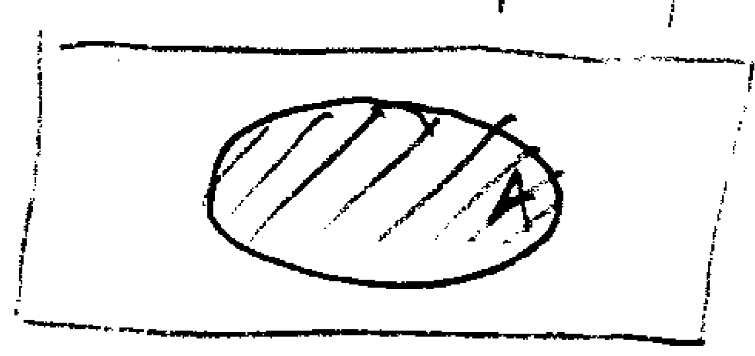
STAT 230 Chapter 4 : Probability Rules and Conditional Probability

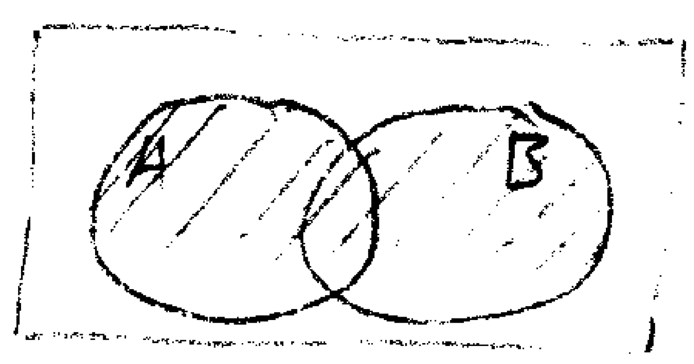
4.1 General Methods:

- $P(S)=1$
- for any event A, $0 \le P(A) \le 1$
- if A and B are two events with $A \subseteq B$ then $P(A) \le P(B)$

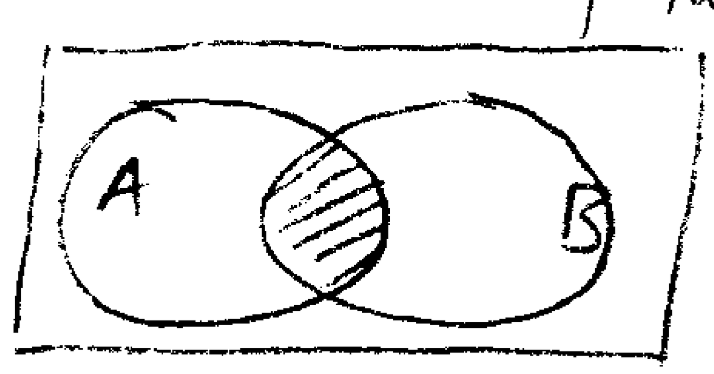
- Event A is sample space S:



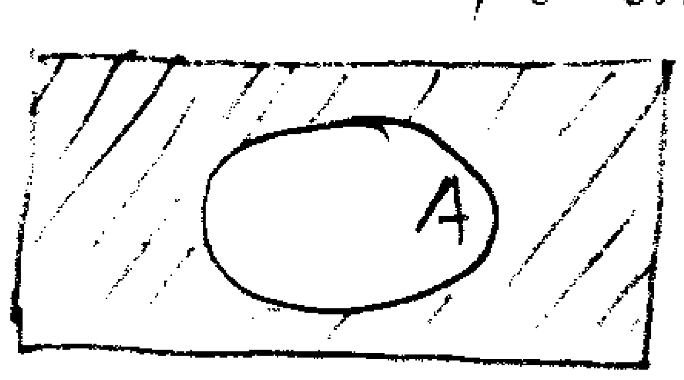
- The union of two events $A \cup B$



- The intersection of two events $A \cap B$



- $\bar{A}$ = the complement of the event A



- De Morgan's Laws:

a) $\overline{A \cup B} = \bar{A} \cap \bar{B}$

b) $\overline{A \cap B} = \bar{A} \cup \bar{B}$

4.2: Rules for Unions of Events

- Addition Law of Probability or the Sum Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Probability of the Union of Three events:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$$

- Probability of the Union of n events

$$P(A_1 \cup A_2 \cup A_3 \cdots A_n) = \sum P(A_i) - \sum_{i<j} P(A_i A_j) + \sum_{i<j<k} P(A_i A_j A_k) \cdots$$

- Defn: Event A and B are mutually exclusive if $A \cap B = \phi$

Since mutually exclusive events A, B have no common points $P(A \cap B) = P(\phi) = 0$

- Probability of the Union of Two Mutually Exclusive Events.

Let A and B be mutually exclusive events.

Then $P(A \cup B) = P(A) + P(B)$

- Probability of the Union of n Mutually Exclusive Events.

In general let $A_1 A_2 \cdots A_n$ be mutually exclusive events, then

$$P(A_1 \cup A_2 \cdots A_n) = \sum P(A_i)$$

- Probability of the Complement of an event

$$P(A) = 1 - P(\bar{A})$$

4.3 Intersections of Events and Independence

Defn: Event A and B are independent events if and only if
$P(A \cap B) = P(A)P(B)$
If they are not independent, we call the events dependent

Defn: The events $A_1 \cdots A_n$ are mutually independent if and only if
$P(A_1 \cap \cdots \cap A_n) = \prod P(A_i)$

note: if A and B are indep events, then $\bar{A}$ and B are also indep

4.4 Conditional Probability

Defn: The conditional probability of event A given B is
$P(A|B) = \frac{P(A \cap B)}{P(B)}$ provided $P(B) > 0$

note: if A and B are independent then
$P(A \cap B) = P(A)P(B)$
so $P(A|B) = \frac{P(AB)}{P(B)} = P(A)$

thm Suppose A and B are two events defined on a sample space S such that $P(A) > 0$ and $P(B) > 0$, then A & B are indep. events iff either of the following statements is true:

- $P(A|B) = P(A)$ or
- $P(B|A) = P(B)$

4.5 Product Rules, Law of Total Prob. and Bayes' Theorem

rule Product rule: Let A B C D... be arbitrary events in a sample space. Assume that $P(A) > 0$, $P(AB) > 0$, $P(ABC) > 0$, then

- $P(AB) = P(A)P(B|A)$
- $P(ABC) = P(A)P(B|A)P(C|AB)$

$\vdots$

rule: Law of Total Probability
Let $A_1 \cdots A_k$ be a partition of the sample space S into disjoint events.
let B be an arbitrary event in S, then:
$P(B) = P(BA_1) + P(BA_2) \cdots$
$= \sum P(B|A_i)P(A_i)$

Bayes' Theorem: suppose A and B are events defined on a sample space S. Suppose also that $P(B) > 0$. then

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$
$$= \frac{P(B|A)P(A)}{P(B|\bar{A})P(\bar{A}) + P(B|A)P(A)}$$