# ORF 555 HW 3

(Q.1) oil producer: $\frac{dx}{dt} = -q(x(t))$ as long as $x(t) > 0$

$x$ = initial reserves $= x(0)$.

market price $= 1 - q(x(t))$

objective function $= J(q) = \int_0^{\tau_x^q} e^{-rt} q(x(t))(1 - q(x(t)))\,dt$

where $\tau_x^q$ is the exhaustion time from initial reserves $x$ under the strategy $q$ i.e. the time at which reserves run out.

(a) Time in yrs.

$$90 = 100e^{-r} \Rightarrow e^r = \frac{100}{90} = \frac{10}{9} \Rightarrow r = \ln\left(\frac{10}{9}\right)$$

$$\Rightarrow \boxed{r = 10.5361\%} \qquad \Rightarrow \boxed{r = 0.105361}$$

from here, $r = 0.1$

(b)

$$q = \frac{1}{2} \Rightarrow \frac{dx}{dt} = -\frac{1}{2}$$

$$x(0) = 10$$

$$\Downarrow$$

$$x(t) - x(0) = -\frac{t}{2}$$

$$\Rightarrow x(t) = x(0) - \frac{t}{2} = 10 - \frac{t}{2}$$

~~t at which~~ $0 = x(\tau_x^q) = 10 - \frac{\tau_x^q}{2}$

$$\Rightarrow \boxed{\tau_x^q = 20 \text{ yrs}}$$

$$J(q) = \int_0^{20} e^{-0.1t} \frac{1}{2} \cdot \left(1 - \frac{1}{2}\right) dt$$

$$= \frac{1}{4} \int_0^{20} e^{-0.1t} dt$$

$$= \frac{1}{4 \times 0.1} \left[e^{-0.1t}\right]_{20}^{0}$$

$$= \frac{1}{0.4}\left(1\cancel{0} - e^{-2}\right)$$

$$= \frac{1}{0.4}(1 - 0.1353)$$

$$= \frac{0.8647}{0.4}$$

$$= \boxed{2.1617}$$

(c) $V(x) = \frac{1}{4r}\left(1 + W(-e^{-2rx-1})\right)^2$ (Eq$^n$(45) of Notes)

$V'(x) = \frac{1}{\cancel{4r}} \cdot \cancel{2} \cdot (1 + W(-e^{-2rx-1}))(W'(-e^{-2rx-1}))(+e^{-2rx-1})(+\cancel{2r})$

$= e^{-2rx-1} W'(-e^{-2rx-1})(1 + W(-e^{-2rx-1}))$

Let $y = W(x) \Rightarrow x = W^{-1}(y) = ye^y$

$\Rightarrow W(ye^y) = y$

$W'(x) = \frac{dW(x)}{dx} = \frac{dy}{dx}$

$\frac{1}{W'(x)} = \frac{dx}{dy} = \frac{d(ye^y)}{dy} = ye^y + e^y = x + \frac{x}{y} = \frac{xy+x}{y}$

$\Rightarrow W'(x) = \frac{y}{x(1+y)} = \frac{W(x)}{x(1+W(x))}$

At $t = \tau^{q^*}$, $x(t) = 0$, $\cancel{q(t) = 0 \Rightarrow V'(t) = 1}$

$q(0) = 0 \Leftarrow q(x(t)) = 0 \Rightarrow V'(x(t)) = 1 \Rightarrow V'(0) = 1$ ✓

$\left(\because q(x(t)) = \frac{(1 - V'(x(t)))}{2}\right)$ $(W(0) = 0)$

$\Rightarrow V'(x) = \cancel{e^{-2rx-1}}_{-\Delta} W'(\Delta)(1 + W(\Delta))$

$= -\cancel{\Delta} \frac{W(\Delta)}{\cancel{\Delta}\,\cancel{(1+W(\Delta))}} \cancel{(1+W(\Delta))}$

