

# ORF 555 : HW2

$$(3+1)^2 P = 1P$$

$$(Q-1)(a) C_1 = 0, C_2 = \frac{1}{4}$$

$$P(Q) = 1 - Q = 1 - (q_1 + q_2)$$

$$(q_1^*, q_2^*) \in [0, 1]^2$$

$$\Pi_1^* = \max_{q_1 \geq 0} q_1(1 - q_1 - q_2^* - C_1)$$

$$\Pi_2^* = \max_{q_2 \geq 0} q_2(1 - q_2 - q_1^* - C_2)$$

$$\frac{\partial \Pi_1^*}{\partial q_1} = 0 \Rightarrow 1 - q_1 - q_2^* - C_1 - q_1 \Big|_{q_1=q_1^*} = 0$$

$$\Rightarrow 1 - 2q_1 - q_2^* - C_1 \Big|_{q_1=q_1^*} = 0$$

$$= 1 - 2q_1^* - q_2^* - C_1 = 0 \quad \text{--- (1)}$$

$$\text{I.I by } \frac{\partial \Pi_2^*}{\partial q_2} = 0 \Rightarrow 1 - q_1^* - 2q_2^* - C_2 = 0 \quad \text{--- (2)}$$

$$(1) + (2) \text{ gives :- where } Q^* = q_1^* + q_2^*$$

$$2 - 3Q^* - (C_1 + C_2) = 0$$

$$\Rightarrow \boxed{Q^* = \frac{2 - (C_1 + C_2)}{3}} \quad \text{--- (4)}$$

$$(1) - (2) \text{ gives :-}$$

$$q_1^* - q_2^* + C_1 - C_2 = 0$$

$$\Rightarrow \boxed{q_1^* - q_2^* = C_2 - C_1} \quad \text{--- (5)}$$

$$(2) \text{ & (5) gives :-}$$

$$q_1^* = \frac{2 - C_1 - C_2 + 3C_2 - 3C_1}{6} = \frac{2 + 2C_2 - 4C_1}{6}$$

$$\Rightarrow \boxed{q_1^* = \frac{1 + C_2 - 2C_1}{3}}$$

$$\text{I.I by } q_2^* = \frac{2 - C_1 - C_2 - 3C_2 + 3C_1}{6} \quad \text{--- (6)}$$

$$\text{So, } (q_1^*, q_2^*) = \left( \frac{1 + C_2 - 2C_1}{3}, \frac{1 + C_1 - 2C_2}{3} \right)$$

$$(3+1)^2 P = 1P \rightarrow P = \frac{1}{3+1} = \frac{1}{4}$$

Plugging in values,

$$\boxed{(q_1^*, q_2^*) = \left( \frac{5}{12}, \frac{1}{6} \right) = \frac{9b}{3b}}$$

$$P = 1 - Q^* = 1 - (q_1^* + q_2^*)$$

$$\begin{aligned} &= 1 - \left( \frac{5}{12} + \frac{1}{6} \right) \\ &= 1 - \frac{7}{12} = \frac{5}{12} \end{aligned}$$

$$= \frac{5}{12}$$

$$\Rightarrow P = \frac{5}{12} \cdot 3 \cdot 1 \text{ more}$$

Afterwards of sub. beginning sing  
T. more now  
• pd satisfies 3390 ft  
sing in parent ent  
2U of reheat at 1116 K  
sing air expands with decrease of  
stresses of heat flow  
at ground at room temp  
are being ent of heat sing  
sing ent kinetics of J. i  
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bottomless well & ent of will be H =

$$\Rightarrow \boxed{q_2^* = \frac{1 + C_1 - 2C_2}{3}}$$

$$(b) q_1 = q_1^* (1 + \epsilon)$$

$$\Rightarrow P_n = 1 - q_1^* - q_2^*$$

$$\therefore P = 1 - q_1^* - q_2^*$$

$$\Delta P = P_n - P = q_1^* - q_1 = q_1^* - q_1^*(1 + \epsilon)$$

$$\Rightarrow \Delta P = -q_1^* \epsilon$$

$$\Rightarrow \frac{dP}{d\epsilon} = -q_1^* = -\frac{5}{12}$$

(c) By similar logic,

$$\frac{dP}{d\epsilon} = -q_2^* = -\frac{1}{6}$$

(d) Price impact due to deviation from N.E. qty :-

OPEC > US

$\Rightarrow$  If OPEC deviates by  $\epsilon$ , some amount  
the change in price,  
it will be harder for US  
to control the change in price  
~~they~~  $\Rightarrow$  US  $\Rightarrow$  will need to deviate  
 by a lot more to bring the  
 price back to the original one  
 i.e. to control the price  
 $\Rightarrow$  harder for ~~US~~ US  
 $\Rightarrow$  Headline is true & hence validated.

