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Assignment 3

① What do you mean by homogeneous coordinate transformation? Explain about shear transformation.

→ Homogeneous coordinate transformation is defined as a transformation such that it provides a uniform framework for transformation of all the coordinates using multiplication of matrices.

↳ To perform more than one transformation, coordinate transformation is used.

Shear transformation: Shear transformation is a non-rigid body transformation which consists of distortion of a shape of a body in a particular direction.

Here, let x, y be initial vertices while x' y' be vertices after transformation.

on S_x shear transformation on x and S_y on y

$x' = x + S_x \cdot y$

$y' = y + S_y \cdot x$.

Representing in matrices

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & S_x \\ S_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$ ← for shearing in both directions

For only one direction,

S_x or $S_y = 0$.

ie for X direction

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & S_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Y direction

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ S_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

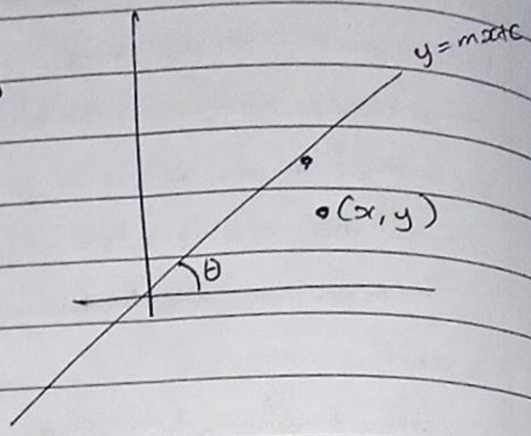
② How do you perform a 2D reflection about line $y = mx + c$? Explain the required steps with ~~the~~ derivation.

Solution.

To reflect (x, y) about $y = mx+c$.

Now

→ Here, let the slope of line be given by m.



→ $\tan\theta = m.$

$\theta = \tan^{-1}(m).$

Also

$\tan\theta = m,$ $\qquad \cos^2\theta = \dfrac{1}{\sec^2\theta}$

$\dfrac{\sin\theta}{\cos\theta} = m$ $\qquad \cos^2\theta = \dfrac{1}{\tan^2\theta + 1}$

$\cancel{\dfrac{\sin\theta}{1-\sin^2\theta} = m,}$

$\cos^2\theta = \dfrac{1}{m^2+1}$

$\cos\theta = \dfrac{1}{\sqrt{m^2+1}}$

$\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \dfrac{1}{m^2+1}$

$= \dfrac{m^2+1-1}{m^2+1}$

$\sin^2\theta = \dfrac{m^2}{m^2+1} = \dfrac{m}{\sqrt{m^2+1}} = \sin\theta$

Now, • Displace the line by c to bring it to origin.

• Step 1: Rotating the line $y = mx+c$ by $\theta = \tan^{-1}(m)$ to reach x axis.

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{m^2+1} & -m/\sqrt{m^2+1} & 0 \\ m/\sqrt{m^2+1} & 1/\sqrt{m^2+1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

• Reflect the object about the axis

$$Ref = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

• Perform reverse rotation

• Perform reverse trasformation

Now,

$P = P(x) \, P'$

$$P(x) = R_{ev}(x) \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{m^2+1} & -m/\sqrt{m^2+1} & 0 \\ m/\sqrt{m^2+1} & 1/\sqrt{m^2+1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}$$

$$\because R_{ev}(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{m^2+1} & m/\sqrt{m^2+1} & 0 \\ -m/\sqrt{m^2+1} & 1/\sqrt{m^2+1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P(x) = \begin{bmatrix} \frac{1-m^2}{m^2+1} & \frac{2m}{m^2+1} & \frac{-2mc}{m^2+1} \\ \frac{2m}{m^2+1} & \frac{m^2-1}{m^2+1} & \frac{2c}{m^2+1} \\ 0 & 0 & 1 \end{bmatrix}$$

(5) Solution

$A(8,7) \quad B(10,6) \quad C(8,4)$

(a) Traslate so that $A(1,9)$

Here, $T_x = 8 \to 1 = -7$

$T_y = 7 \to 9 = 2$

Now

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 & 10 & 8 \\ 7 & 6 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 \\ 9 & 8 & 6 \\ 1 & 1 & 1 \end{bmatrix}$$

(5) $A(5,5)\ B(7,3)\ C(3,3)$

Translating by $(1,0)$

Followed by reflection

$$\cancel{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \Big[} \qquad P = \begin{bmatrix} 5 & 7 & 3 \\ 5 & 3 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

Now $P(x) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$P(x)\cdot P = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 7 & 3 \\ 5 & 3 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

(a) A(8,7) B(10,6) C(8,4)

Now,

$(8,7) \longrightarrow (1,9)$

$T_x = -7 \quad T_y = 2$

Now,

$$T = \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 & 10 & 8 \\ 7 & 6 & 4 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 \\ 9 & 8 & 6 \\ 1 & 1 & 1 \end{bmatrix}$$

Now, Reflection about $y=x+3$.

Now,

$P' = P(x) \cdot P$

$$P(x)_{y=x+3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & +1/\sqrt{2} & 0 \\ -\frac{1}{\sqrt{2}} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P(x) = \begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 \\ 9 & 8 & 6 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 5 & 3 \\ -2 & 0 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

Then,

(c) $C(x) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

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