

Truth Tables

The Truth of the Matter

SUGGESTED LEARNING STRATEGIES: Close Reading, Activating Prior Knowledge, Group Discussion, Interactive Word Wall, Vocabulary Organizer, Think/Pair/Share

My Notes

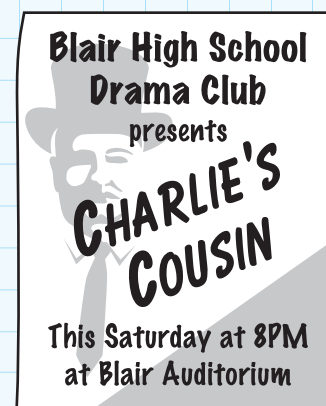
Symbolic logic allows you to determine the validity of statements without being distracted by a lot of text. You can use symbols, such as p and q , to represent simple statements. A **compound statement** is formed when two or more simple statements are connected as a conditional (if-then) a **biconditional** (if and only if), a **conjunction** (and), or a **disjunction** (or).

1. The symbol \rightarrow represents a conditional. You read $p \rightarrow q$ as, “if p , then q ,” or “ p implies q .” Let p represent the statement “you arrive before 7 PM,” and let q represent the statement “you will get a good seat.” Use the information in the play poster to the right to write the statement that is represented by $p \rightarrow q$.

Truth tables are used to determine the conditions under which a statement is true or false. This truth table displays the truth values for $p \rightarrow q$, which are dependent on the truth values for p and q .

	p	q	$p \rightarrow q$
Row 1	T	T	T
Row 2	T	F	F
Row 3	F	T	T
Row 4	F	F	T

2. Row 1 addresses the case when both p and q are true. Refer to the play announcement that you completed in Item 1. Explain why $p \rightarrow q$ would be true if both p and q are true.
3. Row 2 addresses the case when p is true and q is false. In terms of the play announcement, explain why $p \rightarrow q$ would be false if p is true and q is false.
4. Refer to Rows 3 and 4. In terms of the play announcement, explain why $p \rightarrow q$ is true if p is false.



My Notes

SUGGESTED LEARNING STRATEGIES: Close Reading, Create Representations, Think/Pair/Share

The symbol $\sim p$ is the **negation of p** and can be read as “not p .” This truth table shows the conditions under which $\sim p$ is true or false. When p is true, $\sim p$ is false, and when p is false, $\sim p$ is true.

p	$\sim p$
T	F
F	T

- Let p represent the simple statement “Triangles are convex,” which is a true statement. Write the statement denoted by $\sim p$ and state whether it is true or false.
- Create a simple statement p that you know to be false. Write the statement $\sim p$ and state whether it is true or false.
- Let p represent “you are not in the band” and let q represent “you can go on the trip.” Write the compound statement represented by $\sim p \rightarrow q$.

EXAMPLE

Make a truth table for $\sim p \rightarrow q$.

Step 1

Write down all possible T and F combinations for p and q .

p	q
T	T
T	F
F	T
F	F

Step 2

Add a column for $\sim p$ and negate p .

p	q	$\sim p$
T	T	F
T	F	F
F	T	T
F	F	T

Step 3

Add a column for $\sim p \rightarrow q$ and evaluate $\sim p \rightarrow q$.

p	q	$\sim p$	$\sim p \rightarrow q$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

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ACTIVITY 1.4

continued

SUGGESTED LEARNING STRATEGIES: Close Reading, Think/Pair/Share, Create Representations, Identify a Subtask, Activating Prior Knowledge, Interactive Word Wall, Vocabulary Organizer

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8. Follow the steps and complete the truth table for $\sim(p \rightarrow q)$.

Step 1

Find all possible T and F combinations for p and q .

Step 2

Evaluate $p \rightarrow q$.

Step 3

Negate $p \rightarrow q$.

Step 1		Step 2	Step 3
p	q	$p \rightarrow q$	$\sim(p \rightarrow q)$

9. Write the steps and complete the truth table for $\sim q \rightarrow \sim p$.

Step 1		Step 2		Step 3
p	q	$\sim p$	$\sim q$	$\sim q \rightarrow \sim p$

10. The symbol \leftrightarrow represents a **biconditional**. You read it as “if and only if.” Let p represent “A chord in a circle contains the center,” and let q represent “The chord is a diameter.” Write the statement that is represented by $p \leftrightarrow q$.

My Notes

MATH TIP

Because biconditionals are true only when both parts have the same truth value, many definitions are written in the form of a biconditional.

MATH TERMS

A **rectangle** is a parallelogram with a right angle.

A **rhombus** is a parallelogram with consecutive congruent sides.

SUGGESTED LEARNING STRATEGIES: Close Reading, Think/Pair/Share, Create Representations, Identify a Subtask, Discussion Group, Group Presentation, Interactive Word Wall, Vocabulary Organizer

The truth table for $p \leftrightarrow q$ is shown to the right. Notice that $p \leftrightarrow q$ is true only when p and q are both true or both false.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

11. Construct a truth table for $(p \rightarrow q) \leftrightarrow \sim q$.

12. The symbol \wedge represents a **conjunction**. You read it as “and.” Let p represent “the figure is a rectangle,” and let q represent “the figure is a rhombus.”

a. Write the statement that is represented by $p \wedge q$.

b. Write the statement that is represented by $p \wedge \sim q$.

The truth table for $p \wedge q$ is shown to the right. Notice that $p \wedge q$ is only true when both p and q are true.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

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ACTIVITY 1.4

continued

SUGGESTED LEARNING STRATEGIES: Close Reading, Create Representations, Identify a Subtask

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13. The symbol \vee represents a **disjunction**. You read it as “or.” Let p represent “the figure is not a rectangle,” and let q represent “the figure is a rhombus.”

a. Write the statement that is represented by $p \vee q$.

b. Write the statement that is represented by $\sim p \vee q$.

The truth table for $p \vee q$ is shown to the right. Notice that $p \vee q$ is only false when both p and q are false.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

14. Construct a truth table for $p \vee (p \wedge q)$.

My Notes

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Identify a Subtask, Create Representations, Identify a Subtask

A **tautology** is a statement that is true for all cases of p and q . The corresponding truth table will have only T's in the last column.

15. Construct a truth table for $(p \rightarrow q) \leftrightarrow (p \wedge q)$. Is this a tautology?

16. Construct a truth table for $\sim(p \vee q) \leftrightarrow (\sim p \wedge \sim q)$. Is this a tautology?

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

Construct a truth table for the compound statements. Identify which statements, if any, are tautologies.

1. $\sim p \wedge q$
2. $q \rightarrow (p \vee \sim p)$
3. $p \vee (q \rightarrow p)$

4. $(p \rightarrow q) \wedge p$
5. $q \wedge (p \rightarrow \sim q)$
6. $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
7. **MATHEMATICAL REFLECTION** How could a truth table help you to analyze a campaign promise made by a person running for office? Think of a conditional statement starting, "If I am elected, ..."