\textcircled{1}}{dq_1} = 0 \Rightarrow q_1 \left( -\frac{1}{q_1 + q_2} \right) + (-\log Q - c) = 0$$

$$\downarrow \quad \Rightarrow \frac{q_1}{Q} + \log Q + c = 0 \quad \textcircled{3}$$

After this,  
we don't do  $\frac{d\textcircled{1}}{dq_2}$   $\because$   $\textcircled{1}$  represents player 1  
+ he/she is optimising only for  $q_1$ ,

$$\text{likewise, } \frac{d\textcircled{2}}{dq_2} = 0 \Rightarrow \frac{q_2}{Q} + \log Q + c = 0 \quad \textcircled{4}$$

$\textcircled{3} + \textcircled{4}$  gives :-

$$\frac{Q}{Q} + 2\log Q + 2c = 0$$

$$\Rightarrow 2\log Q + 2c + 1 = 0$$

$$\Rightarrow \log Q = \underline{\underline{-c - \frac{1}{2}}}$$

$$\therefore Q^* = \underline{\underline{Q^*}} = \exp(-c - \frac{1}{2}) \quad \textcircled{5}$$

$\textcircled{3} - \textcircled{4}$  gives :-

$$q_1 = q_2 \Rightarrow q_1^* = q_2^*$$

$$\text{So, } \underline{\underline{q_1^* = q_2^* = \frac{Q^*}{2} = \frac{1}{2} \exp(-c - \frac{1}{2})}}$$

$$P(Q^*) = -\log Q^* = \boxed{C + \frac{1}{2}}$$

② per barrel savings =  $P(Q_M^*) - P(Q_D^*)$  | Note that  $P(Q) = -\log Q$   
 $= (c+1) - (c + \frac{1}{2})$  is ~~zero~~ (assumed to be)  
 $= \frac{1}{2}$  price per barrel only

	# barrels sold (in the optimal scenario)
Monopoly	$q^* = e^{-(c+1)}$
Duopoly	$Q^* = e^{-(c+\frac{1}{2})}$

$$\Rightarrow \boxed{Q^* > q^*} \\ \Rightarrow \boxed{Q_D > Q_M}$$

(g)

	Revenue (R)
M	$R = q^* (-\log q^* - c) = e^{-(c+1)} (e+1 - e) = e^{-(c+1)}$
D	$R_{\text{Total}} = Q^* (-\log Q^* - c) = e^{-(c+\frac{1}{2})} (e^{\frac{1}{2}} - e) = \frac{1}{2} e^{-(c+\frac{1}{2})}$
	$R_{\text{1 player}} = q_i^* (-\log q^* - c) = \frac{1}{2} R_{\text{Total}} = \frac{1}{4} e^{-(c+\frac{1}{2})}$

Now,  ~~$e < 2$  since~~  $e < 4 \Rightarrow \sqrt{e} < 2$

$$\Rightarrow e^{-(c+1)} > \frac{1}{2} e^{-(c+\frac{1}{2})}$$

$$\therefore R_M > R_{D(\text{Total})} > R_{D(1 \text{ player})} = \frac{R_{D(\text{Total})}}{2}$$

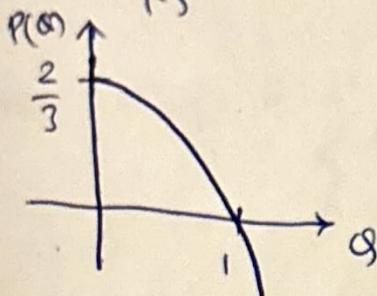
$$(Q2) P(Q) = \begin{cases} \frac{1}{1-\beta} (1-Q^{1-\beta}) & ; \beta \neq 1 \\ -\log Q & ; \beta = 1 \end{cases} \quad (\beta \in \mathbb{R} = \text{parameter})$$