$$\frac{1}{P} = \frac{1}{1 - q_1^* - q_2^*} \cdot \epsilon = \frac{1}{1 - q_1^* (1 + \epsilon)} \cdot \epsilon = \frac{\epsilon}{1 + \epsilon}$$

$$(1 - q_1^*)^{-1} = D - 1 = \frac{D}{D}$$

$$S(1, 0) \Rightarrow (1 - q_1^*, 1)$$

$$(D - 1)^{-1} P = \frac{1}{D} P = \frac{1}{1 + \epsilon}$$

$$(1 - q_2^*)^{-1} P = \frac{1}{1 + \epsilon}$$

$$\frac{1}{1 - q_2^*} = \frac{1}{1 - q_2^* (1 + \epsilon)} \cdot \epsilon = \frac{\epsilon}{1 + \epsilon}$$

$$O = \frac{1}{1 - q_2^*} P = \frac{1}{1 - q_2^* (1 + \epsilon)} \cdot \epsilon = \frac{\epsilon}{1 + \epsilon}$$

$$O = D - 1 = \frac{1}{1 + \epsilon}$$

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$$O = (1 + D) - \frac{1}{1 + \epsilon} = \frac{1}{1 + \epsilon}$$

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$$(Q.2) \frac{dS_t}{S_t} = (\mu - \delta) dt + \sigma dW_t$$

↳ Brownian Motion  
 $= W_t \sim N(0, t)$

$$\Rightarrow S_t = S_0 \exp \left( \left( \mu - \delta - \frac{\sigma^2}{2} \right) t + \sigma W_t \right)$$

$$\Rightarrow S_T = S_t \exp \left( \left( \mu - \delta - \frac{\sigma^2}{2} \right) (T-t) + \sigma (W_T - W_t) \right)$$

(a) Now, if  $W = \exp(Z)$  where  $Z \sim N(\mu, \sigma^2)$ ,

$$E(W) = \exp(\mu + \frac{\sigma^2}{2}) \quad | \rightarrow \phi_z(t) = E(e^{tZ}) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$$

$$E(W^2) = \exp(2\mu + 2\sigma^2) \quad \Rightarrow E(e^{2Z}) = e^{2\mu + 2\sigma^2}$$

$$\begin{aligned} \Rightarrow \text{Var}(W) &= E(W^2) - (E(W))^2 \\ &= \exp(2\mu + 2\sigma^2) - \exp(2\mu + \sigma^2) \\ &= \exp(2\mu + \sigma^2) [\exp(\sigma^2) - 1] \end{aligned}$$

In our case, ~~given~~  $S_T = \underbrace{S_t \exp((\mu - \delta - \frac{\sigma^2}{2})(T-t))}_{C} \exp(\sigma(W_T - W_t)) \underbrace{N}$

$\Rightarrow$  Given  $S_t$ ,  $C$  is known

$$\Rightarrow \frac{S_T}{C} = \exp(\sigma(W_T - W_t)) = \exp(A) \text{ where } A \sim N(0, \sigma^2(T-t))$$

$$\begin{aligned} \Rightarrow \text{Var}\left(\frac{S_T}{C}\right) &= \exp(2 \cdot 0 + \sigma^2(T-t)) (\exp(\sigma^2(T-t)) - 1) \\ &= \exp(2\sigma^2(T-t)) - \exp(\sigma^2(T-t)) \end{aligned}$$

$$\Rightarrow \text{Var}(S_T) = C^2 (\exp(2\sigma^2(T-t)) - \exp(\sigma^2(T-t)))$$

$$= S_t^2 \exp(2(\mu - \delta)(T-t) - \cancel{\sigma^2(T-t)}) \exp(\sigma^2(T-t)) (\exp(\sigma^2(T-t)) - 1)$$

$$= S_t^2 \exp(2(\mu - \delta)(T-t)) (\exp(\sigma^2(T-t)) - 1)$$

$$\Rightarrow \text{Var}(S_T | S_t) = \underline{S_t^2 \exp(2(\mu - \delta)(T-t)) (\exp(\sigma^2(T-t)) - 1)}.$$

