Truth Tables
The Truth of the Matter

ACTIVITY 1.4

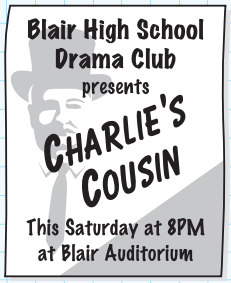
SUGGESTED LEARNING STRATEGIES: Close Reading, Activating Prior Knowledge, Group Discussion, Interactive Word Wall, Vocabulary Organizer, Think/Pair/Share

My Notes

Symbolic logic allows you to determine the validity of statements without being distracted by a lot of text. You can use symbols, such as p and q, to represent simple statements. A **compound statement** is formed when two or more simple statements are connected as a conditional (if-then) a **biconditional** (if and only if), a **conjunction** (and), or a **disjunction** (or).

1. The symbol $\rightarrow$ represents a conditional. You read $p \rightarrow q$ as, "if p, then q," or "p implies q." Let p represent the statement "you arrive before 7 PM," and let q represent the statement "you will get a good seat." Use the information in the play poster to the right to write the statement that is represented by $p \rightarrow q$.

If you arrive early, then you will get a good seat.



Truth tables are used to determine the conditions under which a statement is true or false. This truth table displays the truth values for $p \rightarrow q$, which are dependent on the truth values for p and q.

	p	q	$p \rightarrow q$
Row 1	T	T	T
Row 2	T	F	F
Row 3	F	T	T
Row 4	F	F	T

2. Row 1 addresses the case when both p and q are true. Refer to the play announcement that you completed in Item 1. Explain why $p \rightarrow q$ would be true if both p and q are true.

The statement on the poster is an agreement. If you do arrive before 7 PM and you do get a good seat, then the agreement has been kept.

3. Row 2 addresses the case when p is true and q is false. In terms of the play announcement, explain why $p \rightarrow q$ would be false if p is true and q is false.

If you do arrive early, and, yet, you do not get a good seat, then the agreement on the poster has not been kept.

4. Refer to Rows 3 and 4. In terms of the play announcement, explain why $p \rightarrow q$ is true if p is false.

If p is false, then you do not get there early. The poster says nothing about not arriving early, so for all we know, the statement is true.

ACTIVITY 1.4 Guided

Truth Tables

Activity Focus
- Truth tables

Materials
- Whiteboard or chart paper and markers (optional)

Chunking the Activity

#1–4	#8–9	#12–14
#5–7	#10–11	#15–16

TEACHER TO TEACHER This activity has many new concepts presented in paragraph form. Remind students to apply the Close Reading strategy and to use their Vocabulary Organizers throughout the activity to aid their comprehension.

1 Activating Prior Knowledge, Group Discussion, Interactive Word Wall Students should be familiar with writing conditional statements in if-then form.

TEACHER TO TEACHER It may be helpful for some students to see the construction of the truth table for $p \rightarrow q$ modeled step by step.

Step 1: Write down all possible T and F combinations for p and q.

Step 2: Evaluate $p \rightarrow q$.

2-4 Think/Pair/Share, Group Discussion, Debriefing These items are intended to give students a context for the truth values for $p \rightarrow q$.

5-7 Close Reading, Create Representations, Think/Pair/Share, Debriefing The symbol ~ will be new for students, but not the concept of negating a simple statement. Many students will enjoy sharing their responses to Item 6.

TEACHER TO TEACHER The steps in the example on this page are shown. However, it may be helpful for some students to see this process modeled in a step-by-step manner.

Differentiating Instruction

To extend students' reasoning skills, you can introduce a third simple statement, *r*. Use the information below right to develop a mini lesson for constructing a truth table for three simple statements.

My Notes

SUGGESTED LEARNING STRATEGIES: Close Reading, Create Representations, Think/Pair/Share

The symbol $\sim p$ is the **negation of *p*** and can be read as "not *p*." This truth table shows the conditions under which $\sim p$ is true or false. When *p* is true, $\sim p$ is false, and when *p* is false, $\sim p$ is true.

p	$\sim p$
T	F
F	T

5. Let *p* represent the simple statement "Triangles are convex," which is a true statement. Write the statement denoted by $\sim p$ and state whether it is true or false. Triangles are not convex. False.

