CS215 Assignment3 Problem 1

Parth Pujari and Anish Kulkarni

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1 Question

Using the MATLAB function randn() to generate a data sample of N = $[5, 10, 20, 40, 60, 80, 100, 500, 10^3, 10^4]$ sample gaussian random variables with mean = 10 and variance = 16.
Using two priors:

1. A Gaussian pdf with mean = 10.5 and standard deviation = 1
2. A uniform distribution over [9.5, 11.5]

To calculate the Maximum likelihood estimate ($\hat{\mu}^{\mathrm{ML}}$) for the mean (assuming known variance) and the maximum a -posteriori for the mean using both of the aforementioned priors ($\hat{\mu}^{\mathrm{MAP1}}$, $\hat{\mu}^{\mathrm{MAP2}}$) on these data sets over **100 trials** and plot their errors with respect to the true mean.

2 Finding the estimates

The expression randn()*4 + 10 generates a gaussian pseudo-random variable with mean 10 and standard deviation 4.

2.1 The maximum likelihoood estimator

As is known, the maximum likelihood estimator for the mean of a gaussian distribution is unbiased and is equal to:

$$arg\ max_{\mu} \prod_{n=1}^{N} G(X = x_i, \mu, \sigma_o)$$

Which turns out to be the sample mean .

2.2 The first prior

The first prior is a gaussian with mean = 10.5 and standard deviation = 1.
Thus the maximum a-posteriori for the mean is calculated as follows:

$$arg\ max_{\mu} \prod_{n=1}^{N} G(X = x_i, \mu, \sigma_o) \times G(X = \mu, \mu_{prior}, \sigma_{prior})$$

$$\hat{\mu}^{\mathrm{MAP1}} = \frac{\bar{x}\sigma_0^2 + \mu_o \sigma_{prior}/N}{\sigma_0^2 + \sigma_{prior}/N}$$

2.3 The second prior

The second prior is a uniform distribution over 9.5 to 11.5, thus the maximum a-posteriori for the mean is calculated as

$$\hat{\mu}^{\mathrm{MAP2}} = \begin{cases} 9.5 & \bar{x} < 9.5 \\ \bar{x} & 9.5 \leq \bar{x} \leq 11.5 \\ 11.5 & \bar{x} > 11.5 \end{cases}$$

Where $\bar{x}$ is the sample mean.

3 Plotting the errors

For each sample size N, 100 trials were calculated and their mean $\bar{x}$ was used to find the estimates as above. The error was calculated as

$$|Est - \mu_{true}|/\mu_{true}$$

The box plots for each estimate (maximum likelihood, MAP1 and MAP2 in that order) for each sample size are shown on the next page.

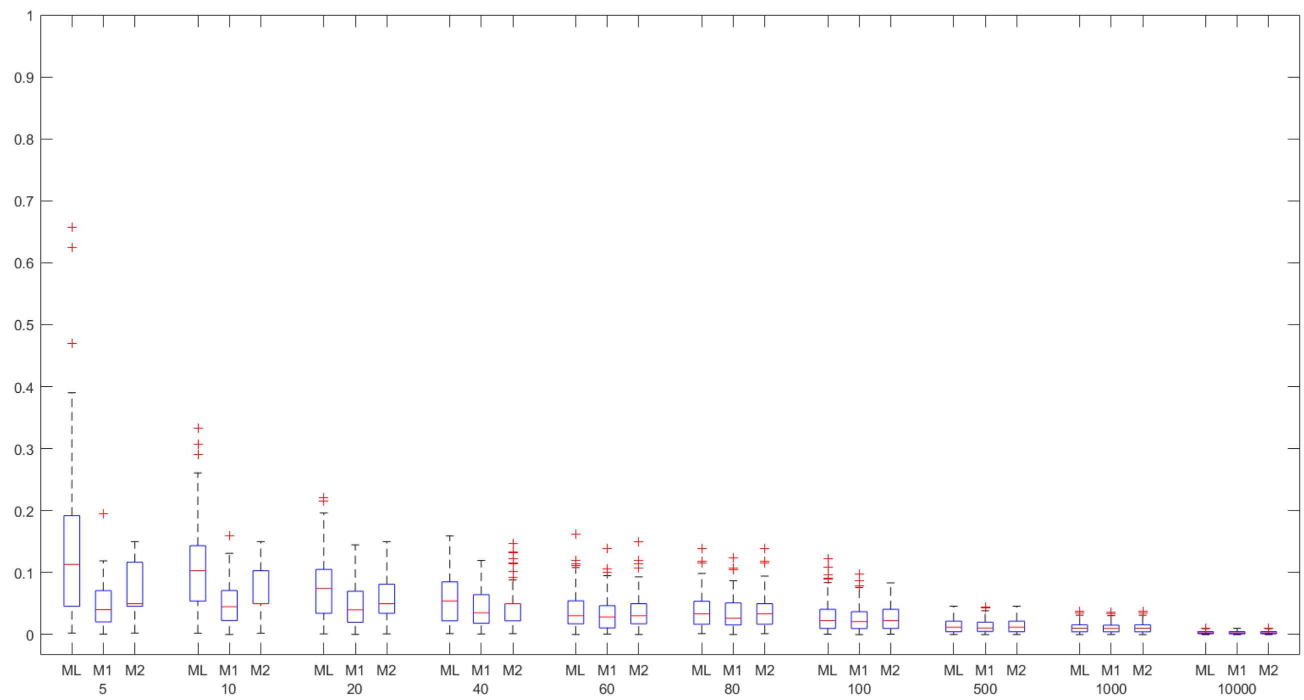

Figure 1: Boxplots for Estimators
X-axis: Estimator type and Sample size
Y-axis: Error in estimate

4 Interpretation

As the value of N increases, owing to the law of large numbers, the error of the estimates decrease. Further, we observe that the posterior estimate for the mean for the first prior is significantly more accurate and precise, i.e., it converges to 0 faster and it has a smaller standard deviation.
This shows that the second prior is a better estimator for μ.