(Q.2)

⑥  $P\left(\frac{S_T - S_t}{S_t} \geq \frac{\Delta}{100}\right)$  cannot control at  $\Delta$  (with  $S_t$  known to  $t$ )

$$= P\left(\frac{S_t \exp((\mu - \delta - \frac{\sigma^2}{2})(T-t) + \sigma(w_T - w_t))}{S_t} \geq \frac{\Delta}{100}\right)$$

↓  
Ind of  $S_t$  words now (but know  $w_t$  before  $T$ )

$$= P\left(\exp((\mu - \delta - \frac{\sigma^2}{2})(T-t) + \sigma(w_T - w_t)) \geq \frac{\Delta}{100} + 1\right)$$

$$= P\left(\exp(\sigma(w_T - w_t)) \geq (\frac{\Delta}{100} + 1)c\right)$$

(where  $c = \exp(-(\mu - \delta - \frac{\sigma^2}{2})(T-t))$ )

$$= P\left(\sigma(w_T - w_t) \geq \log\left(\frac{\Delta}{100} + 1\right)c\right)$$

$$= P\left(w_T - w_t \geq \frac{1}{\sigma} \log\left(\frac{\Delta}{100} + 1\right)c\right)$$

$$= P(N(0, T-t) \geq c_1)$$

(where  $c_1 = \frac{1}{\sigma} \log\left(\frac{\Delta}{100} + 1\right)c$ )

$$= P(N(0, 1) \geq \frac{c_1}{\sqrt{T-t}})$$

$$= 1 - P(N(0, 1) < \frac{c_1}{\sqrt{T-t}})$$

$$= 1 - \Phi\left(\frac{c_1}{\sqrt{T-t}}\right)$$

$$= 1 - \Phi\left(\frac{1}{\sigma\sqrt{T-t}} \log\left(\frac{\Delta}{100} + 1\right)c\right)$$

$$= 1 - \Phi\left(\frac{1}{\sigma\sqrt{T-t}} \log\left(\frac{\Delta}{100} + 1\right) \exp(-(\mu - \delta - \frac{\sigma^2}{2})(T-t))\right)$$

$$= 1 - \Phi\left(\frac{\log(\frac{\Delta}{100} + 1)}{\sigma\sqrt{T-t}} - \frac{(\mu - \delta - \frac{\sigma^2}{2})\sqrt{T-t}}{\sigma}\right)$$

The answer does NOT depend on St because we are talking in relative terms (change is  $\Delta \frac{V}{V}$ ) ~~is not~~

$\downarrow$   
required to measure.  $(\text{fw} - \text{rw})_0 + (\text{ft} - \text{rt})(\frac{\text{fw} - \text{rw}}{\text{fw}})_0 =$

If the change requested (or mentioned) was absolute, then  
 $\text{St}$  would have been there in the picture.

$(1 + \frac{\Delta}{\text{fw}}) \ll (\text{fw} - \text{rw})_0 + (\text{ft} - \text{rt})(\frac{\text{fw} - \text{rw}}{\text{fw}})_0 =$

$$\left( \rightarrow \left( 1 + \frac{\Delta}{\text{fw}} \right) \ll (\text{fw} - \text{rw})_0 \quad \cancel{\text{ft} - \text{rt}} \right) q =$$

$$((\text{ft} - \text{rt})(\frac{\text{fw} - \text{rw}}{\text{fw}})_0 - )q \approx 0 \text{ m/s}$$

$$\left( (\rightarrow \left( 1 + \frac{\Delta}{\text{fw}} \right))_{\text{rel}} \ll (\text{fw} - \text{rw})_0 \right) q =$$

$$\left( (\rightarrow \left( 1 + \frac{\Delta}{\text{fw}} \right))_{\text{rel}} \frac{1}{\phi} \ll \text{fw} - \text{rw} \right) q =$$

$$(\rightarrow \ll (\text{ft}, 0) \text{ N}) q =$$

$$((\rightarrow \left( 1 + \frac{\Delta}{\text{fw}} \right))_{\text{rel}} \frac{1}{\phi} = 1) \text{ m/s}$$

$$\left( \frac{10}{\text{FTV}} \ll (1, 0) \text{ N} \right) q =$$

$$\left( \frac{10}{\text{FTV}} > (1, 0) \text{ N} \right) q = 1 =$$

$$\left( \frac{10}{\text{FTV}} \right) \phi^{-1} =$$

$$\left( (\rightarrow \left( 1 + \frac{\Delta}{\text{fw}} \right))_{\text{rel}} \frac{1}{\frac{10}{\text{FTV}} \phi} \right) \phi^{-1} =$$