6. Create a simple statement *p* that you know to be false. Write the statement $\sim p$ and state whether is true or false.
Statements may vary, but their opposite should be true.

7. Let *p* represent "you are not in the band" and let *q* represent "you can go on the trip." Write the compound statement represented by $\sim p \rightarrow q$.
If you are in the band, then you can go on the trip.

EXAMPLE

Make a truth table for $\sim p \rightarrow q$.

Step 1 *Write down all possible T and F combinations for p and q.*

p	*q*
T	T
T	F
F	T
F	F

Step 2 *Add a column for ~p and negate p.*

p	*q*	$\sim p$
T	T	F
T	F	F
F	T	T
F	F	T

Step 3 *Add a column for $\sim p \rightarrow q$ and evaluate $\sim p \rightarrow q$.*

p	*q*	$\sim p$	$\sim p \rightarrow q$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

MINI LESSON: Truth Tables for Three Simple Statements

- Create table headings for *p*, *q*, and *r*. Do not show rows of cells below the headings
- Develop the first few rows with students, adding one row at a time. Then ask students to add and fill in rows to complete the table.
- Students should discover that the number of possible T and F combinations for three statements is 8.
- The table to the right models one way to organize the eight different combinations.

p	*q*	*r*
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Truth Tables

The Truth of the Matter

ACTIVITY 1.4
continued

SUGGESTED LEARNING STRATEGIES: Close Reading, Think/Pair/Share, Create Representations, Identify a Subtask, Activating Prior Knowledge, Interactive Word Wall, Vocabulary Organizer

My Notes

8. Follow the steps and complete the truth table for $\sim(p \rightarrow q)$.

Step 1
Find all possible T and F combinations for p and q.

Step 2
Evaluate $p \rightarrow q$.

Step 3
Negate $p \rightarrow q$.

Step 1		Step 2	Step 3
p	q	$p \rightarrow q$	$\sim(p \rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

9. Write the steps and complete the truth table for $\sim q \rightarrow \sim p$.

Step 1
Find all possible T and F combinations for p and q.

Step 2
Negate p and q

Step 3
Evaluate $\sim p \rightarrow \sim q$.

Step 1		Step 2		Step 3
p	q	$\sim p$	$\sim q$	$\sim q \rightarrow \sim p$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

10. The symbol $\leftrightarrow$ represents a **biconditional**. You read it as "if and only if." Let p represent "A chord in a circle contains the center," and let q represent "The chord is a diameter." Write the statement that is represented by $p \leftrightarrow q$.

A chord in a circle contains the center if and only if the chord is a diameter.

ACTIVITY 1.4 *Continued*

8-9 Think/Pair/Share, Identify a Subtask, Debriefing For students who are struggling with Item 9, refer them to the example and Item 8.

TEACHER TO TEACHER Definitions are considered to be biconditionals.

10 Close Reading, Activating Prior Knowledge, Interactive Word Wall, Vocabulary Organizer, Create Representations The terms *circle, chord, center* and *diameter* were reviewed in Activity 1.1.

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ACTIVITY 1.4 *Continued*

11 Identify a Subtask, Create Representations, Group Presentation, Debriefing You may or may not require students to write the steps (as in item 9).

12-13 Close Reading, Think/Pair/Share, Discussion Group, Interactive Word Wall, Vocabulary Organizer
A class discussion concerning the *inclusive* (as opposed to *exclusive*) definition for "or" may be necessary. The meaning of "or" is understood to be "one or the other or both."

TEACHER TO TEACHER It may be helpful for some students to see the construction of the truth tables for $p \wedge q$ and $p \vee q$ modeled in a step-by-step manner.

SUGGESTED LEARNING STRATEGIES: Close Reading, Think/Pair/Share, Create Representations, Identify a Subtask, Discussion Group, Group Presentation, Interactive Word Wall, Vocabulary Organizer

My Notes

MATH TIP
Because biconditionals are true only when both parts have the same truth value, many definitions are written in the form of a biconditional.

The truth table for $p \leftrightarrow q$ is shown to the right. Notice that $p \leftrightarrow q$ is true only when p and q are both true or both false.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

11. Construct a truth table for $(p \rightarrow q) \leftrightarrow \sim q$.

Step 1		Step 2	Step 3	Step 4
p	q	$p \rightarrow q$	$\sim q$	$(p \rightarrow q) \leftrightarrow \sim q$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

MATH TERMS
A **rectangle** is a parallelogram with a right angle.