$\nwarrow$

$Q = \underline{\text{Total Quantity}}$

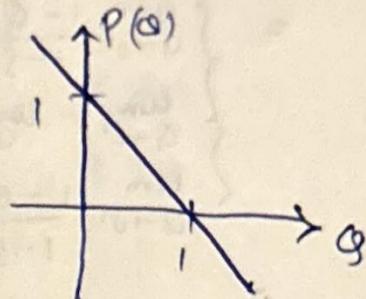
$$(a) (i) \beta = -0.5$$

$$\Rightarrow P(Q) = \frac{1}{1.5} (1-Q^{1.5})$$

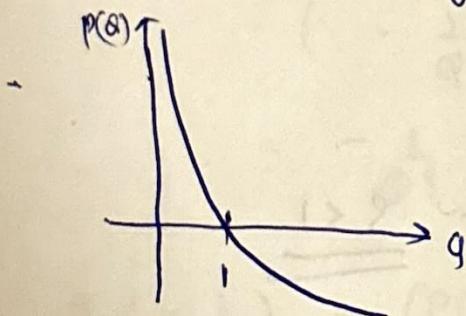


$$(ii) \beta = 0$$

$$\Rightarrow P(Q) = 1-Q$$

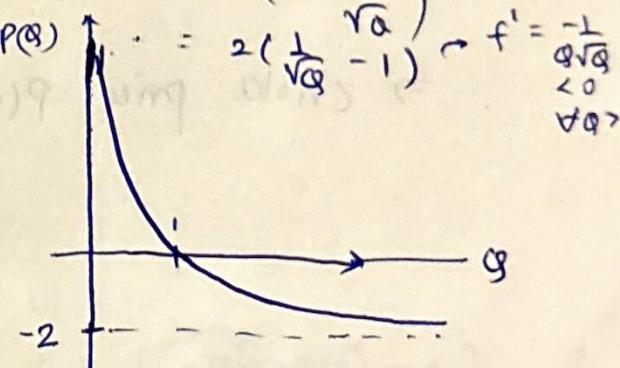


$$(Q) (iii) \beta = 1 = P(Q) = -\log Q$$



$$(iv) \beta = 1.5 = P(Q) = \frac{1}{-0.5} (1-Q^{-0.5})$$

$$= -2(1 - \frac{1}{\sqrt{Q}})$$



$$(b) \text{ price} = 0 \Rightarrow P(Q) = 0$$

$$\Rightarrow \beta \neq 1 \Rightarrow Q^{1-\beta} = 1 \Rightarrow Q = 1$$

$$\beta = 1 \Rightarrow \log Q = 0 \Rightarrow Q = 1$$

So, saturation demand level  $Q$  (at which price becomes 0)  $= 1$

$$(c) P(Q) = \begin{cases} \frac{1}{1-s} (1 - Q^{1-s}) & ; s \neq 1 \\ -\log Q & ; s = 1 \end{cases}$$

~~for P~~

$$P(0^+) = \begin{cases} \lim_{Q \rightarrow 0^+} \left( \frac{1 - Q^{1-s}}{1-s} \right) & ; s \neq 1 \\ \lim_{Q \rightarrow 0^+} (-\log Q) & ; s = 1 \end{cases}$$

$$= \begin{cases} \lim_{Q \rightarrow 0^+} \frac{1 - Q^{1-s}}{1-s} & ; s < 1 \\ \lim_{Q \rightarrow 0^+} -\log Q & ; s = 1 \\ \lim_{Q \rightarrow 0^+} \frac{1 - Q^{-s}}{1-s} & ; s > 1 \end{cases}$$

$$= \begin{cases} \frac{1}{1-s} & ; s < 1 \\ \infty & ; s = 1 \\ \infty & ; s > 1 \end{cases}$$

$\Rightarrow$  choke price  $P(0)$  is finite for  $\underline{s < 1}$

(d) cost (of production) =  $c$   $\theta \in [0, P(0^+)]$

~~To solve:  $f(q^*) = \max_{q \geq 0} q(P(q) - c)$~~

(or) where  $q^* = \arg\max_{q \geq 0} q(P(q) - c)$

~~Let  $f(q) = q(P(q) - c)$~~

~~$\Rightarrow \frac{df}{dq} = 0 \Rightarrow q(P'(q)) + (P(q) - c) \cdot 1 = 0 \quad \text{--- (1)}$~~