$= -W(\Delta)$

$= \boxed{-W(-e^{-2rx-1})}$

$\Rightarrow V'(0) = -W(-e^{-1}) = -(-1) = 1$

$\Rightarrow q^*(x) = \frac{1 - V'(x)}{2} = \boxed{\frac{1 + W(-e^{-2rx-1})}{2}} \Rightarrow \boxed{V(x) = \frac{q^2(x)}{r}}$

Eq$^n$(43)

All in all, we have :-

$$\boxed{q^*(x) = \frac{1 + w(-e^{-2rx-1})}{2}}$$ where $r = 0.1$

~~$q^*(x) = 1 + w$~~

for time, we know that :-

$v'(x(t)) = v'(x(0))\, e^{rt}$ (By Hotelling's rule)

$\Rightarrow v'(x(\tau)) = v'(x(0))\, e^{r\tau}$ (let $\tau = \tau_{10}^{q^*}$ of the $q^n$)

$\Rightarrow v'(0) = v'(10)\, e^{r\tau}$

$$\Rightarrow \tau = \frac{1}{r} \ln\left(\frac{v'(0)}{v'(10)}\right) = \frac{1}{r} \ln\left(\frac{1}{v'(10)}\right)$$

$$= -\frac{\ln v'(10)}{r}$$

Now, $v'(x) = -w(-e^{-2rx-1})$

$\Rightarrow v'(10) = -w(-e^{-2r\cdot 10-1}) = -w(-e^{-2-1}) = -w(-e^{-3})$

$$\Rightarrow \boxed{\tau = -\frac{\ln(-w(-e^{-3}))}{r}}$$

$\Rightarrow$ From the computation, $\boxed{\tau = 29.4753}$ yrs

(d) $$v(x) = \frac{(1 + w(-e^{-2rx-1}))^2}{4r}$$

$$\Rightarrow v(10) = \frac{(1 + w(-e^{-2-1}))^2}{0.4} = \frac{(1 + w(-e^{-3}))^2}{0.4} = \boxed{2.2445}$$

~~$\Rightarrow v(10) = 2.2445$~~

(Q.2) P1 → oil P2 → solar, cost $c > 0$

$x(t)$ = oil reserves remaining at time t.

P1 → extract @ $q_1(x(t))$ P2 → extract @ $q_2(x(t))$

$$\frac{dx}{dt} = -q_1(x(t)) \text{ as long as } x(t) > 0$$

$q_1(0) = 0$

Price $= 1 - q_1 - q_2$

$\eta$ = oil exhaustion time ⇒ $x(\eta) = 0$, $q(x(\eta)) = q(0) = 0$

P1 → $u(x) = \ldots$

P2 → $w(x) = \ldots$

$r = 0.1$

$x(0) = 10$

(a) cost → P1 : $u'(x)$ | P2 : $w'(x) + c$

$$q_1(x) = \frac{1 - u'(x)}{2}$$

Now, from proposition 4.4 (eqn (59))

$$x_b = \frac{4}{9r}\left((1+c)\log\left(\frac{1+c}{2(2c-1)}\right) - 3(1-c)\right)$$

$x_b = 5$ ⇒ $$5 = \frac{4}{0.9}\left((1+c)\log\left(\frac{1+c}{2(2c-1)}\right) - 3(1-c)\right)$$

$$\Rightarrow \frac{4.5}{4} = (1+c)\log\left(\frac{1+c}{4c-2}\right) - 3 + 3c$$

1.125

$$\Rightarrow 4.125 = (1+c)\log\left(\frac{1+c}{4c-2}\right) + 3c$$

So, to solve:-

$$\boxed{4.125 = (1+c)\log\left(\frac{1+c}{4c-2}\right) + 3c}$$

$$f(c) \rightarrow f'(c) = \left(\frac{\cancel{1+c}}{\cancel{1+c}}\right) \times \cancel{(4c-2)} \left(\frac{4c-2-(1+c)4}{(4c-2)^{\cancel{2}}}\right) + \log\left(\frac{1+c}{4c-2}\right) + 3$$