$$(((\text{ft} - \text{rt})(\frac{\text{fw} - \text{rw}}{\text{fw}})_0 - )q \approx (1 + \frac{\Delta}{\text{fw}}))_{\text{rel}} \frac{1}{\frac{10}{\text{FTV}} \phi} \phi^{-1} =$$

$$\left( \text{FTV} \frac{(\frac{\text{fw} - \text{rw}}{\text{fw}})_0}{\phi} - \frac{(1 + \frac{\Delta}{\text{fw}}) \text{ rel}}{\text{FTV} \phi} \right) \phi^{-1} =$$

(Q.2) (c)

when real world changes to Q,  
replace  $\mu$  by  $r$  (in real world formula)

$$\Rightarrow \frac{df(t, T)}{f(t, T)} = (r - r) dt + \sigma dW_t^Q$$

$$= \cancel{\sigma dW_t^Q}$$

$$= \underline{0} dt + \sigma dW_t^Q$$

$$\begin{aligned} \text{So, } df_t &= \frac{\partial f}{\partial S_t} dS_t + \frac{\partial f}{\partial t} dt + \frac{1}{2} \left( \frac{\partial^2 f}{\partial S_t^2} \right) (dS_t)^2 \\ &= e^{(r-\delta)(T-t)} dS_t + -(r-\delta) S_t e^{(r-\delta)(T-t)} dt \\ &= \frac{f_t}{S_t} dS_t - (r-\delta) f_t dt \end{aligned}$$

$$\Rightarrow \frac{df_t}{f_t} = \frac{dS_t}{S_t} - (r-\delta) dt$$

$$= \sigma dW_t^Q$$

$$\Rightarrow \boxed{\frac{df_t}{f_t} = \underline{0} dt + \underline{\sigma} dW_t^Q}$$

$$(d-2)(d) \quad d(\ln F) = \frac{\partial \ln F}{\partial F} dF + \frac{\partial \ln F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 \ln F}{\partial F^2} (dF)^2$$

$$= \frac{1}{F} dF + 0 dt + \frac{1}{2} \left( -\frac{1}{F^2} \right) \cancel{\frac{\partial^2 \ln F}{\partial F^2}} (dF)^2$$

~~$\frac{\partial^2 \ln F}{\partial F^2}$~~

current time

$$= \sigma dW_t^Q + \frac{1}{2} \sigma^2 \left( \frac{dF}{F} \right)^2$$

↑

Assuming  $T < T'$ ,

$$= \sigma dW_t^Q - \frac{\sigma^2}{2} dt$$

$\Rightarrow \ln F_{T'} - \ln F_T = \sigma (W_{T'}^Q - W_T^Q) - \frac{\sigma^2}{2} (T' - T)$

Assuming  $t < T < T'$ ,

(6)  $\Rightarrow \frac{F(t, T')}{F(t, T)} = \exp \left( -\frac{\sigma^2}{2} (T' - T) + \sigma (W_{T'}^Q - W_T^Q) \right)$

(Assumption  $T < T'$ )

$$\sim N(0, T' - T)$$

$\Rightarrow E(F(t, T')) = F(t, T) \exp \left( -\frac{\sigma^2}{2} (T' - T) \right) \exp \left( \sigma (W_{T'}^Q - W_T^Q) \right)$

$\Rightarrow E(F(t, T') | F(t, T)) = F(t, T) \exp \left( -\frac{\sigma^2}{2} (T' - T) \right) \exp \left( \frac{\sigma^2 (T' - T)}{2} \right)$

$$= F(t, T)$$

$\Rightarrow E(F(t, T') | f(t, T)) = F(t, T)$

Hence,  $F(t, T)$  is a martingale.

Yes, it is the time  $t$  price of a derivative security expiring at time  $\Theta T$ .