~~Now,  $P'(q) = \begin{cases} \frac{1}{1-s}(-1-s)q^{-s} & , s \neq 1 \\ -\frac{1}{q} & , s = 1 \end{cases}$~~

~~= \begin{cases} -\frac{1}{q^s} & , s \neq 1 \\ -\frac{1}{q} & , s = 1 \end{cases}~~
  
~~= -\frac{1}{q^s}~~

~~$\Rightarrow q\left(-\frac{1}{q^s}\right) + (P(q) - c) = 0$~~

~~$\Rightarrow q^{1-s} + P(q) - c = 0 \quad \text{--- (2)}$~~

Case(i)  $s \neq 1$

~~$\Rightarrow q^{1-s} + \frac{1}{1-s}(1-q^{1-s}) - c = 0$~~

~~$\Rightarrow \frac{1}{1-s} - c = q^{1-s}\left(1 + \frac{1}{1-s}\right)$~~

~~$\frac{1-c+s}{1-s} = q^{1-s}\left(\frac{2-s}{1-s}\right)$~~

~~$\Rightarrow q^* = \left(\frac{1-c+s}{2-s}\right)^{\frac{1}{1-s}}$~~

~~$f(q^*) = q^* \cdot \cancel{(q^* + 1)} (P(q^*) - c)$~~ 
  
~~= q^\* \left(\frac{1}{1-s}(1-q^{1-s}) - c\right)~~

~~$= q^* \left(\frac{1}{1-s}(1-\frac{1-c+s}{2-s}) - c\right)$~~

~~$= q^* \left(\frac{2-s-1+c-s}{(1-s)(2-s)} - c\right)$~~

~~$= q^* \left(\frac{1-s+c(1-s)}{(1-s)(2-s)} - c\right)$~~

~~$= q^* \left(\frac{4-c}{(2-s)}\right)$~~

~~? =  $q^*$~~

~~= \left[\frac{(1-c+s)}{2-s}\right] \cdot \frac{1}{1-s}~~

~~So,  $f(q^*) = q^* = \left(\frac{1-c+s}{2-s}\right)^{\frac{1}{1-s}}$~~

(Q.2) (a)

case (i):  $s \neq 1$

$$\Rightarrow -s^{1-s} + \frac{1}{1-s}(1-q^{1-s}) - c = 0$$

$$\Rightarrow \frac{1}{1-s} - c = q^{1-s} \left(1 + \frac{1}{1-s}\right)$$

$$\Rightarrow \frac{1 - c + cs}{1-s} = q^{1-s} \left(\frac{2-s}{1-s}\right)$$

$$\Rightarrow q^s = \left(\frac{1 - c + cs}{2-s}\right)^{\frac{1}{1-s}}$$

$$f(q^s) = q^s (P(q^s) - c)$$

$$= q^s \left( \frac{1}{1-s} (1 - (q^s)^{1-s}) - c \right)$$

$$= q^s \left( \frac{1}{1-s} \left(1 - \left(\frac{1-c+cs}{2-s}\right)\right) - c \right)$$

$$= q^s \left( \frac{(2-s-1+c-cs)}{(1-s)(2-s)} - c \right)$$

$$= q^s \left( \frac{(1-s)(1+c)}{(1-s)(2-s)} - c \right)$$

$$= q^s \left( \frac{1+c-2c+sc}{2-s} \right)$$

$$= q^s \left( \frac{1-c(1-s)}{2-s} \right)$$

$$\Rightarrow f(q^s) = \left( \frac{1-c(1-s)}{(2-s)} \right)^{\frac{1}{1-s}} \left( \frac{1-c(1-s)}{2-s} \right)$$