$$= \frac{\cancel{4c}-2-4-\cancel{4c}}{4c-2} + \log\left(\frac{1+c}{4c-2}\right) + 3$$

$$= -\frac{3}{2c-1} + 3 + \log\left(\frac{c+1}{4c-2}\right)$$

$$= 3 - \frac{3}{2c-1} + \log\left(\cancel{(c+1)}\frac{1}{4}\left(\frac{4c+4}{4c-2}\right)\right)$$

$$= 3 - \frac{3}{2c-1} + \log\left(\frac{1}{4}\right) + \log\left(1 + \frac{6}{4c-2}\right)$$

$$= 3 - \frac{3}{2c-1} - \log 4 + \log\left(1 + \frac{3}{2c-1}\right)$$

At $c = 1$, $f'(1) = 0$ ( & $f(1) = 3$)

After solving using computational tools, we get:-

$\cancel{\boxed{c = 2.154}}$ $\boxed{c = 0.59}$ $(c < 1)$

(b) $u(x) = \frac{1}{4r}\left(1 + W(\theta(x - x_b))\right)^2$ | $\theta(x) = (1-2c)e^{1-2c-2rx}$

└ Eq$^n$ 60

$u(10) = \frac{1}{0.4}\left(1 + W(\theta(10-5))\right)^2$

$= 2.5\left(1 + W(\theta(5))\right)^2$

$r = 0.1$

↓

$\theta(x) = -e^{-2rx-1}$

(previous (Q1) case)

when $c = 1$

& $x_b = 0$

$\theta(5) = (1-2c)\, e^{1-2c-2r \times 5}$

$= (1-2c)\, e^{\cancel{1}-2c-\cancel{1}}$ $(\because r = 0.1)$

$= (1-2c)\, e^{-2c}$

On solving computationally, we get :-

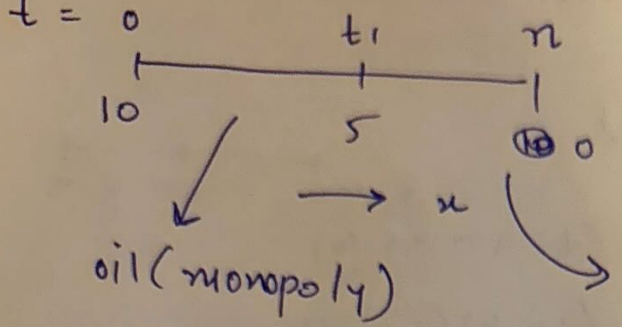
~~$\theta(5) = -0.0445$~~

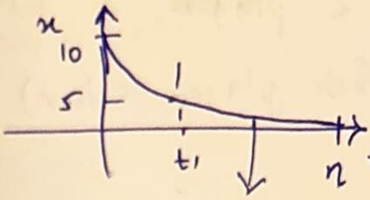
& ~~So, $u(10) = 2.27$~~

$\boxed{u(10) = 2.2139}$

(c) $x$ goes from 10 to 0

i.e. $x(0) = 10$, $x(10) = 0$




$\left|\frac{dx}{dt}\right|_{t>t_1} < \left|\frac{dx}{dt}\right|_{t<t_1}$

(True otherwise also) - i.e. when solar is not there)

So, from ~~to~~ $t = 0$ to $t_1$, since there is a single player, Hotelling rule holds

$\Rightarrow v'(x(t)) = v'(x(0))\, e^{rt}$

$\Rightarrow v'(x(t_1)) = v'(x(0))\, e^{rt_1}$

$\Rightarrow v'(5) = v'(10)\, e^{rt_1}$ $\qquad v'(x) = -w(-e^{-2rx-1})$

$\Rightarrow t_1 = \frac{1}{r} \ln\left(\frac{v'(5)}{v'(10)}\right)$

$\cancel{=} \frac{1}{r} \ln\left(\frac{+w(-e^{-2r\cdot 5-1})}{+w(-e^{-2r\cdot 10-1})}\right)$

