CS215 Assignment1 Problem 1

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1 The Laplace Distribution

Laplace.mlx generates the PDF and the CDF for a Laplace distribution on a set of 100 uniformly distributed numbers from -40 to +40.
The Laplace PDF (x) for the location parameter μ (equal to 2) and the scale parameter $b > 0$ (also equal to 2) is as follows

$$PDF : P(x) = \frac{1}{2b} e^{(-\frac{|x-\mu|}{b})}$$

The CDF is generated by adding the PDF stepwise in accordance with the Riemann sum. Theoretically, the CDF is:

$$C(x) \int_{-\infty}^{x} P(x)\, dx \; = \int_{-\infty}^{x} \frac{1}{2b} e^{(-\frac{|x-\mu|}{b})}\, dx \; = \frac{1}{2} + \frac{1}{2} sgn(x-\mu) \left(e^{(-\frac{|x-\mu|}{b})} \right)$$

The variance was calculated to be 8.0000 (theoretical variance = $2b^2 = 8$).
The following are the plots for the PDF and the CDF (Figure 1 and Figure 2).

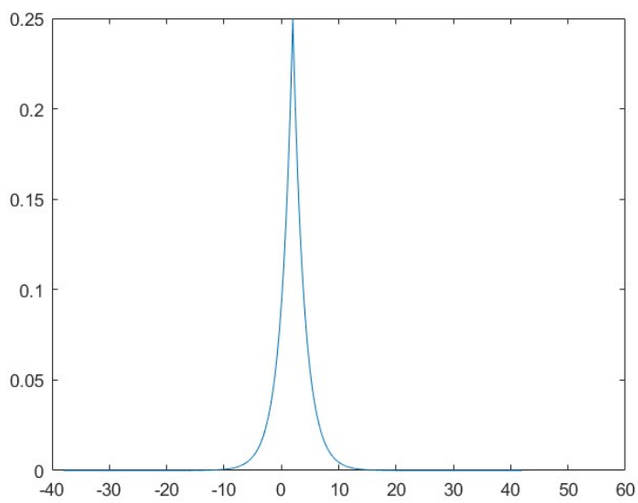

Figure 1: Laplace PDF

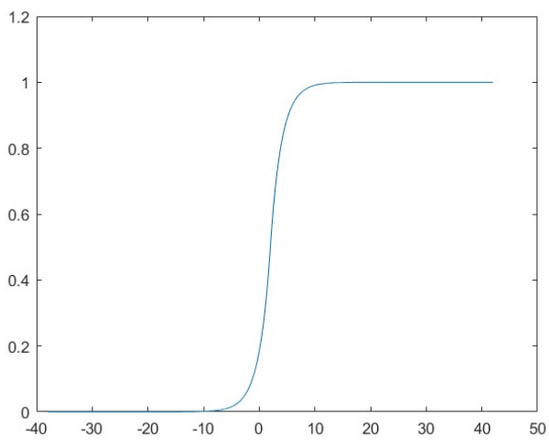

Figure 2: Laplace CDF

2 Gumbel Distribution

Gumbel.mlx generates the PDF and the CDF for a Gumbel distribution on a similar set of uniform points. The PDF is calculated setting the location parameter μ to 1 and the scale parameter β to 2.
The generated PDF should take the form:

$$PDF : P(x) = \frac{1}{\beta} e^{-(z+e^{-z})}$$

where z is:

$$z = \frac{(x-\mu)}{\beta}$$

The CDF is:

$$C(x) = e^{e^{-\frac{x-\mu}{\beta}}}$$

The variance was calculated to be 6.5797 (theoretical variance $= \pi^2\beta^2/6 = 6.5797$
The following figures (Figure 3 and 4) show the PDF and the CDF:

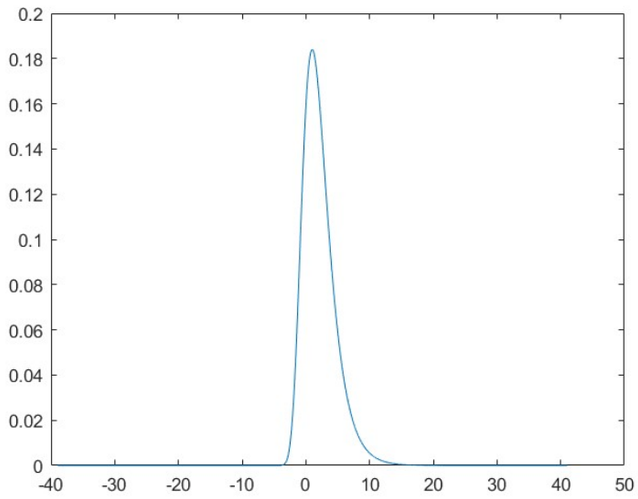

Figure 3: Gumbel PDF

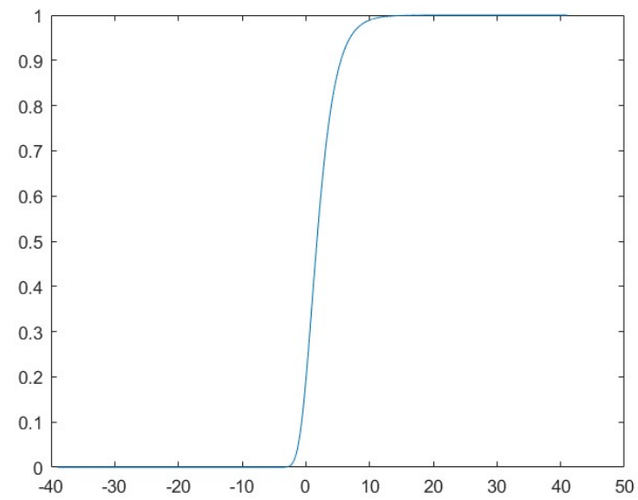

Figure 4: Gumbel CDF

3 Cauchy Distribution

Cauchy.mlx generates the PDF and the CDF for a Cauchy distribution on a similar set of uniform points. The PDF is calculated setting the location parameter x_o to 1 and the scale parameter γ to 1.
The generated PDF should take the form:

$$PDF : P(x) = \frac{1}{\pi}\left(\frac{\gamma}{x^2 + \gamma^2}\right)$$

The CDF is:

$$C(x) = \frac{1}{\pi} tan^{-1}\left(\frac{x}{\gamma}\right) + \frac{1}{2}$$

The variance is undefined The following figures (Figure 5 and 6) show the PDF and the CDF:

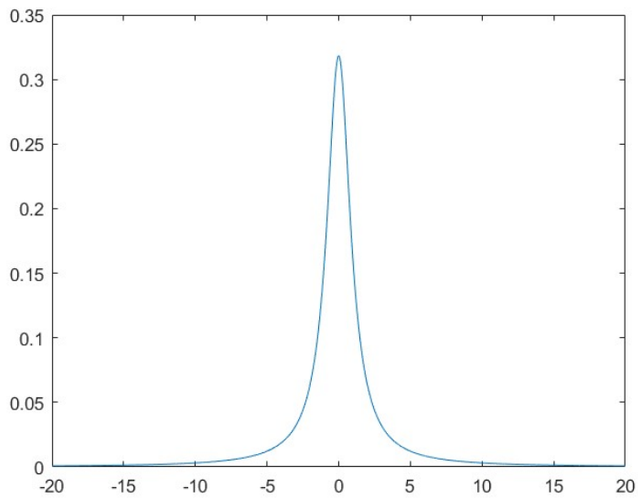

Figure 5: Cauchy PDF

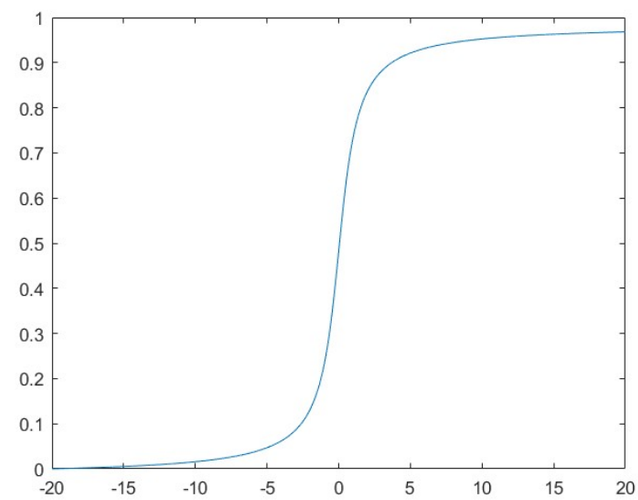

Figure 6: Cauchy CDF