A **rhombus** is a parallelogram with consecutive congruent sides.

12. The symbol $\wedge$ represents a **conjunction**. You read it as "and." Let p represent "the figure is a rectangle," and let q represent "the figure is a rhombus."

a. Write the statement that is represented by $p \wedge q$.
The figure is a rectangle and the figure is a rhombus.

b. Write the statement that is represented by $p \wedge \sim q$.
The figure is a rectangle and the figure is not a rhombus.

The truth table for $p \wedge q$ is shown to the right. Notice that $p \wedge q$ is only true when both p and q are true.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

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SUGGESTED LEARNING STRATEGIES: Close Reading, Create Representations, Identify a Subtask

My Notes

13. The symbol ∨ represents a **disjunction**. You read it as "or." Let p represent "the figure is not a rectangle," and let q represent "the figure is a rhombus."

a. Write the statement that is represented by $p \vee q$.
The figure is not a rectangle or the figure is a rhombus.

b. Write the statement that is represented by $\sim p \vee q$.
The figure is a rectangle or the figure is a rhombus.

The truth table for $p \vee q$ is shown to the right. Notice that $p \vee q$ is only false when both p and q are false.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

14. Construct a truth table for $p \vee (p \wedge q)$.

Step 1		Step 2	Step 3
p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

ACTIVITY 1.4 *Continued*

14 Identify a Subtask, Create Representations, Debriefing For students who are struggling, suggest that they think about the steps, whether or not you ask students to write them.

Additional Answers for Check Your Understanding, p. 36

6. This is a tautology.

Step 1		Step2	Step3	Step 4
p	q	$p \rightarrow q$	$\sim q \rightarrow \sim p$	$(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

7. Answers may vary. Sample answer; Using the conditional "If I am elected, then I will not raise taxes," assign p to the hypothesis and q to the conclusion, set up a truth table, and then translate each statement back to the candidate's promise.

ACTIVITY 1.4 *Continued*

15-16 **Think/Pair/Share, Identify a Subtask, Create Representations, Debriefing** For students who are still struggling, suggest that they write out each step.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 36, #1–7

UNIT 1 PRACTICE
p. 84, #22–25

CHECK YOUR UNDERSTANDING

1.

Step 1		Step 2	Step 3
p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

2. See below right. This is a tautology.

3.

Step 1		Step 2	Step 3
p	q	$q \rightarrow p$	$p \vee (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	T	T

4.

Step 1		Step 2	Step 3
p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge p$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

5. See below right.
6–7. See p. 35.

My Notes

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Identify a Subtask, Create Representations, Identify a Subtask

A **tautology** is a statement that is true for all cases of p and q. The corresponding truth table will have only T's in the last column.

15. Construct a truth table for $(p \rightarrow q) \leftrightarrow (p \wedge q)$. Is this a tautology? no

Step 1		Step 2	Step 3	Step 4
p	q	$p \rightarrow q$	$p \wedge q$	$(p \rightarrow q) \leftrightarrow (p \wedge q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	F	F

16. Construct a truth table for $\sim(p \vee q) \leftrightarrow (\sim p \wedge \sim q)$. Is this a tautology? yes

Step 1		Step 2	Step 3	Step 4		Step 5	Step 6
p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$\sim(p \vee q) \leftrightarrow \sim p \wedge \sim q$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

Construct a truth table for the compound statements. Identify which statements, if any, are tautologies.

1. $\sim p \wedge q$
2. $q \rightarrow (p \vee \sim p)$
3. $p \vee (q \rightarrow p)$
4. $(p \rightarrow q) \wedge p$
5. $q \wedge (p \rightarrow \sim q)$
6. $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
7. MATHEMATICAL REFLECTION How could a truth table help you to analyze a campaign promise made by a person running for office? Think of a conditional statement starting, "If I am elected, …"

2.

Step 1		Step 2	Step 3	Step 4
p	q	$\sim p$	$p \vee \sim p$	$q \rightarrow (p \vee \sim p)$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	T	T

5.

Step 1		Step 2	Step 3	Step 4
p	q	$\sim q$	$p \rightarrow \sim q$	$q \wedge (p \rightarrow \sim q)$
T	T	F	F	F
T	F	T	T	F
F	T	F	T	T
F	F	T	T	F