This security is forward. So, payoff =  $S(T) - F(t, T)$

$\underbrace{S(T)}$   
 Stock price at time  $T$   
 (pre)determined at time  $t$   
 forward price of a fwd expiring  
 at time  $T$

$$(Q.2)(d) \quad \text{Now, } F(t, T) = \frac{S_t}{W_t} + \frac{\sigma^2}{2} dt = S_T$$

$$\text{we have: } d(\ln F) = \sigma dW_t + \frac{\sigma^2}{2} dt$$

consider a forward expiring at time  $T \Rightarrow f(\cdot, \cdot) = F(\cdot, T)$

we look at its price at 2 different times  $s \leq t < T$

$$\Rightarrow \ln F(t, T) - \ln F(s, T)$$

$$= \sigma \int_s^t dW_u - \int_s^t \frac{\sigma^2}{2} dt \quad |T > T \text{无关}$$

$$= (\bar{W}_T - \bar{W}_s) - \frac{\sigma^2}{2}(t-s) \quad |T > T > \text{无关}$$

$$\Rightarrow F(t, T) = F(s, T) \exp\left(-\frac{\sigma^2}{2}(t-s)\right) \exp\left(\sigma(\bar{W}_t - \bar{W}_s)\right) \quad |T > T \text{无关} \sim N(0, t-s)$$

$$E(F(t, T) | F(s, T)) = F(s, T) \exp\left(-\frac{\sigma^2}{2}(t-s)\right) \exp\left(\frac{\sigma^2}{2}(t-s)\right)$$

$$= F(s, T)$$

$$\Rightarrow E(F(t, T) | F(s, T)) = F(s, T) \quad (s < t)$$

$\Rightarrow$  so,  $F(t, T)$  is a martingale

Yes, it is the time  $t$  price of a derivative security

expiring at time  $T$ .

This derivative is a forward.

$$\text{so, payoff} = S(T) - F(t, T)$$

Stock price at time  $T$

→ fwd price (pre-determined)  
at time  $t$  (for a  
fwd expiring at time  $T$ )

+ wait for binomial (exp)

unique but a few branching points  
( $T$  wait to)

(Q.2)

(e) Given:-  $t < T < T'$

$$\text{So, } F(t, T') = S_t \exp$$

$$F(t, T') = F(t, T) \exp\left(-\frac{\sigma^2}{2}(T' - T) + \sigma(w_T^q - w_t^q)\right)$$

But we want  $F(t, T')$  in terms of  $F(T, T')$  (or reverse).  
Let's write  $F(t, T)$  in terms of  $S(T)$ .

$$\text{So, } F(t, T') = S_t e^{(r-\delta)(T'-t)}$$

$$F(T, T') = S_T e^{(r-\delta)(T'-T)}$$

$$\Rightarrow \frac{F(T, T')}{F(t, T')} = \frac{S_T}{S_t} e^{(r-\delta)(T-t)}$$

$$= e^{(r-\delta)(T-t)} + \cancel{\sigma(w_T^q - w_t^q)} \cancel{(r-\delta)(T-t)}$$

$$= e^{(r-\delta-\frac{\sigma^2}{2})(T-t)} + \sigma(w_T^q - w_t^q) e^{-(r-\delta)(T-t)}$$

$$= e^{-\frac{\sigma^2}{2}(T-t)} + \sigma(w_T^q - w_t^q)$$

$$\Rightarrow F(T, T') = \underline{F(t, T')} e^{-\frac{\sigma^2}{2}(T-t)} \underline{e^{\sigma(w_T^q - w_t^q)}}$$

(Q.5) F(1), we know that:-

$$F(t, T) = S_t e^{(r-\delta)(T-t)} \quad (\text{atleast under the risk neutral world})$$