Note that  $\lim_{s \rightarrow 1} \left( \frac{1-c(1-s)}{2-s} \right)^{\frac{1}{1-s}} \left( \frac{1-c(1-s)}{2-s} \right)$

$$= \exp \left( \frac{1-c(1-s)}{(2-s)} - 1 \right) \frac{1}{1-s}$$

$$= \exp \left( -\frac{c(c+1)}{2-s} \cdot \frac{1}{1-s} \right)$$

$$= \exp \left( -\frac{c(c+1)}{1} \right)$$

$$= \exp(-c-1)$$

Case (ii):  $s = 1$

$$\Rightarrow P(q) = -\log_2$$

$$\Rightarrow -q^{1-1} - \log_2 q - c = 0$$

$$\Rightarrow 1 + \log_2 q + c = 0$$

$$\Rightarrow \log_2 q = -c-1$$

$$\Rightarrow q = \exp(-c-1)$$

$$\Rightarrow q^s = \exp(-c-1)$$

$$f(q^s) = q^s (P(q^s) - c)$$

$$= q^s (-\log_2 q - c)$$

$$= \exp(-c-1) (c+1-x)$$

$$= \exp(-c-1)$$

$$\Rightarrow f(q^s) = \exp(-c-1)$$

---

f(q) = Profit at qty q

- (e) ① Unique NE  $\rightarrow$  2 player Cournot game  
 • with  $s < 3$   
 • players' cost  $\rightarrow c_1, c_2 \in [0, P(0^+))$

$$P_1 \rightarrow \max_{q_1 \geq 0} q_1 (P(Q) - c_1) \quad P_2 \rightarrow \max_{q_2 \geq 0} q_2 (P(Q) - c_2)$$

where  $Q = q_1 + q_2$

$$\frac{\partial f_1}{\partial q_1} = 0 \Rightarrow q_1 P'(Q) + (P(Q) - c_1) = 0 \quad | \quad \frac{\partial f_2}{\partial q_2} = q_2 P'(Q) + P(Q) - c_2 = 0$$

L ①    L ②

① + ② gives :-

$$Q P'(Q) + 2P(Q) - C = 0 \quad (\text{when } C = c_1 + c_2) \quad - \textcircled{3}$$

① - ② gives :-

$$(q_1 - q_2) P'(Q) = c_1 - c_2 \quad - \textcircled{4}$$

}

from ④,

$$\text{case i: } s = 1 \Rightarrow P(Q) = -\log Q = \frac{1}{Q}$$

$$P'(Q) = -\frac{1}{Q^2}$$

- ④ becomes  $\Rightarrow$

$$Q\left(-\frac{1}{Q}\right) + 2\left(-\log Q\right) + C = 0$$

$$\Rightarrow 2\log Q = -\frac{C-1}{2}$$

$$\boxed{Q^2 = \exp\left(-\frac{C-1}{2}\right)}$$

To find  $q_1, q_2$  using ②,

$$(q_1 - q_2)\left(-\frac{1}{Q}\right) = c_1 - c_2$$

$$\Rightarrow (q_1 - q_2) = (c_2 - c_1)Q$$

$$\Rightarrow q_1 - q_2 = (c_2 - c_1)Q^2$$

$$\because q_1^* + q_2^* = Q^2,$$

$$q_1^* = \frac{1}{2}(c_2 - c_1 + 1)Q^2, \quad q_2^* = \frac{1}{2}(-c_2 + c_1 + 1)Q^2$$

$$\boxed{q_1^* = \frac{1}{2}(c_2 - c_1 + 1)\exp\left(-\frac{C-1}{2}\right)} \quad \textcircled{5}$$

$$q_2^* = \frac{1}{2}(c_1 - c_2 + 1)\exp\left(-\frac{C-1}{2}\right)$$

$$\text{case ii: } s \neq 1 \Rightarrow P(Q) = \frac{1}{1-s}(1-Q^{1-s})$$

$$= P'(Q) = -\frac{1}{Q^s}$$

= ④ becomes :-

$$Q\left(-\frac{1}{Q^s}\right) + \frac{2}{1-s}(1-Q^{1-s}) - C = 0$$

$$\Rightarrow -Q^{1-s} - \frac{2}{1-s}Q^{1-s} = C - \frac{2}{1-s}$$

$$\Rightarrow Q^{1-s}\left(\frac{1-s+2}{1-s}\right) = \frac{2}{1-s} - C$$

$$\Rightarrow Q^{1-s}\left(\frac{3-s}{1-s}\right) = \left(\frac{2-C+sC}{1-s}\right)$$