$= \frac{1}{r} \ln\left(\frac{w(-e^{-2})}{w(-e^{-3})}\right)$

$\cancel{= \boxed{11.06}}$

Here, $v'(x) \neq -w(-e^{-2rx-1}) \because c \neq 1$.

computed afterwards

Now, when $t > t_1$, solar comes into picture

$\Rightarrow$ 2 players $\Rightarrow$ But $\frac{dx}{dt} = -q_1(x(t))$ still since the other player (solar) does not extract oil ~~but just prov~~

Now, $q_1^*$ has changed a bit

$t \rightarrow$ 0 $t_1$ $\eta$

Earlier (from $q_1$),

$$q_1^* = \frac{1 + W(\cdot)}{2}$$

$$= \frac{1 + W(-e^{-2rx-1})}{2}$$

Now ($t > t_1 \rightarrow x < x_b$)

$$q_1^* = (1 + W(\cdot))\left(\frac{1+c}{3}\right)$$

$$= \left(\frac{1+c}{3}\right)(1 + W(\cdot))$$

$$\cancel{= \frac{1.59}{3}\left(1 + W\left(-e^{-\frac{9rx}{4 \cdot 1.59}-1}\right)\right)}$$

$$= \left(\frac{1+c}{3}\right)\left(1 + W\left(-e^{-\frac{9rx}{4(1+c)}-1}\right)\right)$$

So, ~~$q_1^* = 1$~~

By hotelling's rule,

$$u'(x(t_2)) = u'(x(t_1))\, e^{r(t_2 - t_1)}$$

$$u'(x(\eta)) = u'(x(t_1))\, e^{r(\eta - t_1)}$$

$$u'(0) = u'(5)\, e^{r(\eta - t_1)}$$

$$\Rightarrow \eta - t_1 = \frac{1}{r} \ln\left(\frac{u'(0)}{u'(5)}\right)$$

Now, from eq$^n$ (55),

$$u(x) = \frac{(1+c)^2}{9r}\left(1 + W(\theta(x))\right)^2 \quad \text{where} \quad \theta(x) = -\exp(-krx - 1)$$

$$\& \; k = \frac{9}{4(1+c)}$$

Q.2(c) continued

$$\text{So, } u'(x) = \frac{(1+c)^2}{9r} \cdot 2\,(1+W(\theta(x)))\,W'(\theta(x))\,\theta'(x)$$

$$= \frac{(1+c)^2}{9r}\,(1+\cancel{W(\theta(x))})\,\frac{W(\theta(x))}{\theta(x)\,\cancel{(1+W(\theta(x)))}} \times \theta'(x)$$

$$= \frac{(1+c)^2}{9r}\,\frac{W(\theta(x))}{\cancel{\theta(x)}} \cdot \cancel{\theta(x)}\,(-kr) \qquad : k = \frac{9}{4(1+c)}$$

$$= \frac{(1+c)^{\cancel{2}}}{\cancel{9r}}\,W(\theta(x))\left(\frac{-\cancel{9r}}{4\cancel{(1+c)}}\right)$$

$$= -\frac{(1+c)}{4}\,W(\theta(x))$$

( $\because$ $\theta(x) < 0$, $W(\theta(x)) < 0 \Rightarrow u'(x) > 0$ which makes sense because as $x \uparrow$, $u'(x) \uparrow$ )

$$\text{So, } \eta - t_1 = t_2 = \frac{L}{r}\ln\left(\frac{-\cancel{\frac{(1+c)}{4}}\,W(\theta(0))}{-\cancel{\frac{(1+c)}{4}}\,W(\theta(5))}\right)$$

$\Rightarrow \cancel{\boxed{t_2 = 0.37}}$ $\boxed{t_2 = 14.80}$

$\Rightarrow$ $\boxed{\text{Total time}}$ = $\eta = t_1 + t_2 = \overset{11.22}{\cancel{11.06}} + \cancel{0.37}\,14.80$ → shown afterwards