$$\Rightarrow F(0, T) = S_0 e^{(r-\delta)T} \quad \downarrow \\ w^q_T$$

$$80 = S_0 e^{(r-\delta) \cdot \frac{1}{6}} \quad \text{where } r = 0.05$$

$$75 = S_0 e^{(r-\delta) \frac{2}{6}}$$

$$70.5 = S_0 e^{(r-\delta) \frac{3}{6}}$$

$$67 = S_0 e^{(r-\delta) \frac{4}{6}}$$

$$64.5 = S_0 e^{(r-\delta) \frac{5}{6}}$$

$$63 = S_0 e^{(r-\delta) \frac{6}{6}}$$

$$e^{\cancel{(r-\delta) \frac{1}{6}}} \sim \frac{15}{16}$$

$$(Q.4) P(q) = 1 - q$$

cost of production =  $c$

$$(a) \Pi = \max_{q \geq 0} q(1 - q - c)$$

$$\frac{d\Pi}{dq} = 0 \Rightarrow q(-1) + (1 - q - c) \Big|_{q=q^*} = 0$$

$$\Rightarrow 1 - 2q - c = 0 \quad |_{q=q^*}$$

$$\Rightarrow q = \frac{1-c}{2} \quad |_{q=q^*}$$

$$\Rightarrow q^* = \boxed{\frac{1-c}{2}}$$

(b) Reserves  $\rightarrow x(t)$ ,  $t \geq 0$

extraction rate  $\rightarrow q_t > 0$

$$\text{so, } \frac{dx}{dt} = -q_t$$

$$\Rightarrow dx = -q_t dt$$

$$\Rightarrow \int_0^t dx = \int_0^t -q_t dt$$

$$\Rightarrow x(t) - x(0) = - \int_0^t q_p dp$$

$$\Rightarrow \boxed{x(t) = x(0) - \int_0^t q_p dp} \quad -\textcircled{1}$$

finite time horizon ( $T > 0$ )

beyond which no selling

$\Rightarrow$  ~~Given~~ Given  $x(0) = x > 0$ ,

$$\text{Revenue} = \boxed{v(x_t) = \int_0^T e^{-rt} q_t (1 - q_t - q_{at}) dt}$$

In this part,  $q_t = q^*$   $\forall t \in [0, T]$

$$\Rightarrow x(t) = x(0) - \int_0^t q^* dp$$

$$= \boxed{x(0) - q^* t} \quad -\textcircled{2}$$

for reserves to be exhausted by time  $T$ ,

$$\text{~~Given~~} x(T) = 0 \text{ & } x(t) = 0 \text{ for some } t \leq T$$

$$\Rightarrow x(0) - q^* t = 0 \text{ for } t \leq T$$

$$\Rightarrow x(0) = q^* t \leq q^* T = (\frac{1-c}{2}) T$$

$$\Rightarrow \text{for } \boxed{x \in [0, (\frac{1-c}{2}) T]},$$

reserves will be exhausted by time  $T$ .

$$(\frac{1-c}{2}, T)_{min} = (x_0, T)_{min}$$

$$(\frac{1-c}{2}, T)_{max} =$$

$$((\frac{1-c}{2}, T)_{min} =$$

$$(c) \text{ For } q_t = \frac{x}{q^*} \text{ & } \alpha = \frac{x}{q^*} < T$$

$$V(x) = \int_0^T e^{-rt} \left(\frac{1-c}{2}\right)^2 dt$$

$$= \int_0^\alpha e^{-rt} \left(\frac{1-c}{2}\right)^2 dt + \int_\alpha^T e^{-rt} (1-\alpha-c) dt$$

where  $\alpha = \alpha \left( \frac{1-c}{2} \right)$  ( $\because$  after this,  $x=0 \Rightarrow q_t=0$ )

$$\Rightarrow \alpha = \frac{2x}{1-c}$$

$$\Rightarrow V(x) = \left(\frac{1-c}{2}\right)^2 \left( \frac{1-e^{-rx}}{r} \right)$$

$$V(x) = \left(\frac{1-c}{2}\right)^2 \left( \frac{1-e^{-\frac{2rx}{1-c}}}{r} \right)$$

This is true when  $\alpha < T$ ,

If  $\alpha \left( = \frac{x}{q^*} \right) > T$ ,

$$V(x) = \int_0^T e^{-rt} \left(\frac{1-c}{2}\right)^2 dt$$

$$= \left(\frac{1-c}{2}\right)^2 \frac{1}{r} (1-e^{-rT})$$

$$\Rightarrow V(x) = \left(\frac{1-c}{2}\right)^2 \frac{1-e^{-rT}}{r}$$

So,  $V(x) = \left(\frac{1-c}{2}\right)^2 \frac{1-e^{-rT_f}}{r}$

where  $T_f = \min(T, \frac{2x}{(1-c)})$

$$\left(\frac{1-c}{2}\right)$$

$$p - t = (3)^2 / (4 \cdot 2)$$

$$(3-p-1) p^{2A/11} = 71 \quad (N)$$

$$p = (3-p-1) + (1-p) \Leftrightarrow p = 71$$

$$p = (3-p-1) + (1-p) \Leftrightarrow p = 71$$

$$\left(\frac{2x}{1-c}\right) = p \quad F$$

$$\left(\frac{2x}{1-c}\right) = p \quad F$$

$$\min(T, \alpha) = \min(T, \frac{2x}{q^*})$$

$$= \min(T, \frac{x}{(1-c)/2})$$

$$= \min(T, \frac{2x}{1-c})$$