$$\Rightarrow Q^{1-s} = \frac{2+C(s-1)}{3-s} \xrightarrow{s < 3 \Rightarrow Nm^r < 2+2C}$$

$$\boxed{Q^* = \left(\frac{2+C(s-1)}{3-s}\right)^{\frac{1}{1-s}} \xrightarrow{s < 3 = Dm^r > 0}$$

$$(q_1 - q_2)\left(-\frac{1}{Q^s}\right) = (c_1 - c_2)$$

$$\Rightarrow q_1 - q_2 = (c_2 - c_1)Q^{1-s} \quad ] \Rightarrow q_1 = \frac{Q + (c_2 - c_1)Q^s}{2}$$

$$q_1 + q_2 = Q^s$$

$$q_2 = \frac{Q - (c_2 - c_1)Q^s}{2}$$

$$S=1$$

$$q_1^+ = \frac{1}{2} (1 + c_2 - c_1) Q^+$$

$$q_2^+ = \frac{1}{2} (1 - (c_2 - c_1)) Q^+$$

where  $Q^+ = \exp\left(-\frac{(c_1 + c_2) - 1}{2}\right)$

$$q_1^+ = \frac{Q^+ + (c_2 - c_1) Q^{+^3}}{2}$$

$$q_2^+ = \frac{Q^+ - (c_2 - c_1) Q^{+^3}}{2}$$

where  $Q^+ = 2 + C(S+1)$

where  $Q^+ = \left(\frac{2 + (c_1 + c_2)(S-1)}{3 - S}\right)^{\frac{1}{1-S}}$

(Q.3)

(b)  $C_1 = C_2$  2 player static Cournot game ( $\varphi$  determines  $P$ ).

$$C_1 = 0 \quad q_1^* \in [0, 1]$$

$$C_2 = 0 \quad q_2^* \in [0, 1]$$

$P_1 \rightarrow \max_{\substack{q_1 \geq 0 \\ q_2 \geq 0}} q_1(1 - q_1 - q_2^*)$

$P_2 \rightarrow \max_{q_2 \geq 0} q_2(1 - q_2 - q_1^*)$

R1  
other player's eq/b strategies

$$\frac{\partial P_1}{\partial q_1} = 0 \quad \text{at } q_1 = q_2^*$$

$$\text{likewise } \frac{\partial P_2}{\partial q_2} = 0 \quad q_2 = q_1^*$$

$$\Rightarrow q_1(-1) + (1 - q_1 - q_2^*)(1) = 0 \mid q_1 = q_2^* \Rightarrow q_2(-1) + (1 - q_2 - q_1^*)(1) = 0 \mid q_2 = q_1^*$$

$$\Rightarrow -q_1 + 1 - q_1 - q_2^* = 0 \mid q_1 = q_2^*$$

$$\Rightarrow 1 - 2q_1 - q_2^* = 0 \mid q_1 = q_2^*$$

$$\Rightarrow 1 - 2q_1^* - q_2^* = 0 \quad \text{--- (1)}$$

$$\Rightarrow 1 - 2q_2 - q_1^* = 0 \mid q_2 = q_1^*$$

$$\Rightarrow 1 - 2q_2^* - q_1^* = 0 \quad \text{--- (2)}$$

$$\text{--- (1)} + \text{--- (2)} = 2 = 3Q^* \Rightarrow Q^* = q_1^* + q_2^* = \frac{2}{3}$$

$$\text{Also, } q_1^* = q_2^* \Rightarrow q_1^* = q_2^* = \frac{1}{3} \quad \text{--- (3)}$$

$$\boxed{\text{Eq/b mkt oil price}} = 1 - Q^* = 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$$

