Definitions, Laws, and Axioms

Tanisha Gupta, Aidan Nguyen, Jennifer Jiang, Parth Parikh

Spring 2026

This document contains the official list of definitions, laws, and axioms that students are permitted to reference and use for CS 2050 assignments and examinations.

1 Definitions

1. **Negation ($\neg$):** the negation of p is the statement "not p."
2. **Conjunction ($\wedge$):** the conjunction of p and q is the statement "p and q." For the statement to be true, both p and q must be true.
3. **Disjunction ($\vee$):** the disjunction of p and q is the statement "p or q." For the statement to be true, at least one of p or q must be true.
4. **Exclusive-or ($\oplus$):** the exclusive-or of p and q is the statement "p XOR q." For the statement to be true, exactly one of p and q must be true.
5. **Conditional statement ($\rightarrow$):** "If p, then q." Written $p \rightarrow q$. This is false when p is true and q is false, and true in every other instance.
6. **Converse:** $q \rightarrow p$.
7. **Inverse:** $\neg p \rightarrow \neg q$.
8. **Contrapositive:** $\neg q \rightarrow \neg p$ (logically equivalent to the given conditional).
9. **Bi-conditional statement ($\leftrightarrow$):** "p if and only if q." One cannot exist without the other. Statement is true when both statements are true or both statements are false. q is sufficient for p and p is sufficient for q.
10. **Definition of Biconditional:**
$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$
11. **Tautology:** a compound proposition that is always true. Examples: $p \rightarrow p, p \vee \neg p$.
12. **Contradiction:** a compound proposition that is always false. Example: $\neg p \wedge p$.
13. **Contingency:** a compound proposition that is neither a tautology nor a contradiction. Most statements are contingencies. Example: $p \wedge q$.
14. **Logical Equivalence:** Two compound propositions p and q are logically equivalent if $p \leftrightarrow q$ is a tautology. Notation: $p \equiv q$.
15. **Predicate (Propositional Function):** A predicate is a statement involving one or more variables that becomes either true or false when values are assigned to the variables. Written like $P(x)$.
16. **Universal Quantifier ($\forall$):** The universal quantifier $\forall$ is read as "for all" or "for every." The statement $\forall x P(x)$ means that the predicate $P(x)$ is true for every element x in the specified domain.

17. **Existential Quantifier ($\exists$):** The existential quantifier $\exists$ is read as "there exists." The statement $\exists x P(x)$ means that there is at least one element x in the domain for which $P(x)$ is true.

18. **Unique Existential Quantifier ($\exists!$):** The unique existential quantifier $\exists!$ is read as "there exists exactly one." The statement $\exists! x P(x)$ means that there is exactly one element x in the domain for which the predicate $P(x)$ is true.

19. **Sets of Numbers:**

    - $\mathbb{N}$: Natural numbers $\{0, 1, 2, 3, \ldots\}$
    - $\mathbb{Z}$: Integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
    - $\mathbb{Q}$: Rational numbers $\left\{\frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0\right\}$
    - $\mathbb{R}$: Real numbers (all points on the number line)
    - $\mathbb{C}$: Complex numbers $\{a + bi \mid a, b \in \mathbb{R}, i^2 = -1\}$
    - $\mathbb{I}$: Irrational numbers (reals not in $\mathbb{Q}$, e.g., π, $\sqrt{2}$)
    - $\mathbb{Z}^+$: Positive integers $\{1, 2, 3, \ldots\}$
    - $\mathbb{Z}^-$: Negative integers $\{-1, -2, -3, \ldots\}$

20. **Nested Quantifier:** When one quantifier is within the scope of another. Ex: $\forall x \exists y (x - 2y = 5)$

21. **Argument:** Is a collection of premises and a conclusion. An argument is valid if and only if the truth of the premises guarantees the conclusion. Or, in the language of the class (where t is the conclusion and $p_1, p_2, \cdots, p_k$ are premises):
$$(p_1 \wedge p_2 \wedge \ldots p_n) \to t$$

2 Laws & Axioms

- **De Morgan's Laws:**
  1. $\neg(p \wedge q) \equiv \neg p \vee \neg q$
  2. $\neg(p \vee q) \equiv \neg p \wedge \neg q$
- **Conditional Disjunction Equivalence (CDE):** $p \to q \equiv \neg p \vee q$.

Table 1: **Only equivalences present on this table will be accepted.**

Equivalence	***Name***
$p \to q \equiv \neg p \lor q$	Conditional Disjunction Equivalence
$p \to q \equiv \neg q \to \neg p$	Contrapositive Law
$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$	Definition of Biconditional
$\neg(\neg p) \equiv p$	Double Negation Law
$p \land \mathbf{T} \equiv p$ $p \lor \mathbf{F} \equiv p$	Identity Laws
$p \lor \mathbf{T} \equiv \mathbf{T}$ $p \land \mathbf{F} \equiv \mathbf{F}$	Domination Laws
$p \lor p \equiv p$ $p \land p \equiv p$	Idempotent Laws
$p \lor q \equiv q \lor p$ $p \land q \equiv q \land p$	Commutative Laws
$(p \lor q) \lor r \equiv p \lor (q \lor r)$ $(p \land q) \land r \equiv p \land (q \land r)$	Associative Laws
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Distributive Laws
$\neg(p \land q) \equiv \neg p \lor \neg q$ $\neg(p \lor q) \equiv \neg p \land \neg q$	De Morgan's Laws
$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption Laws
$p \lor \neg p \equiv \mathbf{T}$ $p \land \neg p \equiv \mathbf{F}$	Negation Laws

- **De Morgan's Laws for Quantifiers:**
  1. $\neg(\forall x\, P(x)) \equiv \exists x\, \neg P(x)$
  2. $\neg(\exists x\, P(x)) \equiv \forall x\, \neg P(x)$

Table 2: **Only equivalences and rules of inference present on this table will be accepted.**

Rule of Inference	***Name***
p $p \to q$ ――――― $\therefore q$	Modus Ponens
$\neg q$ $p \to q$ ――――― $\therefore \neg p$	Modus Tollens
p ――――― $\therefore p \vee q$	Addition
$p \wedge q$ ――――― $\therefore p$	Simplification
p q ――――― $\therefore p \wedge q$	Conjunction
$p \to q$ $q \to r$ ――――― $\therefore p \to r$	Hypothetical Syllogism
$p \vee q$ $\neg p$ ――――― $\therefore q$	Disjunctive Syllogism
$p \vee q$ $\neg p \vee r$ ――――― $\therefore q \vee r$	Resolution