$= \cancel{\boxed{11.43}} = \cancel{\boxed{25.86}}$ $= \boxed{26.02}$

Note :-

$\theta_{small}(x) = -e^{-\frac{9rx}{4(1+c)}-1}$ , $\theta_{large}(x) = \cancel{e^{-2rx-1}}$ ,

$(1-2c)e^{1-2c-2rx}$

~~For a given cost c~~ & r fixed

Things to compare:-

For a given cost c, $\theta_{small}$ vs $\theta_{large}$

( ~~we are In both Q1 & Q2, from x = t0 to x (= $x_b$) = 5, only 1 player. However, in Q1, we use $\theta_{large}$~~

For $\theta_{small}$, $c = \cancel{0.54}$ 1 vs $c = 0.59$

(Q1) (Q2)

Solar in action when $x < x_b$

oil will be consumed more slowly

$\Rightarrow t_2^{Q1} < t_2^{Q2}$

$\Rightarrow \eta^{Q1} < \eta^{Q2}$

| Duopoly | $c > \frac{1}{2}$ | $c < \frac{1}{2}$ |
|---|---|---|
| $x \geq x_b$ | Blockaded | NB |
| $x < x_b$ | NB | NB |

Consider $x \in (x_b, \infty) \Rightarrow$ Q1: Monopoly $\xrightarrow{c=1}$ $\theta_{large}$ with $c = 1$

Q2: Duopoly but since $x > x_b$ & $c = 0.59 > \frac{1}{2}$,

$\Rightarrow$ ~~Monopoly~~ $\Rightarrow \theta_{large}$ with $c = 0.59$

Now, for $x > x_b \Rightarrow t \in [0, t_1]$

$$t_1 = \frac{1}{r} \ln\left(\frac{v'(5)}{v'(10)}\right) = \frac{1}{r} \ln\left(\frac{u'(5)}{u'(10)}\right)$$

$$\Rightarrow u(x) = \frac{1}{4r}\left(1 + W(\theta(x-5))\right)^2 \quad \text{where } \theta(x) = (1-2c) \times e^{1-2c-2rx}$$

$$u'(x) = \frac{1}{4r} 2 (1 + W(\theta(x-5))) W'(\theta(x-5))\, \theta'(x-5)$$

$$= \frac{1}{2r} \cancel{(1 + W(\theta(x-5)))} \frac{W(\theta(x-5))}{\theta(x-5)\cancel{(1+W(\theta(x-5)))}} \theta'(x-5)$$

$$= \frac{1}{\cancel{2r}} * \frac{W(\theta(x-5))}{\cancel{\theta(x-5)}} * \cancel{(1-2c)e^{1-2c}} \cancel{(-2r)}\, \cancel{\theta(x-5)} (-\cancel{2r})$$

$$= -W(\theta(x-5))$$

This $\theta$ is $\theta$ - large

$$\Rightarrow t_1 = \frac{1}{r} \ln\left(\frac{W(\theta(5-5))}{W(\theta(10-5))}\right)$$

$$= \frac{1}{r} \ln\left(\frac{W(\theta(0))}{W(\theta(5))}\right)$$

$$= \boxed{11.22}$$

Finally, $\boxed{\text{total time}} = \boxed{26.02}$ $(11.22 + 14.80)$

So, Monopoly $\Rightarrow$ Duopoly with $c = 1 \Rightarrow x_b = 0$

| Monopoly | $c > \frac{1}{2}$ | ~~$c < \frac{1}{2}$~~ |
|---|---|---|
| $x > x_b$ | $B(\theta_l(c=1))$ | |
| ~~$x < x_b$~~ | | |

| Duopoly | $c > \frac{1}{2}$ | $c < \frac{1}{2}$ |
|---|---|---|
| $x > x_b$ | $B(\theta_l(c))$ | $NB(\theta_{small})$ |
| $x < x_b$ | $NB(\theta_{small})$ | $NB(\theta_{small})$ |