(c) When profits are being maximised jointly,

$$P = \max_{q_1, q_2 \geq 0} q_1(1 - q_1 - q_2) + q_2(1 - q_1 - q_2)$$

$$\frac{\partial P}{\partial q_1} = 0 \Rightarrow q_1(-1) + (1 - q_1 - q_2)(1) + q_2(-1) = 0 \quad \left| \begin{array}{l} \text{Total qty produced} \\ = \boxed{Q^* = \frac{1}{2}} \quad (< Q^*_{\text{earlier}} = \frac{2}{3}) \end{array} \right.$$

$$\Rightarrow -q_1 + 1 - q_1 - q_2 - q_2 = 0$$

$$\Rightarrow 1 - 2q_1 - 2q_2 = 0$$

$$\Rightarrow 1 - 2Q = 0$$

$$\Rightarrow Q = \frac{1}{2}$$

$$\Rightarrow Q^* = \frac{1}{2}$$

$$\boxed{\text{Mkt oil priu} = 1 - \frac{1}{2} = \frac{1}{2}}$$

$$> \text{price earlier} = \boxed{\frac{1}{3}}$$

$$\text{likewise } \frac{\partial P}{\partial q_2} = 0 \Rightarrow q_1(-1) + q_2(-1) + (1 - q_1 - q_2)(1) = 0$$

$$\Rightarrow Q^* = \frac{1}{2}$$

$\Rightarrow$  If both players produce equally,

$$\underline{q_1^* = q_2^* = \frac{1}{4}}$$

(a) Date  $\rightarrow$  April 2020  $\rightarrow$  Just after covid hit  
Cournot model (of oil prices)  $\rightarrow$  Quantity determines price  
Slashing  $\rightarrow$  cutting (oil production)

So, if slashing is done (when mutually agreed upon),  
Lower quantity ( $Q$ ) produced  $\Rightarrow$  Price per unit increases  
overall / total

$\downarrow$   
Price is stabilized

Intuitively, if the players decide to maximise joint revenue, they can perhaps earn more (total) revenue (than / as compared to adding indiv revenue together by maximising indiv profit) by slashing ~~the~~ production. Hence, they may want to do slashing.

~~In the context in which this headlines came out (just after covid), players may also be interested in raising the steeply fallen prices + maximizing their indiv revenues which would require raising steeply fallen price  $\Rightarrow \downarrow$  supply  $\Rightarrow$  cutting prodn.~~

Stabilizing oil prices  $\rightarrow$  Not fluctuating much  $\rightarrow$  will  $\downarrow$  if more quantity (of oil) is produced  $\rightarrow$  even if the players have the capacity to produce more (oil), just to avoid falling & hence fluctuating of oil prices, players won't produce more & possibly slash the production.

All this happened just after covid was announced. Covid led to  $\downarrow$  in demand steeply, thus  $\downarrow$  oil prices suddenly. So, to stabilize prices  $\Rightarrow$  prices were required to rise  $\Rightarrow$  demand won't change much, supply had to reduce  $\Rightarrow$  Slashing would help (If this wouldn't have been done, price would fall further  $\Rightarrow$  more volatility / losses / instability in the mkt).

### (Q.4) Electricity production (single producer)

- $C = 0$
- $P(q) = 1 - q \quad \rightarrow q^* = \frac{1}{2}$

with shortfall (trembling hand),  $q \rightarrow q - \varepsilon$

$$\text{So, } q_{us}^* = \frac{1 + \mu c}{2} \Big|_{c=0} = \frac{1 + \mu}{2} \text{ (Supply)}$$

$$\varepsilon \sim \text{EXP}(10) \quad \rightarrow f_\varepsilon(x) = 10e^{-10x}$$

$$\Rightarrow \mu = E(\varepsilon) = \frac{1}{10} = 0.1$$

$$\Rightarrow q_{us}^* = \frac{1 + 0.1}{2} = 0.55$$

$$\textcircled{2} \Rightarrow \text{Supply} = 0.55$$

→ Demand

$$D \sim \text{EXP}(2) = f_D(x) = 2e^{-2x}$$

$$\Rightarrow E(D) = \frac{1}{2} = 0.5$$

Demand = 0.5

$$D \perp \varepsilon$$

(a)  ~~$P(P(q^* - \varepsilon) < D)$~~

~~$P(D + \varepsilon > q^*)$~~

Now, if  $X \sim \text{EXP}(\lambda_1)$ ,  $Y \sim \text{Exp}(\lambda_2)$  &  $X \perp\!\!\!\perp Y$ ,

