Digital Image Processing HW1 Parth Pujari and Aayushi Barve

1 Question 5

The following are the three figures for the nearest neighbour interpolation:



Figure 1: Nearest Neighbour Interpolation

The following is the same for bilinear interpolation:



Figure 2: Bilinear Interpolation

Assume the n points used as references for physically corresponding points were colinear. Referring to our affine transformation calculations we would get the following: Let the affine transformation be A. Let P_1 be

the matrix
$$\begin{bmatrix} x_1^1 & x_2^1 & \dots & \dots & x_{12}^1 \\ y_1^1 & y_2^1 & \dots & \dots & y_{12}^1 \\ 1 & 1 & \dots & \dots & 1 \end{bmatrix}$$
 and $P_2 \begin{bmatrix} x_1^2 & x_2^2 & \dots & \dots & x_{12}^2 \\ y_1^2 & y_2^2 & \dots & \dots & y_{12}^2 \\ 1 & 1 & \dots & \dots & 1 \end{bmatrix}$

Then we have

$$A \cdot P_1 = P_2 \tag{1}$$

$$A = (P_2 \cdot P_1^T) \cdot (P_1 \cdot P_1^T)^{-1} \tag{2}$$

$$A = P_2 \cdot P_1^{-1} \tag{3}$$

If P_1^{-1} exists.

Since P_1^T has non 0 nullity, $(P_1 \cdot P_1^T)^{-1}$ does not exist. If we use a pseudo inverse we get a matrix P^{-1} (and hence A) that maps points to arbitrarily large values that could be out of range. For example for a singular matrix B, if Bv = 0 and a small perturbation gives $B'v = \epsilon$ where ϵ is a vector slightly perturbed from 0, then $B'^{-1}\epsilon = v$ which means large enough vectors proportional to ϵ in the image coordinate ranges are mapped to arbitrarily large values. This lack of accurate transformation power is evident from the fact that there simply isn't enough information in P_1 to calculate A. Further, we use reverse transform A^{-1} which may or may not exist for A.