Note :- $\theta_l(c=1)$ $\downarrow$ ~~11.06~~ $t_1 = 11.06$

$\theta_l(c = 0.59)$ $\downarrow$ $t_1 = 11.22$

$Q_1$: Monopoly

$x$:

$\to 5$ | $t_1$ with $\theta_l(c=1) \to 11.06$

$\to 0$ | $t_2$ with $\theta_l(c=1) \to 18.41$

$Q2$: Duopoly

$t_1$ with $\theta_l(c = 0.59) \to 11.22$

$t_2$ with $\theta_s(c = 0.59) \to 14.80$

$\Rightarrow$ As $c \downarrow$, $t_1 \uparrow$ (Not entirely true because we get 11.22 when arguments to $\theta_l$ from $x$ side are $x - x_b$ i.e. 0 to 5

So, 11.22 should be compared with 18.41

$\Rightarrow$ As $c \downarrow$, $t_1 \downarrow$

Ideally, what should happen?

As $c \downarrow$, solar $\uparrow \Rightarrow$ player coming in but blockaded before $x_b$. So, it depends on how cost affects $q^*_{oil}(x)$

contain integral over $t$ $\Rightarrow$ does not depend on $t$ naively

Note that :-

* $\boxed{v(x) = \frac{(1 + w(\theta_{large}(1, r, x)))^2}{4r}}$

$\boxed{v'(x) = -w(\theta_1(1, r, x))}$

* $\boxed{q^*(x) = \frac{1 - v'(x)}{2} = \frac{1 + w(\theta_1(1, r, x))}{2} = \sqrt{r\, v(x)}}$

* $\boxed{\text{Exhaustion time } \tau}$

parameters of interest :-

① optimal strategy $\rightarrow q^*(x(t))$

② Exhaustion time $\rightarrow \tau$

③ value earned till that ($t$) $\rightarrow v(x)$

In general,

when $x > x_b$ & $c > 1$, $v'(x) = -w(\theta_1(c, r, x))$

$$\Rightarrow q^*(x) = \frac{1 + w(\theta_1(c, r, x_0 - x_b))}{2}$$

Note that $\theta_1(\ ) = (1-2c)e^{1-2c-2rx}$

$$= (1-2c)e^{1-2c}\, e^{-2rx}$$

$\theta_1(0, r, x) = e^{1}e^{-2rx}$

$\theta_1(1, r, x) = -e^{-1}e^{-2rx}$

$$\Rightarrow \frac{\partial \theta_1}{\partial c} = (1-2c)\, e^{1-2c}(-2) + e^{1-2c}(-2)$$

$$= e^{1-2c} \times (-2)(1 - 2c + 1)$$

$$= -2e^{1-2c} \cdot 2(1-c)$$

$$= -4(1-c)e^{1-2c}$$

$$< 0 \quad (\because c < 1)$$

$\Rightarrow$ As $c \uparrow$, $\theta_1 \downarrow \Rightarrow w(\theta_1) \downarrow$ ($\because w$ is $\uparrow$) $\Rightarrow q^*(x) \downarrow$

~~So, when~~

$$t_1 = \frac{1}{r} \ln \left( \frac{-V'(\delta)}{-V'(1\delta)} \right)$$

$w'(\theta(x - x_b, S))$

$= w'(\theta(0))$

$w'(\theta(S))$

has $x_b$ which also depends on $c$ !

So, when $c \to 1$, $q^*$ is smallest.