$$P(X+Y < \alpha) = \int_0^\alpha \int_0^{\alpha-x} P(X=x, 0 < Y < \alpha-x) dx dy$$

$$= \int_0^\alpha f_X(x) \left( \int_0^{\alpha-x} f_Y(y) dy \right) dx$$

$$= \int_0^\alpha f_X(x) \left( \int_0^{\alpha-x} \lambda_2 e^{-\lambda_2 y} dy \right) dx$$

$$= \int_0^\alpha f_X(x) (1 - e^{-\lambda_2(\alpha-x)}) dx$$

$$= \int_0^\alpha \lambda_1 e^{-\lambda_1 x} (1 - e^{-\lambda_2(\alpha-x)}) dx$$

$$= \int_0^\alpha (\lambda_1 e^{-\lambda_1 x} - \lambda_1 e^{-\lambda_2 \alpha} e^{(\lambda_2-\lambda_1)x}) dx$$

(Assuming  $\lambda_1 \neq \lambda_2$   
which is our case)

$$= [e^{-\lambda_1 x}]_0^\alpha - \frac{\lambda_1 e^{-\lambda_2 \alpha}}{\lambda_2 - \lambda_1} [e^{(\lambda_2 - \lambda_1)x}]_0^\alpha$$

$$= 1 - e^{-\lambda_1 \alpha} - \frac{\lambda_1 e^{-\lambda_2 \alpha}}{\lambda_2 - \lambda_1} (e^{(\lambda_2 - \lambda_1)\alpha} - 1)$$

$$= 1 - e^{-\lambda_1 \alpha} + \frac{\lambda_1 e^{-\lambda_2 \alpha}}{\lambda_2 - \lambda_1} (1 - e^{(\lambda_2 - \lambda_1)\alpha})$$

$$= 1 - e^{-\lambda_1 \alpha} + \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_2 \alpha} - e^{-\lambda_1 \alpha})$$

$$= 1 - e^{-\lambda_1 \alpha} \left( 1 + \frac{\lambda_1}{\lambda_2 - \lambda_1} \right) + \left( \frac{\lambda_1}{\lambda_2 - \lambda_1} \right) e^{-\lambda_2 \alpha}$$

$$= 1 - e^{-\lambda_1 \alpha} \left( \frac{\lambda_2}{\lambda_2 - \lambda_1} + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 \alpha} \right)$$

$$= 1 - \left( \frac{\lambda_2 e^{-\lambda_1 \alpha} - \lambda_1 e^{-\lambda_2 \alpha}}{\lambda_2 - \lambda_1} \right)$$

$$\therefore F_z(\alpha) = \text{f } (z = x+y)$$

$$f_z(\alpha) = \frac{df}{d\alpha} = + \frac{\lambda_2 e^{-\lambda_1 \alpha}}{\lambda_2 - \lambda_1} (+\lambda_1) + \frac{\lambda_1 e^{-\lambda_2 \alpha}}{\lambda_2 - \lambda_1} (-\lambda_2)$$

$$\Rightarrow f_z(\alpha) = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} (e^{-\lambda_1 \alpha} - e^{-\lambda_2 \alpha}) \quad | \quad \alpha > 0$$

In our case,  $\lambda_1 \sim \text{Exp}(10) \leftarrow \lambda_2$

$D \sim \text{Exp}(2)$ ,  $E \sim \text{Exp}(10)$

Let  $D+E = Z \Rightarrow$

$f_z(z) = \frac{2 \cdot 10}{10-2} (e^{-2z} - e^{-10z}) ; z > 0$ 
 $= \frac{5}{2} (e^{-2z} - e^{-10z}) ; z > 0$

$$(a) P(Q^+ - \varepsilon < D) = P(Q^+ < D + \varepsilon)$$

$$= P(Q^+ < z)$$

$$= P(z > Q^+)$$

$$= P(z > 0.5)$$

$$= \frac{5}{2} \left( \frac{e^{-1}}{2} - \frac{e^{-5}}{10} \right)$$

$$= \frac{5}{2} \left( \frac{0.3679}{2} - \frac{0.0067}{10} \right)$$

$$= \frac{5}{2} (0.18395 - 0.00067)$$

$$= \frac{5}{2} (0.18328)$$

$$= \frac{0.91640}{2}$$

$$= 0.45820$$

$$= 0.4582$$

$$\Rightarrow \boxed{\text{Prob} = 0.4582}$$

In other case,

$D \sim \text{Exp}(2)$ ,  $E \sim \text{Exp}(10)$  (10 is 10 made to be 2 & not 2 for  $f_Z$  to be  $> 0$ )

