

Definitions, Laws, and Axioms

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Spring 2026

This document contains the official list of definitions, laws, and axioms that students are permitted to reference and use for CS 2050 assignments and examinations.

1 Definitions

1. **Negation (\neg):** the negation of p is the statement “not p .”
2. **Conjunction (\wedge):** the conjunction of p and q is the statement “ p and q .” For the statement to be true, both p and q must be true.
3. **Disjunction (\vee):** the disjunction of p and q is the statement “ p or q .” For the statement to be true, at least one of p or q must be true.
4. **Exclusive-or (\oplus):** the exclusive-or of p and q is the statement “ p XOR q .” For the statement to be true, exactly one of p and q must be true.
5. **Conditional statement (\rightarrow):** “If p , then q .” Written $p \rightarrow q$. This is false when p is true and q is false, and true in every other instance.
6. **Converse:** $q \rightarrow p$.
7. **Inverse:** $\neg p \rightarrow \neg q$.
8. **Contrapositive:** $\neg q \rightarrow \neg p$ (logically equivalent to the given conditional).
9. **Bi-conditional statement (\leftrightarrow):** “ p if and only if q .” One cannot exist without the other. Statement is true when both statements are true or both statements are false. q is sufficient for p and p is sufficient for q .
10. **Definition of Biconditional:**
$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$
11. **Tautology:** a compound proposition that is always true. Examples: $p \rightarrow p, p \vee \neg p$.
12. **Contradiction:** a compound proposition that is always false. Example: $\neg p \wedge p$.
13. **Contingency:** a compound proposition that is neither a tautology nor a contradiction. Most statements are contingencies. Example: $p \wedge q$.
14. **Logical Equivalence:** Two compound propositions p and q are logically equivalent if $p \leftrightarrow q$ is a tautology. Notation: $p \equiv q$.
15. **Predicate (Propositional Function):** A predicate is a statement involving one or more variables that becomes either true or false when values are assigned to the variables. Written like $P(x)$.
16. **Universal Quantifier (\forall):** The universal quantifier \forall is read as “for all” or “for every.” The statement $\forall x P(x)$ means that the predicate $P(x)$ is true for every element x in the specified domain.

17. **Existential Quantifier (\exists):** The existential quantifier \exists is read as “there exists.” The statement $\exists xP(x)$ means that there is at least one element x in the domain for which $P(x)$ is true.
18. **Unique Existential Quantifier ($\exists!$):** The unique existential quantifier $\exists!$ is read as “there exists exactly one.” The statement $\exists!xP(x)$ means that there is exactly one element x in the domain for which the predicate $P(x)$ is true.
19. **Sets of Numbers:**
- \mathbb{N} : Natural numbers $\{0, 1, 2, 3, \dots\}$
 - \mathbb{Z} : Integers $\{\dots, -2, -1, 0, 1, 2, \dots\}$
 - \mathbb{Q} : Rational numbers $\{\frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0\}$
 - \mathbb{R} : Real numbers (all points on the number line)
 - \mathbb{C} : Complex numbers $\{a + bi \mid a, b \in \mathbb{R}, i^2 = -1\}$
 - \mathbb{I} : Irrational numbers (reals not in \mathbb{Q} , e.g., $\pi, \sqrt{2}$)
 - \mathbb{Z}^+ : Positive integers $\{1, 2, 3, \dots\}$
 - \mathbb{Z}^- : Negative integers $\{-1, -2, -3, \dots\}$
20. **Nested Quantifier:** When one quantifier is within the scope of another. Ex: $\forall x \exists y (x - 2y = 5)$
21. **Argument:** Is a collection of premises and a conclusion. An argument is valid if and only if the truth of the premises guarantees the conclusion. Or, in the language of the class (where t is the conclusion and p_1, p_2, \dots, p_k are premises):

$$(p_1 \wedge p_2 \wedge \dots p_n) \rightarrow t$$

2 Laws & Axioms

- **De Morgan’s Laws:**

1. $\neg(p \wedge q) \equiv \neg p \vee \neg q$
2. $\neg(p \vee q) \equiv \neg p \wedge \neg q$

- **Conditional Disjunction Equivalence (CDE):** $p \rightarrow q \equiv \neg p \vee q$.

Table 1: **Only equivalences and rules of inference present on this table will be accepted.**

<i>Equivalence</i>	<i>Name</i>
$p \rightarrow q \equiv \neg p \vee q$	Conditional Disjunction Equivalence
$p \rightarrow q \equiv \neg q \rightarrow \neg p$	Contrapositive Law
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Definition of Biconditional
$\neg(\neg p) \equiv p$	Double Negation Law
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity Laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination Laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent Laws
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative Laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive Laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's Laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption Laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation Laws

• **De Morgan's Laws for Quantifiers:**

1. $\neg(\forall x P(x)) \equiv \exists x \neg P(x)$
2. $\neg(\exists x P(x)) \equiv \forall x \neg P(x)$