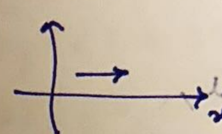
$\Rightarrow$ In monopoly, rate of extraction is the smallest

As the other player comes into picture ($c \downarrow$),

$q^* \uparrow$ (& $x_b \uparrow$). ( $c \to \frac{1}{2} \Rightarrow x_b \to \infty \Rightarrow$ Never blockaded)

~~(i) for $x > x_b$~~ as $c \downarrow$, $x_b \uparrow$

(i) for $x > x_b$, $q^*_{oil} \uparrow$



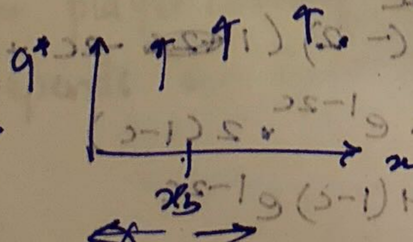
(ii) for $x < x_b$, $q^* = \dfrac{1 + w(\theta_s(c, r, x))}{2}$

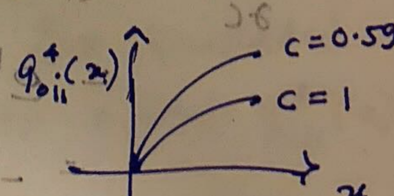
$$\theta_s(c, r, x) = -e^{-\frac{9rx}{4(1+c)} - 1}$$

$$\Rightarrow \frac{\partial \theta_s}{\partial c} = -e^{-\frac{9rx}{4(1+c)} - 1} \cdot \frac{9rx}{4(1+c)^2} < 0 \Rightarrow q^*_{oil} \uparrow$$

~~$\Rightarrow$ as $c \downarrow$, $\theta_s \uparrow \Rightarrow q^*_{oil} \uparrow$~~

$\Rightarrow$ As $c \downarrow$, $x_b \uparrow$ & $\forall x$, $q^*_{oil} \uparrow$




$\Downarrow$

$\tau_{0.59} < \tau_1$

(d) $q_1^{(0)*}(x) = \frac{1}{2}(1 + w(\theta(x - x_b)))$

$= \frac{1}{2}(1 + w(\theta(x - 5)))$

$\cancel{= \frac{1}{2}(1 + w(\theta(5)))}$

$= \frac{1}{2}(1 + w(\theta(x - 5)))$

$q_2^{(0)*}(x) = 0$

Eq^n (60)

The above is for $x \geq x_b$.

For $x < x_b$,

$q_1^{(0)*}(x) = \frac{(1+c)}{3}(1 + w(\theta(x)))$ — Eq^n (57)

$q_2^{(0)*}(x) = \frac{1}{3}\left(1 - 2c - \frac{(1+c)}{2} w(\theta(x))\right)$ — Eq^n (58)

Here $\theta(x) = -\exp\left(-\frac{grx}{4(1+c)} - 1\right)$

(e) After $t = \eta$, oil is exhausted $\Rightarrow q_1^*(x) = 0$

$\Rightarrow$ price $= \underline{\underline{1 - q_2^*(x(t))}}$.

$= 1 - q_2^*(0) \qquad (\because x(t) = 0 \ \forall \ t \geq \eta)$

$= 1 - \left(\frac{1-c}{2}\right) = \frac{1+c}{2} = \frac{1.59}{2} = \boxed{0.795}$

Here, $x^*(t)$ = Nash Eq$^{lb}$ when P1 is using $q_1^*(x)$

~~$x^*(t)$~~ Situation : $c > \frac{1}{2}$ | $x \gtrless x_b$

$$\Rightarrow \frac{dx}{dt} = -q_1^*(x(t)) = -q_1^*(x)$$

$$\Rightarrow \frac{dx}{-q_1^*(x)} = dt$$

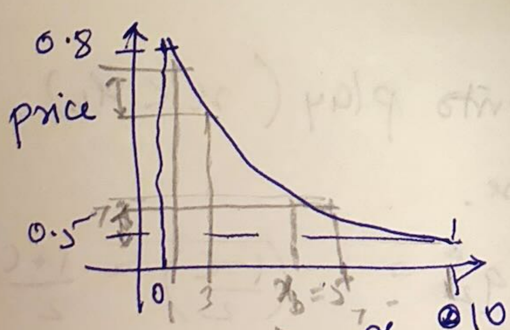
$$\int \frac{dx}{-q_1^*(x)} = dt$$

Roughly, as $t \uparrow$ ($0 \to t_1 \to \eta$), $x \downarrow$ & so, $q_1^*(x)$ will $\downarrow$