Let  $D+E = Z$

$$f_Z(z) = \frac{2e^{-2z}}{10-2} (e^{-10z} - e^{-10z}) \cdot z > 0$$
$$= \underline{\underline{2(e^{-2z} - e^{-10z})}}$$

$$(b) \text{ So, } P(q_{us}^L - \epsilon < 0) = P(q_{us}^L < D + \epsilon)$$

$$= P(q_{us}^L < Z)$$

$$= P(Z > q_{us}^L)$$

$$= P(Z > \frac{1+0.1}{2} = 0.55)$$

$$= \int_{0.55}^{\infty} f_Z(z) dz$$

$$= \underline{\underline{\frac{5}{2} \left( e^{-\frac{11}{2}} - e^{-5.5} \right)}}$$

$$= \underline{\underline{\frac{5}{2} \left( \frac{0.3329}{2} - \frac{0.0041}{10} \right)}}$$

$$= \underline{\underline{\frac{5}{2} (0.16645 - 0.00041)}}$$

$$= \underline{\underline{\frac{5}{2} \cdot 0.16604}}$$

$$= \underline{\underline{0.83020}}$$

$$= \underline{\underline{0.41510}}$$

$$= \underline{\underline{0.4151}}$$

$$\rightarrow \boxed{\text{Prob} = 0.4151}$$

$$(C) \text{ USD} \Rightarrow 1 - 2q_{\text{USD}}^+ + \mu + \alpha \bar{F}(q_{\text{USD}}^+) = 0 \quad \dots \textcircled{D}$$

Now,  ~~$F(z)$~~   $F_z(z) = P(Z \leq z) \hookrightarrow \alpha = \frac{2q_{\text{USD}}^+ + \mu}{\bar{F}(q_{\text{USD}}^+)} \uparrow \text{in } q_{\text{USD}}^+$

$$\begin{aligned} \Rightarrow \bar{F}_z(z) &= P(Z > z) \\ &= \frac{5}{2} \left( \frac{e^{-2z}}{2} - \frac{e^{-10z}}{10} \right) \\ q_{\text{USD}}^+ \uparrow &\Rightarrow \frac{Nmr \uparrow}{DmL} \Rightarrow RHS \approx LHS \text{ s.t.} \\ \text{So } \alpha &\downarrow \Rightarrow q_{\text{USD}}^+ \downarrow \quad \text{---} \textcircled{D} \end{aligned}$$

$\Rightarrow \textcircled{D}$  becomes :-

$$1.1 - 2q_{\text{USD}}^+ + \alpha \cdot \frac{5}{2} \left( \frac{e^{-2q_{\text{USD}}^+}}{2} - \frac{e^{-10q_{\text{USD}}^+}}{10} \right) = 0$$

$$q_{\text{USD}}^+ > 0,$$

$$\text{if } \alpha > 0,$$

$$\text{Term 3} > 0$$

$$2q_{\text{USD}}^+ = 1.1 + \cancel{\text{Term 1}}$$

$$= 1.1 + > 0$$

$$> 1.1$$

$$\therefore q_{\text{USD}}^+ > 0.55$$

$$\begin{aligned} P(q_{\text{USD}}^+ - \Delta E < D) &= P(q_{\text{USD}}^+ < \varepsilon + D) \\ &= P(q_{\text{USD}}^+ < z) \\ &= P(Z > q_{\text{USD}}^+) \\ &= \bar{F}(q_{\text{USD}}^+) \\ &= \frac{2q_{\text{USD}}^+ - 1.1}{\alpha} \end{aligned}$$