Definitions, Laws, and Axioms

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This document contains the official list of definitions, laws, and axioms that students are permitted to reference and use for CS 2050 assignments and examinations.

1 Definitions

1. **Negation ($\neg$):** the negation of p is the statement "not p."
2. **Conjunction ($\wedge$):** the conjunction of p and q is the statement "p and q." For the statement to be true, both p and q must be true.
3. **Disjunction ($\vee$):** the disjunction of p and q is the statement "p or q." For the statement to be true, at least one of p or q must be true.
4. **Exclusive-or ($\oplus$):** the exclusive-or of p and q is the statement "p XOR q." For the statement to be true, exactly one of p and q must be true.
5. **Conditional statement ($\rightarrow$):** "If p, then q." Written $p \rightarrow q$. This is false when p is true and q is false, and true in every other instance.
6. **Converse:** $q \rightarrow p$.
7. **Inverse:** $\neg p \rightarrow \neg q$.
8. **Contrapositive:** $\neg q \rightarrow \neg p$ (logically equivalent to the given conditional).
9. **Bi-conditional statement ($\leftrightarrow$):** "p if and only if q." One cannot exist without the other. Statement is true when both statements are true or both statements are false. q is sufficient for p and p is sufficient for q.
10. **Definition of Biconditional:**
$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$
11. **Tautology:** a compound proposition that is always true. Examples: $p \rightarrow p, p \vee \neg p$.
12. **Contradiction:** a compound proposition that is always false. Example: $\neg p \wedge p$.
13. **Contingency:** a compound proposition that is neither a tautology nor a contradiction. Most statements are contingencies. Example: $p \wedge q$.
14. **Logical Equivalence:** Two compound propositions p and q are logically equivalent if $p \leftrightarrow q$ is a tautology. Notation: $p \equiv q$.
15. **Predicate (Propositional Function):** A predicate is a statement involving one or more variables that becomes either true or false when values are assigned to the variables. Written like $P(x)$.
16. **Universal Quantifier ($\forall$):** The universal quantifier $\forall$ is read as "for all" or "for every." The statement $\forall x P(x)$ means that the predicate $P(x)$ is true for every element x in the specified domain.

17. **Existential Quantifier ($\exists$):** The existential quantifier $\exists$ is read as "there exists." The statement $\exists x P(x)$ means that there is at least one element x in the domain for which $P(x)$ is true.

18. **Unique Existential Quantifier ($\exists!$):** The unique existential quantifier $\exists!$ is read as "there exists exactly one." The statement $\exists! x P(x)$ means that there is exactly one element x in the domain for which the predicate $P(x)$ is true.

19. **Sets of Numbers:**
    - $\mathbb{N}$: Natural numbers $\{0, 1, 2, 3, \dots\}$
    - $\mathbb{Z}$: Integers $\{\dots, -2, -1, 0, 1, 2, \dots\}$
    - $\mathbb{Q}$: Rational numbers $\left\{\frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0\right\}$
    - $\mathbb{R}$: Real numbers (all points on the number line)
    - $\mathbb{C}$: Complex numbers $\{a + bi \mid a, b \in \mathbb{R}, i^2 = -1\}$
    - $\mathbb{I}$: Irrational numbers (reals not in $\mathbb{Q}$, e.g., π, $\sqrt{2}$)
    - $\mathbb{Z}^+$: Positive integers $\{1, 2, 3, \dots\}$
    - $\mathbb{Z}^-$: Negative integers $\{-1, -2, -3, \dots\}$

20. **Nested Quantifier:** When one quantifier is within the scope of another. Ex: $\forall x \exists y (x - 2y = 5)$

21. **Argument:** Is a collection of premises and a conclusion. An argument is valid if and only if the truth of the premises guarantees the conclusion. Or, in the language of the class (where t is the conclusion and $p_1, p_2, \cdots, p_k$ are premises):
$$(p_1 \wedge p_2 \wedge \dots p_n) \to t$$

22. **English Conditionals (the non-intuitive ones have been bolded):**
    - If p, then q.
    - p is sufficient for q.
    - If p, q.
    - **q if p.**
    - q when p.
    - A necessary condition for p is q.
    - q unless $\neg p$.
    - p implies q.
    - **p only if q.**
    - A sufficient condition for q is p.
    - q whenever p.
    - q is necessary for p.
    - q follows from p.
    - q provided that p.

23. **Even**: An integer is even if it can be represented as two times another integer, or in other words, an integer x is even if $x = 2a, a \in \mathbb{Z}$.

24. **Odd**: An integer is odd if it can be represented as two times another integer plus 1, or in other words, an integer x is odd if $x = 2a + 1, a \in \mathbb{Z}$.

25. **Perfect Square**: An integer x is a perfect square if it can be written as $x = a^2, a \in \mathbb{Z}$.

26. **Prime:** A prime number x is a natural number greater than 1 that has exactly two distinct factors: 1 and itself.

2 Laws & Axioms

- **De Morgan's Laws:**
  1. $\neg(p \land q) \equiv \neg p \lor \neg q$
  2. $\neg(p \lor q) \equiv \neg p \land \neg q$
- **Conditional Disjunction Equivalence (CDE):** $p \to q \equiv \neg p \lor q$.

Table 1: **Only equivalences present on this table will be accepted.**

Equivalence	***Name***
$p \to q \equiv \neg p \vee q$	Conditional Disjunction Equivalence
$p \to q \equiv \neg q \to \neg p$	Contrapositive Law
$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$	Definition of Biconditional
$\neg(\neg p) \equiv p$	Double Negation Law
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity Laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination Laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent Laws
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative Laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive Laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's Laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption Laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation Laws

- **De Morgan's Laws for Quantifiers:**
  1. $\neg(\forall x\, P(x)) \equiv \exists x\, \neg P(x)$
  2. $\neg(\exists x\, P(x)) \equiv \forall x\, \neg P(x)$

Table 2: **Only rules of inference present on this table will be accepted.**

Rule of Inference	***Name***
p $p \to q$ ――――― $\therefore q$	Modus Ponens
$\neg q$ $p \to q$ ――――― $\therefore \neg p$	Modus Tollens
p ――――― $\therefore p \vee q$	Addition
$p \wedge q$ ――――― $\therefore p$	Simplification
p q ――――― $\therefore p \wedge q$	Conjunction
$p \to q$ $q \to r$ ――――― $\therefore p \to r$	Hypothetical Syllogism
$p \vee q$ $\neg p$ ――――― $\therefore q$	Disjunctive Syllogism
$p \vee q$ $\neg p \vee r$ ――――― $\therefore q \vee r$	Resolution

Axioms:

- Closure: Multiplication and Addition are closed under the domains of $\mathbb{Z}, \mathbb{Q}, \text{and}, \mathbb{R}$.