{ $q_2^*(x)$ will $\uparrow$

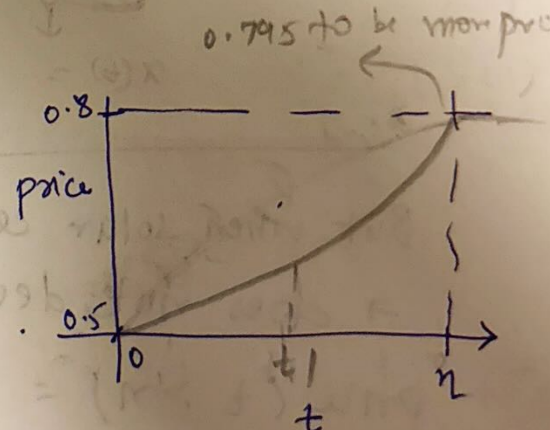
But overall, $q_1^* + q_2^* \downarrow \Rightarrow$ price$(1 - q_1^* - q_2^*) \uparrow$

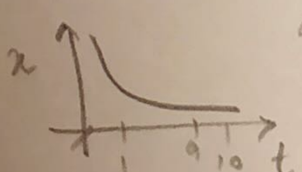
overall,




$\Delta P_2 > \Delta P_1$

$\Rightarrow$




$t \to x \to q_1^* \to P$
$q_2^*$

$\Delta = -3 \quad \Delta = -2$
$10 \to 7 \quad 3 \to 1$
$\Delta P_1 < \Delta P_2$

~~In our case~~. Ideally, after solar enters, we expect price to $\downarrow$ / atleast not $\uparrow$ too much

(continued on next page)

__Mathematical way__

In our case, (Lemma 4.2, 4.3 & around)

$$r v_0 = q_0(1 - q_0 - q_1 - v_0') = g(v_0')$$

$$\Rightarrow g(x) = q_0(1 - q_0 - q_1 - x)$$

$$g'(x) = -q_0$$

$$h(z) = \int_{g^{-1}(0)}^{z} \frac{g'(w)}{w}\,dw = \int_{g^{-1}(0)}^{z} -\frac{q_0}{w}\,dw = q_0 \ln\left(\frac{g^{-1}(0)}{z}\right)$$

$$\Rightarrow \frac{h(z)}{q_0} = \ln\left(\frac{g^{-1}(0)}{z}\right)$$

$$\Rightarrow e^{h(z)/q_0} = \frac{g^{-1}(0)}{z}$$

$$\Rightarrow z = g^{-1}(0)\exp\left(-\frac{h(z)}{q_0}\right) = \cancel{h^{-1}(z)}$$

$$\Rightarrow \boxed{h^{-1}(y) = g^{-1}(0)\exp\left(-\frac{y}{q_0}\right)}$$

So, $P(b) = 1 - q_1^*(x(t)) - q_2^*(x(t))$

$\Theta_{long}(x(t-\tau))$ $\Theta_{sun}(x)$

$$x(t) = \frac{1}{r} h(e^{rt} h^{-1} r x(0))$$

---

But when solar comes into play ($x < x_b$), price still ↑

⇒ does not decrease.

$$\text{price}(t > n) = 1 - q_2^* = 1 - \left(\frac{1-c}{2}\right) = \frac{1+c}{2} = 0.795 \sim \underline{\underline{0.8}}$$

(consistent with the plot)

(Q. 4)

PJM

(c)

| Type \ Time (AM) → | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|
| Forecast | 75425 | 79257 | 85700 | 90678 | 91481 |
| Actual | 74840 | 76479 | 81793 | 88363 | 91411 |
| $\Delta$ | 585 | 2778 | 3907 | 2315 | 70 |
| % (of forecast) | 0.8 | 3.4 | 4.5 | 2.5 | ~0 |