

Key to Exercise on Lesson 31

Assume in the following *boolean* expressions that a , b , c , and d are *boolean* quantities.

1. Show the algebraic simplification of $ab(!c) + !ab(!c)$ to $b(!c)$.

$$\begin{aligned} & ab(!c) + !ab(!c) \\ &= !cb(a + !a) \\ &= !cb(\text{true}) \\ &= !cb = b(!c) \end{aligned}$$

2. Show the algebraic simplification of $a(!b)c + abc + !abc + !ab(!c) + a(!b)$ to $!ba + !ab + ac$

$$\begin{aligned} & a(!b)c + abc + !abc + !ab(!c) + a(!b) \\ &= ac(!b + b) + !ab(c + !c) + a(!b) \\ &= ac(\text{true}) + !ab(\text{true}) + a(!b) \\ &= ac + !ab + a(!b) \end{aligned}$$

3. Show the algebraic simplification of $!(a * !c) + !b + !(ad)$ to true .

$$\begin{aligned} & !(a * !c) + !b + !(ad) \\ &= !a + !c + !b + !a + !d \\ &= a + c + !b + !a + !d \\ &= a + !a + (c + !b + !d) \\ &= \text{true} + (c + !b + !d) \\ &= \text{true} + (\text{anything}) = \text{true} \end{aligned}$$

4. Show the algebraic modification of $!(a + bc)(a + c)$ to $!a(!b + !c) + !a(!c)$.

$$\begin{aligned} & !(a + bc)(a + c) \\ &= !(a + bc) + !(a + c) \\ &= !a(!bc) + !a(!c) \\ &= !a(!b + !c) + !a(!c) \end{aligned}$$

5. Express $ab(!c) + !ab(!c)$ using proper Java syntax ($\&\&$, $\|$, and $!$).

$$a \&\& b \&\& !c \parallel !a \&\& b \&\& !c$$

6. Express $a(!b)c + abc + !abc + !ab(!c) + a(!b)$ using proper Java syntax ($\&\&$, $\|$, and $!$).

$$a \&\& !b \&\& c \parallel a \&\& b \&\& c \parallel !a \&\& b \&\& !c \parallel a \&\& !b$$

7. Express $!(a * !c) + !b + !(ad)$ using proper Java syntax ($\&\&$, $\|$, and $!$).

$$!(a \&\& !c) \parallel !b \parallel !(a \&\& d)$$

8. Simplify $x + (x * y)$ where x and y are *boolean* variables.

$$x \text{ (law of absorption)}$$

9. Express $!(a + bc)(a + c)$ using proper Java syntax ($\&\&$, $\|$, and $!$).

$$!(a \parallel b \&\& c) \&\& (a \parallel c)$$

10. Express $!(a \ \&\& \ b \ \&\& \ c)$ using **ORs** instead of **ANDs**.

$$!a \ || \ !b \ || \ !c$$

11. Express $(a \ \&\& \ b \ \&\& \ c)$ using **ORs** instead of **ANDs**.

$$\begin{aligned} & !!(a \ \&\& \ b \ \&\& \ c) \\ & = !(\ !a \ \&\& \ b \ \&\& \ c \) \\ & = !(\ !a \ || \ !b \ || \ !c \) \end{aligned}$$

12. Illustrate the equivalence of $!(a * b)$ and $!a + !b$ using truth tables.

a	b	a * b	!(a * b)
false	false	false	true
false	true	false	true
true	false	false	true
true	true	true	false

a	b	!a	!b	!a + !b
false	false	true	true	true
false	true	true	false	true
true	false	false	true	true
true	true	false	false	false

Notice the two gray sections are identical. Also, notice that the two black sections are the same.

13. Which of the following is the equivalent of $((p > 3) \ || \ (q < b))$?

- $!(p > 3) \ \&\& \ !(q < b)$
- $!(p \leq 3) \ \&\& \ (q \geq b)$
- $!(p > 3) \ || \ (q < b)$
- $!(p \leq 3) \ || \ (q \geq b)$
- More than one of these

Notice that $!(p > 3)$ is the same as $(p \leq 3)$.

Also, $!(q < b)$ is the same as $(q \geq b)$.

$$!!((p > 3) \ || \ (q < b)) = !(\ !(p > 3) \ || \ !(q < b) \) = !((p \leq 3) \ \&\& \ (q \geq b))$$

14. Write a Boolean expression that produces the following truth table:

A	B	C	D
(input)			(output)
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$$(!A)(!B)(!C) + (!A)(B)(!C) + (A)(!B)(!C)$$

Key to Boolean Algebra and DeMorgan's Theorem... Contest Type Problems

1. B
2. B
3. B
4. A
5. D